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Operating Costs at the Well Level for Natural Gas Wells in Alberta

by

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ABSTRACT

This thesis examines the relationship between operating costs of natural gas production, and both quantity produced and remaining reserves. The focus is at the individual well level for wells in the Countess region of the Province of Alberta. The results support the theory that operating costs are rising with quantity produced, and decreasing in the level of remaining reserves. By looking at the components of operating costs, it is possible to see that some costs are better explained by quantity and reserves.

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LIST OF SYMBOLS

a	hyperbolic constant
α	constant
B	discount factor
b	natural log of initial production (i.e. q_0)
β	coefficient
C	cost
c	operating costs
γ	function indicator
D	net depreciation
d	annual percentage decline
DE	depth of producing zone
Δ	costate variable
δ	coefficient
E	quantity of exploratory effort
e	random error variable
EP	electric pump
e	natural exponent
ξ	Elasticity
F	physical upper bound on production
f	function indicator
Φ	binary variable
G	exogenous factors
g	function indicator
GL	gas lift
H	royalty rates
h	height
θ	costate variable
IW	Injection well
j	an individual well

k	vector of capital
K	stock of capital
λ	costate variable
m	water injected
MP	mechanical pump
n	number of wells
P	net change in pressure
p	price
pm	production month
Π	profit
ρ	pressure
Q	cummulative production
$\bar{q}$	leasewide production
q	quantity produced or extracted
r	discount rate
R	remaining reserves
R'	ratio of remaining reserves
RA	reserve additions
S	saturation
T	terminal time
t	time
τ	optimal abandonment date
φ	a parameter
U	level of cumulative discoveries
u	gross change in pressure
ϕ	porosity
ω	radius
V	vector of gross past investment
v	gross investment
W	vector of error terms
w	well radius

X	total resource stock
x	independent variable
Z	pay thickness
z	severance taxes
ψ	costate variable

Chapter One: Introduction: Upstream Natural Gas Production

The focus of this thesis is on the determinants of operating costs of natural gas reservoirs. In order to address the question of what factors affect operating costs, it is necessary to have an understanding of the technical aspects of a natural gas reservoir, and extraction of gas from a reservoir. The introduction to this thesis sets down a brief overview of a gas reservoir and the upstream natural gas industry. From there, a review of the economic literature follows, and then the empirical work of this thesis.

1.1 Natural Gas Reservoirs

Conventional natural gas is a depletable natural resource. A depletable natural resource is a resource that does not renew itself within a "reasonable" time frame. By reasonable time frame, what is meant is within a time scale that is relevant to economic decision making. Most geologists argue that conventional natural gas originated from organic life millions of years ago, although there is some dispute. Hobson and Tiratsoo (1981) state that the dispute revolves around whether natural gas comes from inorganic or organic sources. The inorganic argument is based on the fact that methane can be formed by a non-organic chemical reaction. Hobson and Tiratsoo argue that a reaction of water with a metallic carbide (such as calcium or iron), or with an alkali metal and carbon dioxide, can produce methane. According to Scelley (1998), other planets (namely Jupiter and Saturn) have methane in their atmosphere, and methane is often found in volcanic eruptions. These arguments support the supposition that at least some of the methane found in the earth is formed inorganically. However, most geologists do not believe that this inorganic origin of natural gas would account for most of the concentrated accumulations that have so far been found in nature, especially given their apparent geological age and their coexistence with liquid petroleum.

The organic argument involves the breakdown of organisms. Scelley (1998) argues that there are two ways of getting methane from an organic source, through either a thermal or a bacterial breakdown. Thermal breakdown occurs below the surface in an oxygen free environment, according to Abdulin (1985). The lack of oxygen keeps bacteria from breaking down the organic compound and allows time for the thermal breakdown. Millions of years of heat and pressure transformed the organic remains into hydrocarbons. With this length of time involved in the adjustment process, it is fair to treat any specific unit of natural gas as a resource that is available only once. Moreover, natural gas is a non-recyclable resource; once used it is transformed into non-energy components of less value, mostly carbon dioxide and water vapor. The organic theory is also given support by the fact that coal has a definite organic origin. Both coal and methane are hydrocarbons, and methane is often found with coal. Scelley (1998) cites the U.S. Department of Energy, that nearly 250 million cubic feet of methane is vented from U.S. coal mines on a daily basis.

Gray (1995) argues that life began millions of years ago in seas and lakes. As the organisms died their remains were buried in silt and mud at the bottom of the water. This silt and mud were continually being deposited in greater quantities by the erosion of other areas through water currents. When enough of this sediment was deposited, the weight pushed the seafloor down. This pressure, and the increased heat from being closer to the earth's core, aided in the formation of both sedimentary rocks, and oil and gas. The oil and gas then started to migrate upwards and laterally through the most porous and permeable rocks, until they were trapped by a layer of impermeable rock. The rocks where the oil and gas were formed are termed source rocks, and those to which the oil and gas migrated are termed reservoir rocks.

Berger and Anderson (1992) argue that four conditions must be present for oil or gas to accumulate in a reservoir. First, there must be an original source of carbon and hydrogen that underwent heat and pressure. Second, the rock that later buried this source must have been porous. Thirdly, the pores must have been aligned to allow for flow. Finally, a barrier must have been present to prevent further flow, and entrap the petroleum in the reservoir rock.

The bulk of natural gas consists of methane. According to Hobson and Tiratsoo (1981), methane typically makes up 84 to 96 percent of the natural gas in a deposit. Methane itself is a hydrocarbon that falls into the category of paraffins. The identifying characteristic of a paraffin is the formula C_nH_{2n+2}, where C is the elemental symbol for carbon and H is the elemental symbol for hydrogen. Methane is the lightest of the paraffins and is represented as CH_4. Associated with methane, other paraffins are often found. These include ethane (C_2H_6), propane(C_3H_8), and butane(C_4H_{10}). More complex molecules are less common, though "wet" gas reservoirs include heavier liquid hydrocarbons. Other hydrocarbons, such as benzene, are also sometimes found with methane. In addition to hydrocarbons, other gases are often found in natural gas, including carbon dioxide, hydrogen sulfide, and helium.

As a resource, natural gas is found in underground reservoirs. All rocks include tiny holes, or pores that can hold fluids. Geologists group rocks into three types, igneous, metamorphic, and sedimentary. According to Roberts (1989) igneous rocks are formed from molten magma that cools. This can happen either below the surface, or at the surface through volcanic eruptions. Igneous rocks are generally hard and crystalline in structure. Metamorphic rocks are formed by changing existing rocks. Birkeland and Larson (1989) state that heat, pressure, and fluids acting on pre-existing rocks form metamorphic rocks. These changes alter the chemical composition of the rock. Due to their formation, igneous and metamorphic rocks usually have very little pore space, and as such are unlikely to hold petroleum stocks of commercial value. Birkeland and Larson argue that igneous and metamorphic rocks are dense, having porosities of one percent or less, with larger porosities being determined by joints in the rock According to Birkeland and Larson sedimentary rocks are composed of bits and pieces of either organic or inorganic material that are cemented together. The term sedimentary comes from the fact that these pieces or sediments are deposited in an area, usually by water. This created the potential for large pores because if the grains of sediment were of varying size and shape, they may not fit together that well. Scelley (1998) argues that most major accumulations of petroleum are found within sedimentary rocks.

The larger the pores in the rock, the greater the porosity, and therefore the greater the amount of fluids that the rock can hold. As well, larger pores are associated with an easier flow through the rocks. The ability of a fluid to flow through the pores of a rock is known as permeability. Other factors influence the permeability as well as the porosity. Items such as viscosity of the fluid, the other fluids occupying the pores, and how the pores interconnect also impact how substances travel within the rocks.

Hydrocarbons within a reservoir are driven by pressure. Pressure increases the deeper into the ground that you go, due to gravity. In a reservoir, the hydrocarbons cannot escape toward the surface, and are compressed. When water is also in the reservoir, it acts to further maintain this pressure. Water moves into the pores that the gas vacates and thus drives the gas. Other factors contribute to pressure in the reservoir. Gas within the reservoir has its own energy, which adds to the reservoir pressure. Gas achieves its energy by its tendency to expand, its elasticity.

Pressure is the key to production. Petroleum is produced, or lifted to the surface, by a difference in pressure between the well bore and the reservoir. Since gas in the reservoir is held at a higher pressure than the pressure initially in the well bore, the pressure is released by a flow of petroleum into the well bore and up to the surface. As the reservoir contents are removed, the pressure in the reservoir declines. As the pressure declines, there is less of a pressure differential between the reservoir and the well bore. This leads to a smaller flow, and therefore less production. This is known as production decline.

Porosity and permeability differ among reservoirs, and even within reservoirs. Other factors also differ among reservoirs. The volume of the reservoir rock, the depth at which natural gas may be found, sulfur content, the amount of associated water, the presence of other hydrocarbons, and the pressure at which the gas is being held are some of the items that are not constant across all reservoirs. This means that natural gas, as a resource, is heterogeneous. This heterogeneity implies that different stocks of natural gas are more valuable than other stocks.

The fact that deposits of petroleum are underground implies uncertainty. Natural gas resource stocks are not known with certainty. Exploration to locate deposits is a fundamental activity in this industry. There is no way of knowing for certain what exists in the ground at a particular location until a hole is drilled. Once a reservoir of petroleum has been found, estimates of the size can be made, but it is not possible to measure the exact size of the reservoir, nor how much of the reservoir will eventually be produced. However, development of the reservoir improves knowledge, as does the experience of production.

Natural gas is almost never found in isolation from other resources. As discussed above, items such as oil, sulfur, and water exist in reservoirs along with natural gas, and each reservoir is unique. Thus the exploration process itself entails more than one product. This leads to problems of joint products, or how to deal with an activity (exploration) that generates more than one output. During the process of exploration, a well that is drilled can access volumes of petroleum that are commercially significant, or insignificant, or may yield no petroleum. A reservoir that has petroleum stocks large enough to be commercially worth producing, may contain crude oil, natural gas, or more typically some combination of the two as well as other components like water or hydrogen sulfide. Therefore, drilling an exploratory well yields one or more items: information, oil, gas, and perhaps other by products. Given this fact of joint products, how do you allocate expenses from exploratory drilling among these products?

The joint product nature extends beyond exploration. Once a well has been found to contain hydrocarbons, production of one of these may entail producing other hydrocarbons as well. Moreover, as noted, producing hydrocarbons may yield other products, such as sulfur or helium, which can be sold as well. Since the producers usually cannot control the contents of the hydrocarbons that are extracted, it is difficult to assign costs to the individual products that are drawn from the reservoir. For this reason, joint product problems exist in the lifting phase as well.

1.2 The Upstream Natural Gas Industry

The upstream natural gas industry can be divided into three activities: exploration, development, and production or lifting. Exploration refers to the search for natural gas reservoirs. Development is the process of installing the necessary infrastructure to produce the natural gas, once the reservoir has been found. Production involves bringing the natural gas to the surface and preparing it for transportation. Transportation is considered part of the midstream gas industry, and will not be examined here.[1] Before these three activities are investigated, some other terminology will be defined.

First, the distinction between surface and mineral rights must be made. Owning the surface rights does not imply access to the resources below. Within Canada the provincial governments own most of the mineral rights. Private individuals own mineral rights in some areas. The privately owned mineral rights are called freehold mineral

[1] Within the production component some transportation may be included in the case of a field plant where the gas is moved to the plant and processed to meet pipeline specifications.

rights, and those owned by the government are Crown mineral rights. A similar distinction exists for surface rights, freehold surface rights and Crown surface rights.

Exploration for oil and gas involves geological methods. The three main methods are land surveys, geophysical techniques, and through drilling. These are in order of cost, with land surveys being the least expensive and drilling being the most expensive.

Land surveys involve looking at the surface for signs of oil and gas and formations that may be likely to contain them. Geologists can tell a certain amount about the underlying rock from surface exploration. For instance, Cullen (1993) suggests that an area of raised ground could be due to sedimentary layers being deposited over a reef, and therefore may be a good candidate for oil or gas reservoirs.

Geophysical techniques include seismic, gravity meters, and magnetometers. Seismic surveys use sound waves to examine the densities of rocks below the surface. An energy source at the surface is released that sends shock waves into the earth. These shock waves reflect off the rock surfaces and give a two-dimensional picture of the subsurface. Gravity meters measure the gravitational pull of the earth. Large masses of dense rock affect the earth's gravitational field. Magnetometers measure variations in the earth's magnetic field. This identifies rocks that have varying magnetic intensity.

Drilling provides much more information both about the specific drilling site and also about the geology of the surrounding region. Measurements of electrical, sonic, thermal and compositional properties are taken when drilling. Moreover, rock chips and core samples are taken to give the exact composition of the rock formations.

These methods identify possible prospects for drilling (or further drilling). Decisions must then be made as to which of these prospects will be drilled. It is suggested by Cullen (1993) that most of these decisions come down to "gut feelings". However, more formal methods of evaluation are commonly applied. The most common of these are net present value analysis and rate of return analysis. More complicated techniques are also available to explicitly include the risk involved with exploration, future resource prices, taxes, and recoverability of the resource. These typically involve the use of subjective probabilities.

The company must obtain the mineral rights once a prospect has been selected. If the government owns the rights, the company needs to participate in a public land sale to obtain the mineral rights. This process involves an auction for the land in which companies can place their bids. A property goes up for auction once an interested party requests it. There is a reservation price for land in Alberta.

When a private individual or company owns the property, then the mineral rights must be gained through negotiation with the owner. The company may purchase the lease rights outright, or they may get a working interest under which the company shares in activities with another producer who holds the mineral rights. For freehold land, Cullen argues that most agreements with private companies contain provisions giving access for three years to five years. If oil or gas is found then the lease will be extended.

Why would a company that purchased a lease sell it? Cullen (1993) suggests three reasons. The first is that it is possible that a company is not interested in the contents of a discovered reservoir. In the case of finding natural gas, the company may only be interested in developing oil producing wells. Second, the lease may not be consistent with the company's strategies. The company may not be interested in risking capital by drilling new wells on this lease, but rather may be interested in focusing on enhancing existing production elsewhere. Finally, the company may be willing to trade the lease for another in an area that is more appealing to the company at the time.

The company has other options besides selling the entire lease. A working interest may be given up, as mentioned above. The company may enter a joint venture to share costs. A joint venture will set out explicitly each company's share of the costs of the well, and the production resulting from the well. Joint venture agreements do not necessarily set up profit sharing. Two common joint venture agreements are farmin/farmout agreements, and joint operating agreements.

In the case of a farmin/farmout agreement, the leaseholder may "farmout", to another company, a portion of the exploration, testing, drilling, and/or development cost. By doing this, the leaseholder may meet the lease requirements for when a well must be drilled, despite not being willing, or able to drill at the time. A second possibility is that the leaseholder may be wishing to decrease risk by having another company share some of the costs. Conversely, the farmee, a company wishing to get access to land that it does not have mineral rights to, gains this access under a farmout agreement. The most common form of a farmout agreement has the farmee receiving one half of the production from the well, while assuming all the costs of drilling.

A joint operating agreement occurs when more than one company has jointly purchased the lease, or a company has transferred a share of its interest to other companies (perhaps through a farmin/farmout agreement). This agreement involves choosing an operator and how costs and production will be allocated. According to Cullen (1993) the agreement usually sets a date for drilling and the responsibilities of each party.

The next step is to arrange for surface access rights for land that will be "used" in exploration, development, and lifting. In Alberta, when the Crown owns the land permission must be obtained from several agencies. The Alberta Forest Service, Land Management and Development Branch, and Fish and Wildlife all need to consent before access rights can be granted. For freehold lands an agreement must be made with the surface landowner. This agreement will include an entry fee and an annual fee. These amounts are to compensate the landowner for the loss of the use of a portion of their land, as well as any inconvenience that arises because of having a well on their property. If the interested company can not reach an agreement with the landholder, then it is possible to obtain an Order-of-Entry from the Alberta Energy and Utilities Board. This order mandates the terms of access.

At this point the company is ready to drill its first well. If the well is successful in finding oil or gas then development can take place.

The first stage in development is the process of completing the successful exploratory well to get it ready to produce oil, gas or both. The drilling of infill wells is another form of development. An infill well is a well that is drilled within the proved area of an existing reservoir to increase the current productivity of the reservoir. It allows for more rapid recovery of petroleum. This is in contrast to outpost wells that are drilled to define the boundaries of an existing pool. Outpost wells are often credited to the exploration phase of the industry. (This is not a clear distinction since they do occur after a reservoir is located. It is sometimes not immediately clear whether a successful well has discovered a new reservoir or has tapped a more distant part of a previously discovered pool.)

A company must be mindful of well spacing requirements when drilling development wells. especially infill wells. Well spacing requirements set up the minimum distance between wells, and between a well and the lease boundary or property line. These were set up to achieve optimal production in two ways. The first is to promote the optimal rate of take from the reservoir by protecting the resource from wasteful production. By limiting the number of wells that can be drilled in an area. it is possible to limit reservoir-damaging overproduction from a pool in a given time period. Second. well spacing helps to protect the rights of other leaseholders adjacent to the well. (It is more difficult for a producer to drain the resources of another leaseholder when the producer is unable to increase production by adding more wells.) In Southern Alberta well spacing requirements are such that there can be only one well in a legal subdivision (sixteen hectares). unless it can be shown that more are needed to drain the reservoir. This is in contrast to the 258 hectares. or one section. that is the norm in the rest of the province for well spacing. Cullen (1993) argues that this is because pools are generally "tighter". or less permeable in Southern Alberta. More wells are needed to drain reservoirs in a way that allows producers to generate sufficient revenues to make the ventures worthwhile.

Once a well is drilled it must be completed. This involves alternating "strings". or vertical layers of pipe between concrete. The first. or outside. string of pipe is called the conductor pipe. This pipe can be pounded into the ground. or cemented in place. The purpose of the conductor pipe is to prevent the bore from collapsing at the surface. The next string is the surface casing. This is cemented. The surface casing is designed to protect the ground water from contamination. and helps to keep unwanted substances (such as sand) from entering the well bore. On deeper wells there is often an intermediate casing. This string is to protect the integrity of the well bore and intermediate formations. Finally, a production casing is installed. The production casing is cemented as well. and it extends the length of the bore. The production casing. as its name implies. is to allow production.

Once the casings are in place, Cullen (1993) suggests that there are five ways to finish completing a well. The first is an open hole completion. This is uncommon. contains no production casing. and is used only in low pressure formations with only one producing zone. Next is a perforated completion, which is the most common. This involves perforating the production casing and cement in the gas bearing formation to allow for production. Third is a wire-wrapped completion. This is used when there is a lot of sand in the reservoir. A short pipe is covered with a mesh screen and inserted into the perforated production casing. The perforations are then packed with gravel. The gas can flow through the gravel and screen, but the sand is kept out of the well bore. Fourth, the well can be completed as a tubingless completion. This involves no tube in the production casing, and is used when there is low pressure gas, no natural gas liquids, and a small diameter bore. Finally, a multiple zone completion may be done. This is where more than one pool is tapped at a time, but each has a separate production tubing.

According to Berger and Anderson (1992) it is often necessary to assist the flow of gas into the well bore. This is done by fracturing the reservoir rock around the well. This can be done by injecting acid or a fluid with a propping agent into the reservoir. The acid dissolves part of the formation around the well bore, making existing pores larger. Injecting fluids causes the formation to crack open more. The propping agent then prevents the formation from collapsing back. Berger and Anderson state that silica sand, glass beads, and epoxy are often used as propping agents.

The next step is the production, or lifting, of the natural gas. As mentioned in the previous section on natural gas reservoirs, gas is driven by a pressure differential. This pressure differential between the reservoir, the well bore, and the surface drives the gas to the surface. Natural gas production rarely involves the use of energy that is external to the reservoir. However, in some cases, where the reservoir size is large, compressed gas is added to the reservoir to reduce the pressure decline.

Production involves pressure decline. By removing gas from the reservoir, there is less left in the reservoir, and therefore more space for the remaining gas to occupy in the reservoir. To test the pressure of a well, producers run an absolute open flow test, or AOF. This test uses a recorder that is lowered into the well bore. The recorder measures the pressure while the well is produced at varying flow rates. After this the well is shut in to allow the pressure to stabilize. The recorder can then be removed, and the open hole flow rate can be estimated based on the pressure. This is the rate the well would produce at without any valves to restrict flow. By performing the AOF test, the MER can be calculated. The MER, or maximum efficient rate, as defined by Berger and Anderson (1992), is the greatest rate of flow of oil or gas that can be produced from a well without damaging the reservoir. Drawing the reservoir down too quickly can damage the reservoir. For example, producing too rapidly may pull water into the space that the oil or gas is occupying. This water impedes the flow of the hydrocarbons, and reduces the amount that is then recoverable. However, producers typically attempt to draw a well down as quickly as permitted by the MER so as to generate cash flow for other investments, or to maximize the present value of earnings. (Unless significant increases are expected in gas prices, higher production, with a given capital investment, will almost always raise the present value of the net operating revenue.)

Production decline in petroleum reservoirs typically exhibits hyperbolic decline. In this case, using Nind (1981)[2], the relationship between the rate of change in current production and time is:

$$\frac{\partial q(t)}{\partial t} = \frac{-q(t)}{(a+bt)} \tag{1}$$

where: q (t) is production at time t

a is the hyperbolic constant

b is the natural log of initial production (ln q_0).

This allows us to integrate (1) to get the production at any time t as:

$q(t) = q_0 e^{\frac{-t}{a}}$ if b = 0 (in this case a is the constant production decline rate), and

$$q(t) = q_0 \left(\frac{a}{a+bt} \right)^{\frac{1}{b}} \text{ if } b \neq 0 \tag{2}$$

Similarly cumulative production from the start of production up to the year t, Q(t), can be calculated by integrating q(t)[3]:

$$Q(t) = \left(\frac{a}{1-b} \right) q_0 \left[1 - \left(\frac{a}{a+bt} \right)^{\frac{1-b}{b}} \right] \text{ if } 0 < b < 1 \tag{3}$$

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[2] The presentation follows Black and LaFrance (1998) as they have presented the material in a much more convenient way for the purposes of this thesis. However the original source is Nind (1981)

$$Q(t) = aq_0(1 - e^{\frac{-t}{a}}) \text{ if b=0}$$

$$Q(t) = aq_0 \log\left(\frac{a+t}{a}\right) \text{ if b=1}$$

Besides the problems associated with producing above the MER, other problems may be encountered during production. Care must be taken to prevent water coning. Water coning is when the well bore is surrounded by water, which prevents most of the hydrocarbons from entering the well bore. Water coning is associated with the MER, as noted above, but if the well bore is completed nearer to the bottom of the producing zone, it is more likely for water coning to be a problem as the water has a shorter distance to travel to reach the well bore. Therefore, care must be taken to ensure that the rate of take from the reservoir is not such that it draws the water too close to the well bore.

In addition to water coning, fluids (i.e. water and oil) within the reservoir may cause other problems. When fluids enter the production casing, they can cause the flow of gas to be choked off. This happens when pockets of gas get trapped between layers of fluid within the string of pipe. To prevent this problem the lines must be blown or purged randomly.

Water is more difficult to lift than gas. This is because it is heavier and more viscous than gas. Sometimes it is difficult to lift the water, particularly in lower pressure formations. Often this problem is prevented by the use of soap sticks. The soap sticks mix with the water and form suds. The soap suds are less viscous and therefore easier to lift. Another way of dealing with water is to use the production tubing. The production tubing is a string of pipe inside the production casing. It is obviously of lesser diameter than the production casing. Gas is produced through the production casing, and water through the production tubing. The smaller diameter pipe requires less pressure to lift the water.

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[3] Notice that b=0 and b=1 are special cases of hyperbolic decline termed exponential decline and harmonic decline respectively

Quantities of salt water that are produced with natural gas need to be disposed of. Salt water can be removed either at the well or at a processing plant. Salt water must be disposed of by dumping it into a trap beneath the surface. This is to prevent contamination of the surface water, as well as to protect other reservoirs. Often producers will use one of their producing wells, or drill a specific water disposal well, to inject the water back into the bottom of the formation that they are producing from. In this way they can maintain reservoir pressure.

Hydrates are another concern. A hydrate is a combination of a hydrocarbon and water. Hydrates can freeze up and inhibit or stop flow within a well. Methanol can be injected into the well to prevent hydrates.

Valves control the rate of production from a gas well. There are two valves on a typical gas well, the master valve and the wing valve. The master valve is the closest to the well bore, and is used to control the flow from the well. The wing valve is closest to the transmission lines, and is used to reduce strain and protect the master valve. The master valve is very difficult to replace; replacement involves plugging the well bore with cement that then must be removed by drilling to begin production again. For this reason the wing valve is added to the system. If the wing valve fails, it can be replaced by closing the master valve. When production is not needed from a well the valves are closed, shutting the well in, starting with the wing valve. When production is required, the valves are opened this time beginning with the master valve.

There are two common measures of a well's ability to produce. The first of these is production capacity, or flow capacity, which is characterized by the well's MER. The production capacity is affected by the reservoir's characteristics, and by the equipment in place at the well. Secondly, deliverability measures the volume of gas that can be supplied to markets taking into account the external effects of regulation, pipeline capacity, and processing plant capacity. Once gas is lifted to the surface it can either be processed, or shipped directly to market. It is unlikely in most cases to find gas that is ready to be shipped to market without processing. This is because of the other components often found within the gas. Sour gas, gas containing significant portions of sulfur, needs to be shipped to a sweetening plant to remove the sulfur. Gas containing large concentrations of water or liquid hydrocarbons needs to have these removed either at the well by installing equipment, or at a field processing plant. before it can shipped in the pipeline.

The gathering pipelines that are involved in the production phase need maintenance. The maintenance takes the form of cathodic protection. This is done to prevent the corrosion of the pipes from the outside. Electrical currents within the ground can transmit the metal from the pipe into the surrounding ground, or on to other parts of the pipe. To prevent this cathodes are added to the pipe. On the inside of the pipe. chemicals are added to prevent corrosion. At the wellhead. maintenance involves the lubrication of the valves to keep them operating within specifications.

1.3 Purpose of this Thesis

The purpose of this thesis is to estimate an operating cost function for natural gas wells. Chermak and Patrick (1995) econometrically estimated a cost function for natural gas wells with data from three U.S. regions. Wyoming, East Texas. and West Texas. They found a relationship between operating costs and both the quantity of gas lifted and the remaining reserves. This thesis is an attempt to replicate and extend their results using data obtained from a region in Alberta. Canada. Their work will be extended by first making a clearer definition of operating costs and what is included in operating costs. Secondly. more sophisticated econometric techniques will be used to attempt to better fit the model with the data.

An estimate of an operating cost function is particularly significant if it is found that there are common factors affecting costs across all reservoirs. This estimate serves many uses. Primarily. an estimate of a cost function is very valuable as a tool for calculating projected costs for a project. A reasonable estimate of operating costs can be combined with expected drilling and infrastructure costs to get a measure of the entire cost of the project. This can then be compared to the expected revenues from the project to determine if the project is worthy of investment. In addition. the cost function helps to determine the optimal output rate each period. as well as when wells should be abandoned. From economic theory. we know that a profit-maximizing producer should produce each unit up until the last unit produced costs as much to produce as it yields in benefits. This will hold for both how much to produce in each time period, as well as when to shut the well in because production in future periods is no longer economic. Without a good idea of what the last unit costs to produce, it is difficult to make a rational decision.

With an estimate of the cost function, and duality theory, it is possible to gain understanding of the production function underlying gas production. Knowing the production function would make it possible to investigate characteristics such as economies of scale in gas pools. This may be useful in determining whether reservoirs should be operated in a unitized fashion, with one firm making production decisions for the entire reservoir. Griffin and Jones (1988) undertook such a study at the lease level and found evidence for unitization. For private industry, it may be useful in determining whether to enter into contracts with other companies who have land rights in the same reservoir.

As was discussed above, the oil and natural gas industry has joint product characteristics in the exploration as well as the production phases. Knowledge, oil reservoirs, natural gas reservoirs, and reservoirs containing both oil and gas all may occur from exploration. Although some regions and geological formations are more likely to contain oil rather than gas, or more likely to have gas with a higher yield of natural gas liquids, there are no ways to target specifically which product you will find when you first drill an exploratory well. Oil, gas, other liquid hydrocarbons, sulfur, carbon dioxide, and other trace compounds may all result from the lifting of resources out of the ground. There may be some control in output properties, for example by varying the depth you produce from. For instance, if you have a reservoir with a gas cap, you may choose to extend your production tubing below the gas and into the crude oil. This however will not eliminate natural gas from your production, only reduce the amount that will be produced with the oil. That is, to some extent the joint product problem is one of fixed rather then vaariable costs. With joint products it is difficult to decide how to allocate costs between the products. This study looks at wells within the Countess region of Alberta, where there are few other products produced from the natural gas. The gas found in this region is primarily "dry" (contains negligible amounts of liquid hydrocarbons), and "sweet" (low concentrations of sulfur). Trace elements such as carbon dioxide are not typically removed at the well, nor are they typically sold by the producer of the natural gas. Hence the joint product problem is negligible in this study of operating costs.

Economic analysis of the crude petroleum industry raises aggregation issues. To what extent, for instance, can a region's gas output be treated as if coming from a single pool, or a pool's output as coming from a single well? The nature of oil and gas reservoirs makes such aggregation difficult. The fact that reservoirs are not homogenous is at the center of this. Different reservoirs have differing levels of porosity, permeability, pressure, and viscosity, among other factors. Even within the same reservoir these differences may exist. Studies such as Livernois (1985) and Livernois and Uhler (1987) have shown that models that are built around individual reservoirs rather then regions make better models. Chermak et al (1999) outlined four conditions for aggregatability from a well to a reservoir level (i.e. conditions under which a reservoir could be treated as if it operated like a single well). These are: homogeneity of a reservoir, identical production paths of the wells, identical time horizons of all wells, and identical production technology. But in order to see how well these assumptions are met, it is necessary to have an understanding of what affects well production and costs.

This thesis is concerned with the econometric estimation of operating costs, for natural gas, at the well level. The well level has been chosen to reduce the issues of aggregatability, since it is the "smallest" physical avenue of production .

Chapter Two: Literature review

The economics literature on depleteable natural resources in general. and petroleum in particular. is quite extensive. The intent is not to delve too deeply into this literature because it is so broad. However. a brief outline of a couple of the issues does help to motivate this thesis. In particular. examples of both theoretical models. and empirical models of petroleum extraction will be discussed. The theoretical section entails a discussion of: (i) natural resource depletion models. (ii) petroleum supply models. and (iii) other issues including uncertainty. joint products. and aggregation. In the Empirical models section. four models which involve oil and gas operating cost estimation will be discussed.

2.1 Theoretical Models

2.1.1 Natural Resource Depletion

The two models discussed in this section are representative of the models economists have built to analyze production from an exhaustible natural resource. They are not tied specifically to natural gas.

According to Sweeney (1993). depletable. or exhaustible natural resources are ones that have a "regeneration process" that is very slow. so slow that they may be treated as only being available once for the decision maker. This makes them different from natural resources that replenish themselves. like forests. rivers. and fish. With depleteble natural resources. the depletion effects are of fundamental concern in most contexts.

For example. knowing that the stock of oil in an oil pool is limited. a profit maximizing decision maker must consider the future value of oil when deciding whether to produce now or to produce later. For example. if oil prices were expected to rise significantly in the future it may be more profitable to leave the oil in the ground now. and extract it later when the prices are higher. This would not be a concern if using the resource now still enabled use in the future.

Economic models of depleteable natural resources typically assume that you start with a fixed stock of the resource in the ground. As the stock is used over time, the stock remaining in the ground decreases. This creates a "user cost" of production. This user cost of a depleteable natural resource is the opportunity cost of producing the resource today instead of tomorrow. For example, by producing the oil now, there would be less to produce later, so the opportunity cost is the net loss to the producer from the loss of the ability to sell the oil in the future if the oil is sold now. Profit maximizing firms would account for this user cost when making decisions. A profit maximizing firm will, at the margin, set the value from extracting the resource equal to the value of not extracting the resource. The value from extracting a unit of the resource is the marginal revenue minus the extraction costs. The value of not extracting the resource is the user cost, or the increase in the present value of future profits which arises from not producing now.

Following Sweeney (1993), resource owners are assumed to maximize the present value of profits by choosing a planned time path of extraction. Define a cost function, $C_t(q_t, R_{t-1})$, such that costs in time t are a function of the extraction rate at time t, R_t, and the stock remaining at the end of last period, R_{t-1}. The interest rate (i.e. the marginal opportunity cost of investment or the normal profit rate) is given as r, the price of the resource at time t is known and given as p_t. Moreover, the firm is assumed to be a price taker, and the extraction takes place over an arbitrary, but known, time period with terminal period denoted as T (i.e. the time horizon over which production takes place is not specified, but the firm knows what this time horizon is). The firm's problem then is to maximize the following:

$$\Pi = \sum_{t=0}^{T} \left(p_t q_t - C_t(q_t, R_{t-1})\right) e^{-rt} \tag{4}$$

subject to:

$$R_t = R_{t-1} - q_t \text{ for all t} \tag{5}$$

$$R_T \geq 0 \tag{6}$$

$$q_t \geq 0 \tag{7}$$

Solving this problem yields the following conditions for profit maximization, i.e. for all t:

$$p_t = \frac{\partial C_t}{\partial q_t} + \lambda_t \text{ if } q_t > 0, \tag{8}$$

$$p_t \leq \frac{\partial C_t}{\partial q_t} + \lambda_t \text{ if } q_t = 0. \tag{9}$$

$$\lambda_t = \lambda_{t-1} e^{-r} + \frac{\partial C_t}{\partial R_{t-1}}, \text{ and} \tag{10}$$

$$\lambda_T R_T = 0 \tag{11}$$

Where λ_t are the Lagrangian multipliers associated with the resource stock constraints. (6).

Looking at (8), we can observe that the price will be the sum of two items. The first of these is the marginal extraction cost, the second is the marginal user cost. The marginal extraction cost is quite straight forward. The user cost states that as the resource is depleted there may exist additional costs that are not captured by the extraction costs. These additional costs are the opportunity costs of producing quantities now rather then waiting to produce them at some point in the future. In the case of petroleum this is largely due to the fact that extraction reduces the pressure of the reservoir. As the pressure is depleted it becomes more costly to produce out of the reservoir: that is costs are higher as the size of the remaining resource stock is smaller. Therefore current extraction reduces future profits. This is known in the literature as a "stock effect" or a "degradation effect".

This model can be further generalized. Krautkraemer (1998) suggests that exploration can be accounted for explicitly in the model by incorporating reserve additions (RA) into the stock equation, and a cost function for reserve additions (C^{RA}). Equation (5) now becomes:

$$R_t = R_{t-1} + RA_t - q_t \text{ for all t} \tag{5'}$$

and the cost function for reserve additions is assumed to be a function of the reserves added (i.e. $C_t^{RA} = C^{RA}(RA_t)$ (i.e. there are no stock effects in reserve additions)). The problem now is to maximize:

$$\Pi = \sum_{t=0}^{\infty} \left(p_t q_t - C_t(q_t, R_{t-1}) - C_t^{RA}(RA_t)\right) e^{-rt} \tag{4'}$$

subject to:

$R_t = R_{t-1} + RA_t - q_t$ for all t (5')

$q_t \geq 0, q_t \leq R_t + RA_t$ (7')

Solving this problem yields the following conditions for profit maximization. i.e. for all T:

$$p_t = \frac{\partial C_t}{\partial q_t} + \sum_{s=t+1}^{\infty} \frac{\partial C_s}{\partial q_t} e^{-rt} + \sum_{t=0}^{\infty} \psi_t \text{ if } q_t > 0, s = t+1, t+2, t+3, \ldots, \infty; \quad (8')$$

$$p_t \leq \frac{\partial C_t}{\partial q_t} + \sum_{s=t+1}^{\infty} \frac{\partial C_s}{\partial q_t} e^{-rt} + \sum_{t=0}^{\infty} \lambda \psi \text{ if } q_t = 0. \quad (9')$$

$$\sum_{t=0}^{\infty} \psi_t = \frac{\partial C_t^{RA}}{\partial RA_t} - \sum_{s=t+1}^{\infty} \frac{\partial C_t}{\partial q_t} e^{-rt} \quad . \quad (10')$$

where ψ_ts are the Lagrangian multipliers associated with the constraint (5').

From these first order conditions, the following rule for exploration is achieved:

$$p_t - \frac{\partial C_t}{\partial q_t} = \frac{\partial C_t^{RA}}{\partial RA_t} \quad (11')$$

Equation (11') states that a profit maximizing firm will explore for more resources up to the point where marginal benefits from adding reserves (price less the marginal extraction cost) are equated with the marginal cost of adding reserves. Clearly the extraction cost function, C_t, is one of the factors which determines the level of current production and the amount of activity in adding new reserves.

This model separates out the effects of reserve additions from the extraction costs, and therefore is more applicable to the analysis here. By separating the exploration and development reserve additions from extraction, this model sets the stage for the analysis of this study. The focus of this thesis is to estimate the operating costs captured by the marginal extraction cost term, and we abstract from the exploration and development costs contained in the marginal reserve addition costs.

2.1.2 Petroleum Supply Models

From the various models of petroleum production, I shall discuss a general one.

Kuller and Cummings (1974) developed a model of petroleum reservoir management. Their purpose was to build an economic model that included the interaction between current production rates, investment, and reservoir characteristics.

This model helps to identify some key elements to consider when analyzing petroleum production.

To build their model, they assumed that there were n price taking firms using a given petroleum reservoir. Agents are assumed to have certainty with regards to both economic and physical variables. Moreover, they assumed that there was a finite, T period, horizon, and that all prices and costs over this period were known.

The authors formulated firm j's problem as to maximize the value of a stream of discounted profits according to the formula:

$$\sum_{t=1}^{T}\left[p_t q_{jt} - C_{jt}\left(Q_t, V_t, K_{jt}\right)\right]B_t \qquad (12)$$

subject to the following restrictions:

$$K_{jkt+1} = K_{jkt} - D_{jkt}(q_{jt}, v_{jkt}, K_{jkt}) \qquad (13)$$

$$q_{jt} \leq F_{jt}(Q_t, V_t, K_{jt}) \qquad (14)$$

$$\sum_{\tau=1}^{T}\sum_{j=1}^{n} q_{j\tau} \leq X(Q_T, V_T) \qquad (15)$$

for all j, k, and t; τ = 1, ..., t

where the symbols are defined by Kuller and Cummings as:

p_t = the unit price at time t

q_{jt} = the amount that firm j produces at time t

q_t = the total volume of petroleum produced from the reservoir in period t

$C_{jt}(Q_t, V_t, K_{jt})$ = the cost function for firm j at time t

Q_t = the vector of past production for all firms for all periods up to and including period t

(i.e. $Q_t = (q_{11}, q_{21}, \ldots, q_{n1}, q_{12}, q_{22}, \ldots, q_{nt-1}, \ldots, q_{nt})$)

v_{jt} is the gross investment for all capital components by firm j during period t

V_t = the vector of gross past investment for all capital components, for all firms, for all periods up to period t (i.e. $V_t = (v_{11}, v_{21}, \ldots, v_{n1}, v_{12}, v_{22}, \ldots, v_{nt-1}, \ldots, v_{nt})$

K_{jkt} = is the stock of capital component k for firm j at the start of period t

K_{jt} = the vector of stocks of capital for firm j at the start of period t: $(K_{j1t}, \ldots, K_{jqt})$

D_{jkt} = net depreciation of capital component k during period t for firm j

F_{jt} = the physical upper bound on production in period t for firm j

B_t = the discount factor $(1+r)^{-t}$, where r is the appropriate discount rate

X = the total amount of petroleum recoverable from the reservoir

Equation (1) states that firm j will maximize, over all periods, the present discounted value of revenues minus costs. The cost function in equation (12) $C_{jt}(Q_t, V_t, K_{jt})$, states that costs for firm j are a function of past reservoir production, past gross investment by all firms, and the capital stock of firm j. Equation (13) states that the stock of capital component k for firm j at time t+1 is equal to the initial stock of component k minus net depreciation. Kuller and Cummings assert that the following relationships hold:

$$\frac{\partial D_{jkt}}{\partial v_{jkt}} \leq 0 \qquad \frac{\partial D_{jkt}}{\partial q_{jt}} \geq 0 \qquad \frac{\partial D_{jkt}}{\partial K_{jkt}} \geq 0$$

$\frac{\partial D_{jkt}}{\partial v_{jkt}} \leq 0$ states that net depreciation is nonincreasing with gross investment and this relationship should be equal to -1 (i.e. $\partial D_{jkt} = |\partial v_{jkt}|$). $\frac{\partial D_{jkt}}{\partial q_{jt}} \geq 0$ asserts that net depreciation is nondecreasing in production, and $\frac{\partial D_{jkt}}{\partial K_{jkt}} \geq 0$ states that net depreciation is a nondecreasing function of the stock of capital of firm j.

Equation (14) gives a physical upper bound on current production. It states that current production cannot exceed a certain level that is influenced by aggregate reservoir production, aggregate reservoir investment, and the existing capital stock of the firm. The following are expected to hold:

$\frac{\partial F_{jt}}{\partial q_{i\tau}} \leq 0 \qquad \frac{\partial F_{jt}}{\partial v_{i\tau}} \geq 0 \qquad \frac{\partial F_{jt}}{\partial K_{jt}} \geq 0$ for all i, j, k, and t; τ = 1, ..., t; Where i is an index of all other firms. $\frac{\partial F_{jt}}{\partial q_{i\tau}} \leq 0$ states that the physical upper bound on production for firm j in period t is nonincreasing in the production of any firm i. $\frac{\partial F_{jt}}{\partial v_{i\tau}} \geq 0$ shows that the physical upper bound on production for firm j in period t is nondecreasing in the gross investment by any firm i, $\frac{\partial F_{jt}}{\partial K_{jt}} \geq 0$ asserts that the physical upper bound on production for firm j in period t is nondecreasing in the capital stock of firm j.

Equation (15) states that the production from the reservoir must not exceed the total amount of recoverable petroleum in the reservoir. It is expected that $\frac{\partial X_t}{\partial Q_T} \leq 0$ and $\frac{\partial X_t}{\partial V_T} \geq 0$, or that the remaining stock in any period is non-increasing in the cumulative quantity extracted and non-decreasing in the amount of cumulative investment.

If you are seeking to maximize pool profits the problem then is to solve the following:

$$\text{Maximize} \sum_{t=1}^{T}\sum_{j=1}^{n} \left[p_t q_{jt} - C_{jt}\left(Q_t, V_t, K_{jt}\right)\right] B_t \tag{16}$$

Subject to:

$$K_{jkt+1} = K_{jkt} - D_{jkt}(q_{jt}, v_{jkt}, K_{jkt}) \tag{13}$$

$$q_{jt} \leq F_{jt}(Q_t, V_t, K_{jt}) \tag{14}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{n} q_{jt} \leq X(Q_T, V_T) \tag{15}$$

$$q_{jt} \geq 0,\ v_{jkt} \geq 0, \tag{17}$$

for all j, k, and t

Note that non zero pool profits implies that common property problems in the reservoir are somehow offset. Solving this problem yields the following conditions (Kuller and Cummings, 1974 page 71):

$$\left(p_t - \frac{\partial C_{jt}}{\partial q_{jt}}\right)B_t = \begin{array}{l} \lambda B_t\left(1 - \frac{\partial X}{\partial q_{jt}}\right) + \psi_{jt}B_t - \sum_{\tau=1}^{T}\sum_{i=1}^{n}\psi_{i\tau}\frac{\partial F_{i\tau}}{\partial q_{jt}}B_\tau \\ + \sum_{k=1}^{q}\Delta_{jk,t+1}B_{t+1}\frac{\partial D_{jkt}}{\partial q_{jt}} + \sum_{\tau=t+1}^{T}\frac{\partial C_{j\tau}}{\partial q_{jt}}B_\tau + \sum_{\tau=t}^{T}\sum_{\substack{i=1 \\ i\neq j}}^{n}\frac{\partial C_{i\tau}}{\partial q_{jt}}B_\tau \end{array} \tag{18}$$

$i, j = 1, \ldots, n;\ 1 \leq t \leq T$

and

$$\frac{\partial C_{it}}{\partial v_{jkt}}B_t = -\Delta_{jk,t+1}B_{t+1}\frac{\partial D_{jkt}}{\partial v_{jkt}} + \lambda B_T\frac{\partial X}{\partial v_{jkt}} + \sum_{\tau=t}^{T}\sum_{i=1}^{n}\psi_{i\tau}B_\tau\frac{\partial F_{i\tau}}{\partial v_{jkt}} \tag{19}$$

$$-\sum_{\substack{i=1 \\ i\neq j}}^{n}\frac{\partial C_{it}}{\partial v_{jkt}}B_t - \sum_{\tau=t+1}^{T}\sum_{i=1}^{n}\frac{\partial C_{i\tau}}{\partial v_{jkt}}B_\tau$$

$i, j = 1, \ldots, n;\ k = 1, \ldots, q;\ 1 \leq t \leq T$

where ψ, and λ are Lagrangian multipliers.

From equation (18), we observe that a firm will produce each period where the present value of net income (revenue (p_t) minus operating costs ($\frac{\partial C_{jt}}{\partial q_{jt}}$)) is equated with the "marginal user costs". The marginal user cost is defined as the present value of foregone future profits from producing a unit now. This user cost is made up of six terms in this model. According to Kuller and Cummings, these terms fall into stock user costs, boundary user costs, a user cost of capital consumption, and production user costs.

The stock user costs exist because the resource is scarce. Producing a unit now eliminates the possibility of producing that unit at a later time period. This user cost is shown as the first term in equation (18). λB_t shows the present value of an increase in the stock of the resource available for production. $\frac{\partial X}{\partial q_{jt}}$ gives the change in the stock of petroleum for a change in production. It is important to note that Kuller and Cummings did not restrict this to be one, which allows for the possibility that there may exists some additional impacts of production on the stock of petroleum. For instance, if you extract one unit of petroleum, the stock of recoverable petroleum may decrease by more (or less) than one unit. If it equals one then user cost is what is often called the user cost of timing. Additional losses generate what is often referred to as a marginal user cost of degradation.

Boundary user costs are the next two terms in equation (18) containing ψ_{it}. These are associated with the upper bound on production in any given period, for any firm i, F_{it}. ψ_{it} measures the increase in net income that would result from relaxing this bound, and $\frac{\partial F_{it}}{\partial q_{jt}}$ is a measure of the relaxation of this bound caused by a change in the production of firm j in period t. These boundary user costs are a type of externality. If one firm chooses to produce more in a given period, then this means that all firms have less stock with which to produce from in that period.

The user cost of capital consumption is associated with $\Delta_{jk,t-1}$. This multiplier measures the marginal productivity of capital component k used by firm j in future periods. This term will be expanded on below. The term $\frac{\partial D_{jkt}}{\partial q_{jt}}$ measures the capital used up in period t because of a change in output by firm j. Together these two terms show the user cost of capital consumption by firm j. In the oil industry, it would appear normal for companies to maintain the productive capability of their capital equipment by periodic maintenance expenses. These are incurred as operating costs and have the effect of keeping $\frac{\partial D}{\partial q} = 0$.

However, even though the capital equipment itself is maintained, the ability to lift oil typically changes overtime as a result of variation in the pressure of the reservoir. These effects are captured by final two terms in equation (18) that show the user cost of production. These terms measure the impact of current production on the future variable costs of firm j. By producing an extra unit now, costs in future time periods may be affected, a user cost of degradation.

Equation (19) characterizes the optimum investment rate. The left hand side is the marginal cost of investment in capital component k to firm j at time t. expressed in present value terms. The right hand side represents the aggregate benefits to the reservoir as whole with such investment. The term $\Delta_{jk,t+1}B_{t+1}$ represents the following:

$$\sum_{\tau=t+1}^{T}\left(-\frac{\partial C_{j\tau}}{\partial K_{jk\tau}}+\psi_{j\tau}\frac{\partial F_{j\tau}}{\partial K_{jk\tau}}\right)B\prod_{s=t+1}^{\tau-1}\left(1-\frac{\partial D_{jks}}{\partial K_{jks}}\right)+\prod_{s=t+1}^{T}\left(1-\frac{\partial D_{jks}}{\partial K_{jks}}\right)\Delta_{jk,T+1}B_{T+1}$$

$\frac{\partial C_{j\tau}}{\partial K_{jk\tau}}$ represents the direct effects on the extraction costs. for firm j. of having a larger stock of capital component k. $\psi_{j\tau}\frac{\partial F_{j\tau}}{\partial K_{jk\tau}}$ represents the imputed value of capital. shown by relaxing the physical bound F. The sum is the present value of the stream of benefits to firm j of an increment in investment. discounted by a capital survival factor. $1-\frac{\partial D_{jks}}{\partial K_{jks}}$. The final term. $\prod_{s=t+1}^{T}\left(1-\frac{\partial D_{jks}}{\partial K_{jks}}\right)\Delta_{jk,T+1}B_{T+1}$. represents the marginal value of current investment in terms of terminal capital stocks. Kuller and Cummings argued that this term would be zero in most applications.[4] Moreover. they argued that $\frac{\partial D_{jkt}}{\partial v_{jkt}}$ should equal to negative one (i.e. if you invest one dollar into capital. all else equal. then your net capital should increase by one dollar therefore decreasing depreciation by one dollar).

The next term in equation (19). $\lambda B_T\frac{\partial X}{\partial v_{jkt}}$. shows the impacts on recoverable stocks for an increment of investment by j. multiplied by the value of additional recoverable stocks. The following term gives the impact of investment by firm j over all firms through changes in the upper bound F. The last two terms measure the impact on operating costs for all firms of changes in investment by firm j.

Therefore at the optimum. firm j will produce at a rate where their marginal net revenue is equated with their user cost of production. However. the same firm will set

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[4] This is presumably because of the sunk specific nature of most of the capital used in petroleum extraction. Firms would attempt to have the present value of the terminal stock of capital as close to zero as possible to avoid wasting investment.

the marginal cost of investment equal to the present value of reservoir wide benefits associated with that investment.

This model shows importance of considering all costs associated with production. In the case of non-renewable resources, this includes a user cost. Optimal production (profit maximizing production) from an oil reservoir involves the complex interaction amongst a variety of factors including expected future market conditions, the nature of the petroleum reservoir, and the nature of the production cost functions, for both investment and operating costs. The present study investigates one of the key components in the petroleum production decision, the nature of the operating cost function, $\frac{\partial C}{\partial q}$.

2.1.3 Other Issues

Several other issues come up that need some attention. Included in this are the uncertainty involved in the natural gas industry, joint products, and aggregation. The first two of these issues, uncertainty and joint products are only of passing interest to this thesis, and therefore will only get minor attention. Aggregation, however, is of more importance.

The models presented thus far have assumed certainty. In the extraction model of the first section, prices, resource stocks, costs, and the time frame over which production takes place were assumed to be known. The model by Kuller and Cummings made assumptions of certainty as well with regard to these same variables plus interest rates. These assumptions are not very valid in practice. For example, Adelman (1992) cites the Kern River reservoir (in California) that was estimated to have 54 million barrels of oil in 1942, produced 736 million barrels between 1943 and 1986, and still had 970 million barrels of expected remaining reserves at the end of 1986. This he argued was not due to conservatism in original estimates. The additional reserves were added with additional development investment, some of which incorporated new technologies. Similarly, future prices, interest rates, and costs are dynamic and subject to external pressures from changes in other industries.

Given that these variables are uncertain, the time frame over which production takes place is itself uncertain. Prices, costs, and the resource stock (or your knowledge of the resource stock) may change over time, and therefore whether you will decide to continue to produce at some later time is not certain. Investment is carried out based on expectations that are derived from the currently available information. The fact that these expectations are subject to revision with new information as it becomes available means that overall plans may change. Moreover, uncertainty gives decision makers the incentive to keep their options open. Thus for example, producers may delay investment in order to be able to reassess the profitability of the venture when some of the uncertainty has been resolved. In another example, if price expectations are low, instead of abandoning a well, a producer may wish to simply shut the well in just in case prices rise. Often shut-in wells cannot readily be reopened, especially if regulation require special sealing. In this case, as Brennan and Schwartz (1985) found it may be optimal to continue to produce even if operating costs are below revenues. They argued that if there is sufficient uncertainty with regard to prices, then the value of the future option to produce is non-negative. It is important to note that many of a petroleum producer's decisions take place at the investment rather then the operation stage. Since this thesis is focusing on the operation activity, the effect of ignoring uncertainty is substantially reduced. However, we must recognize that the current operating decision will reflect in part the operator's reaction to life in an uncertain world.

As alluded to earlier, the gas industry is subject to joint product problems. These appear in both the exploration/development and extraction phases of the industry. Several products are the outcome of exploration. All exploration yields knowledge, and some yields productive reservoirs. The productive reservoirs could contain gas, oil, other hydrocarbons, non-hydrocarbons (e.g. carbon dioxide, water, sulfur), or more likely some combination of all of these. The extraction phase, for a natural gas reservoir, rarely yields pure methane gas, but more likely involves some combination of the components mentioned above. Given that exploration and production often yield more than one product, how does one allocate the costs to each product? Adelman (1992) argues that attempts to allocate these costs to different products can only be done in an arbitrary manner. The reservoirs that comprise this study are comprised largely of sweet and relatively dry gas, thereby eliminating some of the problems associated with joint products. We shall assume that all operating costs are associated with direct gas production.

Non-renewable mineral resources are heterogeneous according to Cairns(1994). This heterogeneity forces the profit maximizing decision maker to account for the full marginal cost of production and the level of reserves instead of just the marginal extraction cost. This was illuminated in earlier sections when we showed that extraction has a 'user cost', and reserves are a function of exploration and investment. Moreover, Cairns argues that the heterogeneity of the resources causes methodological difficulties when attempts are made to study them on an aggregated basis.

The issue of aggregation applies to instances in which one may wish to provide a representation of economic behavior or outcomes at an aggregate level which is the same essential 'form' as an underlying process that is considered to be reasonably accurate at a more disaggregate level. In the economics of depletable natural resources, for example, it is common to build models of aggregate industry supply which "mimic" supply behavior for individual deposits; for example, the costs of production for the industry are assumed to be a negative function of remaining industry reserves in the same way that the costs of production for an individual pool are commonly assumed to relate to existing reserves.

Problems associated with aggregation arise in two areas. First, does aggregation to totals (e.g. reserves and production) or averages (e.g. gas quality, depth) maintain the original "information", or does the information change because of aggregation? Secondly, is aggregation used as an "ad hoc" method (i.e. does the aggregate equation simply "fit well"), or is it seen as an accurate aggregation across the underlying micro processes?

To illustrate the problems associated with the first case a simple model will be used. First, we make several assumptions to keep the model simple. The reservoirs are assumed to be closed (i.e. there exists no pressure maintenance from external sources like a water drive), a constant temperature is assumed, and the reservoir is assumed to contain only water and gas. In this case, Rojey and Jaffret (1997, page 178) assert that production can be characterized by the equation:

$$q = \pi(\omega_e^2 - \omega_w^2)h\phi(1 - S_w)\frac{1}{\rho}\left(-\frac{d\rho}{dt}\right) \tag{20}$$

where: q is the volume of gas produced, ω_e is the radius at which there is no flow in the reservoir, ω_w is the radius of the well bore, h is the height of the reservoir, ϕ is the porosity of the reservoir, S_w is the water saturation percentage, ρ is the pressure of the reservoir, and t is time. Given this equation, if two or more of the variables are not equal across reservoirs then an aggregate function will not adequately capture the production. For instance if ω_e, ω_w, h, ρ, and $\frac{d\rho}{dt}$ are all equal for two reservoirs, but ϕ and S_w are not equal then an aggregate function based on the average values of these will not yield the same value for production as the sum of the two wells. Assume that there are two wells, well one with ϕ_1 and S_{w1} and well two with ϕ_2 and S_{w2}, and the rest of the parameters are equal in both. Adding the production from the two wells yields:

$$q_1 + q_2 = \left(\pi(\omega_e^2 - \omega_w^2)\frac{h}{\rho}\left(-\frac{d\rho}{dt}\right)\right)(\phi_1(1 - S_{w1}) + \phi_2(1 - S_{w2})) \tag{21}$$

Taking an aggregate would yield:

$$\bar{q} = 2\left(\pi(\omega_e^2 - \omega_w^2)\frac{h}{\rho}\left(-\frac{d\rho}{dt}\right)\right)\left(\frac{(\phi_1 + \phi_2)}{2}\frac{((1 - S_{w1}) + (1 - S_{w2}))}{2}\right) \tag{22}$$

Canceling the common factors leaves:

$q_1 + q_2 = \phi_1(1 - S_{w1}) + \phi_2(1 - S_{w2})$ and $\bar{q} = (\phi_1 + \phi_2)((1 - S_{w1}) + (1 - S_{w2}))$, which in general will not be equal.

Another simple model will be used to demonstrate the problems that may arise if the aggregation does not respect the underlying micro process. Assume that the reservoir contains two wells. Each well contains the same reserves, R, and generates the same level of production, q. Now assume that operating costs for the ith well, c_i, are influenced only by reserves and production, such that $c_i = \beta q_i + \delta R_i$, where β is the coefficient on production, and δ is the coefficient on reserves. It is fairly easy to see that the operating costs for well one and well two are equal in this example, and that the reservoir wide costs can be calculated either by adding the operating costs for each well together or by adding the production and reserves together and substituting them into the equation.

(Either gives you costs of $2(\beta q + \delta R)$.) Expressed in other terms, one could estimate the values of β and δ in the equation either by using the data for the two wells separately, or by using the aggregate data and the estimates of β and δ would be the same.

This is quite a simple example, and the results do not hold up well if the assumptions are relaxed. If instead we allow costs to be non linearly related to production, reserves, or both, then the result will again be different costs for the reservoir than the sum of the individual wells. For example, if the cost function is $c_i = \beta q_i + \delta R_i + \phi q_i R_i$, then the corresponding costs for the wells calculated separately is $2\beta q + 2\delta R + 2\phi q_i R_i$, and the aggregated case is $\beta 2q + \delta 2R + \phi 2q_i 2R_i$). This illustrates that linearity in the parameters is a requirement for aggregation. From this example it should be easy to see that similar coefficient are also a requirement (i.e. $\beta_i = \beta_j$ and $\delta_i = \delta_j$).

Chermak and Patrick (1995) argue that restrictions imposed by aggregation across wells in a reservoir include ignoring the well-specific effects of recoverable reserves, reservoir pressure, and the interaction between these. Recoverable reserves are influenced by porosity, water saturation, pay thickness, and drainage area, all of which, they argue, can vary across wells. Moreover, they assert that aggregation does not allow for different starting production times, nor differing technologies to be used across a reservoir.

The model tested in this thesis avoids the issue of reserve additions by using data only on wells that are producing. The well specific nature of the data set used avoids the issues involved with aggregating to the reservoir level or beyond, and allows for factors such as different dates of production start up, the possibility of pressure differences across wells, as well as a general test of whether operating costs include a well-specific component.

2 <u>Empirical Models</u>

In this section four models, as well as a brief review of a study based on aggregate data, will be presented. The first three models deal with the estimation of operating costs for oil production. Two of the models, the ones by Livernois, and Livernois and Uhler are based on reservoir costs. The third model, by Griffin and Jones, is directed at the lease level. The fourth model, by Chermak and Patrick, was designed for natural gas, and focuses on the individual well.

The first three models start with a profit maximizing firm, and build a cost function from an optimization model. The model by Chermak and Patrick is an ad hoc style model, based on their expectations of what might influence costs. The last portion of this section deals with a study by Helliwell et al. (1989) that estimates the operating cost function for Western Canadian natural gas based on industry wide data.

2.2.1 Livernois

Livernois (1985) formulated and tested an operating cost model for oil reservoirs that incorporated water injection. The model was formulated around a material balance equation, which allows for the calculation of the quantity produced. The material balance equation states that for a given drop in reservoir pressure and a given level of reserves, the difference in space that would be occupied by the reserves at the lower pressure would be identical to the volume that is produced, evaluated at reservoir pressure. The following formula represents this process:

$$q = g(\rho_0, -d\rho, R) \tag{23}$$

Where: q is the quantity of oil produced, ρ_0 is the initial reservoir pressure, $d\rho$ is the change in reservoir pressure, R is the reserves, and g is some function. The formula is assumed to satisfy the conditions that $f_i \geq 0$, $f_{ii} \leq 0$ for all $i = \rho, d\rho, R$. This relationship is expected to hold at the well level, and is therefore well specific. Physical characteristics are assumed to be constant throughout the reservoir, and equal sized spacing units are assumed. This allows the flow rate of oil from a reservoir to be calculated by multiplying the number of wells in the reservoir by the flow rate from any given well in that reservoir.

Livernois argues that reserves per well are not directly observable. He therefore assumes a relationship between the pay thickness of the reservoir, the number of wells in the reservoir, and the water saturation level of the reservoir to approximate the reserves per well. Reserves per well are an increasing function of the pay thickness, and a decreasing function of both the water saturation and the number of wells in the reservoir. From this the reservoir production function is:

$$\bar{q} = q \bullet n = f(\rho_0, -d\rho, -S_w, Z, n) \tag{24}$$

Where: $\bar{q}$ is the quantity of oil produced from the reservoir, n is the number of wells in the reservoir. S_w is the water saturation level, and Z is the pay thickness. According to Livernois equation (24) implies that for a given number of wells and pressure decline, the flow rate of oil from a reservoir depends on the values of ρ_0, Z, S_w. Since these factors (ρ_0, Z, S_w) are all exogenously determined and treated as fixed by the producer, the only ways to increase the flow of oil from a reservoir are to increase the number of wells, or to change the pressure.

One of the realities of oil production is the ability of the producer to augment the reservoir's natural pressure by injecting water into the reservoir by means of an injection well. A producer may be able to reduce the net pressure decline from the reservoir by doing this. For this reason, Livernois defined a new pressure decline relationship that included water injections. First, a new variable, m, was defined as the amount of water injection. Then the pressure decline from the reservoir becomes:

$$u = \gamma(m) - dP \tag{25}$$

Where: u is the absolute value of gross reservoir pressure change, γ is a function that measures the amount to which water injections into the reservoir augment reservoir pressure, and dP is now the observed net pressure change. Now it is possible to define the aggregate quantity of oil produced from a reservoir in any period as:

$$\bar{q} = f(\rho_0, \gamma(m) - d\rho, -S_w, Z, n) \tag{26}$$

Livernois argues that equation (26) represents the extraction technology for an individual reservoir. By adding the assumption that the form of this equation is the same across all reservoirs, differences between production possibilities can be explained solely by the differences of the natural factors of the reservoirs (i.e. ρ_0, S_w, and Z).

Livernois further assumes that the reservoir is subject to unitized management and that the output level is fixed by government regulations under market demand prorationing regulations. Then Livernois formulates the producer's problem as the minimization of the present value of the cost of developing and operating the reservoir for a given path of output. Livernois assumes that the development decision occurs only once, when the producer chooses the number of wells to drill in the reservoir. This stock of wells is then assumed to be fixed and non-depreciating. A further assumption is made that equation (4) is invertible. The optimal quantity of water to be injected satisfies:

$$m = m(S_w, \rho_0, -d\rho, \bar{q}, Z, n) \tag{27}$$

and the cost minimization problem is:

$$\min \int_0^T e^{-rt} p_w \cdot m(S_w, \rho_0, \theta, \bar{q}, Z, n)dt + C_d \cdot n \tag{28}$$

with $\bar{q}$(t) being given for all t = 0.....T, $d\rho/dt = -\theta$, $\rho(0) = P_0 > 0$, p_w is the price per unit of water injection, and C_d is the cost of drilling a well. The net change in pressure is now defined as θ and is treated as a control variable in the model. The pressure change is controlled by the use of water injection. Given well characteristics, a large change in the pressure implies that little or no water injection has occurred, and conversely, a small change in pressure requires a large injection of water.

By assuming an interior solution, Livernois arrives at the condition that the marginal cost of changing pressure by water injection is equated with the marginal cost (or price) of pressure. This is the condition that:

$$\frac{\partial m(S_w, \rho, \theta, \bar{q}, Z, n)}{\partial \theta} = \lambda \tag{29}$$

where: λ is a co-state variable that represents the shadow price of pressure. The producer must choose θ (the net pressure change) to make the condition represented by (29) hold.

Livernois argued that an interior solution was more likely to hold for reservoirs of intermediate age. His argument for this was that as production from a reservoir began it usually was produced using the existing reservoir pressure, and the shadow price of pressure is close to zero. As the reservoir is produced, the shadow price of pressure increases, and water is used to maintain pressure, as long as the cost of injecting water is below the shadow price of pressure. As more of the reserves are depleted, and the reservoir nears the end of its productive life, the shadow price of pressure approaches zero. During this last phase there will again be no water injection.

To estimate the cost function, Livernois had to deal with the fact that often there is no water injection. Therefore, a Tobit model was used. The Tobit model is used with data that is censured or truncated allowing for a better fit by reweighting the information given by the zero entries. This required a modification to equation (27). Equation (27) was modified to:

$$m = g(S_w, \rho, \theta, \bar{q}, Z, n) + e \qquad \text{if } g(\bullet) + e \geq 0 \qquad (30)$$

$$m = 0 \qquad \text{otherwise}$$

The equation then estimated was a quadratic of the form:

$$g(x) = \beta_0 + \sum_{i=1}^{6} \beta_i x_i + \frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6} g_{ij} x_i x_j \qquad (31)$$

with the symmetry condition that $g_{ij} = g_{ji}$, and for notational convenience x_i and x_j replace the arguments of the function ($x_1 = S_w$, $x_2 = \rho$, $x_3 = \theta$, $x_4 = \bar{q}$, $x_5 = Z$, $x_6 = n$). Livernois argued that the quadratic function was appropriate because it is a moderately general functional form and it is twice differentiable, and it does not restrict the signs of the arguments. Equation (31) was then estimated using a sample of eighty oil reservoirs within the province of Alberta.

Testing this specification yielded the results that all of the x_i's significantly affected water injection. Livernois argued that this was support for his conclusion that natural factors explained the differences in production possibilities between reservoirs.

Livernois used his model to test the hypothesis that oil production exhibited constant returns to scale. This was a test of whether the production function given by equation (31) was linearly homogeneous in all of its arguments. Since there is a fixed factor of production, the reserves of the reservoir (as captured by Z and S_w), this implies diminishing marginal returns to the variable factors. (Livernois argued that this was analogous to rising long run marginal costs of production[5].) This involves a test of whether the parameters g_{ij} are equal to zero, for all i and j, with the model that he estimated. Livernois rejected this hypothesis, and pointed out that this meant that marginal extraction costs might in fact be nonincreasing. (That is, there might be some areas of increasing returns to scale.)

Livernois also tested whether the marginal extraction costs were a constant function of the extraction rate. This was a test of whether the coefficient on g_{qq} was equal to zero in the model that he estimated. This could not be rejected. Livernois argued that

[5] Given the fact that this model does not include exploration, this can only be seen as a medium run conclusion.

this was only the case because θ was not allowed to vary. To allow θ to vary, he showed that it was possible to calculate another ratio. Livernois showed that $g_{qq} - \frac{(g_{q\theta})^2}{g_{\theta\theta}} = \frac{\partial^2 C}{\partial Q^2}$, and this was calculable from his estimation. This was again tested to see if it equaled zero, which again could not be rejected. With θ allowed to vary, this was a test of whether the full marginal cost (including the user cost of pressure) was increasing in the output rate. This result is somewhat surprising given what the theory argues. The presumption of stock effects normally implies that as production rises so does the user cost of pressure. As was discussed earlier when the model of natural resource depletion from Sweeney was outlined, the theory expects that with depletable natural resources, there is a nonzero user cost that must be included in the marginal costs. This user cost is non-decreasing with the quantity produced. However, Livernois did point out that this test was only valid for the observed extraction rates.

Finally, a test was undertaken for whether an aggregated function could be created for the data that would determine the industry wide production possibilities. For Livernois, this involved a test of whether the individual factor requirement functions were linear, that is $g_{ij} = 0$. This is the same as the test for constant returns to scale, and was rejected.

2.2.2 Griffin and Jones

Griffin and Jones undertook an econometric study in an attempt to investigate possible scale economies in oil production, and to look at the well abandonment decision. Their model focused on aggregation to the lease level. They built a "multiplant" model to investigate scale economies. A multiplant model was built to attempt to capture the reality of oil production. Oil production typically involves production from multiple wells within a lease, and/or from more than one lease within a field. In this way they argued individual wells are analogous to individual plants within a firm.

The multiplant model that Griffin and Jones used was based on a competitive firm. This firm would operate each plant to the point where the marginal cost per unit was equated with the market price (which is the firm's marginal revenue in the case of a competitive firm). If a plant's marginal cost curve lies above the current market price at every point, then that plant would not contribute to the firm's output. This plant might be retained if the price was expected to increase in the future such that expected future profits would at least be sufficient to cover the costs of maintaining the productive capacity of the plant (i.e. the quasi-fixed costs). It is assumed that there are no costs to temporarily shutting a plant down. In other words, there is complete and costless reversibility. In this case the optimal allocation of the firm's output between the individual plants is made by a series of independent decisions at the plant level.

Griffin and Jones argued that this multiplant model is easily extended to oil production by replacing plants with "wells in the lease". In this way a cost function for the lease can be defined as $C_t = C(\bar{q}_t, n_t, G)$. Where C_t is the lease wide cost at time t, $\bar{q}_t$ is the production from all wells in the lease ($\bar{q}_t = \sum_{i=1}^{n_t} q_{it}$, $q_{it} \geq 0$), n_t is the number of producing wells, and G is a vector of exogenous parameters that describes relevant geological factors (e.g. depth of producing zone, type of lift, and age of the lease). $\bar{q}_t$ would be determined by the market price. If $q_{it} = 0$, then this implies that a well is shut in. It is usually argued that a well will be shut in when, due to production decline, its average variable operating cost falls to the level of the crude oil price. This investment decision is often calculated at the lease level be comparing price to $\frac{C_t}{\bar{q}_t}$ or lease wide average operating costs.

In reality, the decision to shut a well in is complicated by the fact that well abandonment may not be costless. A well that is shut in by removing the casing and plugging with cement is no longer available for production in the future. At the time of this study conducted by Griffin and Jones, there were requirements in the United States that wells that had no production for sixty days had to be abandoned by removing the casings and plugging with cement. (This is not the case in Alberta at this time.) Abandonment of the well will result in the loss of the ability to produce from the well in the future. Moreover, if there are other reservoirs accessed through the bore, or deeper, that may be producible in the future, these may less easily be accessed. This defines an option value associated with the decision to produce or shut in. Furthermore, if enough wells within a lease are abandoned then leasewide production may approach zero. If leasewide production approaches zero then the lease may be forfeited. This then adds to the forgone option value associated with the decision to shut a well in.

Griffin and Jones assumed an exponential decline curve in order to examine a rule for well abandonment. This decline curve is $q_{it} = q_{i0}e^{-d_i t}$ (i = 1,2,3, ... Wt). d_i is the annual percentage decline rate for the i^{th} well, and q_{i0} is the initial output from the i^{th} well. Griffin and Jones argued that this would capture the production decline if the well was produced at the MER. The optimal date of abandonment, for each well, (τ_i) is calculated by maximizing the present value of all future quasi-rents (Π_i), from each well, according to:

$$\max_{\tau_i}\Pi_i = \int_0^{\tau_i}\left[p_t q_{i0} e^{-d_i t}(1-H_t)(1-z_t)-\left(\frac{\partial C}{\partial n_{it}}\right)\right]e^{-rt}dt \qquad (32)$$

According to Griffin and Jones a producer must form expectations about oil prices (p_t), future royalty rates (H_t), future severance taxes (z_t), and future marginal operating costs per well, $(\frac{\partial C}{\partial n_{it}})$. Once these expectations are made, the producer must discount to the present to arrive at the optimal abandonment date (τ_i). This disregards any loss of option value associated with either the possible development of deeper pools accessible from the well or or the possibility of future price increases that make more revenues economic. Moreover, the sunk costs of plugging the well with abandonment are also ignored.

Griffin and Jones argued that this should provide a much lower rate of production for abandonment then a static comparison of average revenue (price) with average operating cost per well. There are two reasons for this. The first reason is that if a producer can expect oil prices to rise in the future at a rate that is higher than the well's natural decline rate, then the producer would want to keep the possibility of production open. The second reason is that marginal operating costs per well may be lower than average operating costs (implied by leasewide costs).

Private marginal extraction costs per barrel are not directly observable at the well level according to Griffin and Jones. They argued that marginal extraction costs were likely flat over a wide range up to the MER and then rise quite rapidly. Producer behavior suggested this to them. In the absence of output restrictions, producers seem to produce at the MER. Since producers choose to produce at the MER despite varying prices for oil, their marginal costs must either fall or not rise much as production increases over smaller output levels than the MER, or they would likely adjust production in reference to price changes.

Griffin and Jones argued that the history of oil production in the 1920's in the U.S. was evidence that producers would over produce in an uncontrolled competitive extraction environment, that is when access to reservoirs was shared by a number of companies. Overdrilling and overly rapid depletion of reservoir pressure lead to both economic and physical waste. The regulations in essence treated a pool as common property for those producers with access to it. This gave producers little incentive to restrain current production, since there were no clearly defined property rights associated with the resource in the ground. It made sense to attempt to lift oil as rapidly as possible under these conditions because the only way to ensure that you had rights to the oil was to possess it above ground. Therefore marginal social costs of crude oil production greatly exceeded the marginal private extraction costs since the private costs tended to ignore the user costs of current production. This problem is referred to as the rule of capture. Unitization of the pool leads to the internalization of the externalities. The marginal user cost is included in the decision making process when the resource is operated by a single producer. A single producer can choose whether to extract the resource or to leave it in the ground without the fear of having someone else produce it first. Therefore, a single producer will extract at the point where the marginal extraction cost plus the marginal user cost is equated with the marginal benefit.

The possibility of increasing the present value of total profits from the pool would encourage integration. By unitizing, the firms would be able to increase their profits by increasing their respective number of wells (e.g. if there were three producers with two wells each, by unitizing, they would essentially be one firm with six wells). However, Griffin and Jones argue that integration would probably occur only at the end of the primary production. They cited Wiggins and Libecap (1985) who found that voluntary unitization generally did not occur at the beginning of production. This was argued to be due to disparate expectations by producers in regards to the extent of ultimate recovery. However, unitization is appealing with regards to secondary recovery.[6] Moreover, the uncertainty about potential recovery is reduced as the reserves are depleted.

Voluntary unitization would also be supported, according to Griffin and Jones, if the multiplant cost function of $C_t = C(\bar{q}_t, n_i, G)$ exhibits leasewide economies of scale. To examine the possibilities of scale economies, Griffin and Jones defined the elasticity of cost as being composed of two components, the elasticity of cost with respect to the number of wells and the elasticity of cost with respect to the level of oil production. They introduced an additive notation to measure elasticity of cost, that was derived from their Cobb-Douglas specification of a cost function, such that $\xi_S = \xi_{CW} + \xi_{CQ}$. ξ_S is the elasticity of cost with respect to the number of wells and the level of oil production. ξ_{CW} is the elasticity of cost with respect to the number of wells, and ξ_{CQ} is the elasticity of cost with respect to the level of oil production. (The addition reflects the fact that more wells typically also raises pool output.) If the elasticity of cost were found to be less than one then this would mean that, since costs rose less than proportionally with wells and production, there existed scale economies. Elasticity of cost found to be greater than one would show diseconomies of scale. An elasticity of cost equal to one would give evidence of constant returns to scale. In this way they could determine if there would be additional gains to the producers from voluntary unitization. If the lifting of oil exhibited scale economies, then unitization would be one way for producers to exploit these.

The data used in their sample consisted of data provided by twelve oil producers in Texas for the year of 1983. Each producer provided information on at least five leases for a total of 133 observations. The information provided for each lease included the total oil produced, the number of producing wells, the depth of the pool, the age of the lease, the number of injection wells, the associated water and gas production, and the type of lift. They were disappointed in the fact that they did not have information on the firm's input prices, but assumed that these were constant during the year and across

[6] This point was not elaborated on, but presumably the attractiveness of unitization for the purposes of secondary recovery may have something to do with cost/risk sharing. There may also be some concerns with "free riding." If another firm is willing to install the capital equipment to enhance a reservoir's productive capacity, it would benefit any other producer producing in the reservoir at that time. Therefore,

producers. They argued that this was not too offensive an assumption because they felt that prices would likely not vary too much in the same state, and in the same year[7].

Griffin and Jones estimated a log-linear cost function which related lease operating costs to the variables mentioned in the previous paragraph. They argued that attempts at more flexible forms yielded less satisfactory results. "The introduction of the many second-order and interactive independent variables called for by the translog specification provided no tangible increase in the explanatory power of the model at the expense of the accuracy of the estimates of individual coefficients."(Griffin and Jones ,1988, p.113.) The estimation yielded[8]:

$$\ln c_t = \underset{(2.67)}{2.85} + \underset{(5.98)}{0.53}\ln n + \underset{(3.76)}{0.28}\ln q_o - \underset{(0.83)}{0.01}\ln q_g + \underset{(3.22)}{0.06}\ln q_w + \underset{(0.85)}{0.04}\ln IW + \underset{(3.91)}{0.45}\ln DE -$$

$$\underset{(1.22)}{0.09}\ln A + \underset{(3.38)}{0.82}\ln MP + \underset{(4.51)}{1.43}\ln EP + \underset{(5.03)}{1.27}GL$$

$R^2 = 0.897$

Where n is the number of oil wells, q_o is the quantity of oil produced, q_g is the quantity of gas produced, q_w is the quantity of water produced, IW is the number of injection wells, DE is the depth of the producing zone, A is the age of the well, MP is a dummy variable denoting a mechanical pump, EP is a dummy variable denoting an electric pump, and GL is a dummy variable for gas lift.

The results of their estimation showed that the elasticity of cost with respect to the number of wells was 0.53. This suggested to them that the marginal operating cost per well would be less than the average operating cost per well. The significance of this is that the well abandonment date predicted by the average operating cost would be too soon. Marginal operating cost being less than average operating cost would occur if a significant portion of leasewide operating costs are lease rather then well specific. This

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unless everyone agreed to cooperate, no investment would occur. Whereas primary recovery does take place to some extent without unification.

[7] This assumption may not be justified. From talking to people within the oil and gas industry in the Province of Alberta, input costs are not necessarily constant across the province. For instance, for similar wells, input costs may be five times as high in Northwestern Alberta than in Southeastern Alberta, and oil prices do vary during the year. However, for day to day prices, oil prices tended to be more stable before 1986 then they have been since.

[8] Griffin, James and Clifton Jones, "Economies of Scale in a Multiplant Technology: Evidence From the Oilpatch", Economic Inquiry Vol. 26, 1988 p.115

might be due to fixed capital related costs such as on site oil storage and oil gathering lines.

Elasticity of cost with respect to the level of oil production was estimated to be 0.28. This indicated that there were scale economies at the lease level. (If wells were operating at their MER, then there would be no unexploited scale economies at the well level.) Therefore, to capitalize on the lease wide scale economies could only be achieved by increasing the number of producing wells. Griffin and Jones suggested that this could be done through unitization or merger.

These estimates combined to yield 0.81 as the elasticity of cost with respect to the number of wells and the level of oil production. This suggests that unitizing leases across a reservoir will yield cost savings.

Their results suggest that operating costs are increasing in the quantity of oil production, as is expected by the theory, but decreasing with the quantity of gas produced. However, the sample only included oil wells, and therefore the gas produced was a byproduct of oil production. Their model did not include reserves in the regression.

2.2.3 Livernois and Uhler

Most of the theoretical models of exhaustible resources have been based on the depletion of the region's aggregate stocks of the resource. In these models there are assumed to be costs functions which apply to the aggregate stocks. It is common for instance to include degradation effects by assuming that extraction costs fall as the aggregate reserves in the region rise. In fact, resources of natural gas lie in separate reservoirs. Livernois and Uhler (1987, page 197) built a model to test whether aggregation beyond a single reservoir was appropriate. They argued that the belief that if aggregate resources increased, then extraction costs would decrease, is flawed. Other factors affect extraction costs. For instance, there is a tendency for lower cost deposits to be found first since lower costs deposits are more likely to be larger deposits. Larger deposits are typically found first because there is a better chance of finding something larger rather than something that is smaller. In addition, various exploratory techniques, like seismic, have some ability to direct producers toward lower cost deposits. If the lower cost deposits are found first, then additional finds would serve to increase extraction costs, eventually, as new deposits are more costly to extract than current deposits. Therefore the direction of effect of aggregate resources on extraction costs is ambiguous. If this is the case, then aggregate models built at either a basin level, or perhaps at the national level, would have little predictive power for optimal extraction rates or price paths. To test their hypothesis, Livernois and Uhler built two models, an aggregate model, with many deposits, and a disaggregated pool specific model.

Their first model, the aggregated model, involved "a hypothetical competitive firm that makes optimal extraction and exploration decisions with respect to reserves aggregated over many deposits." Livernois and Uhler (1987). Exogenous prices are assumed as well as that the firm has complete property rights for the pool and maximizes the present value of profits from exploration and extraction according to:

$$\Pi = \int_0^{\infty} [pq - C(q,R) - C_E E] e^{-rt} dt \qquad (33)$$

Subject to: $\dot{R}(t) = \dot{U}(t) - q(t)$; R(0) = R_0 (34)

And $\dot{U}(t) = f[E(t), U(t)]$; U(0) = U_0 (35)

Where p is the price of the extracted resource, C_E is the unit cost of exploratory effort , E is the quantity of exploratory effort expected, r is the discount rate, C[q, R] is the aggregate extraction cost function, R is the reserves at time t, and q is the extraction rate at time t. Equation (34) shows the change in reserves at time t, as the difference between discoveries at time t and extraction at time t. Equation (35) shows the discovery rate at time t, which is a function of the exploratory effort, and the level of cumulative discoveries (X(t)).

Livernois and Uhler give an example of what the price path would look like if the aggregate industry-wide cost function was defined explicitly. They use C(q, R) = q-$\bar{C}$(R). Price will then follow the path determined by $\dot{p} = r[p - \bar{C}(R)] + \bar{C}_R(R) f(E, U)$.

If the assumption of $\bar{C}_R$ < 0 is made, implying that costs rise as reserves fall, Livernois and Uhler suggest a few results. Exploration has a negative impact on extraction costs, and on the market price. If initial reserves are small, then $\bar{C}_R$ will be large in absolute value. In this case, small increases in reserves will lead to large decreases in extraction costs. This gives support to a U-shaped price path. That is, as reserves are added at a faster rate than production they drive down prices, then prices rise as the reserves are depleted.

However, if the opposite assumption of $\overline{C_R} > 0$ is made then the main incentive for exploration is lost, and price rises over time. Livernois and Uhler suggest that the assumption may be more realistic for two reasons. As low cost deposits are used up, and new deposits that are higher cost are found, extraction costs should increase. Moreover, as resources are extracted within a reservoir, extraction costs tend to rise as lower cost reserves are extracted first. In this way exploration is delayed until price equals extraction costs for incremental reserves.

To test these predictions they estimated a model that took the form of:

$$C_t = \alpha + \beta_q q_t + \beta_R R_t + \beta_{qR} q_t R_t + e_t$$

Where C_t is the total extraction costs at time t, q_t is production at time t, and R_t is reserves at time t. In their model the extraction costs include both development and operating costs.

Using this model and data from the Province of Alberta, they strongly reject the hypothesis that the marginal cost of extraction with respect to reserves (i.e. $\frac{\partial C_t}{\partial R_t}$) is decreasing. In fact they found a significant positive relationship between extraction costs and increasing reserves.

The second model built by Livernois and Uhler was a disaggregated model. In this model a firm must choose three things at each point in time. The first, is the rate of exploratory effort to yield a desired number of discoveries. The second, is the extraction rate of each new discovery. Thirdly, the extraction rate of each previous discovery must be chosen. With this model, costs for the k^{th} deposit should depend on the extraction rate for that deposit, q(k,t), the "ratio of remaining reserves", R'(k,t), and some vector of exogenous physical characteristics, G(k). The ratio of remaining reserves (R') was calculated to be one minus the ratio of cumulative extraction over initial deposit size (i.e.

$R' = 1 - \frac{\sum_{t=1}^{t} q_t}{R_0} = \frac{R_0 - \sum_{t=1}^{t} q_t}{R_0}$). However, estimating the initial deposit size is difficult and imprecise as discussed earlier. (The discrepancy over the actual resource size at Kern River in California, discussed in the section on "other issues" page 34, above, is an example.) Livernois and Uhler did not make it explicit how they calculated their initial reserves. From the form of their model it appears that they implicitly assume that the reservoir's entire initial reserves are known with certainty at the start of production. Livernois and Uhler argued that quality of the deposit depended mainly on the initial deposit size, and on some other exogenous physical characteristics that may include items such as reservoir permeability and pressure. Combining this with the fact that the best deposits are usually found first, they concluded that over time extraction costs should rise. Their cost function was then formulated as C = C[q(N,t), R'(N,t),N], where N is an index for the pools such that the first pool found would be N = 1, the second N = 2, and so on.

The instantaneous profit in period t from a given deposit k is given by: $p(t)q(k,t) - C[q(k,t), R'(k,t), k]$; and the profits across all deposits is given by:

$$\int_0^{N(t)} \{p(t)q(k,t) - C[q(k,t), R'(k,t), k]\}dk - C_E(t)E(t) \qquad (37)$$

And the maximization problem becomes:

$$\Pi = \int_0^{\infty} \left(\int_0^{N(t)} \{p(t)q(k,t) - C[q(k,t), R'(k,t), k]\}dk - C_E(t)E(t) \right) e^{-rt} dt \qquad (38)$$

subject to $\dot{R}'(k) = q(k,t)$ for all $k \in (0, N(t))$ (39)

and $\dot{N}(t) = n(t) = f[v(t), N(t)]$ (40)

with n(t) being the number of discoveries in time t. Two costate variables are introduced to solve the problem, λ(k,t) being associated with the reserves remaining in deposit k at time t, and μ(t) being associated with the state variable N(t).

Solution of (38), with (39) and (40), gives the condition that marginal extraction cost plus the marginal user cost for each deposit must be equated. This is shown as:

$$C_q\left[q(i,t),R'(i,t),i\right]+\lambda(i,t)e^{rt}=C_q\left[q(j,t),R'(j,t),j\right]+\lambda(j,t)e^{rt} \qquad (41)$$

for all $i,j\in(0,N(t))$.

Livernois and Uhler argued that a decline in reserves would cause extraction costs to rise. Exploration may yield lower quality (higher cost) deposits, but it still may work to lower the rate of increase in extraction costs. Therefore, it would still be useful to conduct exploration to find new deposits, even when these deposits may be of higher cost then current deposits. This is contrary to the aggregate case where there were no incentives to continue to explore if $C_R > 0$ unless the price covered the costs of exploration plus extraction for the new reserves.

Expected price paths can also be examined using this model. To examine the behavior of prices the market supply needs to be defined as $q_M(t)$, the inverse demand function $p = f(q_M)$ used, and the first order conditions from the above maximization problem need to be totally differentiated for q(k,t). This yields:

$$\dot{P}(t)=A\cdot\int_0^{N(t)}\frac{C_{qR}-r(P-C_q)-C_R}{C_{qq}}dk=q(N,t)\dot{N}(t)\cdot A \qquad (42)$$

with $A=\dfrac{f'(q_M)}{1-f'(q_M)\int_0^{N(t)}C_{qq}^{-1}dk}<0$

Livernois and Uhler argued that equation (42) is ambiguous in sign. This result holds even if there is no exploration (i.e. $\dot{N}(t)=0$). This occurs because costs are decreasing with reserves. Therefore there is a possibility of a U-shaped price path with this model.

The model then estimated using a cross sectional sample from 166 oil pools in the province of Alberta was:

$$C = \alpha + \beta_q q_t + \beta_R R'_t + \beta_N N_t + e \quad (43)$$

Within this model they found that extraction costs, including exploration costs, development costs, and operating costs, were increasing with production, decreasing with reserves, and increasing in N. Having the costs increasing in N captured the effect that they wanted, namely that the later an exploratory well was drilled, the lower the quality of the well. They concluded that modeling the disaggregate setting allows for both an inverse relationship between reserves and cost at the pool level, and for the tendency for low cost deposits to be found first. Hence at the aggregate level, costs could seem to rise as reserves are depleted.

2.2.4 Chermak and Patrick's Model

Chermak and Patrick seem to have been the first to econometrically estimate a cost function for petroleum extraction at the individual well level. Although, as stated in the previous section, others had done estimates of a production cost function, it had always been done on an a more aggregated basis, either for a region or for individual reservoirs. Due to the nature of depleteable natural resources, specifically petroleum, such aggregation may not be appropriate. As discussed previously, the conditions for aggregation to the reservoir level include: identical production technology for all wells, the same start up date of production, and identical reservoir characteristics in the parts of the pool around each well.

Chermak and Patrick's analysis consists of six models which they tested. They used time series data. They had 451 monthly observations from twenty nine tight gas wells. The observations were taken over the period from 1988 to 1990. The wells were located in Wyoming, West Texas and East Texas.

The basic structure of their model is:

$$\ln c_t = \alpha + \beta_q \ln q_t + \beta_R \ln R_t + \beta_t \ln t + \beta_{pm} \ln pm_t + e_{tj} \quad (44)$$

where:

c_t is operating cost at time t

q_t is quantity produced at time t

R_t is remaining reserves at time t

t is a time trend

pm_t is the 'production month' at time t or the number of months since commencement of production

The time trend was included to allow for exogenous and "macro" effects on costs, such as inflation. Production month was included to test the effects of age on costs, since the wells differed quite substantially in their ages. The authors had tested other forms of cost functions including linear, log-linear, and flexible form cost functions. They found that the best performance was obtained using the log-log form that is presented here.

Chermak and Patrick decided to extend their basic model to test further restrictions. They considered a number of other factors that they believed may have an impact on the production costs. These factors included what company the well belonged to, the location of the well, the producing formation of the well, and the well itself. They argued that different companies may influence the production cost through differing levels of efficiency. The location may have an impact because the wells were in three different regions: West Texas, East Texas, and Wyoming, and there may have been cost factors specific to each region. For example, they argued that labour or input costs, regulations, or well densities may not be constant across regions. Geological formations may have differing impacts on the costs as well. Certain formations may be more costly to produce from than others, if they are, for example, less porous. Chermak and Patrick also wanted to test whether individual wells may have different effects themselves. They postulated that it may be possible for individual wells to be quite unique in their production costs, as would be the case, for example, if reservoirs differ significantly in characteristics. Even two wells within the same reservoir could have quite different costs if well spacing in the pool is not even, or if the pool has heterogeneous reservoir characteristics.

They employed the following definitions:

$$\alpha = \bar{\alpha} + \sum_{i=1}^{N-1} p_{ji} D_{jt} \qquad (45)$$

$\bar{\alpha}$ is the "parameter estimate for the intercept variable"

p_{ji} is the ith parameter estimate for the intercept terms of the jth model

D_{ji} is the ith "binary" or "dummy" variable denoting which category a well fit into (e.g. for the location model, two dummy variables were used to denote which of the three locations the well was located in).

$$\beta_k = \overline{\beta_k} + \sum_{i=1}^{N-1} p_{jik} \Phi_{jik} \text{ where k is q, R, t, pm} \qquad (46)$$

$\overline{\beta_k}$ is the parameter estimate for the variable k

p_{jik} is the ith parameter estimate for the first-order interaction terms (i.e. the independent variable is $\Phi_{jik} V_k$ where V_k is the variable k),

Φ_{jik} is the ith binary variable for the jth model

j = r(restricted), c(firm), l(location), f(formation), w(well), or cw(firm/well), denoting which of the six models is being tested.[9]

if j = r, then (N=0) which means there is no interaction and $\beta = \overline{\beta_k}$ and $\alpha = \bar{\alpha}$

if j = c then (N=5)

if j = l then (N=3)

if j = w then (N=29)

e_{tj} is white noise

The results show that operating cost is inversely related to remaining reserves, and directly related to quantity, but the marginal cost of production is decreasing in quantity. These results were consistent across all models, although the parameter values varied according to which other effects were included. For instance the coefficient on q_t varied between 0.05 and 0.48, and on R_t it ranged between -0.05 and -0.35. They further found that costs were negatively related to time, which they argued was consistent with the fact that during the same time frame, costs were decreasing on an industry wide basis.

A surprising finding was that month of production did not have a consistent sign. Chermak and Patrick argued that this could be because of two opposing factors. The first was that as wells age there would be more down time, and hence more costs. The second, opposite effect, was that as well density increased over time there would be economies generated in the "pumper costs" (i.e. the costs associated with pumping the gas to the surface). With more wells in one area, an individual pumper could service more wells.[10] Whether or not the coefficient on production month was positive or negative would depend on which effect dominated.

Out of all the models, they found that the best fit was obtained using the company variable in their specification. All companies showed statistically significant slope and intercept terms. They proposed that accounting and business decisions which differed between companies might explain this However, they had difficulty explaining why two of the firms, which were just separate divisions of the same company, would have different slope and intercept terms. Their conclusion was that reservoir or location differences must be explaining some of the difference.

There were concerns about the robustness of the results. The changes in magnitudes of the coefficients were an indication of this problem. All models were tested for serial correlation, heteroscedasticity, and structural stability. Only the individual well model showed any problems of heteroscedasticity. They corrected it by expanding this specification to include firms. The firm/well specification did not show heteroscedasticity using a White test.

The authors concluded by stating that although there were problems of stability in regards to the magnitudes of the coefficients, the important results of this study were the signs of the coefficients. The signs on production and remaining reserves, as stated above, were constant across models. These results were consistent with that found by Livernois and Uhler for their oil pool (disaggregated) model, namely that, $C_q > 0$ and $C_R < 0$.

2.2.5 Helliwell et al.

Helliwell et al. (1989) undertook a study of the Canadian oil and gas sector in which they examined operating costs for natural gas production in Western Canada with data from 1951-1985. The data used was aggregated to a Western Canada wide basis. They defined operating costs as real operating cost per unit of production. They

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[9] The index for formations was not presented in this paper.

[10] Presumably, more wells may be added as wells aged within an area. It is not clear why they believed that economies from a pumper would enter through the production month, and not the time trend. It seems that the density would be increasing in time as well.

estimated operating costs (c/q) as a function of the level of production (q) and ratio of cumulative production to remaining reserves (Q/R). They found the following results with t-statistics below the estimated coefficients:

$$c/q = 0.15294 - 0.0000954q + 0.56628Q/R \qquad (47)$$

(39.96) (17.86) (21.91)

Multiplying though by quantity yields:

$$c = 0.15294q - 0.0000954q^2 + 0.56628(Q/R)*q$$

These results suggest that for a large range of output, operating costs are increasing with quantity, and decreasing in reserves.

Chapter Three: The Model

This thesis will look at operating costs for a number of Alberta gas wells using an approach similar to that of Chermak and Patrick. The cost equation is not derived from an explicit optimization model, but includes variables which it is expected will affect well costs. Operating costs are assumed to be a function of extraction, reserves, and reservoir physical properties. The quantity of gas extracted from the reservoir is expected to affect costs. That is to say that extracting gas from the reservoir is expected to have some marginal extraction costs. Extraction may affect operating costs through possible scale economies. The direction of effect is possibly ambiguous at the well level. It may be that wells suffer from decreasing returns to scale in that as more is extracted, with reserves held constant, it becomes more costly to extract on average because reservoir pressure is declining more rapidly (i.e. a user cost of degradation). Conversely, it may be that, because there are a number of costs associated with the ability to produce, rather then the amount produced, average operating costs could be decreasing with extraction. That is, at least some of the costs are spread over a larger volume of output, and rise little as output expands. The reserves remaining in the reservoir are expected to impact costs, as the more reserves that are present, the easier it should be to lift a certain quantity of gas from the reservoir without impacting the reservoir much.

Reservoir physical properties are a large category. There are many characteristics that "make up" each reservoir: the depth at which the reservoir is located, the pressure within the reservoir, the thickness of the payzone, the porosity, the permeability, the viscosity, and the other reservoir contents are but a few of these defining characteristics. Within this model the depth the reservoir is located at, the reservoir itself, and the producing zone in which the reservoir is found are included as factors which might affect operating costs. These three variables are chosen to give some weight to the physical properties and at the same time to minimize the relationship with the other variables (i.e. quantity extracted, remaining reserves), which are more interesting to this study. (For instance, the reserves in a reservoir are related to the thickness of the payzone and the reservoir's porosity.) In fact, the inclusion of what reservoir is being produced from should help in removing the other mitigating physical properties from the estimated impacts of the quantity extracted and the remaining reserves on operating costs.

Several models were tested. The first model was a simple model similar to that of Chermak and Patrick, which is formulated as follows:

$$\ln c_t = \alpha + \ln q_t + \ln R_r + \ln t_t + \ln pm_t + e_t \qquad (44)$$

where: $\ln c_t$ is the natural log of operating costs at time t; $\ln q_t$ is the natural log of production at time t; $\ln R_r$ is the natural log of remaining reserves at time t; $\ln t_t$ is the natural log of a time trend at time t; $\ln pm_t$ is the natural log of the well's age at time t; and e_t is a random error. Second, a more flexible model was examined which added the square of production $\ln (q_t^2)$:

$$\ln c_t = \alpha + \ln q_t + \ln q_t^2 + \ln R_r + \ln t_t + \ln pm_t + e_t. \qquad (48)$$

Third, dummy variables were added for companies, formations and pools.

Due to the nature of the data (which will be elaborated on in the following chapter), the model is estimated as a panel data set (i.e. a set of observations across time for a sample "panel" of wells). Using a panel data technique allows for more information to be used in the estimation given the knowledge that the data set includes a mixture of time series and cross-sectional data. Given the available data, differences between wells can be best modeled by using econometric techniques that are designed for use with panel data.

The methods for dealing with panel data typically use the constant term to model the differences between groups (e.g. wells). According to Greene (1994, page 615) the model looks as follows:

$$y_{it} = \alpha_i + \beta' x_{it} + e_{it} \qquad (49)$$

Where α_i is a group effect that is constant over time (t), but not groups (i). In this way, changing output (x) may affect costs (y) in the same way, but the level of costs may be different for different groups. This is done in one of two ways. The first is the fixed effects model. The second is termed the random effects model.

The fixed effects model assumes that $\alpha_i \neq \alpha_j \neq \alpha$, or that differences between groups can be viewed as a parametric shift in the regression function. This model is essentially involves putting in dummy variables for the different groups. In this analysis, this amounts to twenty-one dummy variables for the individual wells. An F-test is done to see if all of these parameters could jointly be equal to zero. The individual relevance of one of the parameters being equal to zero is not of much importance (except with regard to its own interpretation).

The random effects model also assumes that α_i is a group specific disturbance. The difference however, is that it is similar to the error term, rather than a parametric shift. Greene (1994) gives the following modification to the regression equation above:

$$y_{it} = (\alpha + u_i) + \beta' x_{it} + e_{it} \quad (50)$$

Now the differences between groups can be modeled the same as a group specific error term that applies to each observation (i.e. observations at different points in time) in the group identically (i.e. there is no time subscript). With this formulation, the error term can now be written as:

$$w_{it} = u_i + e_{it} \quad (51)$$

and the model is:

$$y_{it} = \alpha + \beta' x_{it} + w_{it}. \quad (52)$$

This adds complexity that ordinary least squares (ols) is not prepared to handle. Ols is based on the assumption that the variance of the error term (e) is σ^2. However, with the random effects model, the variance of the error term (w) is now $\sigma_2\Omega$, where Ω is a positive definite matrix. To deal with this complexity, Greene (1994) suggests that we define P as a vector such that $P'P = \Omega^{-1}$. By doing this, we can premultiply the regression by P to yield:

$$PY = PX\beta + PW \quad (53)$$

(where capital letters denote matrices or vectors, and W is the vector of error terms.)[11] We can now define $Y_* = X_*\beta + W_*$, where Y_* corresponds to PY, X_* corresponds to PX, and W_* corresponds to PW. We can now use ols to estimate this equation. Estimation using this transformation is called generalized least squares, or gls.

Both the fixed and random effects models were chosen to estimate the models. These techniques were chosen to allow greater freedom within the error structure (i.e. the error does not need to be consistent across or within groups, but can vary). The statistical

[11] Note that X includes a column of ones to yield a constant term therefore α is included with the vector of coefficients β

package used for the estimation estimates the following form of equation for fixed and random effects respectively[12]:

$$y_{it} = \alpha + x_{it}\beta + [v_i + e_{it}] \tag{54}$$

$$(y_{it} - \varphi\bar{y}_i) = (1-\varphi)\alpha + (x_{it} - \varphi\bar{x}_i)\beta + [(1-\varphi)v_i + (e_{it} - \varphi e_i)] \tag{55}$$

where φ is a function of the standard deviations of the error terms u_i and e_{it}.

[12] StataCorp. Reference Manual: Volume 4, 1997.

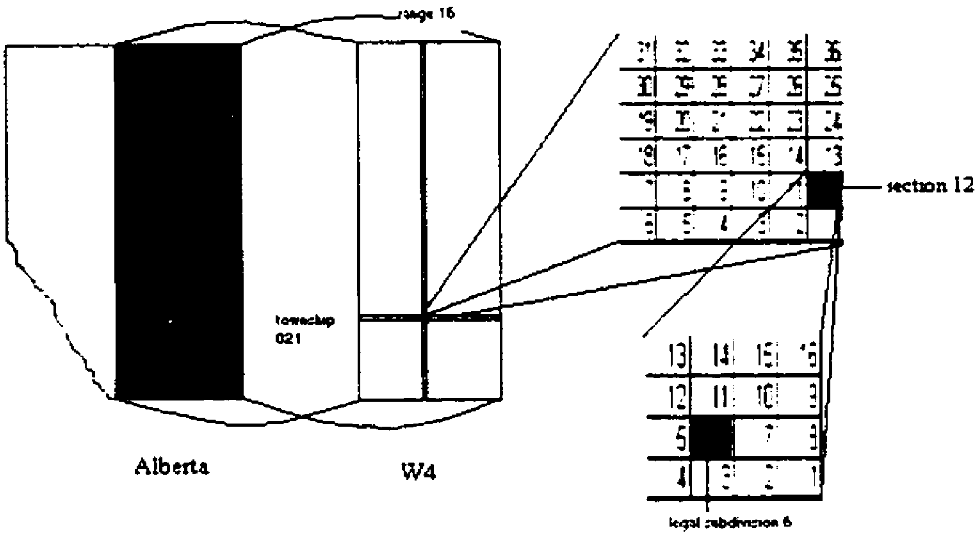
Chapter Four: Data

The data that will be used to carry out statistical tests was provided by an Alberta company. It includes monthly data on twenty-two natural gas wells, starting January, 1996 and ending in December, 1998, from the Countess region of Alberta. This region is mainly a gas producing region, located approximately 70 kilometers east of the city of Calgary. The wells are located in the 16^{th} range west of the forth meridian, in townships 20 and 21.

Wells in the province of Alberta are formally named primarily by their location, but other information is also contained in the name. The first piece of information given is the company name and the strike area. This is shown for example in the name Canor et al Countess. Canor would refer to the name of the company, et al to the fact that Canor has partners in this well, and Countess is the strike area. Continuing with the name, Canor et al Countess 00/06-12-021-16 W4/0 refers to the first well (00) drilled, on the legal subdivision (06), in section (12), of township (021), in range (16), west of the fourth meridian (W4), operated by the company Canor and its partners, and located in the strike area of Countess. The right most number, after the slash, refers to what event this is within the bore. The "event" refers to an encounter of oil or gas. A well denoted 00/06-12-021-16 W4/2 would refer to the same well as the above, but a second occurrence of oil or gas within the bore. A well denoted by 02/06-12-021-16 W4/0 would refer to a second hole drilled at this location. Figure 1 shows the geographical dimensions of the well. The leftmost part shows, in gray, the region of the province denoted as west of the fourth meridian (W4). This is separated into Ranges running from 01 to 30 as vertical separations, with 01 being the eastern most and 30 the westernmost. It is also separated into townships that run from 001 to 126 as horizontal separations, with 001 at the bottom and 126 at the top. The rectangle coming out from the map of Alberta shows how a range (16) and township (021)are used to denote a specific land area. This gives a square that is divided into thirty-six sections as shown coming out of the map of the area known as West of the Fourth Meridian. Each of these thirty-six sections is further divided up into sixteen legal subdivisions, which is shown below the sections. The shaded areas show the plot at 06-12-21-16 W4.

Figure 1

Geographic Location



The data provided by the company consists of monthly observations on operating costs, natural gas production, reserves remaining, time, and production month. Data is also available on the depth of the well, the geological zone, and the pool that is being producing from.

One glaring difficulty is that the raw data consists of 'lumpy' operating costs. That is to say that the time series data has large entries in some months with several observations on either side being very small or, more commonly, equal to zero. This creates difficulties because it is not clear whether this is caused by natural forces, or if it is "artificially" created. For instance, it is not clear if certain months just have large expenditures because that is what happens when producing oil or gas, or if costs to maintain production over a long period of time are arbitrarily chosen to be allocated to one month. Are repairs required in June because something happens in the gas production, only in June, that requires repairs, or is it that once a year it is decided that repairs should be made and it happens that June is chosen for those repairs to take place? Another possibility would be that repairs are required at some, perhaps indeterminate, point over the year, and a billing or accounting convention leads them to be recorded in June.

The operating costs are broken into several categories within the company's records. For example the monthly observations show payments for lease and road, repair and maintenance, chart reading, and overhead, to name a few of these categories. Some of these categories show payments that are incurred sporadically (e.g. repair and maintenance), some are incurred once per year (e.g. AEUB administration fee), and others are incurred at reasonably consistent levels month after month. Moreover, others such as overhead are likely unrelated to production. In an effort to deal with these problems it becomes clear that an understanding of the different costs would be needed. It is important to define those costs that are significant, and describe how they seem likely to have accrued. Then a suggestion will be made as to whether the expense is likely related to production, or not, and whether it is likely "variable" or "quasi-fixed". A variable expense is one that is affected by the amount produced. Quasi-fixed expenses are incurred if production is to take place, but at a set level regardless of the amount of production. Included in the definition of "significant" will be all costs that either are incurred in three or more wells, or make up more than five percent of a single well's annual costs. This accounts for almost all of the costs. The remaining are labeled "insignificant costs". These remaining costs will be grouped as a miscellaneous category, and their timing will be accepted as reported. I will now discuss the significant cost categories

"Contract operator" is an expense related to contracting out the operation of the well. Instead of paying an employee to operate the well, a separate company is contracted to do this work. This is essentially a labour cost at each individual well. Variability between months and wells within this category relate to time and effort put into the well. This expense is likely variable and related to production.

"Chemicals" refer to injections into the well to enhance the well's producing capabilities. These can be either to increase flow, or to prevent hydrates and the freezing of water. Hence this expense is related to and variable with production.

"Lease and road" are expenses directed at road access to the well. Included here are amounts paid for maintaining access roads to the well. Expenses such as snow removal and weed control fall within this category. These are separate from the amounts paid to landholders for the rights to drill, which are the freehold surface lease. The lease and road expense is likely related to production, since it will only be made if production is planned, but is not variable with production.

"Repair and maintenance" refer to expenditures to keep the facilities operating within a 'normal' range. This involves expenditures only on the surface, and is often a very broadly defined category. Included within this category are items such as repairs to the separator, valves, painting, and cleaning. Moreover, this category usually involves a spending cap that is agreed upon by the partners in the well. When the spending exceeds this specified amount then the surplus goes into a new cost category outside of operating expenses. This has not occurred within this sample. This expense is related to production. However, there is some doubt as to whether it will vary with production. It is certainly possible that some maintenance may be required even if no gas is produced, just to keep the facilities ready or able to produce. ("Subsurface repair and maintenance" is a separate cost category that is not considered significant within this data set.)[13]

"Salt water disposal" includes expenditures to dispose of salt water that might be produced with the natural gas, including amounts that are used to pay for accumulating, storage of and a place to dispose of the water. Separate is the cost of "salt water trucking", which is the costs of transporting the salt water to the disposal well. Both of these expenses accrue because of production, and are variable with production.

"Trucking" includes the costs of transporting items to and from the well, excluding those costs associated with salt water disposal. Most notable here would be the transportation of items to the well. This would include the transportation of items such as chemicals for the well, supplies, parts for repairs, as well as any other items that need to be brought to the well site. This expense might or might not be variable with production.

"Municipal Property tax" is a payment made to the municipal government of the region where the well is located. This tax is set based on land values and not related to production, but a cost incurred for the ability to produce.

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[13] Information that I have suggests that subsurface repair and maintenance is an unusual expense within a gas well. This expense is more common in oil wells where you have a pump in the well bore.

The "Production and Alberta mineral tax" is a tax that the Alberta Provincial Government places on mineral production when the mineral rights are not owned by the province. This is set proportional to production of these minerals. and therefore it is variable with production. The Production and Alberta mineral tax is calculated by applying a tax rate to the annual unit value of a field. The annual unit value of a field (V) is calculated according to the following formula: $V = \frac{R-C}{q}$; where R is revenues in a year. C is allowed costs for the year. and q is the quantity of production in the year. Allowed costs include capital cost allowances. gathering, compressing. and processing costs.

"Freehold surface lease" is the payments made to the landholders for the right to have a well on their property. This expense is related to production. since if you choose to abandon the well this expense could be avoided. It is not likely variable with production.

"Freehold p&ng lease" is the payments made to the original mineral rights holder for the right to produce petroleum from their land when the mineral rights holder is not the crown. This expense is not variable with production, but it does accrue because of production for the same reason as the freehold surface lease.

"Small tools and supplies" are just as named. They are items such as cleaning supplies. rags. charts for gas measuring, or equipment and tools for very minor repairs. These are used over time. and not just at the instance of buying. These expenses are therefore a cost of having an operating well. and could vary with production.

"The AEUB administration fee" is an uncontrollable fee that is paid to the Alberta Energy and Utilities Board (AEUB). This fee is based on a year's production. and is a stepwise function with steps at 300. 600. 1200. 2000. 4000. and 6000 cubic meters. so it does vary with production.

"Chart reading" refers to expenses to have the well monitored by a company that specializes in reading production charts. This expense arises because of production. but does not vary with production.

"Cathodic protection" is a treatment to avoid external corrosion of the pipelines. This expense is related to having the ability to produce, and therefore not variable with production.

"Pressure vessel assessment" refers to expenses to have someone inspect the facilities to monitor corrosion, and test the vessels. This expense is again related to the ability to produce not the level of production.

"Testing-pressure survey" is an expense to have a recorder placed in the well bore to record the pressure. This expense is again related to the ability to produce not the level of production.

"Analysis" refers to costs associated with the collection of a sample which is sent away for analysis of the components within the gas. This expense is once again related to the ability to produce not the level of production.

"Overhead" is an accounting convention of allocating a share of company wide costs to individual wells. This is not related to production at the well, and hence will be excluded from further consideration..

Table 1 shows a summary of the three classifications. The categories with asterisks (*) are expenditures that are "lumpy", in the sense that they are incurred only in select months.

Table 1

Summary of Apriori Expense Classifications

Not related to production	Quasi-fixed Operating Costs	Share of Costs	Variable Operating Costs	Share of Costs
overhead	lease and road*	2.8%	contract operator	19.6%
	municipal property tax*	6.7%	chemicals*	2.5%
	freehold surface lease*	14.3%	repair and maintenance*	1.9%
	freehold p&ng lease*	0.6%	salt water disposal*	0.4%
	cathodic protection	0%	trucking*	1.0%
	pressure vessel assessment	0%	small tools and supplies*	0.3%
	testing-pressure survey*	2%	chart reading	0.6%
	analysis*	0.2%	production and Alberta mineral tax*	42.7%
			AEUB administration fee*	3.2%

This "lumpiness" must now be examined. Does this lumpiness reflect the actual economic process of the cost generation, or does it reflect arbitrary contractual or accounting conventions?

The costs that exhibit lumpiness are chemicals. lease and road. repair and maintenance, salt water disposal, trucking, municipal property tax. freehold surface lease. freehold p&ng lease. production and Alberta mineral tax. AEUB administration fee. small tools and supplies. pressure vessel assessment. testing- pressure survey, and analysis. From the discussion above. it should be clear that in all of these cases. the timing of the cost appears to be "arbitrary". in the sense that the specific date (month) at which the cost is recorded bears no apparent relationship to the specific purpose for which the cost was incurred (other then its occurrence in the same year as the associated activity). In the case of a lease. a tax. or the administration fee. the payment for a year is made in an exogenously selected month. With regard to chemicals. repair and maintenance. salt water disposal. trucking. and small tools and supplies. the arbitrariness is generated in the fact that one month is selected for incurring the expense. but other months receive the benefits to production as well. In the other categories. pressure vessel assessment. testing, and analysis. timing of these events are arbitrary. The question now becomes how to deal with this lumpiness.

With regard to most of these categories (lease and road. freehold surface lease. freehold p&ng lease. production and Alta mineral tax. AEUB administration fee. pressure vessel assessment. testing- pressure survey. and analysis) it might seem reasonable to simply average the cost over the period in which they are incurred. thereby distributing the cost over that period evenly.

With regard to the other expenses. more careful thought is required. For instance. chemicals injected into the well bore are more likely to have a larger impact on productive capability close to the time of injection. and a decreasing impact as time becomes more distant. How then should this be modeled? Should the expense be averaged as the above expenses. or should some attempt be made to weight the average to reflect the pattern of productive capability.

In general however. it is not appropriate to smooth out the 'lumpiness' that exists in the data by imposing structure that we do not have. For instance. by assuming that the entry for each month is a simple average of the costs incurred for a year, we are perhaps altering the true relationship of the data. Therefore, to deal with this, the monthly values will be used for those cost categories that do not exhibit this lumpiness, and for those that do exhibit lumpiness, the monthly observations will be summed for the year and this sum will be reported for each month. In other words, where monthly information is believed to exist it will be used, and where yearly information is the most frequent available, it will be used. For example the January cost would include the January value for contract operator and the yearly value for chemicals. The February cost would include the February value for contract operator and the yearly value for chemicals These monthly operating expenses will be regressed on a number of variables.

Table 2 provides summary statistics for the data aggregated to yearly observations. It should be noted that 10 wells were dropped in which remaining reserves became negative yet the well continued to produce.[14] These were excluded because it is likely that additional investment was put into these wells to generate this outcome, and therefore the costs for these wells include an investment component that would bias the results. Alternatively, the reserve estimates may have been incorrect which would also bias the results.

Table 2
Yearly Data Statistics

Variable Name	Variable Symbol	Unit of Measurement	Obs	Mean	Std. Dev.	Min	Max
Operating cost	C	Canadian $/year	96	7456.51	9912.697	0	48098
Quantity of gas extracted	q	10^3m^3/year	96	1687.301	2453.543	0	9919.4
Remaining Reserves	R	10^3m^3	96	11967.64	20084.84	0	93376.8
Time	t	years	96	2	0.820783	1	3
Well age	m	Months	96	86.02083	111.0091	12	441

Table 3 provides a summary of the data used for the estimation using monthly observations (with the lumpy variables averaged on a monthly basis). Again the wells with reserves going below zero were dropped.

[14] The original sample had thirty-two wells, but the revised sample, after wells that incurred production with zero remaining reserves were dropped, included only twenty-two.

Table 3
Monthly Data Statistics

Variable Name	Variable Symbol	Unit of Measurement	Obs	Mean	Std. Dev.	Min	Max
Operating cost[15]	C	Canadian \$/year	708	9006.951	9846.456	1331	44510
Quantity of gas extracted	q	10^3m^3/month	708	170.8413	222.8739	0	888.6
Remaining Reserves	R	10^3m^3	708	17356.22	21353.37	1034.8	92630.8
Time	t	Years	708	20.5339	9.786706	1	36
Well age	m	Months	708	114.9944	127.1953	1	441
Depth	depth	Feet	708	795.6816	267.8487	382	1179.9

[15] Operating costs per month are calculated using the method outlined above (i.e. monthly observations for lumpy costs have the yearly sum reported each month) and are therefore appear quite large in comparison to the annual data.

Chapter Five: Results

The empirical analysis is designed to estimate the main factors that affect the total operating costs for the natural gas wells in the sample. These results will be compared to those obtained for the United States by Chermak and Patrick. Some estimates for disaggregated operating cost components will also be reported. This may allow more understanding of how the explanatory variables affect operating costs, and may allow for assessment of whether the initial division of operating costs into quasi-fixed and variable categories is reasonable.

This section will start with a brief review of the work of Chermak and Patrick. Next, a brief review of log-log estimation will be presented. With this background, an overview of the results will be given. The model was first tested with data aggregated to a yearly basis, then the model was examined using monthly data. Then a more detailed look will be taken at the equations that seem to fit best. This section finishes with an examination of the individual cost categories outlined in the previous section.

5.1 Review of Chermak and Patrick

The model estimated by Chermak and Patrick involved using both cross-sectional and time series observations. They estimated a cost function based on data for twenty-nine individual wells totaling 451 observations. They argued that costs (c) were a function of quantity produced (q), remaining reserves (R), a time trend (t), and the age of the well measured in months (pm). The basic model they estimated was of the form:

$$\ln c_t = \alpha + \beta_q \ln q_t + \beta_R \ln R_t + \beta_t \ln t + \beta_{pm} \ln pm_t + e_{tj} \qquad (44)$$

Using this model they demonstrated that operating costs were inversely related to the reserves remaining, and directly related to quantity being produced. When other variables such as well, company, or formation dummy variables were added, the signs of the coefficients (with the exception of age of the well) remained unchanged, but their estimated coefficients were not very robust. In terms of fit, Chermak and Patrick found that when the company variable was added to their equation the best overall fit was obtained. The results obtained by Chermak and Patrick are presented for comparison in the first columns of Tables 5 and 8.

5.2 Review of Log-Log Estimation

Log-log (or log-linear) models are models of the form: $y_t = \beta_1 x_t^{\beta_2} e^{u_t}$. By taking the natural log the following is obtained: ln y_t = α + β_2 ln x_t + u_t. Taking the partial derivative of ln y_t with respect to ln x_t yields: $\frac{\partial \ln y_t}{\partial \ln x_t} = \beta_2$. Hence β_2 can be interpreted as the elasticity of y with respect to x. Given this formulation, the assumption of constant elasticity is assumed across the model. For cost estimation, differing values of β_2 have differing interpretation. In a long-run model, when all inputs are variable, a value for β_2 of 1 is a technology that exhibits constant returns to scale (CRS). If $\beta_2 > 1$ the technology is said to exhibit decreasing returns to scale (DRS). If $\beta_2 < 1$ then the technology is assumed to have increasing returns to scale(IRS). For instance, if β_2 is less than one, then by increasing production by one unit average costs will decrease (IRS) since the percent increase in costs is less than the percentage increase in output; if β_2 is greater than one, by increasing production by one unit average costs will increase (DRS); if β_2 is equal to one, then increasing production by one unit will leave average costs unchanged (CRS). However, the model being dealt with here is a short run model in that there are fixed factors such as investment (or the capital stock). In this case we can think of $\beta = 1$ as exhibiting constant marginal productivity, $\beta < 1$ as increasing marginal productivity, and $\beta > 1$ as diminishing marginal productivity.

5.3 Total Operating Costs

Several models, with different functional forms, were tested with two forms of the data.

5.3.1 Annual Data

First, data that was aggregated to a yearly basis was used. This data was used because there was uncertainty about the reliability of the monthly data. Results are presented as Table 5. Table 4 shows a correlation matrix for the yearly data. The relatively high correlation between output and remaining reserves should be expected, since higher reserves typically support more production. Two models were tested using this data set. The first of these was a simple log-linear model, of the form used by Chermak and Patrick, shown in Table 4 as model 1. The equation for the estimation was:

$$\ln c_t = \alpha_t + \beta_1 \ln q_t + \beta_2 \ln R_t + \beta_3 \ln t + \beta_4 \ln pm_t + e_t \qquad (44)$$

where:

$\ln c_t$ is the operating costs in period t,

$\ln q_t$ is the log of the quantity extracted in period t,

$\ln R_t$ is the log of remaining reserves in period t,

ln t is the log of a time trend,

$\ln pm_t$ is the log of the age of the well in period t,

e_t is assumed to be a normally distributed error term with mean zero and standard deviation of one.

A more generalized equation was estimated by adding the log of the square of production ($\ln q_t^2$) (model 2). The square of production was added to allow more flexibility in modeling the impacts of production on operating costs.

Using yearly data on the simplest model gives a measure of fit that is larger than that obtained by Chermak and Patrick, 0.52 compared to 0.42, however, the results of this study do not conform to theirs. The coefficient on q has the expected sign, but is substantially lower (less than half the size that Chermak and Patrick obtained), and its t-statistic is well below that of Chermak and Patrick's (2.88 versus 14.83). The low coefficient suggests that average operating costs fall noticeably as production rises. The coefficient on reserves had a sign that was opposite both to what theory suggests, and what was obtained by Chermak and Patrick. Time was not significant in this specification, and the coefficient on age of the well was four times larger than the one obtained by Chermak and Patrick.

When the model was expanded to allow more flexibility in the effect of production on costs, with the addition of a square of production term, the fit improves greatly (R^2 of 0.83). The coefficient for reserves is no longer significantly different from zero, time became significant with a sign opposite to that obtained by Chermak and Patrick, and the effect of the age of the well remained the same with a larger t-statistic. The impact of production becomes more difficult to interpret because of the additional production term. Figure 2 shows a graph of this relationship. The natural log of cost is plotted against quantity for four series. The first shows the linear relationship (equation 44) with the mean values for reserves, well age and time. A second line (Cost linear min) is generated by using the minimum value for reserves with the mean value for well age and time in equation 44. The third line is equation 44, with a quadratic term for quantity included (Cost quadratic), again using the mean values for reserves, well age and time. The fourth series in Figure 2 (Cost quadratic min), is similar to Cost quadratic but uses the minimum value for reserves as well as the mean values for well age and time. Notice from the Figure 2 that the linear and quadratic series are divergent with larger quantities.

Table 4
Correlation Matrix: Annual Data

(obs=96)

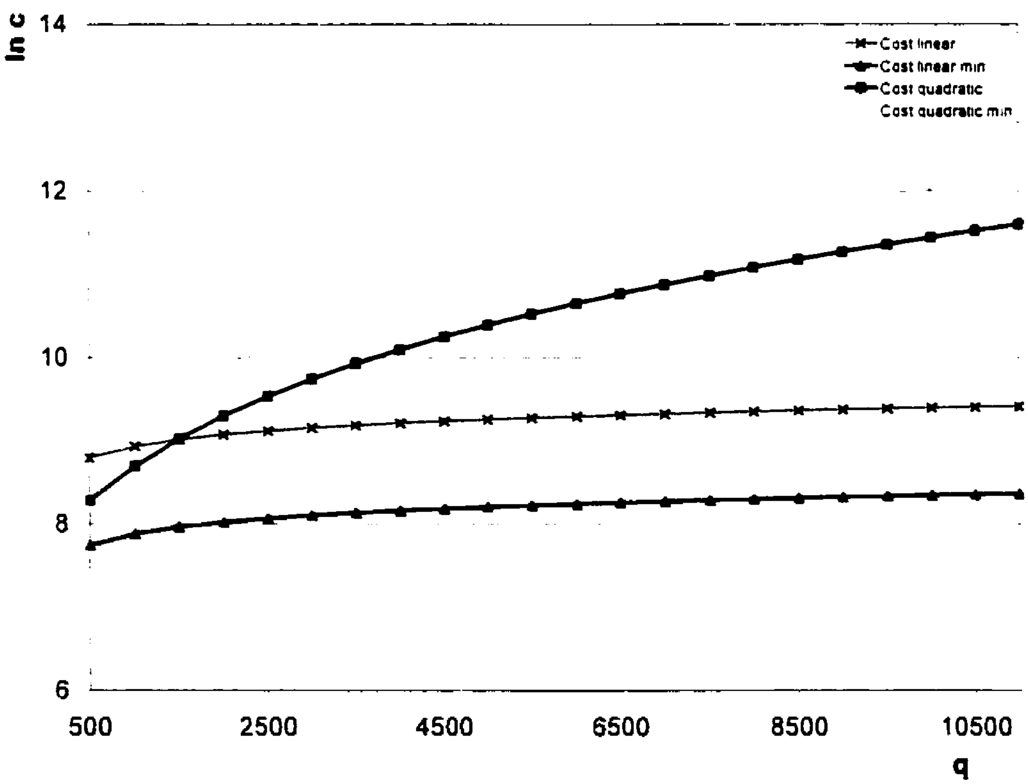
	C	q	R	T	m
C	1				
q	0.6675	1			
R	0.5271	0.8435	1		
t	0.1794	-0.0876	-0.0636	1	
pm	0.2108	-0.0778	0.0989	0.0805	1

Table 5
Yearly Data Results

Variable	Patrick & Chermak	$\ln c = \alpha + \beta_1 \ln q + \beta_2 \ln R + \beta_3 \ln t + \beta_4 \ln pm$	$\ln c = \alpha + \beta_1 \ln q + \beta_2 \ln R + \beta_3 \ln t + \beta_4 \ln pm + \beta_5 \ln q^2$
Constant	8.68 (17.51)	**4.11** **(6.61)**	**11.01** **(12.98)**
ln q	0.48 (14.83)	**0.20** **(2.88)**	**-2.03** **(-8.24)**
ln R	-0.35 (-12.45)	**0.09** **(2.03)**	-0.05 (-1.27)
ln t	-0.50 (-6.83)	0.21 (1.66)	**0.26** **(2.30)**
ln pm	0.13 (6.78)	**0.55** **(5.16)**	**0.55** **(8.91)**
$\ln q^2$			**0.20** **(9.51)**
R^2 adjusted	0.42	0.52	0.83
F statistic		55.74	292.58
Number of Observations	451	96	96

*t or z-statistics are in parenthesis at the 95% confidence level.
Boldface indicates coefficients that are significant at the 95% confidence level

Figure 2
Natural Log of Cost versus Quantity (Annual Data)



5.3.2 Monthly Data

Further results were generated by using monthly data. The results are presented in Table 8. Table 6 shows the correlations between the variables. Again, the high correlation between q and R is notable.

Table 6
Correlation Matrix: Monthly Data

(obs=766)	lC	lq	lR	lt	lpm	ldepth
lC	1					
lq	0.8984	1				
lR	0.8976	0.9378	1			
lt	-0.0933	-0.1399	-0.1968	1		
lpm	0.0835	-0.1430	0.0637	-0.0395	1	
ldepth	0.5079	0.3199	0.3808	-0.2708	0.4767	1

Since the log-log model is somewhat restrictive, the Box-Cox transformation was employed to "test the robustness" of various forms of the estimating equation, and to check if the log-log model was appropriate (Greene 1997, page 486). The Box-Cox transformation takes on the following form: $\frac{x^{\varphi}-1}{\varphi}$, where x is the variable being transformed, and φ is the parameter being used to make the transformation. It is possible to use an iterative process to select the φ that maximizes the log of the likelihood function. Both the linear and the log-log model are nested within this transformation. When φ is equal to 1 this is the linear model, and when φ is equal to zero this is the log-log model. Since φ is not restricted to being positive, it is not possible to use this technique with observations equal to zero for any variable that is transformed[16]. All

[16] When φ is negative this transformation is equivalent to $\frac{\frac{1}{x^{\varphi}}-1}{\varphi}$, and this will not be defined for a value of x = 0.

variables in the equations tested are transformed using the Box-Cox transformation. This created a difficulty because there were one hundred observations containing zeros. To see the significance of these observations, φ was estimated twice, once with the observations that contained zeros dropped, and once with the zeros replaced by 0.01 (i.e. an arbitrarily small value). The results of this are presented as Table 7.

From the table we can see that when these hundred observations are excluded the hypothesis that φ is equal to zero can not be rejected. (The test is a Wald test that uses the log of the likelihood function.) This is equivalent to the log-log model. When the zeros are transformed, both the linear and log-log models are rejected. The expected range for φ is -2 to 2. Figure 3 shows the range of values for the Box-Cox transformation of x with φ allowed to vary over this range. As can be seen from the graph, the log-log model (i.e. limit φ = 0) is quite similar to the model with φ = -0.11. Since φ = 0 cannot be rejected when the zero observations are dropped (as is the case with a negative value for φ since the zero is then in the denominator and therefore undefined), the log-log model will be used.

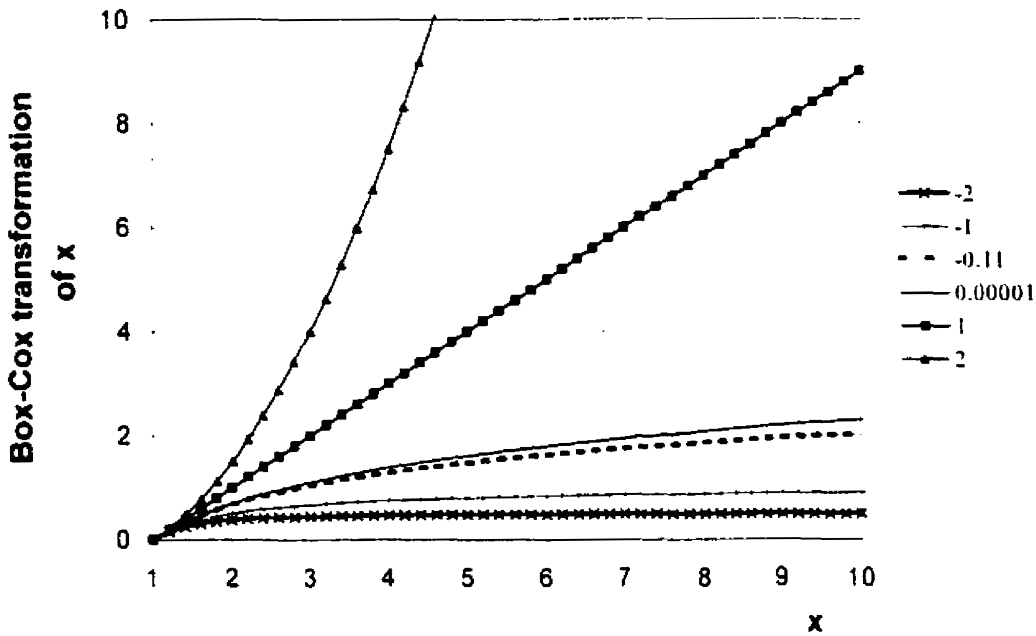
Table 7

Estimation of λ

	Zeros removed	Zeros set to 0.01
Number of Observations	608	708
λ	0.02	-0.11
Log Likelihood (0)	-5560.86	-6480.21
Log Likelihood (1)	-6114.47	-7089.75
Log Likelihood (maximized)	-5550.47	-6468.73
Wald Test λ = 0 $\chi^{2\,a}$	0.39	22.96
Wald Test λ = 1 $\chi^{2\,a}$	563.70	1242.04

[a] The critical value for the χ^2 at the 95% confidence interval with one degree of freedom is 3.84.

Figure 3
Box-Cox Transformation



Initially the simple model of the following form was estimated using a random effects model (Model 1), and a fixed effects model (Model 2): $\ln C_t = \alpha_t + \beta_1 \ln q_t + \beta_2 \ln R_t + \beta_3 \ln t + \beta_4 \ln pm_t + e_t$. A Hausman test was used to test for whether the fixed or random effects model was the most appropriate technique for the panel data. The Hausman test used is a test for whether the errors are correlated within groups. In the fixed effects model, as discussed above (page 53), the constant term models differences between groups, and the random effects model includes a group specific error term. The null hypothesis of the test is that the random effects model is the correct specification. This hypothesis is rejected soundly (χ^2_4 = 629.12). Given this result, the fixed effects model (Model 2) is used for the rest of the econometric estimation.

Several other models were examined. Models that were looked at include: the use of the Box-Cox transformation that was generated, with $\varphi = -0.11$ (Model 3), the addition of a depth of the well variable (Model 4), the addition of a company dummy variable (Model 5), a model with dummy variables for the five different geological zones (Model 6), a model that includes dummy variables for the six pools (Model 7), an equation with the square of production added to allow more flexibility in the impact of production or cuts (Model 8). There were two companies in the data set[17]. The five different geological zones include Belly River as the reference value, zone 1 Bow Island, zone 2 Medicine Hat/Milk River, zone 3 Medicine Hat, and zone 4 Basal Colorado. The six different pools were identified by their code.[18]

All of the models seem to provide a reasonable good fit with pseudo R^2 [19] of between 0.73 and 0.80. Again, the first column of Table 7 includes the coefficients estimated by Chermak and Patrick. (They used ols whereas I use both random effects and fixed effects models.) Since the Hausman test rejected the random effects model (Model 1), only the fixed effects model results are discussed (Models 2-8).

It is interesting to note that the coefficients are reasonably consistent across models. This is especially true of the coefficient on remaining reserves, which ranges from -0.84 to -0.98, and is always significant at the 95 percent level. The coefficient on quantity produced ranged from 0.04 to 0.10 in Models 2 to 7 (i.e. all the models where the square of production is not included in the equation as in Model 8). A range of -0.02 to -0.04 is observed for the coefficient on the time trend. The age of the well had a

[17] More detail is omitted because of the proprietary nature of the information in this data set.
[18] Ibid. This code was established by the Alberta Energy and Utilities Board.

coefficient that ranged from 0.05 to 0.16. All coefficients are of the expected direction, costs are increasing in both quantity produced and age of the well, and decreasing in the remaining reserves. The time trend indicates that operating costs have a tendency to fall. This may be due to technological changes or falling input prices In some of the models, the coefficients on time, well age, and quantity do not hold their statistical significance at the 95 percent confidence level: however, all, but well age in Model 7 and Model 8, remain significant at the 90 percent confidence level. Compared to the results obtained by Chermak and Patrick, these results imply that the wells in this sample have larger elasticity of cost with respect to remaining reserves, and smaller elasticity of costs with respect to quantity produced. The constant is fairly large and significant. This could imply that there is a variable missing from the model, or that there exist significant quasi-fixed costs in the data.

Adding other variables to the model does not seem to alter the results much. The fit, as measured by the R^2, stays in a narrow range of 0.76 to 0.80. Moreover, as mentioned above, the coefficients on production, reserves, time, and age, do not change much with the addition of other variables.

Model 8 adds the square of production to the equation. As with the model for yearly data, the addition of this term makes interpretation difficult. Figure 4 shows a graph of this relationship. Similar to Figure 2, Figure 4 includes both the linear (equation 44) and the quadratic series (equation 44 with a squared quantity term) which are derived using the minimum values for reserves, well age and time. Both series Cost linear and Cost quadratic have two additional lines depicting the minimum value for reserves (Cost linear

[19] The fixed effects model does not provide a true R^2. The test statistic is not bounded by 0 and 1 as is the case with ols. It does however give a "similar indication of fit" in that a higher value indicates that more of

min and Cost quadratic min) and the maximum value for reserves (Cost linear max and Cost quadratic max). From Figure 4 it can be seen that the impact of adding the quadratic term for production is small. The quadratic equations track closely with the linear specifications.

Depth is found to be statistically significant (Model 4). and suggests. as expected, that operating costs increase with the depth of the well. The addition of depth has little influence on the coefficient on quantity, and decreases the coefficient on reserves slightly. Both time and age of the well lose their significance at the 95 percent level with the addition of depth. However. the fit of the model, as measured by the R^2. is not improved.

Model 5 includes a company dummy variable that is found to be significant. implying that one company has higher operating costs than the other. This might be explained by differences in accounting practices. differences in the properties that are being produced from. or possibly that one company is more efficient. The addition of this dummy variable has little impact on the coefficients for quantity, reserves. or well age. The coefficient on time again loses its significance with the addition of a company variable. Again the fit is not improved with the addition of a company dummy variable.

Dummy variables for the different zones (Model 6) did not prove to add much. Only one of the four variables was significant at the 95 percent level. and only the coefficient on reserves remained significant at this level. The results tend to suggests that operating costs are higher for the other zones than the Belly River zone. It is possible that this captures some of the same relationship that depth does. In this sample, the Belly River zone (the reference zone) is the shallowest zone, and the Bow Island zone (Zone 1)

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the variation is explained.

(which is the only zone dummy variable that is significant at the 95 percent level) is the deepest.

Dummy variables for the individual reservoirs were all significant at the 95 percent level (Model 7). This implies that different reservoir characteristics are important in determining operating costs at the well level. The addition of these dummy variables had the impact of reducing the confidence of quantity, time, and production below the 95 percent level, with depth falling below the 90 percent confidence level as well. There may be some correlation between the individual reservoirs and the quantity produced, specifically, some reservoirs may have characteristics that are more or less amiable to production.

The fixed effects regression includes an F test with the null hypothesis that the fixed effects are all equal to zero. The critical value for this test at the 99 percent level is roughly 1.9. All of the tests are rejected soundly at the 99 percent level, with test values ranging from 60.8 to 85.2. This implies that there exists some residual unexplained variance in well operating costs that arises because of individual well differences.

Model 3 involves using the Box-Cox transformation with $\varphi = -0.11$. (This is the value that was generated above (page 72).) This model has the highest R^2 value, suggesting that it has the "best fit." This suggests that the log-log model may be too restrictive. The signs and magnitudes of the coefficients are quite similar to those of Model 2, the log-log model. However, the coefficient on quantity drops below the 95 percent confidence level. This raises the question of how much influence quantity has on operating costs.

Figure 4
Natural Log of Cost versus Quantity (Monthly Data)

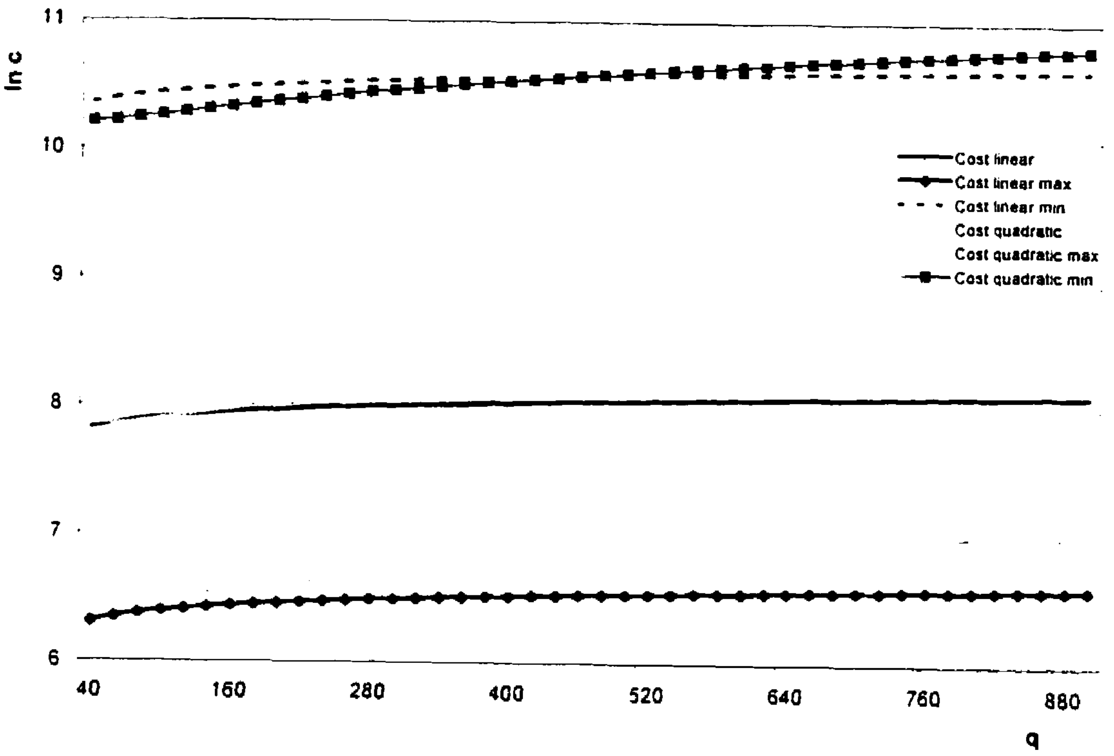

Table 8
Monthly Data Results

Variable	Patrick & Chermak	1	2	3	4	5	6	7	8
Number of Observations	451	610	610	610	610	610	610	610	610
Constant	8.68 (17.51)	**3.31 (8.75)**	**15.87 (15.95)**	**17.12 (19.78)**	**14.36 (14.34)**	**16.17 (16.43)**	**16.12 (16.56)**	**16.53 (17.09)**	**16.21 (16.34)**
ln q	0.48 (14.83)	**0.15 (3.22)**	**0.09 (2.32)**	0.04 (1.77)	**0.09 (2.31)**	**0.10 (2.54)**	0.07 (1.74)	0.07 (1.67)	**-0.44 (-2.66)**
ln R	-0.35 (-12.45)	**0.34 (7.02)**	**-0.90 (-8.86)**	**-0.89 (-10.94)**	**-0.96 (-9.67)**	**-0.93 (-9.33)**	**-0.91 (-9.25)**	**-0.98 (-9.81)**	**-0.84 (-8.16)**
ln t	-0.50 (-6.83)	**0.13 (7.33)**	**-0.04 (-2.31)**	**-0.02 (-2.30)**	-0.03 (-1.71)	-0.03 (-1.78)	-0.03 (-1.76)	-0.03 (-1.88)	**-0.04 (-2.21)**
ln pm	0.13 (6.78)	**0.28 (8.28)**	**0.11 (-2.67)**	**0.05 (2.72)**	0.06 (1.40)	**0.09 (2.09)**	0.07 (1.65)	0.05 (1.24)	**0.16 (3.65)**
ln depth					**0.34 (5.87)**				
ln q^2									**0.06 (3.30)**
ln R^2									
Company						**0.18 (4.16)**			
Zone 1							**0.35 (5.68)**		
Zone 2							0.08 (1.39)		
Zone 3							0.48 (0.55)		
Zone 4							0.09 (1.36)		
Pool 1								**0.61 (5.51)**	
Pool 2								**0.34 (2.97)**	
Pool 3								**0.25 (2.31)**	
Pool 4								**0.32 (3.44)**	
Pool 5								**0.30 (2.55)**	
F test well = 0			85.24	84.37	69.21	60.80	78.08	78.42	72.56
R^2 adjusted	0.42	0.73	0.79	0.80	0.76	0.76	0.77	0.78	0.76
χ^2 or F statistic		290.82	67.24	72.30	63.70	58.74	40.16	37.68	56.89

*t or z-statistics are in parenthesis at the 95% confidence level.
Boldface indicates coefficients that are significant at the 95% confidence level

5.4 Disaggregated Operating Costs

Table 9 presents the results from regressing some (9 of 17) of the individual cost categories against these same four variables using a fixed effects model. and monthly data. The sample size varies due to the fact that observations that contained zeros were dropped. Due to having a large number of zero observations. a number of the cost categories were not tested. The categories not tested were: salt water disposal. trucking. small tools and supplies. cathodic protection, freehold p&ng lease. pressure vessel assessment, testing-pressure survey, and analysis.

Table 9
Disaggregated Results

	Cost Category								
Variable	Contract Operator	Chemicals	Lease and Road	Repair and Maint	Municipal property tax	Production and Alberta Mineral Tax	AEUB Admin Fee	Freehold Surface Lease	Chart Reading
Number of Observations	456	307	382	333	522	429	433	430	456
Constant	**10.33**	**-13.79**	**-6.82**	**-25.03**	**-3.35**	**64.23**	-0.66	**10.20**	3.63
	(6.81)	**(-3.43)**	**(-2.92)**	**(-4.34)**	**(-2.31)**	**(15.18)**	(-0.38)	**(5.24)**	(0.85)
ln q	0.10	-0.13	0.06	**1.19**	-0.05	**0.80**	0.09	**-0.21**	0.17
	(1.73)	(-0.82)	(0.75)	**(6.26)**	(-0.93)	**(6.05)**	(1.12)	**(-3.91)**	(1.40)
ln R	**-0.64**	**2.23**	**1.32**	**2.28**	**0.63**	**-6.21**	0.19	-0.22	-0.23
	(-3.90)	**(5.34)**	**(5.43)**	**(3.86)**	**(4.19)**	**(-15.01)**	(1.03)	(-1.12)	(-0.54)
ln t	0.05	-0.04	**0.32**	-0.14	**0.06**	**-0.53**	-0.01	**0.13**	**0.22**
	(1.73)	(-0.57)	**(8.00)**	(-1.38)	**(2.41)**	**(-7.99)**	(-0.21)	**(4.49)**	**(2.70)**
ln m	-0.02	**-0.22**	-0.15	**0.68**	**0.99**	-0.07	**1.01**	-0.02	-0.14
	(-0.38)	**(-2.12)**	(-1.88)	**(3.90)**	**(19.44)**	(-0.45)	**(16.44)**	(-0.33)	(-1.26)
F test on wells	185.03	116.04	69.03	51.98	86.30	46.88	45.06	28.07	4.18
R^2 adj	0.36	0.01	0.51	0.06	0.09	0.54	0.04	0.00	0.05
F test	24.27	58.26	17.48	30.63	138.91	88.48	103.19	30.00	7.79

*t or z-statistics are in parenthesis at the 95% confidence level.
Boldface indicates coefficients that are significant at the 95% confidence level

The results are somewhat surprising and suggest an absence of consistent relationships across cost categories. From the table it can be seen that only Contract

Operator, Lease and Road, and Production and Alberta Mineral Tax are explained well with these variables. It is difficult to know what to make of the results with such little explanatory power. Only three of the categories, Repair and Maintenance, Production and Alberta Mineral Tax, and Freehold Surface Lease have coefficients for quantity that are significantly different from zero. Our initial division of costs into variable and quasi-fixed categories is not well supported by the disaggregated data. Only two of the "variable" costs had coefficients for quantity that were significantly different from zero. As mentioned above, these are repair and maintenance, and production and Alberta mineral tax. One of the "quasi-fixed" variables, freehold surface lease, had a coefficient for quantity that was different from zero; however, it was negative.

Reserves was significant for all but two of the cost categories (AEUB administration fee, and chart reading), however, its sign and magnitude varied from -6.21 to 2.28 for "variable" cost categories, and -0.21 to 1.32 for "quasi-fixed" cost categories. The time trend was significant in five of the nine equations with coefficients ranging from -0.53 to 0.32. The age of the well was significant in only four of the equations with values ranging from -0.22 to 1.01. An F-test showed that the individual well effects were non-zero across all equations. These results seem to suggest, that as the various cost components are disaggregated, the "random error" aspect becomes more significant. This is probably because very specific factors, related to each cost, become significant.

The categories were then aggregated up to variable cost and quasi-fixed cost variables as per our apriori classification. The results are presented as Table 10 (the results from Model 2, Table 8 are included in the first column as Total Operating Costs for comparison). The quasi-fixed costs are not well explained by these variables (R^2 of

0.01). The variable cost components show a good fit (R^2 of 0.62). The results when using the aggregated "variable" cost are similar in sign to those for the aggregated total costs. Quantity produced has a positive relationship to costs, remaining reserves has a negative relationship with costs, time has a negative relationship to costs, and the age of the well has a positive relationship with costs. In all cases the t-statistic is higher in the variable cost equation. Also, the size of the coefficients increase, with the elasticity of cost with respect to q much closer to that obtained by Chermak and Patrick (0.53 versus 0.48). However, the elasticity of cost with respect to reserves is very high.

This suggests that the definition of costs plays an important role in determining the relationship between costs and both quantity produced and remaining reserves. From the results, it appears that some costs are more readily explained by quantity and reserves, while others are not. This suggests that there may in fact be two components in operating costs as was postulated by dividing costs into quasi-fixed costs and variable costs.

Table 10
Aggregated Monthly Results

Variable	Total Operating Cost	Quasi-Fixed	Variable
Number of Observations	610	610	576
Constant	**15.87** **(15.95)**	**4.97** **(3.02)**	**52.71** **(13.10)**
ln q	**0.09** **(2.32)**	**-0.37** **(-5.71)**	**0.53** **(3.45)**
ln R	**-0.90** **(-8.86)**	**0.64** **(3.85)**	**-5.28** **(-12.80)**
ln t	**-0.04** **(-2.31)**	**0.23** **(7.43)**	**-0.71** **(-9.66)**
ln m	**0.11** **(-2.67)**	**-0.45** **(-6.51)**	**0.48** **(2.95)**
F test on wells	85.24	33.88	26.49
R^2 adj	0.79	0.01	0.62
F test	67.24	27.88	70.64

*t or z-statistics are in parenthesis at the 95% confidence level.
Boldface indicates coefficients that are significant at the 95% confidence level

Chapter Six: Conclusions

The primary focus of this work was an empirical estimate of operating cost for a natural gas well. The results suggest that operating costs are increasing in quantity produced, decreasing in the remaining reserves, and increasing with the age of the well. These results confirm both what depletable natural resource theory has generally assumed about operating costs, as well as the results obtained by both Livernois and Uhler, and Chermak and Patrick.

The results further found support for the notion that operating cost functions for natural gas should be modeled at the individual well level. Using a panel data series, the individual well effects were found to be non-zero across all models tested. This suggests that ignoring the impacts at the individual well by aggregating up to the pool, or the region may not model costs effectively. Important differences in reservoir characteristics, and how they effect costs, make aggregation difficult.

The issue of which costs are included in operating costs was also found to be of importance. Typically producers report operating costs that include both variable and quasi-fixed costs. As should be expected, production and reserves do not do a very good job of explaining the variation in operating costs for quasi-fixed costs. This thesis examined each of the components of the reported operating cost so as to eliminate those costs that were not variable. When this was done, the overall fit of the model improved, the magnitude of the coefficients increased, and the t-statistics improved.

Several suggestions for further work come out of this thesis, the most obvious of which is simply to extend the work into other regions to see how robust the results are in other locations. Another suggestion is to examine the endogeneity between reserves and production, perhaps using a variable for the actual stock of the resource instead of remaining reserves as an explanatory variable. A third area to explore would be the inclusion of some physical characteristic of the reservoir in the equation, for example, the use of permeability and porosity to help explain costs. A fourth extension would be to add an option value component that might help to explain the quasi-fixed costs. All of these might create more confidence around the estimated coefficients for quantity and remaining reserves.

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