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Internet channel entry: retail coverage and entry cost advantage

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Abstract In this research we study how existing market coverage affects the outcome of the Internet channel entry game between an existing retailer and a new entrant. A market is not covered when some consumers with low reservation prices are priced out by existing retailers and do not purchase. In a model with multiple existing retailers and a potential new entrant, we demonstrate that when entry costs are equal, one of the existing retailers enters the Internet channel first. However, if the market is covered by existing retailers before entry, then because of the threat of Internet channel entry by the potential new entrant, retailer entry cannibalizes existing retail profits—cannibalizing at a loss. In addition, if a potential new entrant has a slight advantage in Internet channel entry costs and the market is not covered by existing retailers, then the new entrant enters the Internet channel first. If the market is covered by existing retailers, then the new entrant must have a larger Internet channel entry cost advantage to be first to enter the Internet channel.

Keywords Retail pricing · B2C electronic commerce · Market entry · Stand-alone incentive · Preemption incentive · Timing game · Cost advantage

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1 Introduction

Direct marketing has long been an important business practice. According to a survey conducted by the Direct Marketing Association, direct marketing generated an estimated $2.17 trillion in 2003 sales—$1.17 trillion in consumer and $998.4 billion in the business-to-business market [5]. The traditional direct channel includes catalog mailing and TV advertising. Increasingly firms are using the Internet as a direct market channel to reach consumers. Prior to the advent of the Internet some of these firms already had an existing brick-and-mortar storefront (e.g., Barnes & Noble), while others did not (e.g., Amazon). A puzzling question is why in some industries existing firms with a well-known retail presence and facing significant retail and direct market competition were not first-movers into the Internet channel. In this work we study how the existing market coverage affects the outcome of the Internet channel entry game between an existing retailer and a new entrant, where the existing retail market coverage is whether every consumer purchases from one of the existing retailers (covered) or some consumers with low reservation prices are priced out and do not purchase (uncovered).

Our starting point is a retail market where there are several existing retailers (incumbents) in a location and price equilibrium.[1] Then the technology for an Internet channel becomes available to them and the new entrant. As the entry cost into the Internet channel, mostly technology related, declines over time the existing retailers and the new entrant strategically

[1] Thereafter we use existing retailer and incumbent interchangeably.

decide when to launch their Internet channel. We examine the effect of the existing retail market coverage on the outcome of the entry game—both on the order of entry and on the profitability of entry.

Intuitively we expect that in the uncovered market, both the incumbent and the new entrant would try to move first to grab the uncovered portion of the market and increase market share and profit, while, in the covered market, the incumbent might be reluctant to launch the Internet channel in fear of the possible channel conflict: conflict between the Internet channel and retail channel driving down prices and profits. However, we find that the results run somewhat counter to that intuition. We show that an entry cost advantage over the incumbent is necessary for the new entrant to enter the Internet channel first. In addition, the necessary entry cost advantage is smaller in the uncovered market, so an uncovered market does not favor incumbent entry. Finally, without a sufficient entry cost advantage for the entrant, in the covered market the incumbent enters the Internet channel first but loses money at the margin from entry because of the resulting channel conflict.

Thus, we find that the existing retail market coverage has a significant impact on the entry game, especially in view of the literature that mostly assumes a covered market. Markets that are uncovered have important implications for Internet channels both because of the potential of the Internet to reach anywhere, and the low fixed setup cost relative to incumbents that locate in high-cost shopping areas. An example of an uncovered market is the ethnic book market in the United States. Because the ethnic population is sufficiently scattered, demand in various geographical locations does not justify the opening of a physical bookstore, and in cases where there is a substantial local population that justifies a retail store, the transportation cost prohibits people further away from physically visiting these stores. Therefore, the ethnic book market in the United States is an uncovered market.

The rest of the paper is organized as follows. In the next section, we review the related literature. Then we present our basic model setup. In the subsequent two sections, we discuss separately the price setting game when the market is covered and when it is not, and compare the incentives for the existing retailer and the new entrant in both cases. Our analysis of the timing game, where the incumbent and the new entrant simultaneously decide a time to enter the Internet channel, follows from the comparison of the incentives. In the final section we conclude by discussing the implications and limitations.

2 Related literature and model formulation

The fundamental difference between the existing retailers, and the Internet channel from the consumers' standpoint is that for existing retailers, consumers have to physically travel to the store in order to do the shopping, while in the Internet channel no physical transportation cost is incurred by consumers. Rather, the cost of ''visiting'' a Internet store is mostly a fixed cost, consisting of the shipping cost, the delayed gratification, and so on. We use the Salop's [19] circle model to capture this distance-related retail differentiation. And we place the Internet channel in the center of the circle to capture its ''nowhere-everywhere'' presence, following Balasubramanian [2]. We employ the circular city model because it captures two features simultaneously: the influence of incumbents on each other, and the impact of the Internet channel on each of the incumbents. Assuming that consumers have perfect knowledge about the locations of the firms and the prices the firms charge, consumers decide which firm to buy from based on not only the relative prices, but also the relative distances to each firm.[2]

In Balasubramanian [2], the competition between direct marketers and existing retailers is modeled by distributing the retailers around a circle and putting a third-party direct marketer in the middle of the circle. Thus, each consumer is the same distance from the direct marketer. The presence of the direct marketer alters the market in such a way that with a profitable direct marketer, each retailer is forced to compete against the direct marketer rather than against a neighboring retailer. We borrow the basic modeling idea of placing the Internet channel in the middle of the circle. However, rather than assuming a covered market, we also consider the case of an uncovered market. And we allow the entry by both the new entrant and the existing retailer, while in Balasubramanian [2] only the new entrant can enter the Internet channel.

The previous literature on the timing of entry has mainly focused on the entry into new product markets. Lilien and Yoon [13] argue that the decision to enter the market should be timed to balance the risks of premature entry against the missed opportunity of late entry, and empirically test a set of propositions about the relationship between the market-entry time and

[2] In the spatial differentiation models, the distance between the consumer and the firm can be interpreted both as physical distance and as the degree of the lack of fit between the consumer's ideal product and the firm's actual product offer. In our setting, we take the first interpretation to highlight the importance of the physical presence of the existing retailers.

the likelihood of success for new industrial products. Using data from French firms, they find that firms are more successful when the new product is launched during the introduction or growth stage of the product life cycle. Mitchell [14] also empirically tests the relationship between entry time and the post-entry performance. His argument is that incumbents are likely to possess strong sets of assets required for the commercialization of goods in a new technical subfield, and as a consequence, the effects of being early or late vary with the type of entrant. Our paper, however, investigates the entry into a new marketing channel—the Internet channel. In the Internet channel incumbents' usual entry advantages are mitigated by expertise required to apply new technology—expertise they may not possess. Schoenecker and Cooper [20] study the role of firm resources and organizational attributes in determining entry timing and find that two categories of resources, technological and marketing, are associated with early entry. This is consistent with our results, as we show that an entry cost advantage, which can be the consequence of resources or capabilities, is necessary for the new entrant to enter successfully. In addition, our results are consistent with Nault and Vandenbosch [15, 16], who show that the incumbent may launch prematurely, and sometimes lose at the margin, in order to preempt the entrant, and the incumbent may be preempted by an entrant with a capabilities advantage in the next generation product.

Our paper is also related to the multi-channel literature. Zettelmeyer [23] discusses how, as the Internet expands, firms should price their products and whether they should facilitate consumer search in the retail channel and in the Internet channel. This matters because by varying the amount of product information provided, firms can achieve finer consumer segmentation and increase their market power. In a similar vein, Riggins [18] showed how the digital divide, where high type consumers dominate the Internet channel and low type consumers dominate the retail channel, artificially segments the marketplace thereby mitigating the classic cannibalization problem. Their focus is on the post-Internet-channel-entry decisions regarding to prices, communication strategies, or quality differentiation. Our paper, however, is on the transition from pre-entry to post-entry, focusing on the entry decision.[3]

3 Basic setup

We adopt the circular spatial competition model introduced by Salop [19]. Consumers are assumed to be distributed uniformly on the edge of a circle of unit circumference. Each consumer is in the market for either zero or one unit of a homogeneous good. Consumers have the same reservation price, denoted by R.

The pre-entry market is in equilibrium. We assume that existing retailers are located at equal distances from each other on the circumference, which implies that they are at the maximal distance from each other. This is consistent with the principle of maximal differentiation [22], and Kats [10] who shows that the equal distance is an equilibrium in the circle model. Equilibrium also means that there is no further retail entry. Balasubramanian [2] shows that even though the market is closed to further retail entry, it might still be open to direct market entry as long as the fixed setup cost is low compared to that of the retail entry. In our model we assume equilibrium in the pre-entry market, so the fixed setup cost of retail entry is not relevant, while the Internet channel entry cost is.

Our interest is in an Internet channel entry either by existing retailers or by a new entrant. For simplicity, we assume there are two incumbents (denoted by A and B) in the retail market, and a potential new entrant (denoted by E) not in the retail market. In order to focus our attention on the entry game between the incumbents and the new entrant, we assume that the entry cost for one of the incumbents (say A) is lower than that for the other. And we assume that the entry cost difference for the incumbents is sufficiently large so that we can ignore the other incumbent in our timing game later.[4] In a setting where there are more than two retailers, this is the same as picking the one with the lowest entry cost, or the largest capabilities advantage [16].

Consumers incur travel costs at a (linear) rate t per unit distance when visiting a retailer along the circle. Following Balasubramanian [2], all consumers who buy from the Internet channel incur a fixed shipping and disutility cost of μ. Examples of the disutility are delayed gratification and lack of an opportunity for physical inspection. To concentrate on the impact of market coverage, we assume μ is the same regardless of which firms are in the Internet channel, so the

[3] There are other streams of research that use the circle model for different purposes. For example, Bakos [1] studies the role of buyer search costs for differentiated products in an electronic marketplace, and Dewan et al. [8] study how the distribution of a special commodity—information goods—should be organized through proprietary networks and the Internet.

[4] The purpose of our paper is to investigate whether the incumbent and the new entrant enters the online channel. This assumption allows us to focus on the game between the incumbent and the new entrant and simplifies our analysis. Without this assumption we would have to consider the competition between the incumbents, which is not our main theme.

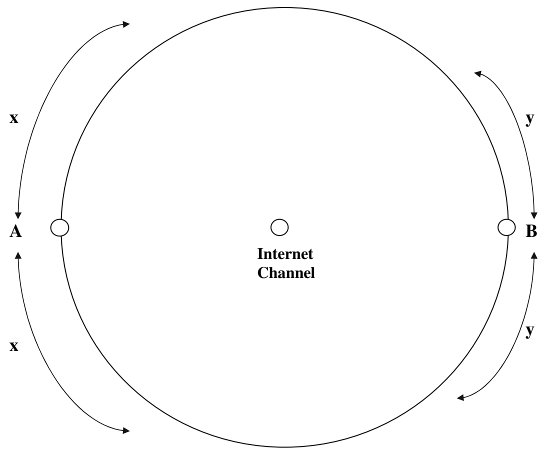

Fig. 1 The model

Internet channel is undifferentiated. Marginal cost of procurement and distribution of the good is equal for the incumbents and entrant, and is normalized to zero. We discuss implications of this in our Conclusion. We take t, μ, and R to be in $\mathcal{R}^+$.

The prices charged by the two incumbents A and B for the retail goods are p_a and p_b, respectively. The price charged for the good in the Internet channel is denoted by p_d. We allow price discrimination by a incumbent if it operates both in the retail channel and in the Internet channel. Profits are π_a and π_b for the two incumbents, and π_e for the entrant. Prior to Internet channel entry, the two retailers market share can be determined using Fig. 1 as $2x$ and $2y$ (x or y on each side of the firm) (Fig. 1).

We assume incumbents and the entrant have full information concerning each other's costs, prices and locations, as well as consumer's disutility of buying online, their distribution around the circle, their travel costs and reservation prices.

3.1 Price setting game

In making the entry decision, the incumbent and the new entrant consider their pre-entry profits, post-entry profits, and the entry cost. Our first step is to solve the price setting game and calculate the equilibrium profit in the pre-entry state where no firm is in the Internet channel (denoted as State 1 or $s1$), and in the following three possible post-entry states: incumbent A in the Internet channel (denoted as State 2 or $s2$), new entrant E in the Internet channel (denoted as State 3 or $s3$), and both incumbent A and new entrant E in the Internet channel (denoted as State 4 or $s4$). State 1 is Salop's [19] model, and when the retail market is covered State 3 is Balasubramanian's [19] model. We follow the convention of using superscripts for states so that, for example, profits of incumbent A in State 1 is denoted by π_a^{s1}. We solve the price setting game and calculate the equilibrium profits in Sect. 4 and 5.

3.2 Timing game

In order to study whether the incumbent or the new entrant enters the Internet channel first, we model the entry game as a timing game with declining entry cost. The timing game is a game in which players decide on the optimal time to enter. Internet channel entry cost is mostly technology-related cost driven by computing and telecommunication devices, and software development. We assume the cost of Internet channel entry is declining because of advances in technology and learning from other applications. Declining technology adoption cost assumptions are common in the literature (e.g., [7, 11, 15–17]). Let $K_i\,(T)$ be the present value of the entry cost at time T, so that the current cost is $K_i(T)e^{rT}$, $i = a, b, e$, where $r > 0$ is the interest rate. Declining entry cost means that $d(K_i(T)e^{rT})/dT < 0$. We also assume that entry cost falls at a decreasing rate, that is, $d^2(K_i(T)e^{rT})/dT^2 < 0$. To make the case interesting, we assume that initially entry is too costly so that no firm enters at time zero. We consider the case where the entry cost is the same for incumbent A and the new entrant (equal entry costs), and the case where the entry cost is not the same (unequal entry costs). The timing game is presented in Sect. 6.

3.3 Market coverage conditions

The market being covered means that the reservation price R is high relative to the transportation cost t. In our setup the market is covered when $R \geq t/2$ and is not covered when $R < t/2$. To see this, suppose the market is not covered. Using incumbent A, the market share of $2x$ can be derived from the limiting equation for the indifferent consumer, $p_a + tx = R$. Incumbent A maximizes profit,

$$\pi_a = 2p_a x = 2p_a\left(\frac{R - p_a}{t}\right),$$

by choosing p_a. First order condition yields $p_a = R/2$ and $x = R/2t$. For the market to be uncovered each incumbent's market share must be less than 1/2, therefore $x < 1/4$ meaning $R < t/2$. If $R \geq t/2$, then the market is covered.[5] We refer to the inequalities $R \geq t/2$ and $R < t/2$ as the *market coverage conditions*.

[5] See Lemma 1 for prices and profits in this case.

3.4 Market comparison condition

For the Internet channel to have a positive market share the reservation price must be greater than the disutility of purchasing from the Internet channel, $R > \mu$. If the market is not covered, then $R > \mu$ is sufficient for the Internet channel to have positive market share. Combining $R > \mu$ with the market coverage condition when the market is not covered gives $t/2 > R > \mu$, which simplifies to $\mu/t < 1/2$. To enable us to compare the covered market and the uncovered market, we restrict our analysis to $\mu/t < 1/2$, and refer to this as the *market comparison condition*. Essentially this means that the disutility of purchasing from the Internet channel is small relative to the travel cost to existing retailers.

3.5 Internet channel participation constraints

The reservation price R plays a central role in the price setting game. Ignoring the incumbents for the moment, for the Internet channel to have positive sales the Internet channel price plus the disutility cost is constrained to be no greater than the consumer's reservation price. We denote this constraint, $p_d + \mu \le R$, by $C1$. Reintroducing competition from the incumbents, we must also constrain the Internet channel to a non-negative market share. We denote this constraint, $x + y \le 1/2$, by $C2$. Although $C1$ and $C2$ are both related to Internet channel participation, the former is a price constraint and the latter is a market share constraint. They are both required as they bind under different conditions. The conditions and constraints are summarized in Table 1.

We first solve the price setting game in the covered market and uncovered market, find the equilibrium profits, and compare the incentives to enter the Internet channel for the incumbent and the new entrant. Then, we analyze the timing game where the incumbent and the new entrant decide on a time to enter the Internet channel.

Table 1 Conditions and constraints

Market coverage conditions	$R \ge t/2$ (covered market) $R < t/2$ (uncovered market)
Market comparison condition	$\mu/t < 1/2$
Internet channel Participation Constraint C1	$p_d + \mu \le R$
Internet channel Participation Constraint C2	$x + y \le 1/2$

4 Price setting game and comparison of incentives in the covered market

4.1 Equilibrium in the price setting game

When the retail market is covered the market coverage condition is $R \ge t/2$. In order to compare this case to the case where the retail market is not covered we add the market comparison condition, $\mu/t < 1/2$. In the following analysis these two conditions are implicitly imposed.

State 1: No Internet channel—Salop's [19] *Model*: State 1 provides the baseline case of strictly retail competition, and the equilibrium depends on the magnitude of the reservation price relative to the transportation cost.[6]

Incumbent A's profit maximization problem is to choose p_a^{s1}, and incumbent B's profit maximization problem is to choose p_b^{s1}:

$$\max_{p_a^{s1}} \pi_a^{s1} = \max_{p_a^{s1}} 2p_a^{s1} x$$

$$\max_{p_b^{s1}} \pi_b^{s1} = \max_{p_b^{s1}} 2p_b^{s1} y$$

$$\ni \quad p_a^{s1} + tx = p_b^{s1} + ty \le R, \quad x + y \le 1/2,$$
$$(p_a^{s1} + tx - R)(x + y - 1/2) = 0, \quad p_a^{s1}, p_b^{s1} \ge 0.$$

Lemma 1 *If there is no firm in the Internet channel, then Nash equilibrium profits are as follows*:

$$\text{If } t/2 \le R \le 3t/4, \text{ then } \pi_a^{s1} = \pi_b^{s1} = \frac{R - t/4}{2}.$$
$$\text{If } R > 3t/4, \text{ then } \pi_a^{s1} = \pi_b^{s1} = t/4.$$

Although $R \ge t/2$ assures the market is covered, $R = 3t/4$ is the break point for competition between the two incumbents. That is, when $R \le 3t/4$, the reservation price is sufficiently low that the incumbents no longer compete with each other. The equilibria when $R < t/2$, when $t/2 \le R \le 3t/4$, and when $R > 3t/4$, correspond to the monopoly equilibrium, the kinked equilibrium, and the competitive equilibrium, respectively, in Salop [19]. Our covered market combines the kinked equilibrium and the competitive equilibrium, and our uncovered market corresponds to the monopoly equilibrium.

State 2: Incumbent A alone in the Internet channel: Referring to Fig. 1, incumbent A's retail market share is $2x$, where x represents the location where the consumer is indifferent between purchasing from

[6] Unless stated below, our proofs are in an appendix that accompanies this manuscript.

incumbent A and from the Internet channel, i.e., $p_a^{s2} + tx = p_d^{s2} + \mu$. Similarly, incumbent B's retail market share is $2y$, where y can be derived from $p_b^{s2} + ty = p_d^{s2} + \mu$. Consequently, incumbent A's Internet channel share is $(1 - 2x - 2y)$.

Moreover, since incumbent A always has the option not to sell in the Internet channel, i.e., charge higher than the reservation price, the equilibrium would be the same as that in State 1 if incumbent A finds that opening an Internet channel is not profitable.

Incumbent A's profit maximization problem is to choose p_a^{s2} and p_d^{s2}, under constraints $C1$ and $C2$:

$$\max_{p_a^{s2}, p_d^{s2}} \pi_a^{s2} = \max_{p_a^{s2}, p_d^{s2}} \left\{ 2p_a^{s2}\left(\frac{p_d^{s2} - p_a^{s2} + \mu}{t}\right) + 2p_d^{s2}\left(\frac{1}{2} - \frac{p_d^{s2} - p_a^{s2} + \mu}{t} - \frac{p_d^{s2} - p_b^{s2} + \mu}{t}\right)\right\} \quad (1)$$

$$\ni \quad p_d^{s2} + \mu \le R \ (C1), \quad x + y \le 1/2 \ (C2), \quad p_a^{s2}, p_d^{s2} \ge 0.$$

Incumbent B's profit maximization problem is to choose p_b^{s2}:

$$\max_{p_b^{s2}} \pi_b^{s2} = \max_{p_b^{s2}} \left\{ 2p_b^{s2}\left(\frac{p_d^{s2} - p_b^{s2} + \mu}{t}\right)\right\} \quad \ni p_b^{s2} \ge 0. \quad (2)$$

And incumbent A always has the option to charge $p_d^{s2} + \mu > R$ and not sell in the Internet channel if the maximizing profit is less than that in State 1. In this case, the profit maximization problem is the same as that in State 1.

Lemma 2 describes the incumbents' equilibrium profits in State 2.

Lemma 2 *If only incumbent A is in the Internet channel, then Nash equilibrium profits are as follows:*

If $\frac{\mu}{t} < \frac{1}{4}$ *and* $\frac{t}{2} \le R < \frac{52\mu^2 - 32\mu t + 25t^2}{36t}$, *or*
$\frac{1}{4} \le \frac{\mu}{t} < \frac{14 - 3\sqrt{3}}{26}$ *and* $\frac{t + 2\mu}{3} \le R < \frac{52\mu^2 - 32\mu t + 25t^2}{36t}$,
then $\pi_a^{s2} = \frac{13\mu^2 - 8\mu t + 4t^2}{18t}$ *and* $\pi_b^{s2} = \frac{(2\mu + t)^2}{18t}$.

If $\frac{1}{4} \le \frac{\mu}{t} < \frac{14 - 3\sqrt{3}}{26}$ *and* $\frac{t}{2} \le R < \frac{t + 2\mu}{3}$, *or*
$\frac{14 - 3\sqrt{3}}{26} \le \frac{\mu}{t} < \frac{1}{2}$ *and* $\frac{t}{2} \le R < \frac{(\sqrt{3} + 1)t - 2(\sqrt{3} - 1)\mu}{4}$,
then $\pi_a^{s2} = \frac{\mu^2 + 2\mu R - 2R^2}{2t} + R - \mu$ *and* $\pi_b^{s2} = \frac{R^2}{2t}$.

If $\frac{\mu}{t} < \frac{1}{4}$ *and* $\frac{52\mu^2 - 32\mu t + 25t^2}{36t} \le R < \frac{3t}{4}$, *or*
$\frac{1}{4} \le \frac{\mu}{t} < \frac{14 - 3\sqrt{3}}{26}$ *and* $\frac{52\mu^2 - 32\mu t + 25t^2}{36t} \le R < \frac{3t}{4}$, *or* $\frac{14 - 3\sqrt{3}}{26} \le \frac{\mu}{t} < \frac{1}{2}$ *and*
$\frac{(\sqrt{3} + 1)t - 2(\sqrt{3} - 1)\mu}{4} \le R < \frac{3t}{4}$,
then $\pi_a^{s1} = \pi_b^{s1} = \frac{R - t/4}{2}$.
If $R > 3t/4$, *then* $\pi_a^{s1} = \pi_b^{s1} = t/4$.

The first equilibrium is an interior solution to (1) and (2). In the second equilibrium $C1$ is binding, which means that incumbent A charges a Internet channel price as high as the consumer reservation price less the disutility of buying online. In the third and fourth equilibria incumbent A finds selling in the Internet channel not profitable, and charges $p_d^{s2} + \mu > R$. In this case the equilibrium is the same as that in State 1. From Lemma 2, we can see that in some cases when the market is covered, because of channel conflict, incumbent A chooses not to sell in the Internet channel.

State 3: New entrant E alone in the Internet channel—Balasubramanian's [2] *model*: In State 3, price competition is between a new entrant in the Internet channel and the two incumbents in the retail market. When the market is covered this is the same analysis as in Balasubramanian [2], resulting in Fig. 1 where the top and bottom segments of the circle represent the entrant's Internet channel share. As we will see later on, Balasubramanian's [2] results are sensitive to the assumption of market coverage.

Lemma 3 *If only the entrant is in the Internet channel, then Nash equilibrium profits are* $\pi_a^{s3} = \pi_b^{s3} = \frac{(t+4\mu)^2}{72t}$ *and* $\pi_e^{s3} = \frac{(t-2\mu)^2}{9t}$.

The new entrant draws some consumers away from the incumbents. As a result, the incumbents' profit is less than the profit before entry. Our formulation of the entrant's profit maximization problem differs from Balasubramanian [2] because we explicitly include $C1$ and $C2$. However the results are the same because an interior solution obtains.

State 4: Both incumbent A and new entrant E in the Internet channel: With both A and E in the Internet channel, because the Internet channel is undifferentiated, Bertrand competition causes the Internet channel price p_d^{s4} to equal the marginal cost.[7] The profit for the new entrant is zero. For the incumbents, $C1$ is automatically satisfied by the market comparison condition. With the Internet channel price set, the incumbents choose p_a^{s4} and p_b^{s4} to maximize their profits. Again using Fig. 1 to illustrate, x is given by $p_a^{s4} + tx = 0 + \mu$, and y is given by $p_b^{s4} + ty = 0 + \mu$. Each incumbent maximizes its profit subject to $C2$, where using A as the example,

$$\max_{p_a^{s4}} \pi_a^{s4} = \max_{p_a^{s4}} \left\{ 2p_a^{s4} \left(\frac{\mu - p_a^{s4}}{t} \right) + 0 \right\} \\ \ni \quad x + y \le 1/2\ (C2), \quad p_a^{s4} \ge 0. \tag{3}$$

Lemma 4 describes equilibrium profits in State 4.

Lemma 4 *If both the incumbent and the new entrant are in the Internet channel, then Bertrand competition causes the Internet channel price to equal marginal cost. Nash equilibrium profits are* $\pi_a^{s4} = \pi_b^{s4} = \mu^2/2t$ *and* $\pi_e^{s4} = 0$.

Competition in the Internet channel not only drives the Internet channel price to marginal cost, but because the Internet channel is a substitute for the retail market, this competition also lowers the retail price. That is, profits are strictly less in State 4 than other states because of the additional competition in the Internet channel. We state this in the following corollary.

Corollary 1 *Incumbent profits in State 1, 2 and 3 dominate incumbent profits in State 4.*

4.2 Incentives for Internet channel entry

Having worked out the profits for the incumbents and the new entrant in each of the states, we are ready to compare their incentives for Internet channel entry. Both stand-alone incentives and preemption incentives are relevant in the timing game which we discuss in Sect. 6. Our definition of stand-alone incentives and preemption incentives follows from Katz and Shapiro [11].

[7] In Bertrand competition, firms compete in prices rather than quantities. The alternative is the Cournot model where firms choose quantities first and the market price is set at a level such that demand equals the total quantity. The Cournot model is a better model if output is difficult to adjust (e.g., hotel rooms) [22]. However, in the online retail industry, output can be easily changed, and Bertrand model is a better model. In Bertrand competition, prices above marginal cost can only be achieved through product differentiation on dimensions like brand, website design and customer service.

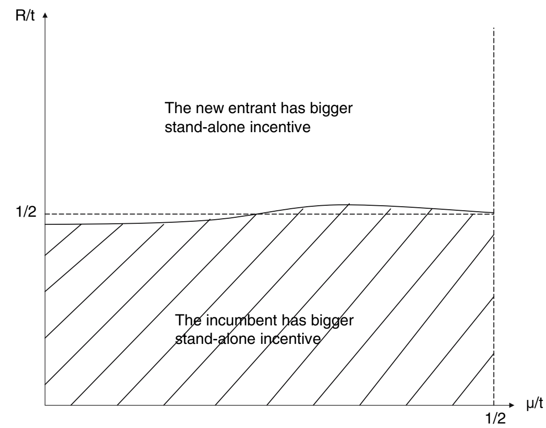

Fig. 2 Comparison of stand-alone incentives. The area below $R/t = 1/2$ is when the market is uncovered. The area above $R/t = 1/2$ is when the market is covered

A firm's *stand-alone incentive* is the difference between its post-entry profit and its profit when no entry has occurred (baseline profit). If a firm believes that its rivals will not enter, the incumbent or the new entrant bases its timing of entry on its stand-alone incentive. However, if a firm believes that its rival will enter the Internet channel if it does not, then it compares its profit as the "winner" in the entry game and its profit as the "loser" where it is preempted by a rival. This difference is called its *preemption incentive*. For the incumbent, its stand-alone incentive is $\pi_a^{s2} - \pi_a^{s1}$, and its preemption incentive is $\pi_a^{s2} - \pi_a^{s3}$. For the new entrant, because it has nothing to lose being preempted, the stand-alone incentive and preemption incentive are the same: $\pi_e^{s3} - 0 = \pi_e^{s3}$. We compare their stand-alone incentives and preemption incentives in the following theorems which we use in our analysis of the timing game. The parameter ranges specified in Theorem 1 only represent a very small proportion of the possible range. See Fig. 2 for an illustration.

Theorem 1 (Comparison of stand-alone incentives) *The new entrant has greater stand-alone incentives, that is,* $(\pi_a^{s2} - \pi_a^{s1}) < \pi_e^{s3}$, *except for the following parameter ranges:*

$$\frac{1}{\sqrt{20}} \le \frac{\mu}{t} \le \frac{6 - \sqrt{11}}{10} \text{ and } \frac{t}{2} \le R \le \frac{20\mu^2 + 17t^2}{36t},$$

$$\text{or } \frac{6 - \sqrt{11}}{10} \le \frac{\mu}{t} < \frac{1}{2} \text{ and } \frac{t}{2} \le R \le \frac{3(t + 2\mu) + \sqrt{11}(t - 2\mu)}{12}.$$

Theorem 2 (Comparison of preemption incentives) *The incumbent has greater preemption incentives, that is,* $(\pi_a^{s2} - \pi_a^{s3}) > \pi_e^{s3}$.

5 Price setting game and comparison of incentives in the uncovered market

5.1 Equilibrium in the price setting game

The retail market is not covered when $R < t/2$. This market coverage condition combined with a reservation price higher than the disutility of purchasing from the Internet channel, $R > \mu$, yields the market comparison condition, $\mu/t < 1/2$. Any entry into the Internet channel leads to a covered market because the cost of buying from the Internet channel, μ, is the same for all the consumers so that if one consumer finds it affordable to buy then all consumers do.

State 1: No Internet channel—Salop's [19] *model*: As before, State 1 provides the baseline case of strictly retail competition. Lemma 5 is proven as part of Lemma 1.

Lemma 5 *If there is no firm in the Internet channel (State 1), then Nash equilibrium profits are* $\pi_a^{s1} = \pi_b^{s1} = R^2/2t$.

When the market is not covered, the Salop [19] model has incumbents pricing based on the consumers' travel costs rather than on competition.

State 2: Incumbent A alone in the Internet channel: When only incumbent A is in the Internet channel, incumbent A chooses p_a^{s2} and p_d^{s2} and its profit maximization is as in (1). Incumbent B chooses p_b^{s2} to maximize its profit as in (2). As in the case when the market is covered, incumbent A always has the option not to sell in the Internet channel, i.e., charge higher than the reservation price. If incumbent A finds that opening an Internet channel is not profitable, then the profit maximization problem would be the same as that in State 1.

Lemma 6 and its proof are similar to the proof of Lemma 2, except that now the market coverage condition is $R < t/2$ and that affects the ranges over which the different equilibrium prices hold.

Lemma 6 *If only incumbent A is in the Internet channel, then Nash equilibrium profits are as follows*

$$\textit{If } \frac{\mu}{t} < \frac{1}{4} \textit{ and } \frac{t+2\mu}{3} < R < \frac{t}{2}, \textit{ then } \pi_a^{s2} = \frac{1}{18t}(13\mu^2 - 8\mu t + 4t^2),\ \pi_b^{s2} = \frac{1}{18t}(2\mu + t)^2.$$

$$\textit{If } \frac{\mu}{t} < \frac{1}{4} \textit{ and } R \le \frac{t+2\mu}{3}, \textit{ or } \frac{1}{4} \le \frac{\mu}{t} < \frac{1}{2} \textit{ and } R < \frac{t}{2}, \textit{ then } \pi_a^{s2} = \frac{1}{2t}(\mu^2 + 2\mu R - 2R^2) + R - \mu,\ \pi_b^{s2} = \frac{R^2}{2t}.$$

Unlike when the market is covered, when the market is not covered the incumbent always chooses to sell in the Internet channel, i.e., charge $p_d^{s2} \le R - \mu$. The intuition is that because the market is not covered prior to the Internet entry, serving the uncovered market niche is always profitable.

In addition, in some of the parameter range, constraint $C1$ is binding (the second equilibrium in Lemma 6) where incumbent A charges $R - \mu$ in Internet channel and just serves the consumers that were not reached before it entered Internet channel, and does not compete with incumbent B for market share. In this case, incumbent B's profit remains the same as before incumbent A enters the Internet channel (i.e., $\pi_b^{s2} = \pi_b^{s1}$). This is consistent with the notion that firms always try to avoid competition. Only when the disutility for the consumers of buying from the Internet channel is small compared to the transportation cost (i.e., $\mu/t < 1/4$), and the reservation price is high (i.e., $R > (t + 2\mu)/3$), will incumbent A find it profitable to use the Internet channel to compete with incumbent B, because the Internet channel is more likely to attract consumers under these conditions. In this case, $C1$ no longer binds and incumbent B earns less profit than it does before incumbent A enters the Internet channel (i.e., $\pi_b^{s2} < \pi_b^{s1}$). When the market is not covered, $C2$ never binds because the Internet channel captures the uncovered market and gets a positive market share.

State 3: New entrant E alone in the Internet channel: In this case, when a new entrant is in the Internet channel and the market is not covered, Balasubramanian's [2] results no longer hold. The entrant chooses p_d^{s3} to maximize its profit yielding the following profit maximization problem:

$$\max_{p_d^{s3}} \pi_e^{s3} = \max_{p_d^{s3}} \left\{ 2p_d^{s3} \left(\frac{1}{2} - \frac{p_d^{s3} - p_a^{s3} + \mu}{t} - \frac{p_d^{s3} - p_b^{s3} + \mu}{t} \right) \right\} \tag{4}$$

$$\ni \quad p_d^{s3} + \mu \le R\ (C1),\quad x + y \le 1/2\ (C2),\quad p_d^{s3} \ge 0.$$

Incumbent A's profit maximization problem choosing p_a^{s3} is

$$\max_{p_a^{s3}} \pi_a = \max_{p_a^{s3}} \left\{ 2p_a^{s3} \left(\frac{p_d^{s3} - p_a^{s3} + \mu}{t} \right) \right\} \quad \ni p_a \ge 0. \tag{5}$$

Incumbent B's profit maximization problem choosing p_b^{s3} is

$$\max_{p_b^{s3}} \pi_b = \max_{p_b^{s3}} \left\{ 2p_b^{s3} \left(\frac{p_d^{s3} - p_b^{s3} + \mu}{t} \right) \right\} \quad \ni p_b \ge 0. \tag{6}$$

The resulting equilibrium profits are given in the lemma below.

Lemma 7 *If only the entrant is in the Internet channel, then Nash equilibrium profits are as follows:*

$$\text{If } \frac{\mu}{t} < \frac{1}{2} \text{ and } \frac{t+4\mu}{6} < R < \frac{t}{2},$$
$$\text{then } \pi_a^{s3} = \pi_b^{s3} = \frac{(t+4\mu)^2}{72t} \text{ and } \pi_e^{s3} = \frac{(t-2\mu)^2}{9t}.$$

$$\text{If } \frac{\mu}{t} < \frac{1}{2} \text{ and } R \le \frac{t+4\mu}{6},$$
$$\text{then } \pi_a^{s3} = \pi_b^{s3} = \frac{R^2}{2t} \text{ and } \pi_e^{s3} = \frac{(R-\mu)(t-2R)}{t}.$$

When the reservation price is small (i.e., $R<(t+4\mu)/6$), $C1$ binds, which means that the new entrant only serves consumers who were not reached before and does not compete with the incumbents. This is because, with low reservation price, the market left uncovered by the incumbent is already enough for the new entrant in the Internet channel to maximize its profit. Even though competing with the incumbents would enlarge the new entrant's market share, it would also drive the Internet channel price down. With these two competing forces, the Internet channel profit is less when the reservation price is low. However, when the reservation price is high (i.e., $R > (t+4\mu)/6$), the Internet channel finds it more profitable to compete with the incumbents, and $C1$ does not bind.

State 4: Both incumbent A and new entrant E in the Internet channel: With both the incumbent and the new entrant in the Internet channel, as before Bertrand price competition causes the Internet channel price to equal marginal cost, $p_d^{s4} = 0$. Within this constraint, incumbents A and B choose p_a^{s4} and p_b^{s4} to maximize their profit. Lemma 8 describes incumbents' profits in State 4, and the proof is then same as for Lemma 4.

Lemma 8 *If both the incumbent and the new entrant in the Internet channel (State 4), then Bertrand competition causes the Internet channel price to equal marginal cost. Nash equilibrium profits are* $\pi_a^{s4} = \pi_b^{s4} = \mu^2/2t$ *and* $\pi_e^{s4} = 0$.

Comparing the profits of both incumbents in State 4 versus other states, we have a similar corollary when the market is not covered as when the market is covered. And as with Corollary 1, from Corollary 2, State 4 is dominated by other states.

Corollary 2 *Incumbent profits in State 1, 2 and 3 dominate incumbent profits in State 4.*

5.2 Incentives for Internet channel entry

We compare the stand-alone incentives and preemption incentives for both the incumbents and the new entrant when the market is not covered. As in Theorem 1, the parameter range specified in Theorem 3 only represents a small portion of the possible range. See Fig. 2 for an illustration.

Theorem 3 (Comparison of stand-alone incentives) *The incumbent has greater stand-alone incentive, that is,* $(\pi_a^{s2} - \pi_a^{s1}) > \pi_e^{s3}$, *except for the following parameter range:*

$$0 < \frac{\mu}{t} \le 1/\sqrt{20} \text{ and } \frac{\sqrt{(5\mu^2+2t^2)}}{3} \le R < t/2.$$

Theorem 4 (Comparison of preemption incentives) *The incumbent has greater preemption incentive, that is,* $(\pi_a^{s2} - \pi_a^{s3}) > \pi_e^{s3}$.

Before we move on to the next section, we briefly compare the relative incentive structure for Internet channel entry when the market is covered and when it is not. The incumbent always has stronger preemption incentive, regardless of whether the market is covered because, as the incumbent, the incumbent suffers more than the new entrant if it is preempted as it faces new competition. However, the stand-alone incentives differ depending on market coverage. When the market is covered, the incumbent has smaller stand-alone incentives except for the small parameter ranges specified in Theorem 1. This is because when the market is covered the incumbents compete with each other. Opening a profitable Internet channel can only intensify competition, driving down prices and the profits. When the market is not covered by incumbents, the incumbents have greater stand-alone incentives, except for the small parameter ranges specified in Theorem 3. The intuition is that the incumbents do not compete with each other when the market is not covered, and there is an unserved market niche. By opening an Internet channel, the incumbent can attract the unserved consumers while leaving the pre-entry monopolistic situation intact. Table 2 shows the relative incentive structure, ignoring the small parameter ranges specified in Theorem 1 and Theorem 3.

With this relative incentive structure, we next turn to our analysis of the timing game.

Table 2 Who has bigger incentives?

	Uncovered market	Covered market
Stand-alone incentive	The incumbent	The new entrant
Preemption incentive	The incumbent	The incumbent

6 Timing game

Consider when incumbent A enters the Internet channel first. Incumbent A's (leader) payoff if it enters the Internet channel first at time T is:

$$L_a(T) = \int_0^T \pi_a^{s1} e^{-rt} dt + \int_T^\infty \pi_a^{s2} e^{-rt} dt - K_a(T) = \frac{1 - e^{-rT}}{r} \pi_a^{s1} + \frac{e^{-rT}}{r} \pi_a^{s2} - K_a(T).$$

The new entrant's (follower) payoff if incumbent A enters the Internet channel first at time T is $F_e(T) = 0$.[8]

Consider when the new entrant enters the Internet channel first. The new entrant's payoff if it enters the Internet channel first at time T is:

$$L_e(T) = \frac{e^{-rT}}{r} \pi_e^{s3} - K_e(T).$$

Incumbent A's payoff is:

$$F_a(T) = \frac{1 - e^{-rT}}{r} \pi_a^{s1} + \frac{e^{-rT}}{r} \pi_a^{s3}.$$

Consider now when both incumbent A and the new entrant enter the Internet channel simultaneously. Incumbent A's payoff if both enter at time T is:

$$M_a(T) = \frac{1 - e^{-rT}}{r} \pi_a^{s1} + \frac{e^{-rT}}{r} \pi_a^{s4} - K_a(T).$$

The new entrant's payoff is $M_e(T) = -K_e(T)$. From Corollary 1 and 2, it can be easily shown that $L_i(T) > M_i(T)$ and $F_i(T) > M_i(T)$, $i = a, e$. Therefore, we drop simultaneous entry from further analysis as neither firm would choose it in equilibrium.

Define $\tilde{T}_i$ by $L_i(T) = F_i(T)$, $i = a, e$. For the incumbent and the entrant $\tilde{T}_i$ defines the time when the payoff of leading is equivalent to the payoff of following.

6.1 Equal entry costs

We first consider the case where entry cost is the same for incumbent A and the new entrant. From the comparison of the preemption incentive, it can be easily shown that $\tilde{T}_a$ occurs prior to $\tilde{T}_e$: the preemption incentive is greater for the incumbent and entry costs are the same. Consequently, from the definition of $\tilde{T}_i$ we get that $K_a(\tilde{T}_a)e^{r\tilde{T}_a} = \frac{1}{r}(\pi_a^{s2} - \pi_a^{s3})$ and $K_e(\tilde{T}_e)e^{r\tilde{T}_e} > \frac{1}{r}\pi_e^{s3}$. Using Theorems 2 and 4, we know that $K_a(\tilde{T}_a) e^{r\tilde{T}_a} > K_e(\tilde{T}_e)e^{r\tilde{T}_e}$, which implies that $\tilde{T}_a < \tilde{T}_e$ by the assumption of declining entry cost.

Define $\hat{T}_a$ as $\hat{T}_a = argmax\ L_a(T)$. This is the time when entry is most profitable. If there is no entry threat from the entrant, the incumbent will enter at this time. However, as shown in the following theorem, the incumbent enters prematurely in order to preempt the entrant. The proof follows from Theorem 1 from Nault and Vandenbosch [15].

Theorem 5 *If $K_a(T) = K_e(T)$, then in both the covered market and the uncovered market, the incumbent enters the Internet channel at time $\tilde{T}_e$, and the new entrant never enters.*

The equilibrium in the competition between incumbent A and the entrant is isomorphic to the incumbent and entrant in Nault and Vandenbosch's [15] Theorem 1. Nault and Vandenbosch's [15] Assumption 1 says that the incumbent always has greater preemption incentive than the entrant. In our setting, the incumbent has greater preemption incentive than the new entrant both when the market is covered and when it is not. See Fig. 3 for an illustration. The intuition is that ideally, the incumbent would like to enter when it is most profitable: at $\hat{T}_a$. But the new entrant makes positive profit from entry any time from $\tilde{T}_e$ onward, and therefore the incumbent must preempt the new entrant at time $\tilde{T}_e$. Even though the new entrant never enters, it still plays a role in the timing game: it alters the time when the incumbent enters the Internet channel.

In addition, if the entry cost is the same for the incumbent and the new entrant, whether the market is covered determines if the incumbent makes a profit at the margin from entry. When the market is covered the

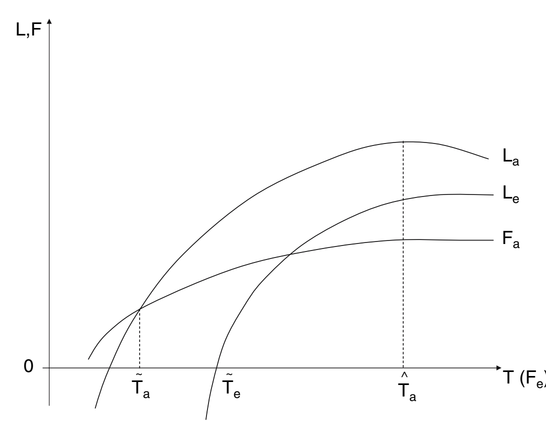

Fig. 3 *L*, *F* functions for incumbent A and the entrant

[8] To be consistent with the literature (e.g., [7, 15]), we use the notation of "leader" and "follower". It is worth noting that in our case follower never "follows", as shown in Theorem 5.

incumbent makes a negative profit at the margin from entry in the covered market, but not in the uncovered market. The following theorem states this result.

Theorem 6 *Assume that $K_a(T) = K_e(T)$. If the market is covered, then the incumbent makes negative profit at the margin from entering the Internet channel except for the parameter range stated in Theorem 1. If the market is not covered, then the incumbent makes a positive profit at the margin from entering the Internet channel, except for the parameter range stated in Theorem 3.*

Proof The incumbent's incremental profit from entering at $\tilde{T}_e$ is

$$\frac{e^{-r\tilde{T}_e}}{r}(\pi_a^{s2} - \pi_a^{s1}) - K_a(\tilde{T}_e).$$

$\tilde{T}_e$ is defined by $L_e(T) = F_e(T)$, which is $(e^{-r\tilde{T}_e}/r)\pi_e^{s3} = K_e(\tilde{T}_e)$. With $K_a(\tilde{T}_e) = K_e(\tilde{T}_e)$, and substituting $K_e(\tilde{T}_e)$ into the incremental profit for the incumbent, we get:

$$\frac{e^{-r\tilde{T}_e}}{r}[(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}].$$

The term in the square bracket is the comparison of the stand-alone incentives of the incumbent and the new entrant. Therefore the comparison of the stand-alone incentives determines the sign of the incremental profit for the incumbent. In the covered market, the incumbent has smaller stand-alone incentive than the new entrant except for the parameter range stated in Theorem 1, so it makes a negative profit at the margin. In the uncovered market, the incumbent has greater stand-alone incentive than the new entrant except for the parameter range stated in Theorem 3, so it makes a positive profit at the margin. □

Whether the profit margin for the incumbent is negative or positive is determined by the relative stand-alone incentive. Making negative profit at the margin is known as "cannibalizing at a loss" in Nault and Vandenbosch [15]. Cannibalizing at a loss happens in the covered market, but not in the uncovered market (except for the parameter range stated in Theorem 3) because in the covered market the incumbents already compete with each other and entry into the Internet channel only intensifies the competition. However, the rationale for the incumbent to cannibalize at a loss is strategic: if the incumbent waits, then it would incur even greater profit loss should the new entrant be first to enter.

6.2 Unequal entry costs

If the new entrant has a lower entry cost, it is possible that it may enter first. The reason is as follows. For the new entrant to enter requires that at some T, $L_e(T) - F_e(T) > L_a(T) - F_a(T)$, from which we get $K_a(T) - K_e(T) > \frac{e^{-rT}}{r}[(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3}]$. Since the incumbent always has greater preemption incentive, that is possible only if $K_e(T) < K_a(T)$. Therefore, a necessary condition for the new entrant to enter first is that the new entrant has a lower entry cost than the incumbents. This result is similar to Corollary 1 in Nault and Vandenbosch [15].

Lower entry cost is defined as "capabilities advantage" by Nault and Vandenbosch [16], a capabilities advantage that results from "a variety of resources, processes, and situations". In the case of Internet channel entry, a capabilities advantage might be the result of more effective web development technology, better consumer database management, or more flexible organizational structure. A perfect example here is the Amazon's technological leadership over Barnes & Noble. Since its founding, Amazon has carried strong reputation for its leading-edge business intelligence, analytics, and database operations. The company has made huge effort in building out and integrating the technology that ran its Web site, customer service unit, payment processing systems, and warehouse operations [12]. Well-known examples of the technological innovations made by Amazon include its one-click buying, which it had patented, Web site personalization, and online recommendations. This technological leadership is critical for Amazon's successful entry into the Internet retailing business.

The following theorem describes the condition under which the new entrant enters and the timing of entry.

Theorem 7 *If the difference in the entry costs satisfies $K_a(T) - K_e(T) > \frac{e^{-rT}}{r}[(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3}]$, then the new entrant enters the Internet channel at time $\tilde{T}_a$, and the incumbent never enters.*

Proof It is easy to show that $\tilde{T}_e < \tilde{T}_a$ under the condition stated in the theorem. For the rest of the proof, see Theorem 1 from Nault and Vandenbosch [15]. □

The condition in Theorem 7 says that the minimum entry cost advantage required for the new entrant to enter is proportional to the difference in the preemption incentives—an entry cost advantage is needed for the new entrant to overcome the incumbent's stronger preemption incentive. Figure 4 shows the curve of the minimum entry cost advantage when

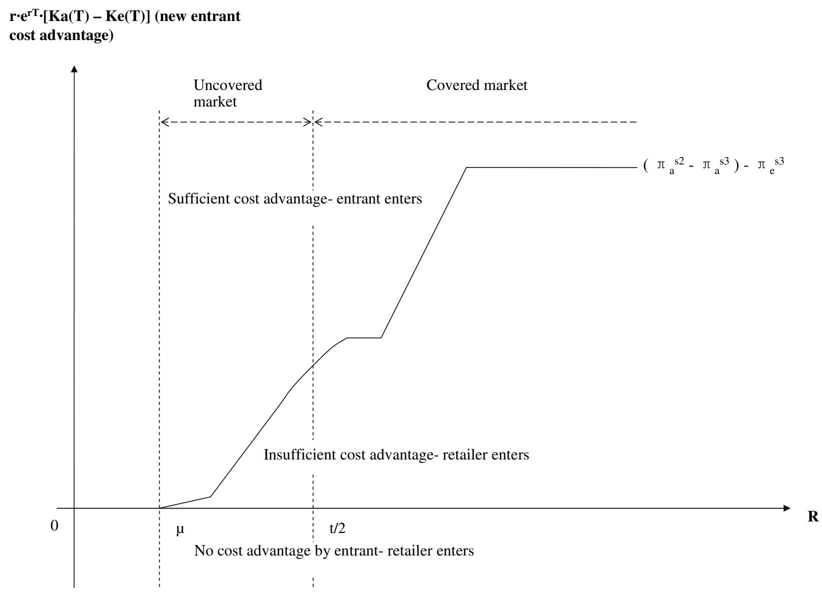

Fig. 4 Minimum entry cost advantages for the new entrant ($1/4 \leq \mu/t < 2/5$)

$\mu/t \in [1/4, \frac{14-3\sqrt{3}}{26})$.[9] We can see that when the new entrant cost advantage is above the curve (i.e., sufficient cost advantage), the new entrant enters, as stated in Theorem 7. When the new entrant cost advantage is below the curve (i.e., insufficient cost advantage), the incumbent enters.

From the non-decreasing pattern of the curve, it is clear that the cost advantage is required to be smaller in the uncovered market than in the covered market.

Corollary 3 *The necessary entry cost advantage for Internet channel entry by the new entrant is smaller in the uncovered market than in the covered market.*

Again we see the impact of market coverage. This result essentially says that it is easier for the new entrant to enter the Internet channel when the market is not covered. It might seem counterintuitive, as one might think that easier entry would occur in the covered market since the entrant could take advantage of the incumbent's reluctance to enter the Internet channel resulted from the negative profit margin. But this is not the case. In the covered market, the entrant makes larger profit compared to the uncovered market, because of larger consumer reservation price R. However, the incumbent is able to make even larger profit compared to the uncovered market. Therefore, the needed entry cost advantage is larger in the covered market to overcome the even stronger preemption incentive of the incumbent.

This result helps to explain the success of Amazon. It is estimated that the number of book titles available at Amazon.com is more than 23 times larger than the number of books on the shelves of a typical Barnes & Noble superstore [3]. By offering obscure book titles (e.g., ethnic books) that are not available at physical bookstores, that is, serving the uncovered market, Amazon.com was successful at preempting Barnes & Noble's entry into the online book retail industry. A contrasting case is the failure of Webvan, which was an online grocery store. The grocery market in general is a covered market, with physical grocery stores offering wide variety of selections in close proximity to most people's homes. When the market is covered, it takes larger entry cost advantage for the new entrant to enter profitably. Unless Webvan possessed this cost advantage over existing retailers, such as Safeway, Webvan was doomed to fail.

7 Conclusion

In this research, we study the Internet channel entry game between an existing brick-and-mortar retailer and a new entrant and analyze how the existing market coverage affects the outcome of the entry game. We use Salop's [19] circle model to capture the distance-related differentiation of existing retailers. We place the Internet channel in the center of the circle to capture the "nowhere-everywhere" presence [2].

We first compare the stand-alone and preemption incentives of the existing retailers and the new entrant for the Internet channel entry and find that market coverage is an important factor determining the relative incentives. In particular, when the market is covered, the new entrant has stronger stand-alone incentives except for the small parameter ranges

[9] The curve has similar shape when μ/t is in other ranges.

specified in Theorem 1. When the market is not covered, the existing retailer has stronger stand-alone incentives except for the small parameter ranges specified in Theorem 3. On the other hand, in both covered and uncovered markets, the existing retailer has stronger preemption incentive because, as the incumbent, it has more to lose if it is preempted.

With this relative incentive structure, we then analyzed the timing game where the existing retailer with the lowest entry cost and the new entrant strategically decide when to launch the Internet channel as the entry cost—mostly technology-related—declines over time. We find that, if the entry costs are equal, then the existing retailer enters the Internet channel first. However, when the market is covered, the incumbent makes a negative profit at the margin from entry. For the new entrant to enter the Internet channel, an entry cost advantage is necessary, and the needed entry cost advantage for the entrant smaller when the market is not covered.

Our results have the following implications. First, when an existing retailer chooses when to launch an Internet channel, it must consider market coverage and the possibility of entry from a new entrant. The existing retailer must be prepared to preempt as the market coverage determines whether entry results in a marginal profit loss. Even though entry might entail a loss at the margin in the covered market, it is justifiable in order to preempt the new entrant and avoid even greater profit loss. When there is an unfilled market niche, each of the incumbents would like to enter the Internet channel to grab the unfilled market niche. The firm with the lowest entry cost—the greatest capabilities advantage—enters first and enjoys the first-mover advantage. This demonstrates that striving to develop the largest capabilities advantage in order to be the first mover among the incumbents is crucial to reap the profit gain derived from the increased market share.

Second, from the standpoint of the new entrant, since the entry cost advantage needed to overcome the existing retailer's stronger preemption incentive is smaller in the uncovered market than the covered market, it is important for the new entrant to "choose the right spot" based on its capabilities advantage relative to the existing retailers. The new entrant should focus more on situations where there is uncovered market niche, as less advantage is required in this kind of market.

Third, our research sheds light on the relationship between the Internet channel and the retail channel. The Internet channel may compete against or complement the firm's own retail outlets, depending on market coverage. When the market is covered, to gain a positive share, the Internet channel has to compete against incumbents for consumers, which would cause profit loss for incumbents. The incumbents may choose not sell in the Internet channel in order to avoid the channel conflict. In the uncovered market, it is possible for the Internet channel to attract the un-served consumers while leaving the pre-entry monopolistic situation intact. Social welfare is increased, both because of higher profit, and more consumers being reached.

We realize that our model is highly stylized. First, we assume undifferentiated competition in the Internet channel, which results in Bertrand competition with more than one firm in the Internet channel. In actuality Internet channel members can differentiate themselves along a variety of dimensions, for example, brand, website design, and suggestion tools.[10] Under product differentiation, there might be a second entry into the online channel. However, in the Internet world, there is always first-mover advantage in building up the customer base. Therefore, who enters first is the most important question. Second, we assume that the firms maximize profit at each time period. If, however, we assume sticky demand (for example, through brand loyalty), the firms can actually maximize market share during the early periods to "lock in" customers and reap greater profit from them later on. If the demand stickiness is carried over to the Internet channel, the incumbent may maximize market share to cover the uncovered market and make the Internet channel less attractive to the new entrant. Demand stickiness and in particular, how the existing retailer should take advantage of the already-established brand to preempt the new entrant are interesting avenues for further research. Related to stickiness are network effects whereby a retailer could be more attractive the more customers it serves. This would provide a greater incentive for earlier entry in order to build up the customer base.

Finally, we assume that the capabilities advantage is linked to lower fixed entry cost, rather than lower marginal cost. Lower marginal cost would always encourage the new entrant to enter because even if the incumbent is in the Internet channel, the new entrant can always enter, undercut the price and drive the incumbent away. However, in the Internet retailing industry we believe a lower marginal cost for the new entrant is unlikely. Marginal cost consists mostly of procurement and distribution cost. If there is a marginal cost advantage, it is most likely owned by the

[10] For a review on price and product differentiation in the Internet channel, see Smith et al. [21].

incumbent because of the ability to leverage pre-existing assets into economies of scale. And this advantage is likely to erode away as the new entrant "gets big fast" [8]. Regarding fixed entry costs, incumbents could spread their fixed costs over the brick and mortar channel and Internet channel, thus reducing fixed entry costs that are common to the two channels. In this case, the capabilities advantage possessed by the entrant would have to be sufficient to overcome the incumbents' advantage from entry costs already spent to enter the brick and mortar channel.

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Appendix of mathematical proofs

Proof of Lemma 1 Referring to Salop's [19] classical circular model with two firms (e.g., [22]), the equilibrium prices are $p_a^{s1} = p_b^{s1} = t/2$. In order for all the consumers on the circle to be served—that is, for the market to be covered—in this symmetric model the most distant consumer from a retailer is $x = 1/4$ and the equilibrium prices have to satisfy $p_a^{s1} + t/4 < R$ and $p_b^{s1} + t/4 < R$, which requires that $R > 3t/4$.

When $R \le 3t/4$, the reservation price is sufficiently low that the retailers no longer compete with each other. In this case, taking retailer A as an example, x is given by $p_a^{s1} + tx = R$. Retailer A's maximization problem is

$$\max_{p_a^{s1}} \pi_a^{s1} = \max_{p_a^{s1}} \left\{ 2p_a^{s1} \left(\frac{R - p_a^{s1}}{t} \right) \right\}$$
$$\ni \; p_a^{s1} + t/4 \ge R\,, \; p_a^{s1} \ge 0.$$

Solving this constrained maximization problem, we get that, if $R \le t/2$, $x < 1/4$ (which means the market is not covered), $p_a^{s1} = p_b^{s1} = R/2$, and $\pi_a^{s1} = \pi_b^{s1} = R^2/2t$. If $t/2 \le R \le 3t/4$, $x = 1/4$ (the constraint is binding), $p_a^{s1} = p_b^{s1} = R - t/4$, and the profits are stated in Lemma 1. □

Proof of Lemma 2 Using (1), we can write the Lagrangian function for retailer A as

$$L = 2p_a^{s2}\left(\frac{p_d^{s2} - p_a^{s2} + \mu}{t}\right) + 2p_d^{s2}\left(\frac{1}{2} - \frac{p_d^{s2} - p_a^{s2} + \mu}{t} - \frac{p_d^{s2} - p_b^{s2} + \mu}{t}\right) - \lambda_1(p_d^{s2} + \mu - R) - \lambda_2\left(\frac{p_d^{s2} - p_a^{s2} + \mu}{t} + \frac{p_d^{s2} - p_b^{s2} + \mu}{t} - \frac{1}{2}\right).$$

The resulting Kuhn-Tucker conditions for retailer A are

$$\frac{\partial L}{\partial p_a^{s2}} = \frac{2}{t}(2p_d^{s2} - 2p_a^{s2} + \mu) + \frac{\lambda_2}{t} \le 0, \tag{7}$$

$$\frac{\partial L}{\partial p_d^{s2}} = \frac{2}{t}(2p_a^{s2} + p_b^{s2} - 4p_d^{s2} - 2\mu) + 1 - \lambda_1 - \frac{2\lambda_2}{t} \le 0, \tag{8}$$

$$\frac{\partial L}{\partial \lambda_1} = -p_d^{s2} - \mu + R \ge 0, \tag{9}$$

$$\frac{\partial L}{\partial \lambda_2} = -x - y + 1/2 \ge 0, \tag{10}$$

$$p_a^{s2} \ge 0, \;\; p_d^{s2} \ge 0, \;\; \lambda_1 \ge 0, \;\; \lambda_2 \ge 0,$$

$$p_a^{s2}\frac{\partial L}{\partial p_a^{s2}} = 0, \;\; p_d^{s2}\frac{\partial L}{\partial p_d^{s2}} = 0, \;\; \lambda_1\frac{\partial L}{\partial \lambda_1} = 0, \;\; \lambda_2\frac{\partial L}{\partial \lambda_2} = 0.$$

Using (2) the Kuhn-Tucker conditions for retailer B are

$$\frac{\partial \pi_b^{s2}}{\partial p_b^{s2}} = \frac{2}{t}(p_d^{s2} - 2p_b^{s2} + \mu) \le 0, \tag{11}$$

$$p_b^{s2} \ge 0, \;\; p_b^{s2}\frac{\partial \pi_b^{s2}}{\partial p_b^{s2}} = 0.$$

Excluding the non-negativity constraints, there are the two possible binding constraints for retailer A's problem, giving rise to four cases which we discuss in turn.

Interior solution. In this case the shadow prices are zero, $\lambda_1 = \lambda_2 = 0$. Assume the three prices are strictly positive so that (7), (8), and (11) hold with equality. From these equations we get the candidate Nash equilibrium prices:

$$p_a^{s2} = \frac{2t + \mu}{6}, \;\; p_d^{s2} = \frac{t - \mu}{3}, \;\; p_b^{s2} = \frac{t + 2\mu}{6}.$$

From the market comparison condition, $\mu/t < 1/2$, the candidate Nash equilibrium prices are strictly positive. Using the strict inequalities from our constraints, $p_d^{s2} + \mu < R$ and $x + y < 1/2$, and our market coverage condition $R \geq t/2$, we derive the additional conditions on μ/t and R: $\frac{\mu}{t}$ and $R \geq \frac{t}{2}$, or $\frac{1}{4} \leq \frac{\mu}{t} < \frac{2}{5}$ and $R > \frac{t+2\mu}{3}$. And the candidate Nash equilibrium profits are $\pi_a^{s2} = \frac{13\mu^2 - 8\mu t + 4t^2}{18t}$ and $\pi_b^{s2} = \frac{(2\mu+t)^2}{18t}$.

*Constraint C*1 *is binding*. In this case the shadow price of $C2$ is zero, $\lambda_2 = 0$. Assume the three prices are strictly positive so that (7), (8), (11) and (9) hold with equality, where the latter equation implies $p_d^{s2} = R - \mu$. From these equations we get the remaining candidate Nash equilibrium prices and the shadow price for $C1$:

$$p_a^{s2} = R - \mu/2, \;\; p_b^{s2} = R/2, \;\; \lambda_1 = \frac{2\mu - 3R + t}{t}.$$

From the market comparison condition, $\mu/t < 1/2$, and the market coverage condition, $R \geq t/2$, the candidate Nash equilibrium prices are strictly positive. Using the strict inequality from $C2$, $x + y < 1/2$, our market coverage condition, and the non-negative shadow price for $C1$, $\lambda_1 \geq 0$, we derive the additional conditions on μ/t and R: $\frac{1}{4} \leq \frac{\mu}{t} < \frac{2}{5}$ and $\frac{t}{2} \leq R \leq \frac{t+2\mu}{3}$, or $\frac{2}{5} \leq \frac{\mu}{t} < \frac{1}{2}$ and $\frac{t}{2} \leq R < t - \mu$. And the candidate Nash equilibrium profits are $\pi_a^{s2} = \frac{\mu^2 + 2\mu R - 2R^2}{2t} + R - \mu$ and $\pi_b^{s2} = \frac{R^2}{2t}$.

*Constraint C*2 *is binding*. In this case the shadow price of $C1$ is zero, $\lambda_1 = 0$. Assume the three prices are strictly positive so that (7), (8), (9) and (10) hold with equality. From these equations we get candidate Nash equilibrium prices and the shadow price for $C2$:

$$p_a^{s2} = \frac{2t}{5}, \;\; p_d^{s2} = \frac{3t - 5\mu}{5}, \;\; p_b^{s2} = \frac{3t}{10}, \;\; \lambda_2 = \frac{2(5\mu - 2t)}{5}.$$

From the market comparison condition, $\mu/t < 1/2$, all candidate Nash equilibrium prices are strictly positive. Using the strict inequality from $C1$, $p_d^{s2} + \mu < R$, our market coverage condition $R \geq t/2$, and the non-negative shadow price for $C2$, $\lambda_2 \geq 0$, we derive the additional conditions on μ/t and R: $\frac{2}{5} \leq \frac{\mu}{t} < \frac{1}{2}$ and $R > \frac{3t}{5}$. And the candidate Nash equilibrium profits are $\pi_a^{s2} = \frac{4t}{25}$ and $\pi_b^{s2} = \frac{9t}{50}$.

*Constraints C*1 *and C*2 *are binding*. Assume the three prices are strictly positive so that (7), (8), (11), (9) and (10) hold with equality. From (9) we get $p_d^{s2} = R - \mu$. From the remaining equations we get other candidate Nash equilibrium prices and the shadow prices for $C1$ and $C2$:

$$p_a^{s2} = \frac{3R - t}{2}, \;\; p_b^{s2} = \frac{R}{2}, \;\; \lambda_1 = \frac{3t - 5R}{t}, \;\; \lambda_2 = 2(\mu + R - t).$$

From our market coverage condition $R \geq t/2$ all prices are strictly positive. Using this condition, together with the two non-negative shadow prices, λ_1, $\lambda_2 \geq 0$, we derive the additional conditions on μ/t and R: $\frac{2}{5} \leq \frac{\mu}{t} < \frac{1}{2}$ and $t - \mu \leq R \leq \frac{3t}{5}$. And the candidate Nash equilibrium profits are $\pi_a^{s2} = \frac{(t-R)(3R-t)}{2t}$ and $\pi_b^{s2} = \frac{R^2}{2t}$.

Our second step is to verify that the Kuhn-Tucker conditions are also the sufficient condition for the maximization problem. It is easy to verify that both the objective functions are concave (the Hessian matrix of the objective function of retailer A is negative definite, and the second derivative of the objective function of retailer B is negative), and the constraints are linear and therefore concave. Thus, what we derive above is the optimal solution to the constrained profit maximization problem.

Next since retailer A always has the option to charge higher than the reservation price and not sell in the Internet channel, we need to verify when profits for retailer A are greater in State 2 than in State 1. By comparing the profits of retailer A in these two states, we derive the conditions under which retailer A chooses to sell in the Internet channel and under which he chooses not to sell in the Internet channel. The conditions are given in the lemma.

Lastly, in order to prove that the optimal solution to the constrained optimization problem is indeed the unique Nash equilibrium, following Friedman [6, p. 152], Gruca and Sudharshan [9], four conditions have to satisfy: (G1) the number of players is finite, (G2) the strategy space of every player is compact and convex, (G3) the payoff functions are continuous and bounded, and (G4) the payoff functions are quasi-concave. It is easy to verify that in our case, the number of players is finite (G1), the constraints insure that the strategy space of every player is compact and convex (G2), the payoff functions are continuous and bounded (G3), and the payoff functions are concave and thus quasi-concave (G4). □

Proof of Lemma 4 The Lagrangian function for (3) is:

$$L = 2p_a^{s4}\left(\frac{\mu - p_a^{s4}}{t}\right) - \lambda\left(\frac{\mu - p_a^{s4}}{t} + \frac{\mu - p_b^{s4}}{t} - \frac{1}{2}\right)$$

The resulting Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial p_a^{s4}} = \frac{2}{t}(\mu - 2p_a^{s4}) + \frac{\lambda}{t} \leq 0,$$

$$\frac{\partial L}{\partial \lambda} = -x - y + 1/2 \geq 0,$$

$$p_a^{s4} \geq 0, \;\; \lambda \geq 0,$$

$$p_a^{s4}\frac{\partial L}{\partial p_a^{s4}} = 0, \;\; \lambda\frac{\partial L}{\partial \lambda} = 0.$$

Excluding the non-negativity constraints, there is the one possible binding constraint for retailer A's problem, giving rise to two cases which we discuss in turn.

Interior solution. In this case the shadow prices are zero, $\lambda = 0$. Following the same analysis as Lemma 2 and the assumption of identical firms, we get candidate Nash equilibrium prices: $p_a^{s4} = p_b^{s4} = \mu/2$. Using the strict inequality from the constraint, $x + y < 1/2$, we get the condition for the candidate Nash equilibrium prices to exist: $\mu/t < 1/2$, which is our market comparison condition.

Constraint C2 is binding. From $x + y = 1/2$, we get $p_a^{s4} = \mu - t/4$. The first Kuhn-Tucker condition holds with equality, since p_a^{s4} is positive because of the market comparison condition. Plugging p_a^{s4} into the first condition, we get $\lambda = 2(\mu - t/2)$. From $\lambda \geq 0$, we get $\mu/t \geq 1/2$, which is contradictory to our market comparison condition. Therefore, in State 4, $C2$ is never binding.

It is easy to verify that both the objective functions are concave (the second derivative is negative), and the constraints are linear and therefore concave. Thus, what we derive above is the optimal solution to the constrained profit maximization problem. Similar as in the proof of Lemma 2, it is easy to verify that in our case, the number of players is finite (G1), the constraints insure that the strategy space of every player is compact and convex (G2), the payoff functions are continuous and bounded (G3), and the payoff functions are concave and thus quasi-concave (G4). Therefore, the optimal solution to the constrained maximization problem is indeed the unique Nash equilibrium. □

Proof of Corollary 1 We first prove the dominance of State 4 by State 1, then by State 2 and State 3.

Dominance of State 4 by State 1: From Lemma 4, $\pi_a^{s4} = \pi_b^{s4} < t/8$ from the market comparison constraint $\mu/t < 1/2$. From Lemma 1, if $R > 3t/4$, then $\pi_a^{s1} = \pi_b^{s1} > \pi_a^{s4} = \pi_b^{s4}$. If $t/2 \leq R \leq 3t/4$, then from the market coverage constraint, $R \geq t/2$, $\pi_a^{s1} = \pi_b^{s1} \geq t/8 > \pi_a^{s4} = \pi_b^{s4}$.

Dominance of State 4 by State 2: Since the profit of retailer A in State 2 is at least as large as that in State 1 (Lemma 2), $\pi_a^{s2} > \pi_a^{s1} > \pi_a^{s4}$. It is also easy to verify that $\pi_b^{s2} > \pi_b^{s4}$.

Dominance of State 4 by State 3: From Lemma 3 and Lemma 4, $\pi_a^{s3} - \pi_a^{s4} = \pi_b^{s3} - \pi_b^{s4} = \frac{(t+4\mu)^2}{72t} - \frac{\mu^2}{2t} = \frac{(t+10\mu)(t-2\mu)}{72t} > 0$. □

Proof of Theorem 1 In order to compare the stand-alone incentives and preemption incentives, we summarize the equilibrium profits under different parameter ranges in State 1 through 3 in Table 3. In this proof, row numbers refer to Table 3.

Row 1: If $\mu/t \in (0, 1/4)$ and $R \in [\frac{t}{2}, \frac{52\mu^2 - 32\mu t + 25t^2}{36t})$,

$$(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} = \frac{(13\mu^2 - 8\mu t + 4t^2)}{18t} - \frac{(R - \frac{t}{4})}{2} - \frac{(t - 2\mu)^2}{9t}$$
$$= \frac{1}{2}\left(\frac{20\mu^2 + 17t^2}{36t} - R\right).$$

Since R has to be in the range of $[\frac{t}{2}, \frac{52\mu^2 - 32\mu t + 25t^2}{36t})$, if $\mu/t \geq 1/\sqrt{20}$, then $t/2 \leq (20\mu^2 + 17t^2)/36t < 3t/4$, so if $R \leq (20\mu^2 + 17t^2)/36t$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} \geq 0$, and if $R > (20\mu^2 + 17t^2)/36t$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} < 0$. If $\mu/t < 1/\sqrt{20}$, then $(20\mu^2 + 17t^2)/36t < t/2 \leq R \leq 3t/4$, and $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} < 0$.

Table 3 Equilibrium profits when the market is covered

μ/t	R	State 1—No direct market	State 2—Retailer A in direct market	State 3—New entrant in direct market
$(0, \frac{1}{4})$	$[\frac{t}{2}, \frac{52\mu^2-32\mu t+25t^2}{36t})$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \frac{13\mu^2-8\mu t+4t^2}{18t},\ \pi_b = \frac{(2\mu+t)^2}{18t}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
	$[\frac{52\mu^2-32\mu t+25t^2}{36t}, \frac{3t}{4})$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
	$(\frac{3t}{4}, \infty)$	$\pi_a = \pi_b = \frac{t}{4}$	$\pi_a = \pi_b = \frac{t}{4}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
$[\frac{1}{4}, \frac{14-3\sqrt{3}}{26})$	$[\frac{t}{2}, \frac{t+2\mu}{3})$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \frac{\mu^2+2\mu R-2R^2}{2t} + R - \mu,\ \pi_b = \frac{R^2}{2t}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
	$[\frac{t+2\mu}{3}, \frac{52\mu^2-32\mu t+25t^2}{36t})$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \frac{13\mu^2-8\mu t+4t^2}{18t},\ \pi_b = \frac{(2\mu+t)^2}{18t}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
	$[\frac{52\mu^2-32\mu t+25t^2}{36t}, \frac{3t}{4})$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
	$[\frac{3t}{4}, \infty)$	$\pi_a = \pi_b = \frac{t}{4}$	$\pi_a = \pi_b = \frac{t}{4}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
$[\frac{14-3\sqrt{3}}{26}, \frac{1}{2})$	$[\frac{t}{2}, \frac{(\sqrt{3}+1)t-2(\sqrt{3}-1)\mu}{4})$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \frac{\mu^2+2\mu R-2R^2}{2t} + R - \mu,\ \pi_b = \frac{R^2}{2t}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
	$[\frac{(\sqrt{3}+1)t-2(\sqrt{3}-1)\mu}{4}, \frac{3t}{4})$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \pi_b = \frac{R-\frac{t}{4}}{2}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$
	$(\frac{3t}{4}, \infty)$	$\pi_a = \pi_b = \frac{t}{4}$	$\pi_a = \pi_b = \frac{t}{4}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t},\ \pi_e = \frac{(t-2\mu)^2}{9t}$

Row 2: If $\mu/t \in (0, 1/4)$ and $R \in [\frac{52\mu^2-32\mu t+25t^2}{36t}, \frac{3t}{4})$,

$(\pi_a^{s2} - \pi_a^{s1}) = 0 < \pi_e^{s3}$.

Row 3: If $\mu/t \in (0, 1/4)$ and $R \in (\frac{3t}{4}, \infty)$,

$(\pi_a^{s2} - \pi_a^{s1}) = 0 < \pi_e^{s3}$.

Row 4: If $\mu/t \in [\frac{1}{4}, \frac{14-3\sqrt{3}}{26})$ and $R \in [t/2, (t+2\mu)/3]$,

$$\begin{aligned}&(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}\\ &= \frac{1}{2t}(\mu^2 + 2\mu R - 2R^2) + R - \mu - \frac{(R - \frac{t}{4})}{2} - \frac{(t-2\mu)^2}{9t}\\ &= \frac{1}{72t}(-72R^2 + 36tR + 72\mu R + 4\mu^2 - 40\mu t + t^2)\\ &= \frac{1}{t}\left(-\left(R - \frac{1}{4}(t+2\mu)\right)^2 + \frac{11}{144}(t-2\mu)^2\right)\\ &= \frac{1}{t}\left(R - \frac{3(t+2\mu) - \sqrt{11}(t-2\mu)}{12}\right)\\ &\quad \times \left(\frac{3(t+2\mu) + \sqrt{11}(t-2\mu)}{12} - R\right).\end{aligned}$$

Note that $(3(t+2\mu) - \sqrt{11}(t-2\mu))/12 < t/2 \le R$. Since $R \in [t/2, (t+2\mu)/3]$, if $\mu/t \ge (6-\sqrt{11})/10$, then $t/2 < (3(t+2\mu) + \sqrt{11}(t-2\mu))/12 \le (t+2\mu)/3$, so if $R > (3(t+2\mu) + \sqrt{11}(t-2\mu))/12$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} < 0$, and if $R \le (3(t+2\mu) + \sqrt{11}(t-2\mu))/12$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} \ge 0$. If $\mu/t < (6-\sqrt{11})/10$, then $(3(t+2\mu) + \sqrt{11}(t-2\mu))/12 > (t+2\mu)/3 \ge R$, and $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} > 0$.

Row 5: If $\mu/t \in \left[\frac{1}{4}, \frac{14-3\sqrt{3}}{26}\right)$ and $R \in \left[\frac{t+2\mu}{3}, \frac{52\mu^2 - 32\mu t + 25t^2}{36t}\right)$,

$$\begin{aligned}(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} &= \frac{(13\mu^2 - 8\mu t + 4t^2)}{18t} - \frac{(R - \frac{t}{4})}{2} - \frac{(t-2\mu)^2}{9t}\\ &= \frac{1}{2}\left(\frac{20\mu^2 + 17t^2}{36t} - R\right).\end{aligned}$$

Since $R \in [\frac{t+2\mu}{3}, \frac{52\mu^2-32\mu t+25t^2}{36t})$, if $\mu/t \le (6-\sqrt{11})/10$, then $(t+2\mu)/3 \le (20\mu^2 + 17t^2)/36t < \frac{52\mu^2-32\mu t+25t^2}{36t}$, so if $R \le (20\mu^2 + 17t^2)/36t$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} \ge 0$, and if $R > (20\mu^2 + 17t^2)/36t$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} < 0$. If $\mu/t > (6-\sqrt{11})/10$, then $(20\mu^2 + 17t^2)/36t < (t+2\mu)/3 < R$, and $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} < 0$.

Row 6: If $\mu/t \in [1/4, \frac{14-3\sqrt{3}}{26})$ and $R \in [\frac{52\mu^2-32\mu t+25t^2}{36t}, 3t/4)$, the analysis is the same as that of Row 2.

Row 7: If $\mu/t \in [1/4, \frac{14-3\sqrt{3}}{26})$ and $R \in (3t/4, \infty)$, the analysis is the same as that of Row 3.

Row 8: If $\mu/t \in [\frac{14-3\sqrt{3}}{26}, 1/2)$ and $R \in [t/2, \frac{(\sqrt{3}+1)t - 2(\sqrt{3}-1)\mu}{4})$,

$$\begin{aligned}(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} &= \frac{1}{2t}(\mu^2 + 2\mu R - 2R^2)\\ &\quad + R - \mu - \frac{(R - \frac{t}{4})}{2} - \frac{(t-2\mu)^2}{9t}\\ &= \frac{1}{t}\left(R - \frac{3(t+2\mu) - \sqrt{11}(t-2\mu)}{12}\right)\\ &\quad \left(\frac{3(t+2\mu) + \sqrt{11}(t-2\mu)}{12} - R\right).\end{aligned}$$

Note that $(3(t+2\mu) - \sqrt{11}(t-2\mu))/12 < t/2 \le R$, and $t/2 < (3(t+2\mu) + \sqrt{11}(t-2\mu))/12 < \frac{(\sqrt{3}+1)t-2(\sqrt{3}-1)\mu}{4}$. So if $R \le (3(t+2\mu) + \sqrt{11}(t-2\mu))/12$, then $(\pi_a^{s2} - \pi_a^{s1}) \ge \pi_e^{s3}$. Otherwise, $(\pi_a^{s2} - \pi_a^{s1}) < \pi_e^{s3}$.

Row 9: If $\mu/t \in [\frac{14-3\sqrt{3}}{26}, 1/2)$ and $R \in [\frac{(\sqrt{3}+1)t-2(\sqrt{3}-1)\mu}{4}, 3t/4)$, the analysis is the same as that of Row 2.

Row 10: If $\mu/t \in [\frac{14-3\sqrt{3}}{26}, 1/2)$ and $R \in (3t/4, \infty)$, the analysis is the same as that of Row 3.

In sum, $(\pi_a^{s2} - \pi_a^{s1}) < \pi_e^{s3}$, except for the following parameter combinations:

(i) $1/\sqrt{20} \le \mu/t < 1/4$ and $t/2 \le R \le \frac{20\mu^2+17t^2}{36t}$, or,
(ii) $\frac{6-\sqrt{11}}{10} \le \mu/t < 2/5$ and $t/2 \le R \le \frac{3(t+2\mu)+\sqrt{11}(t-2\mu)}{12}$, or,
(iii) $1/4 \le \mu/t < \frac{6-\sqrt{11}}{10}$ and $t/2 \le R \le \frac{t+2\mu}{3}$, or,
(iv) $1/4 \le \mu/t \le \frac{6-\sqrt{11}}{10}$ and $\frac{t+2\mu}{3} < R \le \frac{20\mu^2+17t^2}{36t}$, or,
(v) $2/5 \le \mu/t \le 1/2$ and $t/2 \le R \le \frac{3(t+2\mu)+\sqrt{11}(t-2\mu)}{12}$.

Combining (iii) and (iv), we get $1/4 \le \mu/t \le \frac{6-\sqrt{11}}{10}$ and $t/2 \le R \le \frac{20\mu^2+17t^2}{36t}$, which we then combine with (i), and we get the first parameter combination in Theorem 1. Combining (ii) and (v) results in the second parameter combination in Theorem 1. □

Proof of Theorem 2 Again, in this proof, row numbers refer to Table 3.

Row 1: If $\mu/t \in (0, 1/4)$ and $R \in [\frac{t}{2}, \frac{52\mu^2-32\mu t+25t^2}{36t})$,

$$\begin{aligned}(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} &= \frac{(13\mu^2 - 8\mu t + 4t^2)}{18t} - \frac{(t+4\mu)^2}{72t}\\ &\quad - \frac{(t-2\mu)^2}{9t}\\ &= \frac{1}{72t}(4\mu^2 - 8\mu t + 7t^2)\\ &= \frac{1}{72t}(4(\mu - t)^2 + 3t^2)\\ &> 0.\end{aligned}$$

Row 2: If $\mu/t \in (0, 1/4)$ and $R \in [\frac{52\mu^2-32\mu t+25t^2}{36t}, \frac{3t}{4})$,

$$\begin{aligned}(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} &= \frac{R-\frac{t}{4}}{2} - \frac{(t+4\mu)^2}{72t} - \frac{(t-2\mu)^2}{9t}\\ &= \frac{1}{2}\left(R - \frac{8\mu^2-4\mu t+3t^2}{6t}\right)\\ &> \frac{1}{2}\left(R-\frac{t}{2}\right)\\ &\geq 0.\end{aligned}$$

Row 3: If $\mu/t \in (0, 1/4)$ and $R \in (\frac{3t}{4}, \infty)$,

$$\begin{aligned}(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} &= \frac{t}{4} - \frac{(t+4\mu)^2}{72t} - \frac{(t-2\mu)^2}{9t}\\ &= \frac{\mu(t-2\mu)}{3t} + \frac{t}{8}\\ &> 0.\end{aligned}$$

Row 4: If $\mu/t \in [\frac{1}{4}, \frac{14-3\sqrt{3}}{26})$ and $R \in [t/2, (t+2\mu)/3]$,

$$\begin{aligned}(\pi_a^{s2}-\pi_a^{s3})-\pi_e^{s3} &= \frac{1}{2t}(\mu^2+2\mu R-2R^2)+R-\mu\\ &\quad -\frac{(t+4\mu)^2}{72t}-\frac{(t-2\mu)^2}{9t}\\ &= \frac{1}{24t}(-24R^2+24\mu R+24tR-4\mu^2\\ &\quad -16\mu t-3t^2)\\ &= \frac{1}{24t}\left(-24\left(R-\frac{\mu+t}{2}\right)^2+2\mu^2-4\mu t+3t^2\right)\\ &\overset{t/2\leq R\leq\frac{t+2\mu}{3}<\frac{\mu+t}{2}}{>} \frac{1}{24t}\left(-24\left(\frac{t}{2}-\frac{\mu+t}{2}\right)^2+2\mu^2-4\mu t+3t^2\right)\\ &= \frac{1}{24t}(-4\mu^2-4\mu t+3t^2)\\ &= \frac{1}{24t}(t-2\mu)(3t+2\mu)\\ &\overset{\mu/t<1/2}{>} 0.\end{aligned}$$

Row 5: If $\mu/t \in [\frac{1}{4}, \frac{14-3\sqrt{3}}{26})$ and $R \in [\frac{t+2\mu}{3}, \frac{52\mu^2-32\mu t+25t^2}{36t})$, the analysis is the same as that of Row 1.

Row 6: If $\mu/t \in [1/4, \frac{14-3\sqrt{3}}{26})$ and $R \in [\frac{52\mu^2-32\mu t+25t^2}{36t}, 3t/4)$, the analysis is the same as that of Row 2.

Row 7: If $\mu/t \in [1/4, \frac{14-3\sqrt{3}}{26})$ and $R \in (3t/4, \infty)$, the analysis is the same as that of Row 3.

Row 8: If $\mu/t \in [\frac{14-3\sqrt{3}}{26}, 1/2)$ and $R \in [t/2, \frac{(\sqrt{3}+1)t-2(\sqrt{3}-1)\mu}{4})$,

$$\begin{aligned}(\pi_a^{s2}-\pi_a^{s3})-\pi_e^{s3} &= \frac{1}{2t}(\mu^2+2\mu R-2R^2)+R-\mu\\ &\quad -\frac{(t+4\mu)^2}{72t}-\frac{(t-2\mu)^2}{9t}\\ &= \frac{1}{24t}(-24R^2+24\mu R+24tR-4\mu^2\\ &\quad -16\mu t-3t^2)\\ &= \frac{1}{24t}\left(-24(R-\frac{\mu+t}{2})^2+2\mu^2-4\mu t+3t^2\right)\\ &\overset{t/2\leq R\leq\frac{(\sqrt{3}+1)t-2(\sqrt{3}-1)\mu}{4}<\frac{\mu+t}{2}}{>} \frac{1}{24t}(-24(\frac{t}{2}-\frac{\mu+t}{2})^2+2\mu^2-4\mu t+3t^2)\\ &= \frac{1}{24t}(-4\mu^2-4\mu t+3t^2)\\ &= \frac{1}{24t}(t-2\mu)(3t+2\mu)\\ &\overset{\mu/t<1/2}{>} 0.\end{aligned}$$

Row 9: If $\mu/t \in [\frac{14-3\sqrt{3}}{26}, 1/2)$ and $R \in [\frac{(\sqrt{3}+1)t-2(\sqrt{3}-1)\mu}{4}, 3t/4)$, the analysis is the same as that of Row 2.

Row 10: If $\mu/t \in [\frac{14-3\sqrt{3}}{26}, 1/2)$ and $R \in (3t/4, \infty)$, the analysis is the same as that of Row 3. □

Proof of Lemma 6 The Kuhn-Tucker conditions of retailers A and B are the same as in the proof of Lemma 2.

Interior solution. Following the same analysis as what we have in proof of Lemma 2, we get candidate Nash equilibrium prices:

$$p_a^{s2} = \frac{2t+\mu}{6}, \; p_d^{s2} = \frac{t-\mu}{3}, \; p_b^{s2} = \frac{t+2\mu}{6}.$$

Using the strict inequalities from our constraints, $p_d^{s2} + \mu < R$ and $x + y < 1/2$, and our market coverage condition $R < t/2$, we derive the additional conditions on μ/t and R in Lemma 6.

Constraint C1 is binding. Following the same analysis as what we have in proof of Lemma 2, we get candidate Nash equilibrium prices and the shadow price for $C1$:

$$p_a^{s2} = R-\mu/2, \; p_d^{s2} = R-\mu, \; p_b^{s2} = R/2, \; \lambda_1 = \frac{2\mu-3R+t}{t}.$$

Using the strict inequality from $C2$, $x + y < 1/2$, our market coverage condition, $R < t/2$, and the non-negative shadow price for $C1$, $\lambda_1 \geq 0$, we derive the additional conditions on μ/t and R in Lemma 6.

Constraint C2 is binding. Following the same analysis as what we have in proof of Lemma 2, we get candidate Nash equilibrium prices and the shadow price for $C2$:

$$p_a^{s2} = \frac{2t}{5}, \;\; p_d^{s2} = \frac{3t - 5\mu}{5}, \;\; p_b^{s2} = \frac{3t}{10}, \;\; \lambda_2 = \frac{2(5\mu - 2t)}{5}.$$

Using the strict inequality from $C1$, $p_d^{s2} + \mu < R$, we get $R > 3t/5$, which contradicts our market coverage condition $R < t/2$. Therefore, in State 2, when the market is not covered, $C2$ is never binding.

Constraints $C1$ *and* $C2$ *are binding*. Following the same analysis as what we have in proof of Lemma 2, we get candidate Nash equilibrium prices and the shadow prices for $C1$ and $C2$:

$$p_a^{s2} = \frac{3R - t}{2}, \;\; p_d^{s2} = R - \mu, \;\; p_b^{s2} = \frac{R}{2},$$
$$\lambda_1 = \frac{3t - 5R}{t}, \;\; \lambda_2 = 2(\mu + R - t).$$

From $\lambda_2 \geq 0$, we get $R \geq t - \mu$, which is contradictory to our market coverage condition $R < t/2$, since $t/2 < t - \mu$, because of our market comparison condition. Therefore, in State 2, when the market is not covered, $C1$ and $C2$ are never binding simultaneously.

Our second step is to verify that the Kuhn-Tucker conditions are also the sufficient condition for the maximization problem. It is easy to verify that both the objective functions are concave (the Hessian matrix of the objective function of retailer A is negative definite, and the second derivative of the objective function of retailer B is negative), and the constraints are linear and therefore concave. Thus, what we derive above is the optimal solution to the constrained profit maximization problem.

Unlike Lemma 2 for the covered market, retailer A always chooses to sell in the Internet channel in the uncovered market. Similar as in the proof of Lemma 2, it is easy to verify that in our case, the number of players is finite (G1), the constraints insure that the strategy space of every player is compact and convex (G2), the payoff functions are continuous and bounded (G3), and the payoff functions are concave and thus quasi-concave (G4). Therefore, the optimal solution to the constrained maximization problem is indeed the unique Nash equilibrium. □

Proof of Lemma 7 From (4) we can write the Lagrangian function for the new entrant as

$$L = 2p_d^{s3}\left(\frac{1}{2} - \frac{p_d^{s3} - p_a^{s3} + \mu}{t} - \frac{p_d^{s3} - p_b^{s3} + \mu}{t}\right)$$
$$- \lambda_1(p_d^{s3} + \mu - R)$$
$$- \lambda_2\left(\frac{p_d^{s3} - p_a^{s3} + \mu}{t} + \frac{p_d^{s3} - p_b^{s3} + \mu}{t} - \frac{1}{2}\right).$$

The resulting Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial p_d^{s3}} = \frac{2}{t}(p_a^{s3} + p_b^{s3} - 4p_d^{s3} - 2\mu) + 1 - \lambda_1 - \frac{2\lambda_2}{t} \leq 0,$$

$$\frac{\partial L}{\partial \lambda_1} = -p_d^{s3} - \mu + R \geq 0,$$

$$\frac{\partial L}{\partial \lambda_2} = -x - y + 1/2 \geq 0,$$

$$p_d^{s3} \geq 0, \;\; \lambda_1 \geq 0, \;\; \lambda_2 \geq 0,$$

$$p_d^{s3}\frac{\partial L}{\partial p_d^{s3}} = 0, \;\; \lambda_1\frac{\partial L}{\partial \lambda_1} = 0, \;\; \lambda_2\frac{\partial L}{\partial \lambda_2} = 0.$$

From (5) the Kuhn-Tucker conditions for retailer A are

$$\frac{\partial \pi_a^{s3}}{\partial p_a^{s3}} = \frac{2}{t}(p_d^{s3} - 2p_a^{s3} + \mu) \leq 0,$$

$$p_a^{s3} \geq 0, \;\; p_a^{s3}\frac{\partial \pi_a^{s3}}{\partial p_a^{s3}} = 0.$$

From (6) the Kuhn-Tucker conditions for retailer B are

$$\frac{\partial \pi_b^{s3}}{\partial p_b^{s3}} = \frac{2}{t}(p_d^{s3} - 2p_b^{s3} + \mu) \leq 0,$$

$$p_b^{s3} \geq 0, \;\; p_b^{s3}\frac{\partial \pi_b^{s3}}{\partial p_b^{s3}} = 0.$$

Excluding the non-negativity constraints, there are the two possible binding constraints for the new entrant's problem, giving rise to four cases which we discuss in turn.

Interior solution. Following similar analysis as proof of Lemma 2, we get candidate Nash equilibrium prices:

$$p_a^{s3} = p_b^{s3} = \frac{t + 4\mu}{12}, \;\; p_d^{s3} = \frac{t - 2\mu}{6}.$$

From the market comparison condition, $\mu/t < 1/2$, the candidate Nash equilibrium prices are strictly positive. Using the strict inequalities from our constraints, $p_d^{s3} + \mu < R$ and $x + y < 1/2$, and our market coverage condition $R < t/2$, we derive the additional conditions on μ/t and R in Lemma 7.

Constraint $C1$ *is binding*. Following similar analysis as proof of Lemma 2, we get candidate Nash equilibrium prices and the shadow price for $C1$:

$$p_a^{s3} = p_b^{s3} = R/2, \;\; p_d^{s3} = R - \mu, \;\; \lambda_1 = \frac{t - 2(3R - 2\mu)}{t}.$$

From $R > \mu$, the candidate Nash equilibrium prices are strictly positive. Using the strict inequality from $C2$, $x + y < 1/2$, our market coverage condition, $R < t/2$, and the non-negative shadow price for $C1$, $\lambda_1 \geq 0$, we derive the additional conditions on μ/t and R in Lemma 7.

Constraint C2 is binding. Following similar analysis as proof of Lemma 2, we get candidate Nash equilibrium prices and the shadow price for $C2$:

$$p_a^{s3} = p_b^{s3} = \frac{t}{4}, \;\; p_d^{s3} = \frac{t - 2\mu}{2}, \;\; \lambda_2 = 2\mu - t.$$

From the non-negative shadow price for $C2$, $\lambda_2 \geq 0$, we get $\mu/t \geq 1/2$, which is contradictory to our market comparison condition. Therefore, in State 3, when the market is not covered, $C2$ is never binding.

Constraints C1 and C2 are binding. Directly following from the binding constraint $C1$, $p_d^{s3} = R - \mu$. Plugging p_d^{s3} into $\partial\pi_a^{s3}/\partial p_a^{s3} = \partial\pi_b^{s3}/\partial p_b^{s3} = 0$, we get $p_a^{s3} = p_b^{s3} = R/2$. Plugging p_a^{s3}, p_b^{s3}, and p_d^{s3} into the binding constraint $C2$, $x + y = 1/2$, we get $R = t/2$, which contradicts our market coverage condition, $R < t/2$. Therefore, in State 3, when the market is not covered, $C1$ and $C2$ are never binding simultaneously.

It is easy to verify that all the objective functions are concave (the second derivative is negative), and the constraints are linear and therefore concave. Thus, what we derive above is the optimal solution to the constrained profit maximization problem. Similar as in the proof of Lemma 2, it is easy to verify that in our case, the number of players is finite (G1), the constraints insure that the strategy space of every player is compact and convex (G2), the payoff functions are continuous and bounded (G3), and the payoff functions are concave and thus quasi-concave (G4). Therefore, the optimal solution to the constrained maximization problem is indeed the unique Nash equilibrium. □

Proof of Corollary 2 From Lemma 5 and Lemma 8, $\pi_a^{s1} = \pi_b^{s1} = R^2/2t \geq \pi_a^{s4} = \pi_b^{s4} = \mu^2/2t$, because $R > \mu$. Proof of dominance of State 4 by State 2 and State 3 is similar to that in Corollary 1. □

Proof of Theorem 3 Same as in the covered market, we summarize the equilibrium profits under different parameter ranges in States 1 through 3 in Table 4. In this proof, row numbers refer to Table 4.

Row 1: If $\mu/t \in (0, 1/4)$ and $R \in [\mu, (t + 4\mu)/6]$,

$$\begin{aligned}(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} &= \frac{1}{2t}(\mu^2 + 2\mu R - 2R^2) \\ &\quad + R - \mu - \frac{R^2}{2t} - \frac{(R-\mu)(t-2R)}{t} \\ &= \frac{(R-\mu)^2}{2t} \\ &> 0.\end{aligned}$$

Row 2: If $\mu/t \in (0, 1/4)$ and $R \in ((t + 4\mu)/6, (t + 2\mu)/3]$,

$$\begin{aligned}&(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} \\ &= \frac{1}{2t}(\mu^2 + 2\mu R - 2R^2) + R - \mu - \frac{R^2}{2t} - \frac{(t-2\mu)^2}{9t} \\ &= \frac{1}{18t}(-27R^2 + 18\mu R + 18tR + \mu^2 - 10\mu t - 2t^2) \\ &= \frac{1}{18t}\left(-27\left(R - \frac{1}{3}(\mu + t)\right)^2 + (t - 2\mu)^2\right) \\ &\overset{(t+4\mu)/6 < R < t/2}{>} 0.\end{aligned}$$

Row 3: If $\mu/t \in (0, 1/4)$ and $R \in ((t + 2\mu)/3, t/2)$,

$$\begin{aligned}(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} &= \frac{(13\mu^2 - 8\mu t + 4t^2)}{18t} - \frac{R^2}{2t} - \frac{(t-2\mu)^2}{9t} \\ &= \frac{1}{18t}(5\mu^2 + 2t^2 - 9R^2).\end{aligned}$$

Since $R \in ((t + 2\mu)/3, t/2)$, if $\mu/t \leq 1/\sqrt{20}$, then $(t + 2\mu)/3 < \sqrt{5\mu^2 + 2t^2}/3 \leq t/2$, so if $R \geq \sqrt{5\mu^2 + 2t^2}/3$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} \leq 0$, and if $R < \sqrt{5\mu^2 + 2t^2}/3$, then $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} > 0$. If $\mu/t > 1/\sqrt{20}$, then $\sqrt{5\mu^2 + 2t^2}/3 > t/2 > R$, and $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} > 0$.

Table 4 Equilibrium profits when the market is not covered

μ/t	R	State 1—No direct market	State 2—Single retail in direct market	State 3—New entrant in direct market
$(0, \frac{1}{4})$	$[\mu, \frac{t+4\mu}{6}]$	$\pi_a = \pi_b = \frac{R^2}{2t}$	(C1 Binds)$\pi_a = \frac{\mu^2+2\mu R-2R^2}{2t} + R - \mu, \pi_b = \frac{R^2}{2t}$	(C1 Binds) $\pi_a = \pi_b = \frac{R^2}{2t}, \pi_e = \frac{(R-\mu)(t-2R)}{t}$
	$(\frac{t+4\mu}{6}, \frac{t+2\mu}{3}]$	$\pi_a = \pi_b = \frac{R^2}{2t}$	(C1 Binds) $\pi_a = \frac{\mu^2+2\mu R-2R^2}{2t} + R - \mu, \pi_b = \frac{R^2}{2t}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t}, \pi_e = \frac{(t-2\mu)^2}{9t}$
	$(\frac{t+2\mu}{3}, \frac{t}{2})$	$\pi_a = \pi_b = \frac{R^2}{2t}$	$\pi_a = \frac{13\mu^2-8\mu t+4t^2}{18t}, \pi_b = \frac{(2\mu+t)^2}{18t}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t}, \pi_e = \frac{(t-2\mu)^2}{9t}$
$[\frac{1}{4}, \frac{1}{2})$	$[\mu, \frac{t+4\mu}{6}]$	$\pi_a = \pi_b = \frac{R^2}{2t}$	(C1 Binds) $\pi_a = \frac{\mu^2+2\mu R-2R^2}{2t} + R - \mu, \pi_b = \frac{R^2}{2t}$	(C1 Binds) $\pi_a = \pi_b = \frac{R^2}{2t}, \pi_e = \frac{(R-\mu)(t-2R)}{t}$
	$(\frac{t+4\mu}{6}, \frac{t}{2})$	$\pi_a = \pi_b = \frac{R^2}{2t}$	(C1 Binds) $\pi_a = \frac{\mu^2+2\mu R-2R^2}{2t} + R - \mu, \pi_b = \frac{R^2}{2t}$	$\pi_a = \pi_b = \frac{(t+4\mu)^2}{72t}, \pi_e = \frac{(t-2\mu)^2}{9t}$

Row 4: If $\mu/t \in [1/4, 1/2)$ and $R \in [\mu, (t+4\mu)/6]$, the analysis is the same as that of Row 1.

Row 5: If $\mu/t \in [1/4, 1/2)$ and $R \in ((t+4\mu)/6, t/2)$, the analysis is the same as that of Row 2. □

Proof of Theorem 4 Again, in this proof, row numbers refer to Table 4.

Row 1: If $\mu/t \in (0, 1/4)$ and $R \in [\mu, (t+4\mu)/6]$,

$$(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} = (\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} \underset{\text{Theorem 3}}{>} 0.$$

Row 2: If $\mu/t \in (0, 1/4)$ and $R \in ((t+4\mu)/6, (t+2\mu)/3]$,

$$(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} \overset{R>(t+4\mu)/6}{>} (\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3} \underset{\text{Theorem 3}}{>} 0.$$

Row 3: If $\mu/t \in (0, 1/4)$ and $R \in ((t+2\mu)/3, t/2)$, the analysis is the same as that of Row 1 in the proof of Theorem 2.

Row 4: If $\mu/t \in [1/4, 1/2)$ and $R \in [\mu, (t+4\mu)/6]$, the analysis is the same as that of Row 1.

Row 5: If $\mu/t \in [1/4, 1/2)$ and $R \in ((t+4\mu)/6, t/2)$, the analysis is the same as that of Row 2. □

Proof of Corollary 3 The minimum entry cost advantage for the entrant is given by:

$$K_a(T) - K_e(T) > \frac{e^{-rT}}{r}[(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3}].$$

So if $(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3}$ is smaller in the uncovered market than in the covered market, we can prove that $K_a(T) - K_e(T)$ requires to be smaller in the uncovered market than in the covered market. Note that $(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3}$ is actually the difference between retailer A's preemption incentive and the new entrant's. The following proves that $(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3}$ is smaller in the uncovered market than in the covered market.

In the covered market ($R \geq t/2$):

If $\mu/t \in (0, 1/4)$,

$$(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} = \begin{cases} \frac{1}{72t}(4(\mu-t)^2 + 3t^2) & R \in [\frac{t}{2}, \frac{52\mu^2 - 32\mu t + 25t^2}{36t}) \\ \frac{1}{2}(R - \frac{t}{2}) & R \in [\frac{52\mu^2 - 32\mu t + 25t^2}{36t}, \frac{3t}{4}) \\ \frac{\mu(t-2\mu)}{3t} + \frac{t}{8} & R \in (\frac{3t}{4}, \infty) \end{cases}$$

It can be shown that as R varies, $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ is non-decreasing. So the minimum $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ takes is $\frac{1}{72t}(4(\mu-t)^2 + 3t^2)$.

If $\mu/t \in [1/4, \frac{14-3\sqrt{3}}{26})$,

$$(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} = \begin{cases} \frac{1}{24t}(-24(R - \frac{\mu+t}{2})^2 + 2\mu^2 - 4\mu t + 3t^2) & R \in [t/2, (t+2\mu)/3] \\ \frac{1}{72t}(4(\mu-t)^2 + 3t^2) & R \in [\frac{t+2\mu}{3}, \frac{52\mu^2 - 32\mu t + 25t^2}{36t}) \\ \frac{1}{2}(R - \frac{t}{2}) & R \in [\frac{52\mu^2 - 32\mu t + 25t^2}{36t}, \frac{3t}{4}) \\ \frac{\mu(t-2\mu)}{3t} + \frac{t}{8} & R \in (\frac{3t}{4}, \infty) \end{cases}$$

It can be shown that as R varies, $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ is non-decreasing. So the minimum $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ takes is $\frac{1}{24t}(-4\mu^2 - 4\mu t + 3t^2)$.

If $\mu/t \in [\frac{14-3\sqrt{3}}{26}, 1/2)$,

$$(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} = \begin{cases} \frac{1}{24t}(-24(R - \frac{\mu+t}{2})^2 + 2\mu^2 - 4\mu t + 3t^2) & R \in [t/2, \frac{(\sqrt{3}+1)t - 2(\sqrt{3}-1)\mu}{4}) \\ \frac{1}{2}(R - \frac{t}{2}) & R \in [\frac{(\sqrt{3}+1)t - 2(\sqrt{3}-1)\mu}{4}, \frac{3t}{4}) \\ \frac{\mu(t-2\mu)}{3t} + \frac{t}{8} & R \in (\frac{3t}{4}, \infty) \end{cases}$$

It can be shown that as R varies, $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ is non-decreasing. So the minimum $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ takes is $\frac{1}{24t}(-4\mu^2 - 4\mu t + 3t^2)$.

In the uncovered market ($\mu \leq R < t/2$) :

If $\mu/t \in (0, 1/4)$,

$$(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} = \begin{cases} \frac{(R-\mu)^2}{2t} & R \in [\mu, (t+4\mu)/6] \\ \frac{1}{24t}(-24(R - \frac{\mu+t}{2})^2 + 2\mu^2 - 4\mu t + 3t^2) & R \in ((t+4\mu)/6, (t+2\mu)/3] \\ \frac{1}{72t}(4(\mu-t)^2 + 3t^2) & R \in ((t+2\mu)/3, t/2) \end{cases}$$

It can be shown that as R varies, $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ is non-decreasing. So the maximum $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ takes is $\frac{1}{72t}(4(\mu-t)^2 + 3t^2)$.

If $\mu/t \in [1/4, 1/2)$,

$$(\pi_a^{s2} - \pi_a^{s3}) - \pi_e^{s3} = \begin{cases} \frac{(R-\mu)^2}{2t} & R \in [\mu, (t+4\mu)/6] \\ \frac{1}{24t}(-24(R - \frac{\mu+t}{2})^2 + 2\mu^2 \\ -4\mu t + 3t^2) & R \in ((t+4\mu)/6, t/2] \end{cases}$$

It can be shown that as R varies, $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ is increasing. So the maximum $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ takes is $\frac{1}{24t}(-4\mu^2 - 4\mu t + 3t^2)$.

By comparing the minimum that $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ takes in the covered market and the maximum in the uncovered market, it is easy to show that $(\pi_a^{s2} - \pi_a^{s1}) - \pi_e^{s3}$ is smaller in the uncovered market than in the covered market. □

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