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Converting Technology to Mitigate Environmental Damage

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There are many situations where policy makers would like to induce firms to make a major discrete conversion in production technology to help the environment. This paper examines how heterogeneity in the operating condition of firms' plant and equipment, which cannot be observed by policy makers, can affect the choice between incentives to encourage conversion to a cleaner technology. By relating different conditions of firms' plant and equipment to production costs, extent of environmental damage, and cost of conversion to a cleaner technology, we show when a perfectly discriminating incentive to encourage conversion is not feasible. In addition, we show that firms with plant and equipment in better condition will convert their technology to mitigate their environmental damage, and firms with plant and equipment in poorer condition will not. This and a series of additional results lead to conditions under which an administratively simple uniform lump-sum incentive to switch to cleaner technology is preferable to one based on output. These results and conditions extend to cases where there are network externalities in conversion, and where there is strategic timing in firms' choice of when to convert.

Key words: technology conversion; environmental management; taxes and subsidies; capital vintage; incentives
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1. Introduction

This paper considers programs designed to induce firms to make a discrete observable conversion in their production technology to mitigate damage to the environment. Such programs can be targeted at thermal power plants to convert their fuel supply from coal to gas or to install scrubbers, and at hydroelectric-power plants to install salmon escapement ladders. Similarly, they may be designed to induce commercial-vehicle owners to convert from gasoline to natural gas, or to install catalytic converters, and to induce pulp mills to switch from chlorine to oxygen to reduce dioxins from bleaching. Targeted industries typically consist of firms with plant and equipment that differ in type, age, quality of maintenance, and general condition. These plant and equipment differences between firms affect their production costs, the extent of their environmental damage, and their costs of mitigating this damage. This is true, for example, in electric-power generation, where type, age, maintenance, and condition of plant differ markedly, where environmental damage is well-documented, and where pollution abatement costs vary from plant to plant.[1] Similar differences are found in air, rail, and road transportation equipment; manufacturing facilities; mines; iron and steel plants; and so on. The heterogeneity of plant and equipment in use is part of an industry's legacy. For example, the electric-power industry includes different vintages and varieties of gas- and coal-fired generators, hydroelectric facilities, and nuclear-power installations. More importantly, condition and maintenance levels vary between firms using the same type of plant and equipment. This clouds the ability of policy makers to judge the impact of a specific firm on the environment simply from the age or type of its plant.

The principal matter we address is the consequence of alternative subsidy- and tax-based programs designed to induce such conversions. Specifically, we concentrate on programs designed to encourage firms to convert their plant and equipment to mitigate negative externalities, where the condition of plant and equipment varies between firms. In our analysis of

[1] Gray and Shadbegian (1998) indicate the variability of plant and equipment condition in paper mills.

these programs, the policy maker knows the distribution of plant and equipment condition across firms, but not the condition of any individual firm's plant and equipment.[2] Moreover, the condition cannot be inferred because damage caused by individual firms cannot be measured. Whereas the condition of plant and equipment is not observable, a policy maker can tell whether a specific conversion has been made, and also knows the firm's output of the final product. For example, it is possible to tell if scrubbers have been installed in a power plant and to know the amount of power generated, but not the damage.[3]

To induce firms to make conversions—or to take any action to improve the environment—policy makers must choose among alternative programs that include command-and-control policies such as technology mandates and performance standards, and economic instruments such as taxes, subsidies, and tradeable permits. Consequently, the choice between programs is critical, and important work has been done to provide conditions under which one program should be preferred over another for meeting environmental objectives (e.g., Cropper and Oates 1992, Nault 1996). However, the influence of this work on environmental policy has been modest because the development and comparison of these programs has ignored the political economy of environmental policy (Hahn 2000). Indeed, the political setting in which environmental policy is made constrains the form of programs that can be used (Boyer and Laffont 1999). We argue that simplicity of implementation and administration is an important political constraint.

Our focus on technology conversions reflects the fact that adoption of cleaner technologies is critical to substantive long-term improvements in environmental performance (Angell and Klassen 1999). We examine different subsidy and tax programs to motivate firms to convert to cleaner technology. Within each program, we compare the effectiveness of an administratively simple uniform lump sum and a variable incentive based on output. Our comparison of uniform lump-sum and variable incentives incorporates multiple objectives, including environmental damage, as well as costs and benefits facing consumers and producers.

Our model differs from prior work in two important ways. First, we compare different program elements such as lump-sum and output-based components of a subsidy or tax program. Prior work has not considered choices between different program elements, but rather has concentrated on the impact of a variety of circumstances on the efficiency and form of the optimal program. Second, we incorporate the heterogeneity among firms by integrating the condition of a firm's plant and equipment with its production through its profit function, with the environmental damage generated by production, and with the costs of technology conversion. Our significant new finding from integrating these three aspects with the condition of firms' production technology is that under reasonable circumstances, public-policy makers will prefer the uniform lump-sum incentive. That is, we determine specific and easily interpreted conditions under which the policy maker should use only the uniform lump-sum element of a subsidy or tax program. This is important because, in addition to improved social welfare, such a program is simpler to administer and faces fewer political constraints than a more complicated environmental program based on output.

We use "condition" to represent any unobservable aspect of plant and equipment that influences a firm's cost of production, environmental impact, and the cost of converting its plant. We assume firms differ in the conditions of their plant and equipment such that firms with plants in worse condition have higher production costs, inflict greater environmental damage, and face higher costs of conversion. We first show that, not surprisingly, the policy maker prefers a perfectly discriminating incentive—whether it be a tax or subsidy—that is higher for a plant and equipment that is in worse condition. However, because the policy maker can observe only the firm's output and whether it has made the conversion, and cannot depend on the firm revealing its true condition through its choice of output, the policy maker cannot implement such a perfectly discriminating program. We then demonstrate that when faced with a uniform lump sum and an output-based incentive, firms operating plant and equipment in *better* condition convert to mitigate their environmental damage and those with plant and equipment in worse condition do not. Using this result to construct a social welfare function, we show that under reasonable conditions the exclusive use of a uniform lump-sum incentive is the optimal public policy.

[2] We take the condition within each firm as being homogeneous. Many firms have heterogeneous equipment, and consequently may decide to convert some but not all of their equipment. Our model applies to heterogeneous equipment, with decisions being made over each unit of equipment, provided these decisions on whether to convert are independent.

[3] The conversion problem can be illustrated by the conversion of commercial vehicles to alternative, cleaner fuels in Canada, where Natural Resources Canada, in conjunction with local utilities, operates an incentive program. For each vehicle in a fleet that is converted to natural gas or propane, a firm receives a $1,000 lump-sum subsidy from the government and a $500–$1,000 lump-sum subsidy from the utility. The conversion is clearly observable, as is also the vehicle mileage in, for example, taxi fleets, which have made good use of the incentive program. What is not observable, however, is the condition of each vehicle.

We extend our model to show that the main results remain true when conversion costs fall in total and at the margin, with increases in the proportion of firms that convert—a network externality in conversion. We show that this network externality increases the number and output of firms that convert, and these effects reinforce the conditions under which a uniform lump-sum incentive is the optimal public policy. We also extend our model to include strategic timing, where firms enjoy flexibility in when to convert. We incorporate time through conversion costs—costs that fall over time—and derive an intertemporal formulation of the firms' and policy-maker's problems. The output-based incentive encourages earlier conversion, and the conditions under which a uniform lump-sum incentive is the optimal public policy requires a mild additional condition concerning the impacts of earlier conversion.

Related Literature. Other papers have focused on taxes or subsidies related to output in order to reduce firms' production and to thereby *indirectly* reduce externalities, where the principal concern has been with the implication of taxes or subsidies for firms' outputs versus industry output (for example, Baumol and Oates 1988, Polinsky 1979, Burrows 1979). Often, however, policies are analyzed in the context of an individual representative firm in a competitive framework. Because firms are identical, each firm in the industry is similarly affected. Mirrlees (1986) considers firms that differ, but in the absence of externalities such as environmental damage. Spulber (1989) incorporates production externalities, but, unlike our paper, treats output and externalities as separate decision variables.

In our paper, firms have private knowledge about the operating condition of their plant and equipment. Others have addressed information asymmetries about different aspects of the firm. For example, when firms are privately informed about their clean-up costs, Kwerel (1977) finds that a combined license and subsidy mechanism can induce firms to reveal these costs so the planner can minimize clean-up costs and the effects of pollution. Weitzman (1978) analyses regulation when the planner does not know firm costs, and shows that a mixed price and quota system is the optimal social-planning mechanism. Segerson (1988) shows that an efficient solution is achievable when the policy maker can observe overall environmental quality. If the policy maker can observe emissions only by costly monitoring, Swierzbinski (1994) shows how a deposit-refund system motivates firms to report actual emissions. Similarly, when monitoring emissions is costly, but monitoring output is not, Schmutzler and Goulder (1997) demonstrate that a tax on output together with a tax on emissions is optimal when fixed costs of monitoring are low and marginal costs of monitoring increase rapidly with increasing monitoring effort.[4]

Our model implicitly treats the price of output as deterministic, and firm profits depend on output. Others have examined settings where output price is formulated differently. Cortazar et al. (1998) assume output price is stochastic and use real options to evaluate investments in technologies. Klassen and McLaughlin (1996) find that positive and negative environmental events impact market valuations of firms. However, that analysis did not control for output effects, and thus the changes in valuations may have resulted from output effects that were caused by the environmental events. Apart from heterogeneous-condition firms and the information asymmetry concerning condition, we do not consider the further complications introduced when other distortions influence production and consumption. Such complications as preexisting taxes on income, payroll, sales, or factor use can affect the optimality of environmental policies in a general equilibrium setting. Bovenberg and Goulder (1996) and Goulder et al. (1999) show that these distortionary prior taxes can influence the optimal environmental policy instrument.

The set of incentives we consider provides policy makers with the scope to choose the program according to practical considerations such as political acceptability. Environmentalists may object to subsidies on the grounds that it is inappropriate to "bribe" firms not to pollute. On the other hand, there are often difficulties associated with taxation that are not present with subsidies.[5] A subsidy may also be preferred when there are positive network externalities from the conversion. For example, converting more vehicles could reduce conversion costs if the production of cleaner technology enjoys returns to scale. In addition, each vehicle converting from gasoline to natural gas increases the chance that additional natural gas stations will be built because of greater demand, and in this way encourages other vehicle owners to convert. The subsidy to convert, given these positive spillovers, may be viewed as more acceptable than a tax on those that choose not to convert.[6]

[4] For a review of papers that deal with information asymmetries different from ours see Lewis (1996).

[5] For example, as Crandall (1983) has argued, there has been a reluctance to apply a sulfur tax to coal-fired electricity-generating power plants because of the negative job consequences in the mining of high-sulfur coal located in areas of chronic high unemployment. A subsidy to convert from coal to natural gas, or to add emissions scrubbers, may be more acceptable. Similarly, Palmer and Walls (1997) argue that new taxes on solid waste disposal would face political opposition, indicating that an alternative instrument such as a subsidy (deposit-refund system) may be preferable.

[6] Recognizing the mutuality of the positive network externalities of having more convenient supplies of alternative fuels and of vehicle

Another practical consideration concerns the administrative cost of applying any variable incentive that requires that policy makers gather information about a firm's operations—such as the ongoing measurement of emissions. Brock and Evans (1985), for example, employ an administrative cost that is increasing in the marginal tax rate. They reason that because firms would want to misrepresent the condition of their plant and equipment, the policy maker would have to hire accountants to monitor and lawyers to litigate to discourage the avoidance of taxes. Thus, there would be increasingly higher costs of reporting, record keeping, and enforcement the higher the tax rate. A similar argument applies to variable subsidies.

In §2, we provide our main assumptions and notation. In §3, we outline our approach and set up our basic model; §3.1 solves firm production decisions under the conversion and no-conversion options; §3.2 contains the implications of these decisions for feasible incentive schemes; and §3.3 aggregates individual firms' choices to derive industry effects. In §4, we specify our definition of social welfare, solve the social welfare maximization program, and derive our main result. In §5, we extend the analysis to include network externalities in conversion, and in §6, we extend the analysis to incorporate strategic timing in conversion. Section 7 discusses applications and further extensions of our analysis.

2. Assumptions and Notation

Our assumptions relate the condition of firms' plant and equipment to their environmental impact, production costs, and cost of conversion.

Assumption 1. *Firms differ in the condition of their plant and equipment, and an individual firm's condition is not verifiable.*

We allow firms to differ in the condition of their plant and equipment, where condition is influenced by a combination of factors such as type, age, and maintenance level. Let $\theta \in [\underline{\theta}, \bar{\theta}]$ represent the condition of a firm's plant and equipment, where θ follows the density $f(\theta) > 0$. $F(\underline{\theta}) = 0$ and $F(\bar{\theta}) = 1$, where $\underline{\theta}$ and $\bar{\theta}$ are firms in the "best" and "worst" condition, respectively. The condition of an individual firm's plant and equipment cannot be observed by the policy maker and cannot be verified and used as part of a policy program. The condition of a plant depends on the nature of its overall facilities, which may be of a wide range of vintages and maintenance levels. For example, furnaces and boilers might be old and/or run-down, while control/inventory systems are new and in good shape, making it difficult to assign a specific condition.

In addition, a firm's condition cannot be inferred from the level of output, x, because there are many factors that affect output as well as the condition of the plant. For example, previous output may have depended on market conditions and idiosyncratic factors that affected firms' costs in the past. Firms might also have anticipated a tax or subsidy program and made output adjustments in advance. And, finally, the fact that different facilities exist in a single plant makes identifying a firm's condition difficult. The policy maker does know the distribution and range of plant and equipment conditions in the industry.

The production of the final product and the condition of a firm's plant and equipment combine to produce environmental damage from, for example, air or water pollution. Individual firms choose whether to convert to a cleaner production process. Environmental damage from converting and nonconverting firms are represented by $q(x, \theta)$ and $q_n(x, \theta)$, respectively.[7] The following two assumptions specify the effects of output and plant condition on the level of environmental damage. Written in terms of a converting firm's environmental damage, they also apply to damage by nonconverting firms.

Assumption 2. $\dfrac{\partial q(x, \theta)}{\partial x} > 0$ *and* $\dfrac{\partial q(x, \theta)}{\partial \theta} > 0$.

The first condition states that the environmental damage is increasing in output. For example, in electric-power generation the pollution emitted increases with the amount of power generated. Similarly, vehicles that are driven more pollute more. The second condition means that firms with plant and equipment in worse condition produce greater environmental damage for a given level of output; poorly maintained plants or vehicles pollute more.

Assumption 3. $\dfrac{\partial^2 q(x, \theta)}{\partial x\, \partial \theta} > 0$.

Assumption 3 implies that the additional damage resulting from an increase in output is larger for firms with plants in worse condition. In electricity generation, power plants in poorer condition pollute more as power output is increased than do plants in better condition. Similarly, "poorer-condition" vehicles pollute more as they are driven further per period of time.

When not making the conversion to mitigate its damage, the reduced-form profit function for the firm producing x with equipment condition θ is $PR(x, \theta)$. This reduced form represents profits from a finite

conversions, Natural Resources Canada provides a uniform lump-sum subsidy of $50,000 per public gas station that installs natural gas or propane services.

[7] We assume that the functions used are continuous and twice differentiable everywhere as required.

number of individual firms engaged in Cournot competition, where for convenience we suppress industry output as an argument in the notation: $PR(x,\theta) \equiv PR_X(x,\theta) = r(X)\,x - g(x,\theta)$. $r(X)$ is the inverse demand of the final product and is a function of aggregate output, X. Aggregate output is increasing in individual firm output, x, and price is decreasing and concave in aggregate output. $g(x,\theta)$ is the cost of producing x, which is increasing and convex, with negative cross effects so that firms with "poorer-condition" plants have higher marginal costs. Using this exact Cournot form, the first-order condition for profit maximization is

$$\frac{\partial PR(x,\theta)}{\partial x} = \frac{d\,PR_X(x,\theta)}{dx} = r(X) - \frac{\partial g(x,\theta)}{\partial x} + r'(X)\,\frac{dX}{dx} = 0.$$

The second-order condition follows from concave inverse demand and convex costs. Along with these curvature conditions, technical conditions can be assumed to obtain existence and uniqueness of the Cournot equilibrium in pure strategies (see Tirole 1988, pp. 224–226.) We can summarize the assumptions imbedded in the structure of our output market as:

Assumption 4. $\dfrac{\partial^2 PR(x,\theta)}{\partial x^2} < 0$ *and* $\dfrac{\partial PR(x,\theta)}{\partial \theta} < 0.$

That is, the reduced-form profit function is concave in output and is decreasing with declines in the operating condition of plant and equipment. Concavity can be based on increasing marginal costs and/or declining marginal revenue; expanding electricity production at any given plant causes increasing marginal cost and/or a need to reduce electricity prices to sell the extra output. The effect of plant and equipment condition on profit follows if production costs of plants in poorer condition are higher. This coexistence of firms with different equipment conditions and profits can occur in regulated markets and markets where there is product differentiation, locational advantage, friction in entry or capital formation, or where different firms have similar minimum costs at different output levels due to different conditions of plant and equipment.[8]

The fifth assumption is that marginal profit is decreasing with worsening condition of a firm's plant and equipment; i.e.,

Assumption 5. $\dfrac{\partial^2 PR(x,\theta)}{\partial x\,\partial \theta} < 0.$

[8] Our reduced-form profit function can be representative of other structures in the market for the industry output so long as firms make profits and industry demand is downward sloping.

That is, firms with "worse-condition" plant and equipment have greater marginal costs and, consequently, smaller additions to profits from increased output. Thus, our model applies to industries in which higher θ firms face higher marginal costs when expanding output. For example, vehicles in worse condition break down more often when they are driven more.

We represent the cost of conversion to mitigate environmental damage from the level of output x as $C(x,\theta)$. This conversion cost is the incremental flow cost from having converted. Any capital cost of conversion is translated to a flow to include with ongoing incremental operating cost, either by using the rental cost or by allowing for the opportunity cost of the capital expenditure. We assume that this cost is weakly convex in output and increasing with declines in the condition of plant and equipment:

Assumption 6. $\dfrac{\partial^2 C(x,\theta)}{\partial x^2} \ge 0$ *and* $\dfrac{\partial C(x,\theta)}{\partial \theta} > 0.$

In the context of electric-power generation, the marginal cost of emissions reduction does not decrease with increased electricity production, and it is more expensive to reduce emissions at given output levels in "poorer-condition" plants.[9]

Assumption 7. $\dfrac{\partial^2 C(x,\theta)}{\partial x\,\partial \theta} \ge 0.$

Similar to Assumption 5 on the profitability of production, Assumption 7 implies that the marginal cost of conversion is (weakly) higher for firms with poorer-condition equipment. For example, a vehicle in worse condition faces a higher capital or operating cost of conversion.

Our modeling of conversion technology includes the conversion being an upgrade with new equipment whose purchase costs would not differ between firms. Production typically involves a series of facilities/machines for different stages, so conversion means replacing some part of the plant and linking this newly converted part with remaining facilities. This means that installation costs can still differ if, for example, installation of the new equipment is more expensive in a plant in poorer condition. Running the converted facility at higher throughput may also be more costly in plants with old remaining equipment alongside converted equipment. Some remaining parts of plants in poorer condition might be more

[9] No additional assumption is made about the relationship between conversion costs and output. Costs of a discrete conversion may be fixed, in which case conversion costs would not change with output. Alternatively, conversion costs may increase with output, possibly because conversion represents downtime for the firm, and the opportunity cost of the conversion is the profit on the foregone output.

adversely affected by a conversion of some other part of the production process. Breakdown might be more likely from the change somewhere along the production chain, resulting in higher operating/repair costs. Finally, plants in poorer condition might require different conversion equipment that is more expensive than that required by plants in better condition, where condition refers to the parts that are not being converted.

To determine which firms convert, we require that the cross effect of output and the firm's operating condition on the reduced-form profit function is small compared to the effect of condition on the cost of conversion:

Assumption 8. *The effect of the condition of plant and equipment on conversion costs exceeds the difference in the effect of condition on reduced-form profits from different levels of output.*

This assumption, which is necessary for our separation of firms into continuous groups that convert and those that do not, can be expressed as

$$-\frac{\partial^2 PR(x^*, \theta)}{\partial x^* \partial \theta}(x_2 - x_1) < \frac{\partial C(x^*, \theta)}{\partial \theta},$$

where x_1 and x_2 are the two output levels and $x^* \in [x_1, x_2]$. Should $PR(x, \theta)$ be additively separable in x and θ, then $\partial^2 PR(x, \theta)/\partial x\, \partial \theta = 0$, and Assumption 8 would follow directly.

The critical combination of assumptions are Assumptions 3, 5, and 7, reflecting aspects of the firm's plant and equipment condition that are likely to be associated. This formulation is substantially different from others such as Spulber (1989), who employs a quadratic form for costs that associates condition, but not output, with the negative externality. That form results in zero cross effects rather than those in Assumptions 3, 5, and 7.[10]

3. Firm and Industry Behavior

We set up our model as a three-step game of incomplete information, where the firms have private information concerning the condition of their plant and equipment. Our analysis focuses on the subsidy program.[11]

The steps are ordered chronologically: In Step 1, the different subsidy programs are evaluated by the policy maker, who selects an option. At this time the policy maker knows the distribution of firms' plant conditions, but cannot identify an individual firm's condition. In Step 2, firms decide whether to convert to reduce environmental damage (e.g., installation of a scrubber, conversion to natural gas, etc.). In Step 3, firms make output decisions. At this time, output, and whether the firm has converted, is observed by the policy maker. However, individual firms' environmental damage (e.g., emissions) is not observed. Firms electing to convert receive their uniform lump-sum and/or output-based subsidy payments.

We solve these three steps in reverse order, beginning with Step 3, where we solve the individual firm's production decision under the conversion and no-conversion options. We then show that given the structure of the firms' output decisions, a perfectly discriminating subsidy is infeasible, leaving the policy maker with a combination of a uniform lump sum and a variable subsidy based on output. In Step 2, we determine the industry's response to the program—which firms convert and which firms do not. In Step 1, we determine the consequences of the programs for consumers, producers, and social welfare, showing the conditions under which a uniform lump-sum subsidy is preferred to an output-based subsidy.

3.1. Firms' Production Decisions: Step 3

Firms' Responses to Incentives: Firms That Convert. Writing $s(x)$ as the subsidy schedule based on output, $s(0) = 0$, and S as the uniform lump-sum subsidy paid to those firms that convert, the profit of a firm that converts is $PR(x, \theta) + S + s(x) - C(x, \theta)$.[12,13] A profit-maximizing firm will set its output as the x that satisfies the first-order condition

$$\frac{\partial PR(x, \theta)}{\partial x} + s_x - \frac{\partial C(x, \theta)}{\partial x} = \Psi(x, s_x, \theta) = 0, \quad (1)$$

where $s'(x) = s_x$ represents the marginal subsidy, that is, the added revenue associated with a marginal increase in output of the final product.[14] (1) implicitly defines $x(s_x, \theta)$. The third term is the marginal cost of conversion. Indeed, should the marginal cost of conversion be positive, the output of the final product is expanded until the marginal profit on producing and selling the final product is just offset by the net marginal cost of conversion.

[10] Under similar assumptions, Nault (1996) shows that from a set of policy objectives that includes industry output, total negative externalities, and social welfare, any two objectives that are achieved by a given tax regime can also be achieved through a regime of subsidies.

[11] Specification and solution of the alternative tax program, for which the analysis is parallel, is available online at http://mansci.pubs.informs.org/ecompanion.html.

[12] S and $s(x)$ are nonnegative. Otherwise, they would be taxes.

[13] Our model does not suffer from the entry problem that has been associated with subsidies: We allow payment of the lump-sum subsidy only to existing firms that have not previously made the conversion (Cropper and Oates 1992). As a result, our program does not have the drawback of making the industry more attractive to prospective new entrants.

[14] In the special case of a constant unit subsidy, $s(x) = s_x x$, where s_x is a constant.

The output from (1) is a profit-maximizing output if

$$\frac{\partial \Psi(x, s_x, \theta)}{\partial x} = \frac{\partial^2 PR(x, \theta)}{\partial x^2} + s''(x) - \frac{\partial^2 C(x, \theta)}{\partial x^2} < 0. \tag{2}$$

As long as the marginal subsidy is decreasing, (2) is satisfied directly from Assumptions 4 and 6. The following lemmas indicate the effects of the marginal subsidy and the condition of plant and equipment on the output of those firms that convert.[15]

Lemma 1. *For firms that convert, output is increasing in the marginal subsidy.*

We see that the marginal subsidy increases the output of the final product, and therefore from Assumption 2 the environmental damage also increases with the size of the marginal subsidy. In addition, it is clear that for individual firms the uniform lump-sum subsidy does not affect the output of the final product or, consequently, the associated damage.

Lemma 2. *For firms that convert, output is higher for firms with plant and equipment in better condition.*

From Lemma 2, firms with plant and equipment in poorer condition, all things equal, produce less of the final product. Because they generate greater environmental damage at each output, these firms may nevertheless cause higher levels of damage.

Firms' Responses to Incentives: Firms That Do Not Convert. For firms that choose not to convert, $\Pi(x, \theta) = PR(x, \theta)$, and output is set such that

$$\frac{\partial PR(x, \theta)}{\partial x} = 0 = \Psi(x, \theta). \tag{3}$$

This is optimal provided that

$$\frac{\partial \Psi(x, \theta)}{\partial x} = \frac{\partial^2 PR(x, \theta)}{\partial x^2} < 0, \tag{4}$$

which follows from Assumption 4. Lemma 3 indicates that Lemma 2 holds for these firms.

Lemma 3. *For firms that do not convert, output is higher for firms with plant and equipment in better condition.*

Thus, for firms that do not convert, $x'(\theta) < 0$. The output function of the individual firm choosing not to convert, $x(\theta)$, omits s_x because for these firms neither s_x nor S are relevant for output or environmental damage.

3.2. Perfect Discrimination and Revelation

If the policy maker had perfect information on firms' plant and equipment conditions in Step 1, then the policy maker could consider a perfectly discriminating lump-sum subsidy that determines which firms should convert based on the condition of the firms' plant and equipment. Compensation to firms that converted would be just sufficient in each case to make their profits the same as those that would arise otherwise. In this way, the policy maker would achieve a first-best solution. Unfortunately for the policy maker, our assumptions and earlier results rule out this possibility. This is because the level of this perfectly discriminating subsidy would have to be based on the condition of a firm's plant and equipment, which cannot be observed by the policy maker, rather than on output and whether the firm converts, which the policy maker can observe. If with full information about the condition of plant and equipment the perfectly discriminating subsidy is increasing in θ, then a problem arises from using output as a proxy for condition: Firms with plant and equipment in better condition will misrepresent themselves by lowering output, masquerading as "worse-condition" firms to collect the higher subsidy.[16] We show below that the perfectly discriminating subsidy—the full-information solution—cannot be attained given the information constraints, and that any second-best subsidy program must be weakly increasing in output.

Theorem 1. *The perfectly discriminating subsidy is infeasible.*

The perfectly discriminating subsidy would be set to equate a firms' profit from converting and not converting. We find that firms with plant and equipment in worse condition require a greater subsidy to make conversion profitable. The policy maker would infer condition from output, providing a larger subsidy to those firms with lesser output, and hence motivating firms to misrepresent themselves through reducing their output. Consequently, the first-best solution is infeasible. The following corollary follows directly from the revelation principle.

Corollary. *The second-best subsidy scheme must be nondecreasing in output.*

The key result from this subsection is contained in the corollary above: Truthful revelation requires the subsidy be nonincreasing in θ, which, using Lemma 2, means the subsidy must be nondecreasing in output. Otherwise, firms would be motivated to reduce output to collect a higher subsidy. Thus, we can eliminate from further consideration any subsidy program that is decreasing in output.

[15] The proofs of our lemmas and theorems are available online.

[16] This misrepresentation can occur for as long as it is beneficial.

3.3. Industry Response to Subsidy: Step 2

Subsidies and the Marginal Mover. Each firm maximizes profit by deciding whether to convert; that is, it selects $\max\{PR(x(s_x,\theta),\theta)+S+s(x(s_x,\theta))-C(x(s_x,\theta),\theta), PR(x(\theta),\theta)\}$, where $x(s_x,\theta)$ and $x(\theta)$ are the optimal outputs from plant and equipment that are converted and not converted, respectively. Without loss of generality, we assume that $PR(x(\bar{\theta}),\bar{\theta})>0$ so that even the firm with plant and equipment in the worst condition can make profit by producing the final product, while rejecting the subsidy.

We identify the firm with condition $\tilde{\theta}$, which is indifferent between converting and not converting by

$$PR(x(s_x,\tilde{\theta}),\tilde{\theta})+S+s(x(s_x,\tilde{\theta})) - C(x(s_x,\tilde{\theta}),\tilde{\theta})-PR(x(\tilde{\theta}),\tilde{\theta})=0=\phi(S,s_x,\tilde{\theta}), \quad (5)$$

which defines $\phi(S,s_x,\tilde{\theta})$. This leads to our second theorem.

Theorem 2. *Firms with plant and equipment in better condition convert, and firms with plant and equipment in worse condition do not.*

Theorem 2 shows that $\phi(S,s_x,\tilde{\theta})$ is decreasing in the condition of plant and equipment, so that "better-condition" firms make higher profits from converting, and "worse-condition" firms do not. Figure 1, drawn for specific values of S and s_x, shows the derivation of $\tilde{\theta}$ associated with (5). Theorem 2 is important because it separates the continuum of firm plant and equipment conditions into contiguous segments, a version of the "single-crossing property" in the incentives literature (Fudenberg and Tirole 1991). "Better-condition" firms convert because the subsidy program is limited by the corollary to Theorem 1: "Worse-condition" firms cannot receive larger subsidies. It is worth noting that our model does not rule out all firms converting, or the other extreme where no firms convert.[17]

To compare the effects of the uniform lump-sum and output-based subsidies, we need to compare the effects of the two subsidies on the firm just finding it worthwhile to convert, $\partial\tilde{\theta}(S,s_x)/\partial S$ and $\partial\tilde{\theta}(S,s_x)/\partial s_x$, and thereby the effects on the proportion of firms converting.

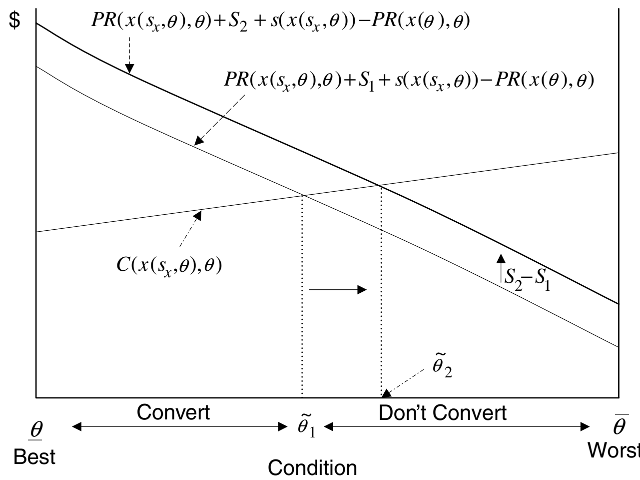
Lemma 4. *The proportion of firms converting is increasing in the uniform lump-sum and output-based subsidy.*

The lemma is illustrated in Figure 1 for an increase from S_1 to S_2 in the uniform lump-sum subsidy.[18]

[17] Theorem 3 provides a condition under which at least one firm will convert.

[18] While similar, the impact of a change in the output-based subsidy is more difficult to show graphically because of additional effects through output.

Figure 1 Theorem 2; Change in Lump-Sum Subsidy



Using the implicit function rule in the proof of Lemma 4, we find that the effect of a change in the output-based subsidy is precisely the effect of a change in the uniform lump sum multiplied by output,

$$x(s_x,\tilde{\theta})\frac{\partial\tilde{\theta}(S,s_x)}{\partial S}=\frac{\partial\tilde{\theta}(S,s_x)}{\partial s_x}>0. \quad (6)$$

The relationship in (6) calibrates the effect of changes in each of our policy variables, which will be important later on. For the remainder of this paper we will drop the arguments of $\tilde{\theta}$ where they are unnecessary.

4. The Policy Problem: Social Welfare

The objective of the policy maker is to maximize social welfare of the chosen regime. Our social welfare function incorporates criteria from multiple stakeholders: consumers, producers, and the public at large. We define the social welfare function to be maximized as the sum of consumer and producer surplus, less the total value of environmental damage,

$$B(S,s_x)=CS(X(S,s_x))+PS(S,s_x) - \omega(Q_n(S,s_x),Q(S,s_x)),$$

where consumer surplus, $CS(X(S,s_x))$, is increasing in aggregate output, $X(S,s_x)$, producer surplus is $PS(S,s_x)$, and the total value of environmental damage is $\omega(Q_n(S,s_x),Q(S,s_x))$; $Q_n(S,s_x)$ and $Q(S,s_x)$ are aggregate damages from nonconverting and converting firms, respectively. We naturally assume that $\omega(\cdot)$ is (weakly) increasing in its arguments. The direct effect of the subsidy program is a transfer and does not directly affect social welfare.

Normalizing the number of firms to unity, aggregate output and producer surplus are

$$X(S,s_x)=\int_{\underline{\theta}}^{\tilde{\theta}(S,s_x)} x(s_x,\theta)f(\theta)\,d\theta+\int_{\tilde{\theta}(S,s_x)}^{\bar{\theta}} x(\theta)f(\theta)\,d\theta$$

and

$$PS(S, s_x) = \int_{\underline{\theta}}^{\tilde{\theta}(S, s_x)} [PR(x(s_x, \theta), \theta)) - C(x(s_x, \theta), \theta)] f(\theta)\, d\theta + \int_{\tilde{\theta}(S, s_x)}^{\bar{\theta}} \Pi(x(\theta), \theta) f(\theta)\, d\theta,$$

respectively. The arguments of $\omega(\cdot)$ are aggregate environmental damages from nonconverting firms and from converting firms, respectively:

$$Q_n(S, s_x) = \int_{\tilde{\theta}(S, s_x)}^{\bar{\theta}} q_n(x(\theta), \theta) f(\theta)\, d\theta \quad \text{and}$$

$$Q(S, s_x) = \int_{\underline{\theta}}^{\tilde{\theta}(S, s_x)} q(x(s_x, \theta), \theta) f(\theta)\, d\theta.$$

The social welfare measure includes the impact of emissions from nonconverting firms as well as emissions from those that do convert, recognizing that environmental damage can also result from the emissions of converting firms. Of course, where conversion removes all environmental damage, $\partial\omega(\cdot)/\partial Q = 0$. Social welfare also includes the producer surplus and the consumer surplus from the final product; if the subsidies result in more output, there is a consumer surplus gain tending to offset the change in producer surplus and the environmental damage. We note that the $q_n(x(\theta), \theta)$ in $Q_n(S, s_x)$ is not impacted by s_x, as only output of firms having converted are affected by the output-based subsidy.

The following two lemmas establish the effects of the uniform lump-sum and output-based subsidies on producer surplus.

Lemma 5. *Producer surplus is decreasing in the uniform lump-sum subsidy.*

Lemma 6. *Producer surplus is decreasing in the output-based subsidy.*

Intuitively, because producer surplus does not include the payments from the subsidy program, and yet explicitly includes the costs of conversion, the effect of either type of subsidy on this surplus is negative. The next two lemmas determine the effects of each subsidy on aggregate damage.

Lemma 7. *Aggregate environmental damage from converting firms is increasing, and aggregate environmental damage from nonconverting firms is decreasing, in the uniform lump-sum subsidy.*

Lemma 8. *Aggregate environmental damage from converting firms is increasing, and aggregate environmental damage from nonconverting firms is decreasing, in the output-based subsidy.*

From Lemma 4, more firms convert as a result of an increase in either subsidy, so the effect through the number of firms converting increases the aggregate damage from converting firms by virtue of having more converting firms. In addition, an increase in the output-based subsidy motivates converting firms to raise output and, consequently, damage.

It is not possible to determine the effect of the uniform lump-sum or output-based subsidies on aggregate output. Differentiating aggregate output with respect to the uniform lump-sum subsidy results in

$$\frac{\partial X(S, s_x)}{\partial S} = x(s_x, \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial S} - x(\tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial S}. \tag{7}$$

The sign of (7) depends on the relative size of outputs under the conversion and no-conversion options, which in turn depends on the size of the output-based subsidy and the marginal conversion cost in (1). Differentiating aggregate output with respect to the output-based subsidy, and using (6), yields

$$\begin{aligned} \frac{\partial X(S, s_x)}{\partial s_x} &= x(s_x, \tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial s_x} + \int_{\underline{\theta}}^{\tilde{\theta}(S, s_x)} \frac{\partial x(s_x, \theta)}{\partial s_x} f(\theta)\, d\theta - x(\tilde{\theta}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}(S, s_x)}{\partial s_x} \\ &= \frac{\partial X(S, s_x)}{\partial S} x(s_x, \tilde{\theta}) + \alpha, \end{aligned} \tag{8}$$

where $\alpha > 0$ represents the term under integration. The opposing influences include the effects of more firms converting, reduced output of those additional firms, and the effect of a larger marginal subsidy increasing output of those firms that already elect to convert.

From the proofs of Lemmas 7 and 8, we find, similar to (6),

$$\frac{\partial Q_n(S, s_x)}{\partial s_x} = \frac{\partial Q_n(S, s_x)}{\partial S} x(s_x, \tilde{\theta}). \tag{9}$$

(9) is an equivalence condition with respect to effects of changes in the two subsidies on the environmental damage from the marginal nonconverting firm, where the $x(s_x, \tilde{\theta})$ term appears because the marginal subsidy is based on output, should the firm convert. For the environmental damage of converting firms, a similar equivalence condition does not hold because the output-based subsidy increases output—and the resulting damage.

4.1. Lump-Sum and Output-Based Subsidies: Step 1

To establish that a subsidy program of the type we propose can be beneficial, we have to show that welfare increases as a result of the subsidy. Thus, we must show that $B(S, s(x)) > B(0, 0)$ for some S or $s_x > 0$. The following theorem provides a sufficient condition for the uniform lump-sum subsidy to be beneficial, $B(S, 0) > B(0, 0)$.

Theorem 3. *A sufficient condition for the subsidy program to improve welfare is that, for the firm with plant and equipment in the best condition, the reduced environmental damage is greater than the foregone profit and reduced consumer surplus from the reduction in output.*

If the subsidy is large enough to encourage production or if costs of conversion are decreasing in output, then consumer surplus is increased and the condition required for Theorem 3 is less stringent. It is worth recognizing that the firm with plant and equipment in the best condition will have the least loss of profit (before the effect of the lump-sum subsidy) and the smallest effect on consumer surplus because it has the lowest cost of conversion. This firm will also have the greatest damage reduction because it has the largest output.

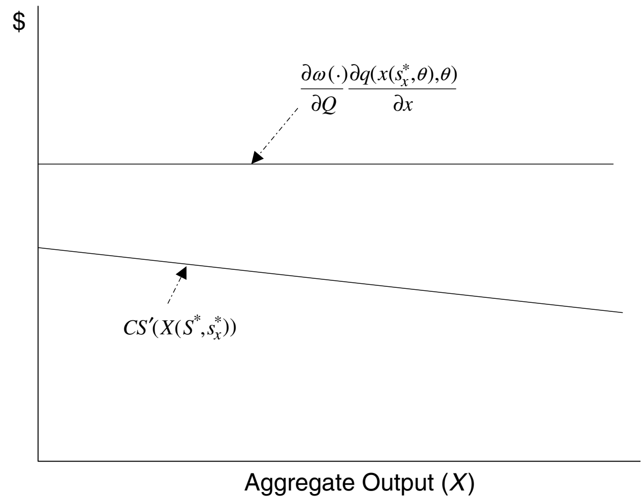
Consider optimizing social welfare, $B(S, s_x)$, by setting the uniform lump-sum subsidy and the schedule of the output-based subsidy, S and $s(x)$. Let the optimal settings that result from this optimal control program be S^* and $s^*(x)$, with the latter schedule yielding s_x^*. The following theorem provides the main result of our paper.

Theorem 4. *A sufficient condition for the optimal subsidy to be a uniform lump sum is that the marginal environmental damage of converting firms is no less than the marginal consumer surplus.*

Uniform lump-sum subsidies are better than output-based subsidies because the latter raise output and, hence, damage. Under the premise of Theorem 4, the proof shows that regardless of whether the uniform lump sum subsidy is positive or zero, the optimal output-based subsidy cannot be positive. Theorem 4 provides a direct way to determine if the optimal subsidy is a uniform lump sum by comparing the relative effects of damage and consumer surplus at the margin. As illustrated in Figure 2, marginal damage must be no less than marginal consumer surplus. Intuitively, the condition precludes the use of the output-based subsidy because the social costs of increased output are higher than the benefits. The lump-sum subsidy does not have a marginal effect on output, but rather determines which firms convert. As we have shown in Theorem 3, and consistent with the condition in this theorem, this conversion can reduce damage—a social benefit—to a greater extent than it negatively impacts producers and consumers.

The condition in Theorem 4 is a binding condition for the optimal output-based subsidy to be zero. Moreover, it is a relationship between two marginal conditions, each of which is an aggregation of primitives of our model. The primitives involved are environmental damage from a converting firm and firm output. Each aggregation involves a sum over individual firm effects and then a monotonic transformation into units that can be compared in the benefit function. The condition in Theorem 4 can apply to many mathematical formulations of firms' damage and profit functions, and to many (monotonic) transformations of the resulting quantities of damage and output into the value of environmental damage and consumer surplus. Functional form assumptions about firms' damage and profit functions would yield a more specific, but no less restrictive, condition.

Figure 2 **Theorem 4; Sufficient Condition for Exclusive Use of Lump-Sum Subsidy**



The condition in Theorem 4 is most likely to be satisfied in industries for which market demand is inelastic—so that consumer surplus diminishes rapidly with consumption—and where marginal environmental damage is increasing in output. For example, it may well occur in the electric-power industry, where demand is relatively price insensitive—people do not turn on more lights because electricity is cheaper—and where atmospheric or water pollution is compounded by exceeding threshold levels.

5. Network Externalities in Conversion

Network externalities may occur if an increase in the number of firms that convert adds additional benefits to conversion, for example, by inducing more natural-gas stations or more service providers.[19] Let y

[19] Alternatively, economies of scale in production of the conversion technology—power cells, pollution scrubbers—might constitute a network externality through cost declines that increase with the number of converting firms.

be the proportion of firms that convert. We add "e" to those assumptions and lemmas that change with network externalities in conversion. Our conversion cost function is then represented by $C(x, \theta, y)$. We assume that this cost is decreasing in the proportion of firms that convert, making the following addition to Assumption 6:

Assumption 6e. $\dfrac{\partial C(x, \theta, y)}{\partial y} < 0.$

In the context of electric-power generation, the network externality means that the cost of emissions reduction decreases with the number of plants that have converted. This would occur if there was learning in the conversion process itself, as well as from other economies of scale in the production and installation of conversion technology. We also require an addition to Assumption 7:

Assumption 7e. $\dfrac{\partial^2 C(x, \theta, y)}{\partial x \partial y} \leq 0$ *and* $\dfrac{\partial^2 C(x, \theta, y)}{\partial \theta \partial y} = 0.$

The first part of Assumption 7e implies that the marginal cost of conversion falls (weakly) with the greater proportion of firms that have converted. That is, the marginal costs of emissions reduction fall the more firms convert. Thus, a network externality makes conversion costs cheaper if more firms convert, both in total and at the margin. These assumptions are standard in the economies of network externalities, namely a downward shift in average and marginal costs at all output levels. The second part of Assumption 7e implies that there is no interaction effect on conversion costs of firm condition and the proportion of firms that have converted. Thus, Assumption 8 holds over all levels of y.

Firm's Production Decisions: Step 3. With network externalities, the analysis of firms' responses to incentives for firms that convert follows extensions of (1), (2), and Lemma 1. (1), as well as optimal output from (1), is redefined to include the proportion of firms converting, $\Psi(x, s_x, \theta, y)$ and $x(s_x, \theta, y)$, respectively. This yields an additional result to Lemma 2:

Lemma 2e. *For firms that convert, output is higher the greater the proportion of firms that convert.*

From Lemma 2e, the more firms convert, the greater the output of each converting firm. As with Lemma 2, because greater output generates greater damage, converting firms may individually cause higher levels of damage the more firms convert. This results as a consequence of lower marginal costs from network externalities increasing the profit-maximizing output. Equations (3) and (4) and Lemma 3 are unaffected by network externalities because they involve firms that do not convert. Theorem 1 and its corollary are also unaffected, although the presence of network externalities requires that the proportion of firms converting be included as arguments to the conversion cost and output of converting firms.

Industry Response to Subsidy: Step 2. Theorem 2 and Lemma 4 are unaffected by the inclusion of network externalities, although in (5) and in Theorem 2 the conversion cost and output of firms that convert require the proportion of firms converting as arguments. However, as a result of Theorem 2 we can formally define the proportion of firms converting, y, as $y(\tilde{\theta}(S, s_x)) = \int_{\underline{\theta}}^{\tilde{\theta}(S, s_x)} f(\theta)\, d\theta$. That is, the proportion of firms that convert is the segment of firms with plant and equipment in better condition. Figure 3 shows graphically that the proportion of firms that convert is greater when there are network externalities lowering conversion costs. To simplify notation, we let $y(\tilde{\theta}(S, s_x)) \equiv y(\cdot)$ for the remainder of the paper. From this definition,

$$y'(\cdot) = dy(\cdot)/d\tilde{\theta} > 0. \tag{10}$$

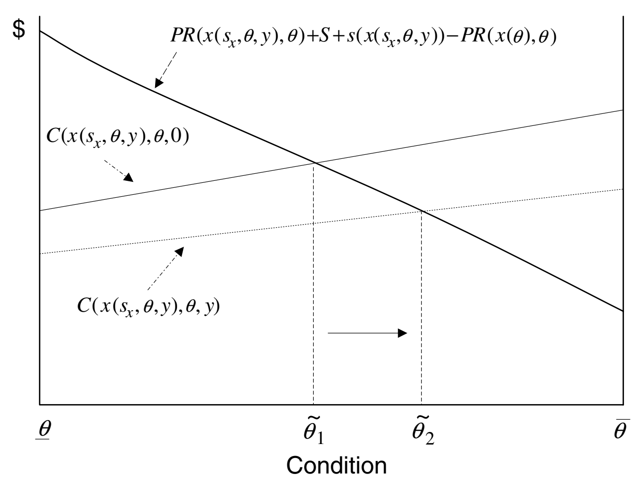
Social Welfare: Step 1. In the presence of network externalities, consumer and producer surplus are redefined to include the proportion of firms converting in the output function of firms that convert and, in the case of producer surplus, the conversion cost: All other things equal, the benefits from conversion cost reductions are a gain in welfare. As a consequence of these redefinitions, Lemmas 5 and 6 must be modified to account for the effect of the subsidy program on producer surplus through the proportion of firms converting.

Lemma 5e. *If the impact of the network externality is greater on output than on conversion costs, producer surplus is decreasing in the uniform lump-sum subsidy.*

Lemma 6e. *If the impact of the network externality is greater on output than on conversion costs, producer surplus is decreasing in the output-based subsidy.*

The producer surplus decreasing in each element of the subsidy program is likely to be true even in

Figure 3 **Effects of a Network Externality**



the absence of the premise condition in Lemmas 5e and 6e. That is, the condition in the premise of Lemmas 5e and 6e is sufficient, but not necessary. For producer surplus to be increasing in either element of the subsidy program would require that the beneficial impact of the network externality on the cost of conversion dominate the detrimental impact of the externality on output plus the negative impact on profit of the marginal firm converting.

In Lemma 7, the effects on aggregate environmental damage of changes in the uniform lump-sum subsidy for converting firms are reinforced by the network externality. That is, as compared to the proof of Lemma 7,

$$\frac{\partial Q(S,s_x)}{\partial S} = q(x(s_x,\tilde{\theta},y(\cdot)),\tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}(S,s_x)}{\partial S} + \int_{\underline{\theta}}^{\tilde{\theta}} \frac{\partial q(x(s_x,\theta,y(\cdot)),\theta)}{\partial x}\frac{\partial x(s_x,\theta,y(\cdot))}{\partial y} \cdot y'(\cdot)\frac{\partial\tilde{\theta}(S,s_x)}{\partial S}f(\theta)\,d\theta. \tag{11}$$

The additional effect is the expression under integration representing the effect of the network externality on output of converting firms. All of the terms under integration are positive from Assumption 2, Lemma 2e, (10), and Lemma 4. Hence, $\partial Q(S,s_x)/\partial S$ is positive. The network externality does not affect the output of those firms that do not convert, so the second part of Lemma 7 remains the same. In Lemma 8, the additional effect is incorporated in $\partial Q(S,s_x)/\partial S$ and the fact that output of converting firms includes the proportion of firms that convert as an argument. Therefore, the effect of the subsidy program for converting firms in Lemma 8 is reinforced by the network externality in the same way as in Lemma 7.

With network externalities, differentiating aggregate output as in (7) yields an additional term:

$$\frac{\partial X(S,s_x)}{\partial S} = x(s_x,\tilde{\theta},y(\cdot))f(\tilde{\theta})\frac{\partial\tilde{\theta}(S,s_x)}{\partial S} - x(\tilde{\theta})f(\tilde{\theta})\frac{\partial\tilde{\theta}(S,s_x)}{\partial S} + \int_{\underline{\theta}}^{\tilde{\theta}(S,s_x)} \frac{\partial x(s_x,\theta,y(\cdot))}{\partial y} \cdot y'(\cdot)\frac{\partial\tilde{\theta}(S,s_x)}{\partial S}f(\theta)\,d\theta. \tag{12}$$

The additional term is the one under integration, and it is positive from Lemma 2e, (10), and Lemma 4. The effect of the network externality on $\partial X(S,s_x)/\partial s_x$ in (8) is contained in the elements of $\partial X(S,s_x)/\partial S$ redefined above, and in the inclusion of the proportion of firms that convert in the argument of the output of converting firms, $x(s_x,\tilde{\theta},y(\cdot))$. Consequently, with positive network externalities the impact of either element of the subsidy program on aggregate output is more likely to be positive.

Theorem 3, which provides a sufficient condition for the uniform lump-sum subsidy to be beneficial, is still true in the presence of network externalities. However, the effect of network externalities can be seen through the change in welfare resulting from a change in the uniform lump-sum subsidy. The expansion of output of converting firms from the presence of network externalities contained in the terms under integration in (11) and (12) makes the change in consumer surplus more likely to be positive, which is in turn offset by a smaller reduction in environmental damage. The change in producer surplus depends on the detrimental network externalities' effect on output relative to the beneficial effect on conversion costs. If the former is larger—the premise of Lemmas 5e and 6e—then the change in producer surplus works against the condition in Theorem 3. However, as we noted in the discussion of Lemmas 5e and 6e, the premise of those lemmas is sufficient but not necessary. Moreover, Lemmas 5e and 6e (or Lemmas 5 and 6) are not needed for Theorem 3.

The main theorem, Theorem 4, is unaffected other than including the proportion of firms that convert as arguments to conversion costs and the output of converting firms. Because the network externality works through the proportion of firms that convert rather than through output, the impacts of each subsidy—lump-sum or output-based—have the same effects via the network externalities, and these effects cancel out in the matching of different forms of subsidy.

6. Strategic Timing in Conversion

Firm decisions about when to convert may be affected if the cost of conversion falls over time and there is a fixed time period during which the subsidy program is offered. This may be the case if, for example, related technologies are improved over time. Let time be continuous and represented by z, and the subsidy program is offered in the interval $[0,\bar{z}]$. We add "z" to those elements of the previous analysis—other than the conversion cost—that change with strategic timing in conversion. We assume that the conversion cost, $C(x,\theta,z)$, is decreasing over time, adding to Assumption 6:

Assumption 6z. $\dfrac{\partial C(x,\theta,z)}{\partial z} < 0.$

The conversion-cost declines in Assumption 6z may be from similar sources as the network externalities, such as learning from related technological developments. To focus on the timing of conversion, we separate the effect of time on fixed conversion costs from the effect of output and of condition on conversion costs. Hence, converting later is less costly, but the conversion technology itself does not change, requiring an addition to Assumption 7:

Assumption 7z. $\dfrac{\partial^2 C(x,\theta,z)}{\partial z\,\partial x} = \dfrac{\partial^2 C(x,\theta,z)}{\partial z\,\partial \theta} = 0.$

From Assumption 7z, conversion time does not affect the marginal effect of condition on conversion costs, so Assumption 8 holds for all z. To clarify the impact of strategic timing on firm decisions, without loss of generality we do not include the time value of money.

Firms' Production Decisions: Step 3. Firms continuously set profit-maximizing output according to (1)–(4), where the arguments to the conversion costs in (1) and (2) include the time of conversion, z. From Assumption 7z, this output is not a function of when firms convert, and Lemmas 1–3 are unchanged. The substance of Theorem 1 and its corollary are also unaffected, although the proof requires equalizing profits from the time of conversion.

Industry Response to Subsidies: Step 2. With strategic timing, a firm's decision is when to convert rather than whether to convert. If firms have not converted by $\bar{z}$, they will not convert. Firms maximize profits when converting at $\hat{z}$ by choosing between converting immediately, where profits are $\int_0^\infty [PR(x(s_x,\theta),\theta)+s(x(s_x,\theta))-C(x(s_x,\theta),\theta,\hat{z})]\,dz+S$; converting in $(0,\bar{z}]$, where profits are

$$\max_{\hat{z}}\left\{\int_0^{\hat{z}} PR(x(\theta),\theta)\,dz + \int_{\hat{z}}^{\infty}[PR(x(s_x,\theta),\theta)+s(x(s_x,\theta)) - C(x(s_x,\theta),\theta,\hat{z})]\,dz+S\right\}; \quad (13)$$

and not converting, in which case profits are $\int_0^\infty PR(x(\theta),\theta)\,dz$. We can identify the firm with condition $\breve{\theta}$ that is indifferent between converting immediately and converting in $(0,\bar{z}]$ by

$$PR(x(s_x,\breve{\theta}),\breve{\theta})+s(x(s_x,\breve{\theta}))-C(x(s_x,\breve{\theta}),\breve{\theta},0) - PR(x(\breve{\theta}),\breve{\theta}) = 0 = \gamma(s_x,\breve{\theta},0). \quad (14)$$

This equation defines when profit flows from conversion are equal to those of waiting at $z=0$, noting that the uniform lump-sum subsidy does not affect the decision of when to convert. If the output-based subsidy is small, (14) may not be satisfied for any $\theta \in [\underline{\theta},\bar{\theta}]$. We can identify the firm with condition $\tilde{\theta}_z$ indifferent between converting at $\bar{z}$ and not:

$$PR(x(s_x,\tilde{\theta}_z),\tilde{\theta}_z)+s(x(s_x,\tilde{\theta}_z))-C(x(s_x,\tilde{\theta}_z),\tilde{\theta}_z,\bar{z})+S - PR(x(\tilde{\theta}_z),\tilde{\theta}_z) = 0 = \phi_z(S,s_x,\tilde{\theta}_z,\bar{z}), \quad (15)$$

similar to (5). Theorem 2 is unaffected other than substituting (15) for (5) in the proof. Therefore, $\theta \le \tilde{\theta}_z$ convert and $\theta > \tilde{\theta}_z$ do not. Allowing firms the choice of when to convert leads to an additional result for Theorem 2.

Theorem 2z. *If $\breve{\theta} < \tilde{\theta}_z$, then for firms that convert, firms with plant and equipment in better condition convert immediately.*

As a consequence of Theorems 2 and 2z, as shown in Figure 4, firms with condition $\theta \le \breve{\theta}$ convert immediately, those with condition $\theta > \tilde{\theta}_z$ do not convert, and those in $(\breve{\theta},\tilde{\theta}_z]$ convert in $(0,\bar{z}]$. If $\breve{\theta} \ge \tilde{\theta}_z$ or if (14) is not satisfied for any $\theta \in [\underline{\theta},\bar{\theta}]$, then we set $\breve{\theta} = \tilde{\theta}_z$ and all converting firms convert at $\bar{z}$.

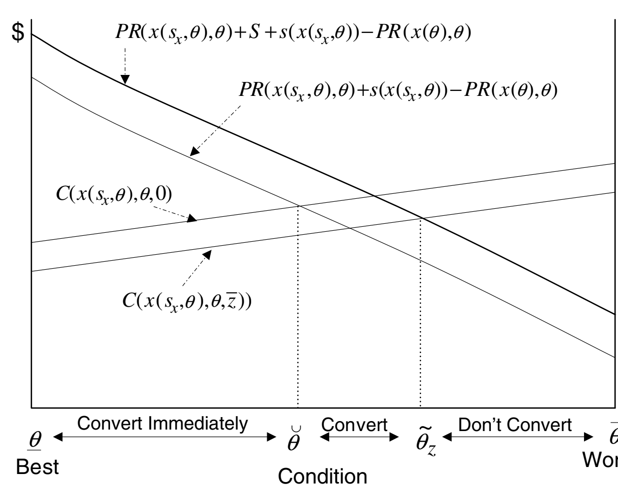
Lemma 4 remains the same except for the inclusion of $\bar{z}$ in the arguments of conversion costs and the subscript z to $\tilde{\theta}$ and ϕ. The calibration relationship in (6) remains the same except for the addition of the subscript z to $\tilde{\theta}$, and the inclusion of $\bar{z}$ in the arguments of $\tilde{\theta}_z$. For those firms that convert in $(0,\bar{z})$, we have the following lemma.

Lemma 9. *For those firms that convert in $(0,\bar{z})$, optimal conversion time is later for firms with plant and equipment in worse condition and earlier for a larger marginal subsidy.*

Social Welfare: Step 1. With firms deciding when to convert, the elements of social welfare must be defined intertemporally. Aggregate output is

$$X_z(S,s_x) = \int_{\underline{\theta}}^{\breve{\theta}(S,s_x)}\int_0^\infty x(s_x,\theta)\,dz f(\theta)\,d\theta + \int_{\tilde{\theta}_z(S,s_x)}^{\bar{\theta}}\int_0^\infty x(\theta)\,dz f(\theta)\,d\theta + \int_{\breve{\theta}(S,s_x)}^{\tilde{\theta}_z(S,s_x)}\left[\int_0^{\hat{z}(s_x,\theta)} x(\theta)\,dz + \int_{\hat{z}(s_x,\theta)}^{\infty} x(s_x,\theta)\,dz\right] f(\theta)\,d\theta.$$

Figure 4 Strategic Timing; Determining When to Convert



The terms on the first line are the outputs from those firms that convert immediately and those that do not convert. The second line is the output from those firms that convert in $(0, \bar{z})$, and includes output pre- and postconversion. Producer surplus is made up of the same elements:

$$\begin{aligned} PS_z(S, s_x) = &\int_{\underline{\theta}}^{\check{\theta}(S, s_x)} \int_0^\infty [PR(x(s_x, \theta), \theta) \\ &\qquad - C(x(s_x, \theta), \theta, 0)]\, dz\, f(\theta)\, d\theta \\ &+ \int_{\tilde{\theta}_z(S, s_x)}^{\bar{\theta}} \int_0^\infty \Pi(x(\theta), \theta)\, dz\, f(\theta)\, d\theta \\ &+ \int_{\check{\theta}(S, s_x)}^{\tilde{\theta}_z(S, s_x)} \bigg[\int_0^{\hat{z}(s_x, \theta)} \Pi(x(\theta), \theta)\, dz \\ &\qquad + \int_{\hat{z}(s_x, \theta)}^\infty [PR(x(s_x, \theta), \theta) \\ &\qquad - C(x(s_x, \theta), \theta, \hat{z}(s_x, \theta))]\, dz\bigg] f(\theta)\, d\theta. \end{aligned}$$

Aggregate damage from nonconverting firms is

$$\begin{aligned} Q_{nz}(S, s_x) = &\int_{\tilde{\theta}_z(S, s_x)}^{\bar{\theta}} \int_0^\infty q_n(x(\theta), \theta)\, dz\, f(\theta)\, d\theta \\ &+ \int_{\check{\theta}(S, s_x)}^{\tilde{\theta}_z(S, s_x)} \int_0^{\hat{z}(s_x, \theta)} q_n(x(\theta), \theta)\, dz\, f(\theta)\, d\theta, \end{aligned}$$

where the first term is damage from nonconverting firms and the second term is preconversion damage from firms that convert in $(0, \bar{z})$. Aggregate damage from converting firms is

$$\begin{aligned} Q(S, s_x) = &\int_{\underline{\theta}}^{\tilde{\theta}_z(S, s_x)} \int_0^\infty q(x(s_x, \theta), \theta)\, dz\, f(\theta)\, d\theta \\ &+ \int_{\check{\theta}(S, s_x)}^{\tilde{\theta}_z(S, s_x)} \int_{\hat{z}(s_x, \theta)}^\infty q(x(s_x, \theta), \theta)\, dz\, f(\theta)\, d\theta, \end{aligned}$$

where the first term is damage from firms that convert immediately, and the second term is postconversion damage from firms that convert in $(0, \bar{z})$.

Lemmas 5 through 8 still hold under strategic timing, although the analysis is more involved and there are additional terms relating to the impact of the marginal subsidy on when to convert. For producer surplus, the additional effect of the output-based subsidy is to encourage earlier conversion, reducing profits of converting firms. The additional effects of the output-based subsidy on aggregate damage are changes in the distribution between damage from nonconverting firms and converting firms caused by encouraging converting firms to convert earlier. These effects reinforce the original effects in Lemmas 5–8.

Using $\hat{z}(s_x, \check{\theta}(S, s_x)) = 0$ and $\hat{z}(s_x, \tilde{\theta}_z(S, s_x)) = \bar{z}$ to simplify the integration limits, the effect of a change in the uniform lump-sum subsidy on aggregate output is

$$\frac{\partial X_z(S, s_x)}{\partial S} = \int_{\bar{z}}^\infty [x(s_x, \tilde{\theta}_z) - x(\tilde{\theta}_z)]\, dz f(\tilde{\theta}_z) \frac{\partial \tilde{\theta}_z(S, s_x)}{\partial S}.$$

The sign depends on the relative size of outputs under the conversion and no-conversion options, as in (7). The effect of a change in the output-based subsidy is

$$\begin{aligned} \frac{\partial X_z(S, s_x)}{\partial s_x} = &\frac{\partial X_z(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) \\ &+ \int_{\underline{\theta}}^{\check{\theta}(S, s_x)} \int_0^\infty \frac{\partial x(s_x, \theta)}{\partial s_x} dz\, f(\theta)\, d\theta \\ &+ \int_{\check{\theta}(S, s_x)}^{\tilde{\theta}_z(S, s_x)} \int_{\hat{z}(s_x, \theta)}^\infty \frac{\partial x(s_x, \theta)}{\partial s_x} dz\, f(\theta)\, d\theta \\ &+ \int_{\check{\theta}(S, s_x)}^{\tilde{\theta}_z(S, s_x)} [x(\theta) - x(s_x, \theta)] \frac{\partial \hat{z}(s_x, \theta)}{\partial s_x} f(\theta)\, d\theta. \end{aligned}$$

The signs of the first term—the effect on the proportion of firms converting—and the last term—the effect of earlier conversion—depend on the relative output under conversion and no-conversion options. The second and third terms representing the output effect are positive from Lemma 1. Using α_z to represent the output effect and η to represent the earlier conversion effect, we can restate the above as

$$\frac{\partial X_z(S, s_x)}{\partial s_x} = \frac{\partial X_z(S, s_x)}{\partial S} x(s_x, \tilde{\theta}_z) + \alpha_z + \eta. \tag{16}$$

Theorem 3 is still true, and even the relative magnitudes of the changes in benefit from an increase in the uniform lump sum are similar as they affect postsubsidy program values. Theorem 4 requires an additional condition to account for the output-based subsidy encouraging earlier conversion:

THEOREM 4z. *A sufficient condition for the optimal subsidy to be a uniform lump sum is that the marginal environmental damage of converting firms is no less than the marginal consumer surplus, and that the decrease in producer surplus from earlier conversion is no less than the change in consumer surplus and mitigation of environmental damage.*

The additional condition in Theorem 4z is the effect of firms that convert in $(0, \bar{z})$. Increases in the output-based subsidy encourage earlier conversion, decreasing producer surplus through higher conversion costs. The effect of earlier conversion on aggregate output and consumer surplus depends on the relative output under the conversion and no-conversion options. If output is greater under conversion, then the effect on consumer surplus runs counter to the effect on producer surplus, and vice versa.

The net effect of mitigation of damage from earlier conversion also depends on the relative output under the conversion and no-conversion options—if output is greater under conversion, damage may be increased, reinforcing the effect on producer surplus. Hence, the effect of earlier conversion on consumer surplus may run counter to the mitigation of damage, making the condition in Theorem 4z more likely.

7. Conclusion

This paper considers incentive programs designed to induce heterogeneous firms to make a major discrete conversion to help the environment. We provide an analysis of uniform lump-sum versus output-based incentives in a realistic setting where firms respond differently because they differ in the conditions of their plant and equipment. Our main result is that under reasonable conditions, exclusive use of a uniform lump-sum incentive is preferred to programs involving variable incentives based on output. This result extends to cases when there are network externalities in conversion and to cases when the incentive program is offered over a period of time and firms make decisions about when to convert.

Our analysis is likely to be viewed favorably by policy makers because when a uniform lump-sum incentive is used exclusively, all that is required is information concerning whether firms make the discrete conversion, rather than recording firms' outputs. For example, it is necessary only to make a visit to a plant to see if the conversion has been made, rather than to continuously monitor the plant. Thus, the uniform lump-sum incentive has an added advantage—it is relatively simple and less costly to administer.[20]

In many industries there may be alternative conversion technologies. In our model, the policy maker is presumed to have computed the social welfare for the alternative conversions and selected the one providing the maximum benefit. The incentive program is offered only to those who make the selected choice. Firms are still self-interested and might make a different conversion choice, but will forego the incentive if they do. Provided there are not other aspects of firms that interfere with the relationships in our assumptions, the results carry through. It is worth noting that this different conversion choice might affect the firm's profit function and maybe its condition. Indeed, choosing alternative technologies instead of or in addition to conversion is what makes it difficult to infer an individual firm's condition. The assumptions in our model, which associate the cost of producing the final product, the cost of converting plant and equipment, and the environmental damage produced, are likely to apply to many industries. Not only does is it apply to energy production, including electricity generation, but also to road, air, and rail transportation, iron and steel, chemical, pulp and paper, and other important industries. For example, in the road transport industry, older, less efficient vehicles generate more air pollution, are less fuel efficient, or require more maintenance, as they are used more intensively, and emissions reduction is more expensive.

In practice, some firms may have already updated their plant and equipment to reduce environmental damage due to social pressure or a prior regulatory program. We interpret our model as dealing with those firms that have not yet converted. Clearly, this raises issues of fairness to firms that converted prior to the subsidy program—issues of fairness not unlike those applying to firms that were in compliance with a previous regulatory program. Including these issues of fairness in the analysis would be a fruitful area for further study. Our modeling approach may also be applied to tradeable permits. A system of tradeable permits combines properties of both taxes and subsidies: The initial endowment of permits is a kind of subsidy, whereas the purchase of permits is like a tax.

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[20] See Russel et al. (1986) for difficulties in ongoing monitoring of pollution-abating firms.

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