THE UNIVERSITY OF CALGARY

ASSESSING COGNITIVE LEVELS IN PRIMARY GRADE GEOMETRY

by

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ABSTRACT

The purposes of this study were: (1) to assess the levels of cognitive response demonstrated in geometry and graphing by Alberta students in Grades 1, 2, and 3, using individual, Piagetian-based interviews; and (2) to construct and administer a Grade 3 paper-and-pencil assessment in geometry and graphing and determine whether such an instrument could serve as an effective alternative to the individual, Piagetian-based interviews for assessing student developmental levels.

The interview tasks were selected from the extensive source of cognitive assessment tasks provided by the work of Piaget and his associates. The cognitive assessment items used in the paper-and-pencil test were drawn primarily from tests developed by the Australian Council for Educational Research. The reliability of the test items, using Cronbach's Coefficient Alpha, was 0.661.

Interview task assessments were made of the responses from 360 Grade 1 to 3 students. The paper-and-pencil test was administered to 112 Grade 3 students.

The data collected from the individual interviews of the students in Grades 1, 2, and 3, and from the paper-and-pencil test administered to Grade 3 students was tabulated and presented in the form of percentage distributions of responses found at each of the cognitive

levels. A Kolmogorov-Smirnov goodness-of-fit test was used to assess the comparability of the distributions of cognitive level ratings of Grade 3 pupil responses to the interview tasks and those produced by the Grade 3 paper-and-pencil test. A decision was made to accept the null hypothesis of no significant difference between the two distributions if the probability of observing the calculated K-S D was greater than 0.05. The probability associated with the observed K-S D of 0.072 was greater than 0.200. Therefore, the null hypothesis was accepted and it was concluded that the two modes of assessment being investigated, interview and paper-and-pencil, were consistent and yielded similar cognitive response level distributions.

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THE NATURE OF THE PROBLEM

INTRODUCTION

Whether one adopts a Piagetian perspective or not, it would be difficult to refute the claim that Jean Piaget has had a significant impact on developmental psychology and educational thought and research. He has helped to improve our perceptions of the ways in which children think and, as Skemp (1979) has indicated, he has focused attention on the common error of treating the mental processes of children as if they were similar to those of adults but less efficient and experienced. Piaget has made us aware that some of the differences between the thinking of children and the thinking of adults are truly qualitative rather than quantitative (Ginsburg and Opper, 1979; Carpenter, 1980). Indeed the child's mind belongs to a different kind of thought which the adult has long since left behind or suppressed (Piaget, 1955). Accepting the importance of Piaget's work, this study is concerned with the qualitative assessment of students' thinking, one of the many implications that Piaget's findings have for educational practice.

The mathematical education literature indicates that the greatest departure from current general teaching practice would be in the closer monitoring by the teacher of the conceptual understanding of the learner. A child's mathematical achievement, it appears, may be aided by instruction that is based on an assessment of the child's developmental level (Suydam and Weaver, 1975; Carpenter, 1980; Bell, Costello and Kuchemann, 1980). Knowledge of students' functioning would help the teacher to present the cognitive conflict situations that are one important mechanism of conceptual growth (Tamburrini, 1975). Therefore teachers should become better informed of their pupils' abilities or modes of cognitive functioning and endeavour to programme their class work both to capitalize on and extend these.

It is important that the teacher be able to make an assessment of the students' capabilities as, once obtained, this knowledge would help in the creation of situations intended to provoke the child to question and experiment (Duckworth, 1964). The teacher needs to know when the child is ready to proceed with, and profit from, experiences or instruction at each step in the mathematical program. The assessment of intellectual level, however, is not an easy task. The evaluation must be different from the usual standard achievement tests which often measure only surface knowledge, rote memory, and other superficial aspects of learning. It must also provide more information than the traditional, group-administered intelligence tests, which furnish scores indicating where students stand in relation to their peers within the normal distribution, but generally cannot explain why specific responses are given. In order to properly assess intellectual level, the teacher would have to evaluate not only the products of thought--correct or incorrect answers--but the process of students' thinking as well.

Piaget has provided a comprehensive, stage-dependent, conceptual framework within which cognitive functioning has been intensively studied. His developmental levels provide a very effective means of determining just how a child conceptualizes and interacts with his world. Thus the principal aim of this study was to develop Piagetian-based cognitive assessment material or procedures which would be practical and efficient for classroom use, by classroom teachers, helping them become more aware of the processes their pupils can or cannot use, and enabling them to provide suitable learning material. Rather than simply fitting children into the appropriate step in a sequence of instruction, teachers would have some means to ensure that students are provided with instruction appropriate for their level of cognitive development.

STATEMENT OF THE PROBLEM

The objectives of this study were:

1. to assess the levels of cognitive response demonstrated in geometry and graphing by Alberta students in Grades 1, 2 and 3, using individual Piagetian-based interviews;
2. to construct a paper-and-pencil test in geometry and graphing, which could be used to determine a student's level of cognitive development;
3. to compare the results obtained by administering the paper-and-pencil test with those obtained using the individual Piagetian-based interviews to determine whether such a test could serve as an effective alternative to the interviews for assessing student developmental levels.

SIGNIFICANCE OF THE STUDY

Children's thinking patterns in mathematics are of utmost importance, yet one of the difficulties encountered in mathematics education has been the problem of adequately assessing children's thinking processes. Often tests which are administered, although objective in nature, only serve to indicate whether a child has learned the "verbalizations" that have been taught. As a result, a teacher may be less than sure of what a child actually understands. Alternatively, the task-related testing situations that Piagetian methods suggest offer a promising approach to assessing children's understanding (Flavell, 1963; Inskeep, 1972).

When considering a means of monitoring the conceptual understanding of young, primary-grade children, especially those in Grades 1 and 2 with their limited reading capabilities, the unique interview techniques developed by Piaget and his collaborators would be most appropriate. However, the clinical interview procedure associated with Piagetian research is generally considered impractical for wide classroom application since it requires a great deal of time and qualified personnel making subjective interpretations of responses to unsystematic questions. Despite this criticism of Piaget's method one cannot overstate the value of personal interviews.

> Every teacher can make a sound investment by interviewing children in her class periodically to determine and study their levels of thinking when dealing with various quantitative situations. By then using the knowledge gained from such interviews to assist her in providing a helpfully differentiated program of teaching . . . every teacher can reap big dividends in the form of increased instructional effectiveness (Weaver, 1955:47).

Much information regarding a child's thinking can be obtained through an individual interview. A well-constructed Piagetian interview provides the teacher with something more than can customarily be derived from standardized test results. For example, global assessments can be made of the ways in which the child organizes, or fails to organize, information. The child's errors and misconceptions are revealed as they occur. From this direct observation of a child's functioning in a problem-solving situation, the teacher can derive many clues as to either readiness for more complex learning or the kinds of experience needed before being able to move ahead (Almy, 1966). Consequently, if several suitable Piagetian tasks could be linked to specific curriculum objectives identified by the Elementary Mathematics Curriculum Guide, Alberta Education, 1982, and if they could be adapted for classroom use in terms of ease of administration and modest time commitments, it would be most beneficial to primary classroom teachers seeking an effective method for assessing student developmental levels.

While cognitive response assessments of students in Grades 1, 2 and 3 can probably best be made with individual Piagetian-based interviews, it is feasible to consider paper-and-pencil cognitive assessments for students in Grade 3 since they are required to complete written provincial achievement tests at that level. If the essentials of Piaget's clinical method were incorporated into a paper-and-pencil instrument, teachers would have a means to assess students' thinking that would combine the efficiency of group-administered written tests with the effectiveness of the interview technique. Responses to items on such an instrument would not be classified as correct or incorrect, but rather would be classified according to stage of cognitive development within a specific content domain. Previous studies have shown that group paper-and-pencil tests, designed to assess Piagetian levels of response, can produce results that are comparable to Piaget's findings (Tisher, 1971; Raven, 1973; Shayer, 1973; Ankney, 1975).

The ultimate goal of cognitive assessment is to furnish information about the thinking of the individual child, whatever assessment procedures are used. Whether using individual Piagetian-based interviews involving some manipulation of objects or materials on the part of the child or group-administered paper-and-pencil assessments, one must insure freedom for each child to reveal personally constructed thoughts, rather than repeating, parrot-like, a response suspected to be the one the teacher wants. It is unlikely that a single, global measure of development would succeed in evaluating children's thinking in all mathematical areas. The implication of Piaget's concept of decalage is that children can be expected to respond at different levels in different contexts, therefore context-specific measures must be employed. Accordingly, this study developed cognitive measures specifically for geometry and graphing contexts.

Cognitive assessment material and procedures which can more accurately determine students' cognitive response levels within a specific content domain would make a significant contribution to mathematics education, particularly if a series of such measures were to be developed in different content areas.

DELIMITATIONS OF THE STUDY

The study sampled student cognitive response levels from Grades 1, 2, and 3 in the mathematics topic strands of geometry and graphing. Students were selected from across the province to ensure that the sample was provincially representative and included a suitable urban/rural balance.

Interview task assessments were made of the responses from 360 Grade 1 to 3 students who were interviewed by 23 Teacher-Interviewers. Grade 3 student response levels were also assessed by means of a paper-and-pencil test which was administered to 112 students.

The developmental framework used to assess student cognitive response levels in this study was based on the work of Jean Piaget.

DEFINITION OF TERMS

Achievement

The term "achievement" in this study is used to refer to learning that concentrates on the mastery of some data, such as facts, skills, concepts, or prescribed problem-solving strategies (Biggs and Collis, 1982). It is learning that is impressed upon a child and taken on in a passive, rote fashion. Skemp (1978) describes such superficial learning as "instrumental understanding" or using "rules without reasons".

Cognition

The term "cognition" is used to refer to a degree and kind of deep learning or understanding, such that the understanding of a principle means that one can identify its appropriateness and usefulness in situations where it has not previously been seen or used, or the understanding of a process implies that one knows when and how to use it effectively. That is, an understanding which will lead to the use of learnings in the normal, out-of-school, life activities of the child. Skemp (1978) regards this as "relational understanding" or "knowing both what to do and why".

Clinical Method

The "clinical method" is a diagnostic tool applied to reasoning in children that takes the form of a dialogue held in an individual session between an adult, the interviewer, and a child, the subject of study. Essentially it constitutes an hypothesis-testing situation, permitting the interviewer to infer rapidly a child's competence in a particular aspect of reasoning by means of observation of his performance at certain tasks. The interviewer presents to the child an "experiment" that involves both a concrete situation with objects placed in front of the child and a verbally presented problem related to this situation. A number of items frequently consisting of physical or spatial manipulations performed on the materials, either by the interviewer or by the child, are presented to the child. For each item the interviewer asks a series of related questions which are aimed at leading the child to predict, observe, and explain the results of the manipulations performed on the concrete objects. The interviewer makes every effort to encourage the child to elaborate on and support judgements made about the different items presented as these verbal explanations are particularly valuable for inferring the child's underlying thought processes. Each successive response of the child guides the interviewer in forming new hypotheses and consequently in the choice of subsequent direction for the experiment. The procedure continues until the interviewer feels that the child's thinking has been explored as far as possible, within the constraints of the particular situation, and that a reasonable explanation of the child's behaviour has been developed (Opper, 1977).

Piagetian-Based Interview

In this study the term "Piagetian-based interview" refers to a standardized version of the clinical method which is an attempt to combine the more structured approach of standardized testing with the flexibility of the clinical method and hence satisfy both the requirements of reasonably standardized systematic observation and those of conducting open-ended research with young children. In this version, the subjects are presented with a standard problem and material, and certain manipulations that are identical in each interview are applied to this material. The subjects are then asked an identical set of questions relating to both the material and the manipulations. The responses of the subject are recorded and rated according to cognitive level by means of predetermined criteria set out on interview procedure/record sheets, copies of which are included in Appendix 1.

Cognitive Levels

The Cognitive Levels criteria which follow have been derived from a summary of "Piaget's Structure" (Lovell, 1974).

Preoperational (PO) thinking is characterized by intuitive and transductive thinking from one particular to another, thinking limited by the particular state of the situation considered, and isolated centrings on one feature only. The child at this cognitive level can only deal with one problem at a time, and is unable to relate one problem to another in the same situation. This is unsystematic, partial, fragmented, inconsistent thinking, lacking reversibility of thought--i.e., an inability to work back from an inconsistency.

> Content-specific criteria (Geometry and Graphing): At this level a child can sort objects but is inconsistent in naming common attribute(s) of each set (Copeland, 1974). A child can locate a point on a sheet of paper similar to a model shown but does so by using visual estimates only (Piaget, Inhelder and Szeminska, 1960).

Early Concrete Operational (EC) thinking is characterized by faulty inductive and deductive logic, generally unsuccessful attempts to consider or relate more than one feature of a situation, attempts at reversibility that end in confusion, incomplete or inconsistent attempts to classify facts, and uncertain judgements.

Content-specific criteria (Geometry and Graphing): The child identifies simple properties and relations in concrete objects only; uses one-way classification, forms simple hierarchies, and orders on the basis of one major attribute; dissociates squares from circles, for example, but is unable to handle class inclusion; sorts objects in one or two ways but is unable to dichotomize using negation (Copeland, 1974); can replicate the order of a set of objects but is unable to make the series in reverse or circular order (Piaget and Inhelder, 1956); relies completely on perception to find only one point equidistant from two given points, the midpoint - chooses other points at random; when locating points equidistant from a given point, chooses locations without measuring and in a row or an irregular ring; locates a point on a sheet of paper similar to a model shown by using visual estimation or inappropriate measuring procedures, usually in one dimension (Piaget, Inhelder and Szeminska, 1960).

Late Concrete Operational (LC) thinking is characterized by inductive and deductive logic limited to concrete situations which involve visual or sensory data, successful classification of tangible data, successful systematic thinking and relating of two or more facts without extension or generalization from one concrete field to another, reversibility when concrete data are being operated with, a tendency to judge purely verbal problems and problem situations in terms of their content as specifically related to personal experience, and concentration on relating things visibly or tangibly present.

Content-specific criteria (Geometry and Graphing): At this level a child uses reversibility of classification and serration, taking into account two major attributes of the objects being considered; uses logical classes, e.g., "circles", "squares", and "blues", and interprets "all" and "some" appropriately; classifies objects in several ways (Copeland, 1974); systematically reverses the order of objects in a row, a circle, or an intertwined arrangement (Piaget and Inhelder, 1956); locates a number of points equidistant from two points or from one point, without using symmetry; locates a point on a sheet of paper similar to a model shown, using trial-and-error two-dimensional measurements (Piaget, Inhelder and Szeminska, 1960).

Early Formal Operational (EF) thinking is characterized by reasonably advanced and consistent inductive and deductive logic limited by the concrete elements in the situation, generally unsuccessful attempts at abstract and propositional thinking, and generally unsuccessful attempts to go outside of known data to form hypotheses.

Content-specific criteria (Geometry and Graphing): The child handles loci by reasoning by recurrence, and immediately locates a point on a sheet of paper similar to a model shown by using coordinated rectangular measurements (Piaget, Inhelder and Szeminska, 1960).

Formal Operational (F) thinking is characterized by hypothetical and deductive thinking, consideration of data in terms of provisionally true or false propositions to be tested out in thought, logical thinking in symbolic and abstract form, recognition of the incompatibility of certain facts with an hypothesis, evidence of a preference to begin consideration of a situation with a theory rather than just the facts, and reasoning by implication at an abstract level.

Cognitive Response Levels

In this study interview tasks and test items were selected and designed to provide cognitive assessments that comply with the criteria included in the preceding Cognitive Levels definitions. In each cognitive assessment the student's responses were rated as Preoperational, Early Concrete Operational, Late Concrete Operational, Early Formal Operational, or Formal Operational.

CHAPTER 2

REVIEW OF THE LITERATURE AND RELATED RESEARCH

INTRODUCTION

One reason for the appeal of Piaget's work is that his descriptions of the development of thinking in children explain so many of the sometimes curious answers children give to a question posed in a classroom. Piagetian theory helps to make sense out of them, for one of Piaget's keen observations is that a child's logic is not the same as an adult's logic. The logic a child uses is not an immature form of adult logic. It is the child's own, and it develops as objects are acted upon, explored with, and manipulated.

Piaget is best known for his description of the various stages of cognitive development. However, if a teacher is to cooperate with children's thinking appropriately, an understanding of these stages of development is necessary but not sufficient. Acquiring information about what a student cannot do, about what conceptual deficits there are in a child, is an inadequate conceptualization of the principle of "starting where the learner is". One must also know what a child can do and where to go from there. It is necessary not just to know the description of different stages of development but also to know about the explanatory part of a theory, the part that deals with how development takes place (Tamburrini, 1975). Accordingly, Piaget's major theoretical notions concerning intellectual development are of central importance.

GENERAL THEORETICAL PERSPECTIVE

Piaget has developed a stage dependent theory of the origins and development of intelligence. At the core of the theory are distinctions between and relationships among three central concepts: function, content, and structure. Simply defined, "function is concerned with the manner in which any organism makes cognitive progress; content refers to the external behaviour which tells us that functioning has occurred; and structure refers to the inferred organizational properties which explain why this content rather than some other content has emerged" (Flavell, 1963, p. 18). Piaget's work has tended to emphasize studying the structure of developing intelligence, as opposed to its function and content (Piaget, 1964, p. 177; Flavell, 1963, pp. 17-19; Ginsburg and Opper, 1979, p. 16). Most of his work concerns the details of structural change, that is, the kind of intellectual organizations encountered in the course of development and the relations among these organizations.

A simple aspect of thought, then, is its manifest content. Content refers to raw, uninterpreted behavioral data - what an individual is thinking about, what interests him at the moment, or what terms he uses in contemplating a given problem. For example, when asked what makes a car go, a mechanic would reply in terms of the explosion of gas, the movement of pistons, and so on. These statements reflect the contents of his thought. Obviously a young child's response to the same question would be quite different because the content of his thought is different from that of the adult. Content represents substantive knowledge of the world; e.g., number, space, time. The physical and/or mental actions performed on these contents form the empirical base of Piaget's theory from which cognitive structures are inferred. Substantive knowledge changes with age and experience, and therefore is manifested in different forms at different stages of development.

Although intellectual content changes with age, cognitive functions do not. Cognitive functions represent the characteristics of mental activity that are invariant throughout development, organization and adaptation. The very essence of intelligent behaviour, cognitive functions are the processes that account for the development, refinement, and transformation of cognitive structures (Flavell, 1963, pp. 41-43). It is through functioning, and only through functioning, that cognitive structures are formed. Piaget postulates that cognitive functions are biological in nature, a part of general heredity. Because they remain essentially constant throughout life, they are referred to as functional invariants. Flavell (1963) described Piaget's general conception of functioning in the following way:

> Intellectual functioning is a special form of biological activity and, as such, possesses important attributes in common with the parent activities from which it derives. In other words, intelligence bears a biological imprint, and this imprint defines its essential characteristics (p. 42).

For Piaget, therefore, intellectual functioning is characterized in terms of the same invariants that hold for more elementary biological processes.

The first principal characteristic of intellectual functioning, organization, is the tendency common to all forms of life to integrate structures, both physical and psychological, into higher-order systems or structures. For example, the very young infant has available the separate behavioral structures of either looking at objects or of grasping them. He does not initially combine the two. After a period of development however, he organizes these two separate structures into a higher-order structure which enables him to grasp something while looking at it. Therefore, in his interaction with the world, an individual tends to integrate his physical and psychological structures into coherent systems.

Adaptation is the second general principle of functioning. All organisms are born with a tendency to adapt to the environment. The ways in which adaptation occurs differ from species to species, from individual to individual within a species, or from stage to stage within any one individual. Nevertheless, the tendency to adapt in some way is an invariant function considered to be inherent in living organisms. Adaptation may be considered in terms of two intimately related but conceptually distinct processes: assimilation and accommodation. The process of accommodation describes the individual's tendency to change in response to environmental demands. Assimilation is the complementary process by which the individual deals with an environmental event in terms of current structures. Thus the individual not only modifies structures in reaction to external demands (accommodation), but also uses existing mental structures to incorporate elements of the external world (assimilation). A new assimilatory structure must always be some variant of the last one acquired, evolving almost imperceptibly. It is this factor which ensures both the gradualness and continuity of intellectual development.

Finally, interposed between the fundamental concepts of function and content, Piaget postulates the existence of cognitive structures. Cognitive structure refers to the inferred organizational properties that underlie a child's thought and behaviour. These properties change, in a qualitative manner, with increasing age and experience and it is these developmental changes that have constituted the major object of study for Piaget.

> These operational structures are what seem to me to constitute the basis of knowledge, the natural psychological reality, in terms of which we must understand the development of knowledge. And the central problem of development is to understand the formation, elaboration, organization, and functioning of these structures (Piaget, 1964, p. 177).

The qualitative changes cognitive structures undergo are at the heart of Piaget's stage theory of intelligence.

<u>The Structure of Developing Intelligence</u>

Basically, the individual tends to organize his behaviour and thought, and to adapt to the environment. These tendencies result in a number of cognitive structures which take different forms at different ages. The child progresses through a series of stages, each characterized by different structures, before attaining adult intelligence. However, although cognitive structures feature different properties at various developmental levels, there are general characteristics of structures that can be identified.

A basic structural concept in Piaget's theory of intellectual development is that of a schema. A schema is a well-defined and organized set of physical or mental actions, which is in some way based on experience. Referring to activity on the part of the child, a schema is used to describe things the child does. Thus, in discussing sensory-motor development, Piaget speaks of the schema of sucking, a grasping schema, and so on. Occasionally, however, schema is used to describe actions which are not immediately obvious. Schema refers to the basic structure underlying the child's overt actions. It is used to designate the essence of the child's behaviour. For example, an infant usually sucks his thumb or a finger, although in examining an infant's behaviour in detail, one will see that no two acts of thumb-sucking performed by one child are exactly the same. The activity may start when the thumb is close to the mouth on one occasion, or farther away on another. The thumb may travel in almost a straight line to the mouth, or take quite an irregular path. There is no one act of thumb-sucking, but many; in fact as many as the number of times the child brings the thumb to the mouth. But what is important, especially for Piaget, is the structure of the behaviour; that is, an abstraction of the features common to a wide variety of acts which differ in detail. In the case of thumb-sucking, what is crucial is that the infant has acquired a regular way of getting the thumb into the mouth. Thus the infant puts the hand into his mouth in many particular ways, no two being identical, and the regularity detected in these specific actions is called a schema. Schema implies that assimilatory functioning has generated a specific cognitive structure, an organized disposition to suck a thumb or fingers, for example, on repeated occasions. It implies that there has been a change in overall cognitive organization so that a new behaviour pattern has become part of the child's intellectual repertoire (Flavell, 1963, p. 53). As another example, in middle childhood there is a schema of intuitive qualitative correspondence which refers to a strategy by which the child tries to assess whether or not two sets of elements are numerically equivalent (Piaget, 1952, p. 88). As Flavell (1963) defines it, "a schema is a cognitive structure which has reference to a class of similar action sequences, these sequences of necessity being strong, bounded totalities in which the constituent behavioral elements are tightly interrelated" (pp. 52-53).

Schemata may be brief and simple in nature, or complex, such as the problem-solving strategies of an adult. But one of the most important characteristics of a schema is its tendency toward repeated application. As schemata are repeatedly applied they are transformed, for functioning not only creates structures, but changes them continually. Schemata continually extend their field of application so as to assimilate new and different situations.

Another basic structural concept in Piaget's description of the development of knowledge is that of an operation. Piaget (1964) has stated:

> Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy or image of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed (p. 176).

Thus, an operation is an interiorized action which can modify objects of knowledge and enable the knower to get at the structures of the transformation. For example, an operation could consist of ordering or putting things in a series, of constructing a classification of objects, of counting, or of measuring. An operation is a reversible action. That is, it can take place in both directions, for instance, adding or subtracting, joining or separating. Also, an operation is never isolated but always linked to other operations. Hence it is always a part of a total structure (Piaget and Inhelder, 1969, pp. 96-97).

Piaget has distinguished four main stages in the child's development of operational structures. The child's rate of progress through the stages is not fixed nor is it the same for all children, but the sequence of the stages is invariant. If knowledge were built by mere absorption or simple input of information, it would be possible to vary the sequence of its acquisition. However, because it is built by a continuous process of construction of structures that are rooted in biological adaptation, the sequence of development is the same for all children regardless of the culture in which they live, and we cannot change the sequence or skip a step in the long process of construction (Kamii, 1973, p. 222).

1. Sensory-Motor Stage

The first stage is a sensory-motor, pre-verbal period, lasting approximately the first eighteen months of life. Built during this sensory-motor stage is the practical knowledge on which later representational knowledge is developed. An example is the construction of the schema of the permanent object. At first objects have no permanence and exist only when in view for an infant. But by the end of this stage an infant will try to find a previously seen object that is hidden from view, having learned that objects do exist outside one's field of vision. Along with the construction of the schema of the permanent object there comes the construction of notions of practical or sensory-motor space, temporal succession, and elementary sensory-motor causality; that is, a series of structures which are indispensable for the structures of later representational thought (Piaget, 1964, p. 177).

2. Preoperational Stage

The preoperational stage, extending from about eighteen months to about seven years of age, is characterized by the beginnings of language, the symbolic function, and therefore of thought, or representation. But at the level of representational thought, there must be a reconstruction of all that was developed on the sensory-motor level, for sensory-motor actions are not immediately translated into operations (Piaget, 1964, p. 177). Thinking becomes possible since knowing is beginning to be dissociated from external actions, but it is still not operational as personal actions still dominate thinking and give it a personal flavour. Thus, the thinking of preoperational children is egocentric; that is, centered on their own personal perspective, on their own personal experience, and they find it difficult to transcend that personal experience (Furth, 1970, p. 36).

Throughout this second stage of preoperational representation, thinking is dominated by immediate perceptions and a general pattern of centration - focusing on a limited amount of the information available. In the conservation of number task, the child judges two sets equal when they are the same length, and ignores another relevant variable, the density. In the conservation of continuous quantity, the child judges two amounts equal when the heights of the columns of liquid are the same and ignores the width. In both problems, the preoperational child focuses on one dimension of a situation, failing to make use of another equally relevant dimension, and, consequently, failing to appreciate the relations between the two. Also, the thought of the preoperational child is static in the sense that it centers on states. In the conservation of substance the focus is on the shape of plasticene (either a ball or a sausage) and changes from one state to another (transformations) are ignored. In conservation of continuous quantity tasks the heights of the columns of liquid are focused upon rather than the act of pouring. There is a lack of adequate representations of an object's shift from one position to another. Generally then, the preoperational child concentrates on the successive states of a situation rather than on the dynamic transformations by which one state is changed into another, and there is an inability to see how invariance of quantity, for example, is insured by the possibility of an inverse transformation to the original state (Flavell, 1963, pp. 156-159).

3. Concrete Operations Stage

In the stage of concrete operations, which occurs from about age seven to about age eleven, the first operations appear along with systems of operations that can be carried out simultaneously. The operations are called "concrete" because they relate directly to real objects and not yet to verbally stated hypotheses. Examples of operations developed in this stage are those of classification, ordering, the construction of the idea of number, spatial and temporal operations, operations of the elementary logic of classes and relations, and operations of elementary mathematics, geometry, and physics (Piaget, 1964, p. 177). The yet incomplete systems of operation are characterized by two forms of reversibility: negation and reciprocity. The concrete operational child, while possessing these two kinds of reversible operations, does not possess a total, integrated system which permits coordination of the two and thereby the ability to solve multivariable problems which require this kind of coordination (Flavell, 1963, p. 204).

4. Formal Operations Stage

The fourth stage, that of formal operations, begins at about eleven or twelve years of age and is characterized by the development of formal, abstract thought operations with which the adolescent can reason in terms of hypotheses and not only in terms of objects in the physical world. Unlike the concrete-operational child, the adolescent begins consideration of a problem by systematically identifying all possible factors relevant to the problem under investigation and forming all possible combinations of these factors, one at a time, two at a time, three at a time, and so on. Then forming hypotheses the adolescent attempts, through a combination of experimentation and logical analysis, to test them against reality and find out which of the possible relations in fact hold true. Thus, the adolescent's thought has a distinctly new orientation, no longer bound to what is real but capable of considering hypotheses that may or may not be true and working through what would follow if they were true (Flavell, 1963, pp. 202-211; Harrison, 1969, pp. 96-97).

Factors Influencing Intellectual Development

According to Piaget, there are four main factors which explain the development from one set of mental structures to another: maturation, experience, social transmission, and equilibration. While he finds each of these insufficient in itself, Piaget considers the fourth, equilibration or self-regulation, to be the fundamental and principal factor (Piaget, 1964, p. 178).

Although maturation of the nervous system plays an indispensable role in development and must not be ignored, it does not explain everything because the average chronological age at which each of the various stages occur (but not the order of occurrence) varies widely from one society to another (Piaget, 1964, p. 178).

Experience of objects, of physical reality, is also a basic factor in the development of cognitive structures but once again this factor does not explain everything. Some of the concepts which appear at the beginning of the stage of concrete operations are such that they cannot be drawn from experience alone. For example, a child becomes aware of conservation of substance at approximately age eight, yet does not assert that weight or volume is conserved until some time later. Both weight and volume can be perceived directly (weighing a plasticene ball can lead to the conservation of weight; immersing it in water can lead to the conservation of volume), however no experiment or physical experience can show the child at this level that there is the same amount of substance. The child comes to understand that when there is a transformation of the shape of some quantity of plasticene for example, something must be conserved because by reversing the transformation the plasticene can be returned to its original condition. Consequently, since conservation of substance is attained before either conservation of weight or volume and there is no experience that can lead to this concept, it is simply a logical necessity. Furthermore, there are two psychologically distinct kinds of experience: physical experience and logical-mathematical experience. Physical experience consists of acting upon objects and drawing some knowledge about the objects by abstraction from the objects. The act of weighing two objects to determine if they have the same weight would be a physical experience. Weight is a property of objects, such as pebbles. Logical-mathematical experience in contrast is not drawn from objects themselves but is drawn from the actions effected upon the objects. For example, a child discovers that no matter how a certain set of pebbles are arranged and no matter what direction they are counted in, there are always the same number. The pebbles themselves have no order and no sum. Action is necessary to order the pebbles and to make a sum. The child has discovered that the action of putting together is independent of the action of ordering - i.e., a property of the actions, not a property of the pebbles. This is quite another form of experience and marks the beginning of mathematical deduction. The subsequent deduction consists of interiorizing the actions carried out on the pebbles so that they can be combined without the need of pebbles. Before the formal operations stage, the coordination of such actions requires the support of concrete material, but later it leads to logical-mathematical structures in which operations are combined through the use of symbols and earlier logical-mathematical structures are used as a point of departure in thinking about new combinations. The source of logic lies in the total coordination of such actions as joining things together, ordering things, and so on. Logical-mathematical experience, an experience of the actions of an individual, is necessary before there can be operations (Piaget, 1964, pp. 178-180; Harrison, 1969, pp. 97-98).

A third basic factor is that of social transmission, linguistic or educational. Social transmission by itself is not adequate to explain development because in order to receive valuable information via language or education directed by an adult, the child must have a structure which enables him to assimilate this information. Ordinarily a five-year-old, for example, cannot be taught higher mathematics because he does not yet have the structures that would enable him to understand. As another example, the relation "brother of" or "sister of" may mean something different to the young child who sees other children as his brothers and sisters but does not see himself as a "brother of" the other children in the family. The reversibility of the "brother of" or "sister of" relation is not yet understood. The child can think in only one direction (Copeland, 1974, pp. 32-33).

The fourth factor, equilibration, serves to coordinate the other three factors. This factor is fundamental in that an individual engaged in the act of knowing is led to react to compensate for external disturbances so that a state of equilibrium can be reached. The process of equilibration leads to operational reversibility, which is characterized by an equilibrated system in which a transformation in one direction is compensated for by a transformation in the other direction. This active process of self-regulation takes the form of a succession of levels of equilibrium. Levels of equilibrium can be identified according to the probability of the occurrence of various possible forms of compensation. In the development of the ability to conserve quantity, for example, the following levels are distinguished: (1) considering one dimension to the neglect of others is most probable in the beginning, (2) emphasizing the second dimension becomes most likely, having used the first strategy, and (3) oscillating between observed compensating changes in the different dimensions becomes most likely as a result of the preceding strategies. Each level is determined as the most probable given that the preceding level has been reached (Piaget, 1964, pp. 181-182).

ASSESSMENT OF STUDENT COGNITIVE RESPONSE LEVELS

Piaget's theory of cognitive development forms a theoretical base which can be related to mathematics and mathematics instruction. Furthermore, the Piagetian method of asking a child probing questions about a carefully chosen situation has proved very powerful in revealing aspects of the child's thinking about which the teacher needs to know in order to successfully help improve the child's mathematical understanding.

Interviews

The work of Piaget and his associates has provided a rich source of cognitive assessment tasks covering a wide range of contexts, including many relevant to school mathematics learning. More specifically, Piaget has explored in detail the understanding that children have of space. The texts entitled The Child's Conception of Space (Piaget and Inhelder, 1956) and The Child's Conception of Geometry (Piaget, Inhelder and Szeminska, 1960) testify to his comprehensive efforts to explore and research many aspects of children's understanding of space. An example of a Piagetian task which can be linked to curriculum objectives set out in the Elementary Mathematics Curriculum Guide, Alberta Education, 1982, is locating a point in two-dimensional space. To find a point in two dimensions a child is given two sheets of plain white rectangular paper, identical in size. The sheets are placed at opposite corners of a table. On one of the sheets a point, P_1, is marked in red about halfway between the centre of the rectangle and its upper right-hand corner. The child is asked to mark a point on the second sheet in the same position as P_1 on the first, so that if the second sheet is placed on top of the first, the two points will coincide. A thirty-centimetre ruler, an unmarked stick, strips of paper, and lengths of string are provided. Children in the first stage of development respond by making no use whatever of the material provided. Instead of attempting to measure they simply place their point by visual estimate. In the following stage children also use visual estimates, although at this level they use rulers and sticks as aids to perception. Their visual estimate may be fairly accurate, or may involve errors of logic and reversals. At the next stage children do begin to measure, however, they are satisfied with one measurement only, and usually the ruler is laid obliquely from one corner of the rectangle. At first children in this stage take little account of the inclination of their ruler when applying it to the second sheet of paper, but later they do try to preserve its slope as they move from the first sheet to the second. At the onset of the final stage of development children start with a single oblique measurement, but show an increasing realization of the importance of the angle at which this is drawn. Gradually they decompose its inclination and express it in terms of two separate measurements along different axes. For children clearly in the final stage of development, there is no trial-and-error behaviour as they respond by immediately coordinating the two rectangular measurements (Piaget, Inhelder and Szeminska, 1960, pp. 153-169).

Most research has confirmed the existence of the stages of development identified by Piaget; that is, they do occur, and they occur by and large in the order Piaget suggested (Pinard and

Laurendeau, 1964; Suydam and Weaver, 1975; Carpenter, 1980). In a study conducted by Lovell, Healey and Rowland (1962) twelve cognitive assessment tasks (among them Locating a Point in Two-Dimensional Space) taken from The Child's Conception of Geometry, sometimes slightly adapted, were undertaken by a population of English Primary and Educationally Subnormal Special School children. All the tasks were undertaken individually by the children in their own schools, and their replies recorded verbatim. General procedures in administering the tasks and the criteria used in assessing the responses were kept as close as possible to that of the Geneva school. The findings broadly confirmed those of Piaget, Inhelder and Szeminska.

A study of spatial concepts in nearly 200 children between the ages of five and eleven years old, reported by Dodwell (1963), also showed that the types of thinking and problem solving described by Piaget for children of this age range occur quite generally. The investigation was an attempt to verify some of Piaget's conclusions concerning spatial concepts on a fairly large sample of children, although Dodwell states it was not possible to characterize many of the children studied as being in any one of the particular stages of spatial concept development described by Piaget. Dodwell found a greater range and variability of responses than in an earlier investigation of the development of number concepts (Dodwell, 1960, 1961).

Paper-and-Pencil Tests

Previous studies have shown that group paper-and-pencil tests, designed to assess Piagetian levels of response, can produce results that are comparable to Piaget's findings. Tisher (1971) reported the successful use of a Piagetian paper-and-pencil test with 232 junior high pupils whose responses he was able to classify as either concrete operational or formal operational. The test items were twenty-four multiple-choice questions based on four tasks developed by Inhelder and Piaget. He also used the clinical interview technique described by Inhelder and Piaget (1958) to test the students' performance on the same tasks. Tisher reported that the findings from the paper-and-pencil test agreed with those of other researchers and that there was a 77 percent agreement in classification between the two techniques used.

The focus of an investigation conducted by Ankney (1975) was the construction of a paper-and-pencil test for the evaluation of concrete reasoning ability. This study sample consisted of 129 children, aged eight to fourteen. Two instruments were developed and administered. The Piagetian Interview Instrument consisted of five concrete reasoning tasks. The same five concepts, plus five others, were then assessed through a paper-and-pencil test. The interview tasks, as well as the majority of the 30 multiple-choice items on the paper-and-pencil test, were adapted from the Piagetian literature. A significant relationship ($r = 0.63$) was found between performance on the Piagetian Interview Instrument and the paper-and-pencil test. Ankney concluded that although the flexibility of the clinical interview cannot easily be built into an objective test, assessment of concrete stage development does appear amenable to objectification.

Similarly, the purpose of Onslow's study (1976) was to provide a valid and reliable multiple-choice test for determining Piagetian developmental levels in the context of ratio and proportion. This study involved 177 students from Grades 5, 7, 8 and 11. A chi-square test of independence was used to compare the developmental classifications derived from the two assessment techniques Onslow administered: a multiple-choice test and Piagetian clinical interviews. The results indicated that there was a 78 percent agreement in classification between the two techniques used.

In each of the cases cited, comparisons between the paper-and-pencil test ratings and those based on individual interviews established that there was a satisfactory level of agreement between the two modes of assessment. Therefore, the development of a paper-and-pencil test which could be used to determine a student's level of cognitive response in the contexts of geometry and graphing was considered feasible.

Levels of Mental Development in Geometry

A central feature of Piaget's characterization of the development of spatial concepts is his distinction between perceptual and conceptual space. Spatial concepts are internalized actions and not merely mental images of external things or events (Piaget and Inhelder, 1956). A young child might be able to perceive the differences between a circle and a triangle but be unable to deal with these differences conceptually. For example, the child may be unable to represent these differences in a drawing or to distinguish between the figures tactically.

Piaget and Inhelder have described three main series of spatial studies--one dealing with topological concepts, one dealing with projective concepts, and one dealing with Euclidean concepts. They have proposed that certain topological properties like proximity, separation, order, enclosure, and continuity are primitive spatial concepts from which projective and Euclidean concepts emerge. These properties are unaffected by a variety of transformations and, therefore, do not require conservation. In projective space, objects are no longer considered in isolation but rather from particular points of view. Thus, the studies in this series characterize children's growing ability to describe objects viewed from a perspective other than their own. From a Euclidean perspective, space is viewed as a common medium containing objects with well-defined spatial relationships between them. At an operational level, distance, area, and volume are conserved and measurement is possible. In addition to concepts of distance, relations between objects depend on a reference system of horizontal and vertical lines. Hence, for Piaget, the ability to conserve and measure and an understanding of the properties of horizontal and vertical lines signify the emergence of an operational view of Euclidean space.

Another characterization of the development of geometric concepts has been proposed by the van Hieles (Freudenthal, 1973; Carpenter, 1980; Hoffer, 1981). They pick up where Piaget leaves off and describe a developmental sequence culminating in abstract geometric systems. They propose that the development of geometry proceeds through five levels. In Level I, recognition, children perceive geometric figures in global terms. Although they recognize and can reproduce shapes such as squares, rectangles, and parallelograms, they cannot isolate specific attributes of the figures. They are unable to identify relationships between different figures and do not recognize that all squares are rectangles, all rectangles are parallelograms, and so on. This is similar to Piaget's observation that young children have difficulty constructing class hierarchies in general.

At Level II, analysis, children can isolate individual attributes of figures, however, these are established empirically, and the child does not see that certain properties imply that other properties must also be present. For example, at this level children may recognize that the opposite sides of a parallelogram are both parallel and congruent but these properties are simply considered to occur concurrently. The child does not recognize that any quadrilateral with opposite sides congruent must be a parallelogram.

Level III, ordering, is a transitional level between the essentially empirical geometry of the first two levels and the formal systems of the next two (Carpenter, 1980). Students at this stage see that certain properties must follow from others and understand the multiple classification of geometric figures, but the student's ability to use deduction is still limited and requires support from the teacher or textbook. At this level a student will understand why every square is a rectangle but may not be able to explain, for example, why the diagonals of a rectangle are congruent.

At Level IV, deduction, the student understands the significance of deduction and the role of postulates, theorems, and proof. But it is not until Level V, rigor, that an understanding of abstract systems divorced from concrete representations is acquired. This most advanced level is rarely reached by high school students (Hoffer, 1981).

The van Hieles have proposed that there are distinct discontinuities between levels and that the levels cannot be skipped. Unlike Piaget, they have held that the levels develop primarily under the influence of school instruction.

The principal aim of the present study was to develop cognitive assessment material or procedures which would be practical and efficient for classroom use, by classroom teachers. Furthermore, a basic premise of the study was that cognitive assessment materials which can more accurately determine students' cognitive response levels within a specific content domain would make a significant contribution to mathematics education, particularly if a series of such measures were to be developed in different content areas. The levels of mental development proposed by the van Hieles refer exclusively to development in geometry and it is unlikely that most teachers would be familiar with these levels. Therefore, it was decided to draw from the extensive work of Piaget and his colleagues, covering a wide range of contexts relevant to school mathematics to provide the theoretical framework for this study.

CHAPTER 3

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

INTRODUCTION

The purpose of this study was threefold: to provide cognitive assessments of students in Grades 1, 2, and 3, using individual Piagetian-based interviews; to explore the possibility of constructing a paper-and-pencil test in geometry and graphing which could be used to assess student cognitive response levels; and to determine whether such a test could serve as an effective alternative to the individual interviews for assessing student developmental levels. This chapter includes a description of the subjects involved in the study, the interview tasks and paper-and-pencil test developed as sources of data, the methodology used, and the statistical procedures followed in processing the data.

It should also be noted that this study was designed as a part of a larger project entitled Assessing Cognitive Levels in Classrooms (ACLIC) in which cognitive assessment procedures were developed for the whole range of mathematics topics in the elementary school grades.

THE NATURE OF THE SAMPLE

The study presently being reported sampled student cognitive response levels from Grades 1, 2, and 3 in the mathematics topic strands

of geometry and graphing. Students were selected from across the province to ensure that the sample was provincially representative and included a suitable urban/rural balance.

Interview task assessments were made of the responses from 360 Grade 1 to 3 students who were interviewed by 23 Teacher-Interviewers. Grade 3 student response levels were also assessed by means of a paper-and-pencil test which was administered to 112 students.

RESEARCH METHODOLOGY

A detailed search of the Piagetian and neo-Piagetian literature resulted in a collection of interview tasks which were well-suited for identifying levels of student cognitive response, in relation to the key concepts contained in the geometry and graphing strands of the 1982 Alberta Education Elementary Mathematics Curriculum Guide, Grades 1 to 3. Once suitable tasks were identified, they were edited, simplified, and adapted for relatively efficient use by the 23 Teacher-Interviewers. The interview tasks were field-tested by the investigator in two Calgary schools, which led to further refinement of the tasks. A total of four interview tasks were eventually included in the study and these were referred to as "Sorting", "Dot", "Loci", and

"Beads". The interview entitled, "Sorting", had two variations, one to be used with the subjects in Grade 1, the other to be used with subjects in Grades 2 and 3. Summative descriptions of the four interview tasks used are given in the Sources of Data section of this chapter, under the heading "Cognitive Response Assessment: Interviews". Also, the "Interview Record" sheets for each of the interviews are contained in Appendix 1.

The 23 Teacher-Interviewers, who were identified by Alberta Education Regional Mathematics Consultants, Mathematics Supervisors of various school districts, and Superintendents, participated in four full days of workshops presented by the investigator and other ACLIC personnel to prepare them for conducting the interviews. During these workshops the teachers were given opportunities to view and analyze video-taped sample interviews, to discuss the specific interview procedures and criteria for rating the cognitive levels of student responses (as defined on the interview procedure/record sheets included in Appendix 1), and to role-play interview situations using the manipulatives required to conduct each interview. Kits which contained the complete sets of manipulatives and materials necessary for conducting each interview were distributed to the Teacher-Interviewers.

In conducting the actual individual interviews with Grade 1 to 3 students, the Teacher-Interviewers were required to complete an interview procedure/record sheet for each student and record the proceedings of the interview on audio cassette. All the interviews were completed during an eight week time interval between October 16 and December 12. After all the materials and record sheets had been collected, ACLIC personnel completed the task of rating the cognitive levels of student responses.

Grade 3 student response levels were also assessed by means of a paper-and-pencil test which was administered to 112 students. To facilitate the construction of this assessment instrument, relevant sources were searched for paper-and-pencil test items that could be used to identify the levels of student cognitive response. The Grade 3 test was ultimately developed by drawing cognitive assessment items primarily from tests developed by the Australian Council for Educational Research (ACER) (Cornish and Wines, 1978). The items used in the assessment instrument were selected according to their degree of difficulty, topic coverage, and cognitive response elicited. They were considered to be compatible with both the individual interview tasks previously identified and with the objectives specified in the geometry and graphing topic strands of the Alberta Elementary Mathematics Curriculum. The paper-and-pencil test was field-tested in Calgary schools not included in later parts of the study.

The Grade 3 paper-and-pencil assessment instruments were administered by regular classroom teachers. ACLIC personnel collected and scored the tests and rated the cognitive level of student responses according to the criteria outlined in Table 1 in the Sources of Data section of this chapter, under the heading "Cognitive Response Assessment: Paper-and-Pencil Test" (p. 47).

SOURCES OF DATA

Cognitive Response Assessment: Interviews

The interview tasks used in this study to assess student levels of cognitive response in the context of geometry and graphing were as follows:

Task	Interview Topic(s)	Reference Source
Sorting	Classification, Inclusion	Copeland, 1974, pp. 39-40
Dot	Graphing	Piaget, Inhelder and Szeminska, 1960, pp. 153-169
Loci	Loci	Piaget, Inhelder and Szeminska, 1960, pp. 209-225
Beads	Order (Linear, Circular)	Piaget and Inhelder, 1956, pp. 80-103

Sorting - Classification, Inclusion (For Grades 1, 2 and 3). In Task One, a set of geometric solids was placed on the table. The child was asked to sort the objects into two groups so that "the things in each group are alike in some way". Then the child was asked, "How are all the objects in this group alike?" The procedure was repeated but with the request that the objects be sorted in another way. For the second task, the interviewer used a specified set of red and blue cardboard cutouts comprised of squares and circles. After ascertaining that the child could distinguish between the squares and circles, the interviewer asked the two questions: (1) Are all the circles blue? and (2) Are all the blue ones circles? Why? Grade 2 and 3 children were given the additional task of repeating Task One, using large and small, single-thickness attribute shapes.

Children's responses were rated as Preoperational if they showed inconsistency in naming an attribute common to all of the objects in a group and inability to consider an entire group of objects simultaneously in order to name a single common attribute. Early Concrete Operational responses were characterized by one or two different groupings of the objects but with an inability to identify the common attributes of each group. Late Concrete Operational responses clearly showed evidence of flexible thinking in the classification tasks and in the identification of attributes in each of the subgroups formed. In addition, Late Concrete Operational responses included correct use of the concept of inclusion in Task Two, whereas Early Concrete Operational responses interpreted the question "Are all of the circles some of the blues?" as "Are all the circles all the blues?" Preoperational responses in Task Two were characterized by an inability to separate the circles as a class from the whole collection. That is, at this level of response "all" can only mean "the whole set of objects" (Copeland, 1974, pp. 39-40).

Interview record sheets for the task called "Sorting" are included in Appendix 1.

<u>Dot</u> - Graphing (For Grade 3). This interview was concerned with locating a point in two-dimensional space. The materials used in the interview were sheets of plain white rectangular paper, a thirty-centimetre ruler (marked in centimetres only), an unmarked stick, strips of paper, lengths of string, pencils and markers. Two identical

sheets of paper were placed at opposite corners of a table. On one of them a point, P, was marked in red about halfway between the centre of the rectangle and its upper right-hand corner. The child was asked to mark a point on the second sheet in the same position as P on the first, so that if the second sheet were placed on top of the first, the two points would coincide. The children were encouraged to use whichever of the measuring tools they wished. After the first attempt the sheets were superimposed and the results evaluated. Children were then given the opportunity to try again if they wished to do so.

Responses to the task were rated as Preoperational if children made no use of the materials provided and placed their point by visual estimate instead of attempting to measure. They were rated as Early Concrete Operational if the point was located visually and measuring devices were used perceptually and inappropriately. Examples of responses at this level include measuring one distance only, either obliquely from one corner of the rectangle to the point or from one edge to the point. When children discovered the need for two-dimensional measurement, through a process of trial-and-error, their responses were rated as Late Concrete Operational. Finally, when there was no trial-and-error behaviour and the child immediately coordinated the two rectangular measurements, the response was rated as Early Formal Operations (Piaget, Inhelder and Szeminska, 1960, pp. 153-169).

The record sheet for this interview task, called "Dot", is included in Appendix 1.

Loci (For Grades 2 and 3). In the first task of this interview, the child watched as the interviewer marked two points on a blank piece of paper, saying, "Let's imagine that these are two trees. Where can you stand so as to be the same distance from either tree?" The child would indicate the positions with beads first and then would draw all possible positions. Next, the child was asked to do the same for two series of trees lying on lines perpendicular to one another. In the third task, a single dot was marked on a sheet of paper and the child was asked to place beads to show where trees might be planted in order to be the same distance or "just as far" from the dot.

A typical Preoperational response in this interview is characterized by random choice of points without regard for distance. Early Concrete Operational responses are distinguished by one or two solutions estimated perceptually, but fairly accurately, or by various responses produced irregularly and at random. Children responding at the Early Concrete Operational level in Task Three, make no attempt to measure. Typical Late Concrete Operational responses show an "inkling" of the "locus" but these are only achieved as extensions of the method used in placing the first bead and placing the others behind one another in a continuous line following the same direction. There are occasional errors in equidistance due to overemphasis on continuing in a chosen direction and to disregard for considerations of symmetry. Early Formal Operational responses demonstrate reasoning by recurrence as when, for example, after determining a few points in the series, a child concludes that all points in the circle or straight line must have the same property (Piaget, Inhelder and Szeminska, 1960, pp. 209-225).

The record sheet for the interview called "Loci" is included in Appendix 1.

Beads - Linear, Circular Order (For Grades 1 and 2). Initially, the child was shown a linear string of nine vari-coloured beads and asked to arrange a duplicate set of loose beads in the same order. Next the child was asked to arrange the loose beads in the reverse order, and finally, to reproduce in linear order a string of twelve vari-coloured beads that were presented in a "figure 8" pattern.

An inability to coordinate a whole row of beads with a given linear ordering was indicative of a Preoperational response. Correct responses to Tasks One and Two, only, were rated as Early Concrete Operational, marking an ability to arrange in reverse order. Correct responses to all of the tasks included in the interview were rated Late Concrete Operational, as this demonstrated an ability to order in both linear and circular arrangements, requiring the concept of reversibility (Piaget and Inhelder, 1956, pp. 80-103).

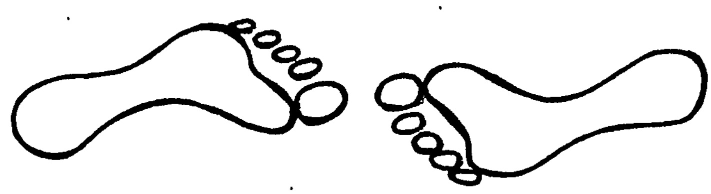
The record sheet for the "Beads" interview is located in Appendix 1.

Cognitive Response Assessment: Paper-and-Pencil Test

Most of the individual items included in the paper-and-pencil test designed for use in this study, were drawn from test materials in the Mathematics Profile Series (MPS) developed by the Australian Council for Educational Research (ACER). The ACER items are based on cognition research by Collis (1972, 1975) and procedures have been established by ACER for connecting item performance with Piagetian levels. Several other items were selected from the Grade 3 Alberta Education Mathematics Achievement Test (1982). Characteristics of the cognitive items of the test are provided in Table 1 which lists the number of cognitive items, the mean item difficulty, the reliability, and the number of students who wrote the test. It should be noted that in the context of the larger provincially-based ACLIC project in which the writer was a team member, it was possible to obtain test item statistics for 202 Grade 3 and Grade 4 students.

In total the test contained 30 items, 25 from the ACER MPS and 5 from the 1982 Alberta Education Grade 3 Mathematics Achievement Test. It calls on students to demonstrate an understanding of order, reversibility, transformations, 3-dimensional visualization, location of objects in a coordinate plane, ability to handle multiple attributes of shapes, and knowledge of symmetry. The following items illustrate the types of questions included in the test:

"These two foot prints were seen on the sand.



They were made by: a) a left foot and a right foot

b) two left feet

c) two right feet"

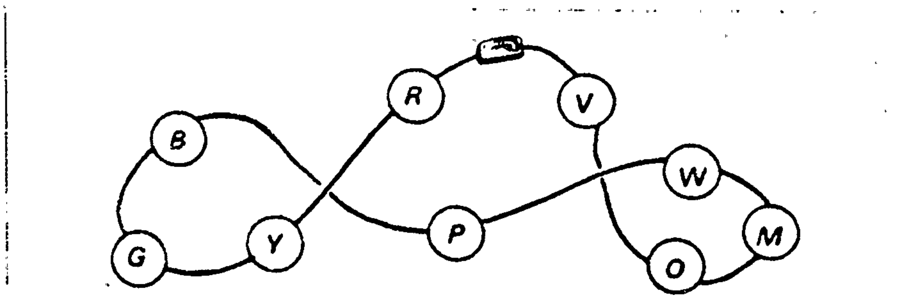
(ACER, 1978, Unit I, p. 4, q. 2)

TABLE 1

PAPER-AND-PENCIL TEST COGNITIVE ITEM CHARACTERISTICS

Number of Items	25
Mean Item Difficulty	0.389
Reliability (Cronbach Alpha)	0.661
Number of Students	202

"If the bead labelled V was threaded first, what was the order for threading the <u>rest</u> of the beads?



a) W,M,O,P,Y,G,B,R

B) W,M,O,P,B,G,Y,R

c) O,M,W,P,Y,G,B,R

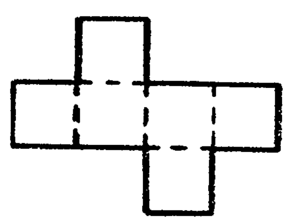
d) O,M,W,P,B,G,Y,R"

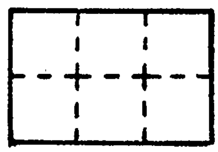
(ACER, 1978, Unit I, p. 4, q.3)

"Which one of the shapes can be folded on the dotted lines to form a cube?"

a)

c)



b) 

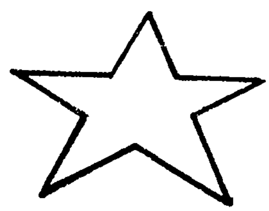
d) 

(ACER, 1978, Unit I, p. 4, q. 4).

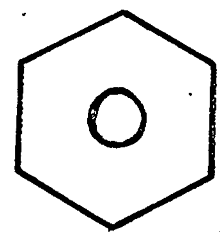
"Steve was asked to feel a shape held behind his back and to describe it. He said, "It has a hole in the middle and I felt <u>at least</u> five corners."

Which of the shapes was it?"

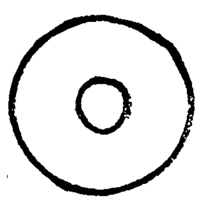
a)



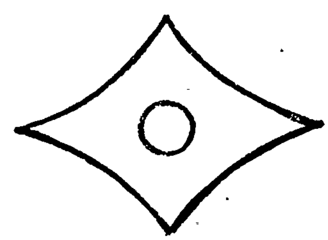
b)



c)



d)



(ACER, 1978, Unit I, p. 6, q. 9)

Table 2 lists the correspondence between raw scores and cognitive level of response ratings on the test. These were derived from tables and charts in the MPS manual (1978). A complete listing of the paper-and-pencil test item characteristics and source references may be found in Appendix 2.

TABLE 2

RAW SCORES CORRESPONDING TO COGNITIVE LEVELS OF RESPONSE

PAPER-AND-PENCIL TEST

Cognitive Level of Response	Raw Score
Items*	25
PO	0-6
EC	7-10
LC	11-16
EF	17-22
F	23-25

*Number of items on cognitive scale; does not include Alberta Achievement Test items

Note: These were derived from tables and charts in the MPS manual (1978).

STATISTICAL PROCEDURE

Data was collected from the individual interviews of the students in Grades 1, 2, and 3, and from the paper-and-pencil test administered to Grade 3 students. The findings were tabulated and presented in the form of percentage distributions of responses found at the Preoperational, Early Concrete Operational, Late Concrete Operational, Early Formal Operational, or Formal Operational levels. A Kolmogorov-Smirnov goodness-of-fit test (Siegel, 1956) was used to assess the comparability of the distribution of cognitive level ratings of Grade 3 pupil responses to the interview tasks and the distribution of the cognitive level ratings produced by the Grade 3 paper-and-pencil test.

CHAPTER 4

ANALYSIS OF DATA

INTRODUCTION

This chapter examines the results obtained from the individual Piagetian-based interviews and the paper-and-pencil test in geometry and graphing, and attempts to demonstrate the feasibility of administering the test as an alternative to the individual interviews for students in Grade 3.

COGNITIVE LEVELS OF STUDENTS' RESPONSES

Cognitive levels of students' responses were observed in interviews in Grades 1, 2, and 3 and were derived from scores obtained on a paper-and-pencil test administered to Grade 3 students. Table 3 provides a detailed account of the number of responses found at the Preoperational, Early Concrete Operational, Late Concrete Operational, Early Formal Operational, or Formal Operational levels, for each of the interview tasks included in the study in Grades 1, 2, and 3. Percentage distributions of the total number of responses found at each of the cognitive levels are also listed by grade level. Similarly, the actual number as well as the percentage distributions of responses observed at the Preoperational, Early Concrete Operational, Late

TABLE 3

DATA SOURCES FOR COGNITIVE LEVELS OF RESPONSE

Grade 1 Interviews	PO	EC	LC	EF	F	Total
BEADS	8	12	10	0	0	30
SORTING (Classification)	45	12	3	0	0	60
(Inclusion)	2	9	49	0	0	60
Interview Totals	55	33	62	0	0	150
	37%	22%	41%	0%	0%	100%
Grade 2 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
BEADS	4	18	35	0	0	57
LOCI (Task 1)	21	33	4	1	0	59
(Task 2)	14	39	3	3	0	59
(Task 3)	10	44	2	3	0	59
SORTING (Classification)	30	27	5	0	0	62
(Inclusion)	2	14	46	0	0	62
Interview Totals	81	175	95	7	0	358
	23%	49%	26%	2%	0%	100%
Grade 3 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
DOT	13	29	12	6	0	60
LOCI (Task 1)	26	21	9	4	0	60
(Task 2)	19	27	9	5	0	60
(Task 3)	12	31	6	11	0	60
SORTING (Classification)	21	25	14	0	0	60
(Inclusion)	2	13	45	0	0	60
Interview Totals	93	146	95	26	0	360
	26%	40%	27%	7%	0%	100%
Grade 3 Paper & Pencil Test	27	48	37	0	0	112
	24%	43%	33%	0%	0%	100%

Concrete Operational, Early Formal Operational, or Formal Operational levels, are provided as observed in the Grade 3 paper-and-pencil data.

Interviews

An objective of this study was to design standardized versions of several suitable Piagetian tasks, creating Piagetian-based interviews which would provide teachers with a means of assessing levels of cognitive response demonstrated in young, primary-grade children. These standardized interviews were designed to be suitable for classroom use in terms of ease of administration and modest time commitments. The successfulness of the four interview tasks adapted and included in this study can be determined by how well the tasks discriminated between students' response levels. As shown in Table 3, the children's responses do not fall within one category, but are appropriately distributed across the levels. As can be expected, very few primary-grade children, 0 in Grade 1, 7 in Grade 2, and only 26 in Grade 3, responded at one of the higher, Early Formal Operational levels in any of the interviews conducted in this study. One task, "Sorting", was administered to students at each of the three grade levels. Preoperational responses were observed for 47 students in Grade 1, 32 students in Grade 2, and 23 students in Grade 3. Although students in each of the three grades responded at the Preoperational level, the greatest number of these responses were recorded for the youngest group of children involved in the study. Furthermore, in all of the tasks reported, at each grade, the greatest concentration of responses were at Concrete Operational levels (Early Concrete and Late Concrete combined). In Grade 1, 55 Preoperational responses were recorded and 95 responses were at Concrete Operational levels. For Grade 2, a total of 81 Preoperational responses were recorded in all the tasks; 270 responses were within the Concrete Operational range. At the Grade 3 level, 93 responses were Preoperational; 241 responses were within the Concrete Operational range. It should be noted that only the two interview tasks, "Beads" and "Sorting", were administered to Grade 1 students, whereas three interview tasks, "Beads", "Loci", and "Sorting", were administered to the Grade 2 students, and the three tasks, "Dot", "Loci" and "Sorting" to the Grade 3 students.

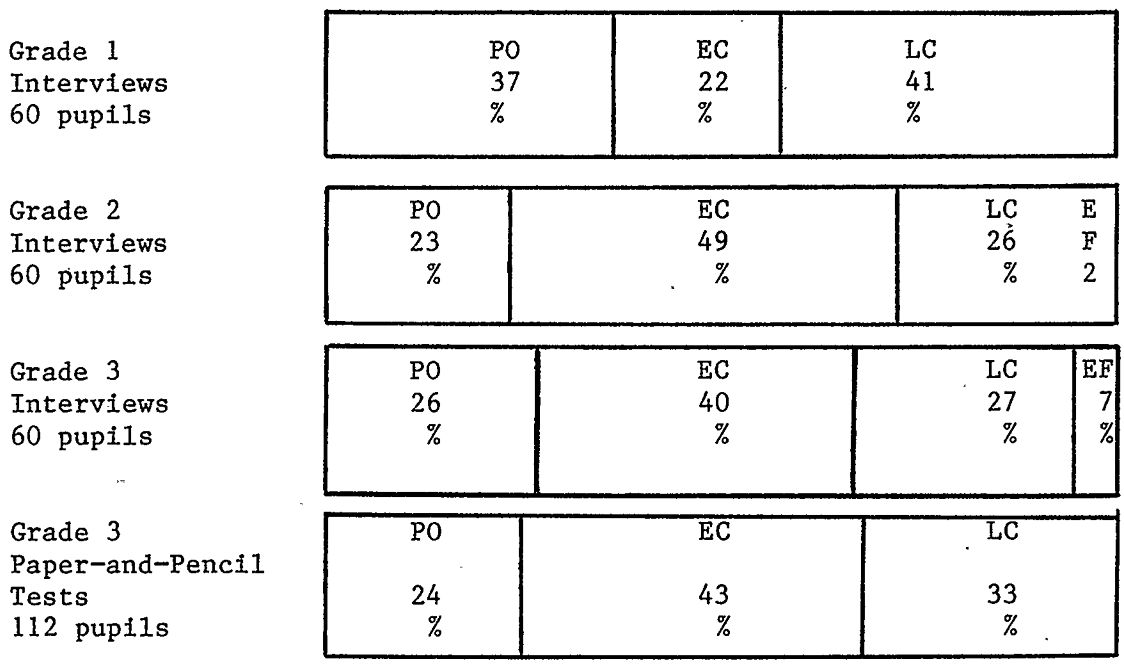
The findings regarding the cognitive levels of students' responses in geometry and graphing in the primary grades are illustrated graphically in Figure 1. The percentage of responses that were Preoperational declined steadily from 37% in Grade 1 to 26% in Grade 3 in the context of individual interviews, while the percentage of Formal Operational responses rose fairly evenly, although almost imperceptibly, from 0 in Grade 1 and 2% in Grade 2 to 7% in Grade 3. Approximately two-thirds or more of the responses were Concrete Operational in the three grades.

An interesting observation that arose from the analysis of the pupils' responses to the Classification-Inclusion tasks (subtasks of the "Sorting" task), was that the additional two-dimensional task assigned to the Grade 2 and 3 children produced more correct responses than did the comparable task with the solids. This raises a question about which is more demanding cognitively in the concrete mode, working with three-dimensional or two-dimensional shapes. The sequence of instruction outlined in the 1982 Alberta Education Elementary

FIGURE 1

COGNITIVE LEVELS OF PUPILS' RESPONSES IN

GEOMETRY AND GRAPHING



Mathematics Curriculum Guide specifies that students in Grade 1 classify three-dimensional objects according to various attributes, and not until Grades 2 and 3 are students required to classify two-dimensional figures as well as three-dimensional objects.

Paper-and-Pencil Test

Another objective of this study was to construct a paper-and-pencil test in geometry and graphing which could be used to determine a student's level of cognitive development. Such a test was developed and administered to 112 Grade 3 students so that the results obtained by administering the paper-and-pencil test could be compared with those obtained using the individual Piagetian-based interviews. An appraisal was made of the effectiveness of the paper-and-pencil test as an alternative to the interviews for assessing student cognitive response levels.

The findings presented in Figure 1 show that the cognitive levels of responses obtained by administering the paper-and-pencil test to Grade 3 students closely parallel those obtained by conducting individual interviews. There are differences of only 2% at the Preoperational level and 3% at the Early Concrete Operational level. Results obtained through the interviews suggest that 27% of the responses were Late Concrete Operational and 7% were Early Formal Operational. According to results obtained by administering the paper-and-pencil test, 33% of students' responses were Late Concrete Operational with no student responses being classified as Early Formal Operational. It is possible that more responses of students completing the paper-and-pencil test were classified at the lower Late Concrete

Operational level because a readability factor may have been involved.

To evaluate the comparability of the cognitive level ratings of the Grade 3 student responses to the Geometry and Graphing interview tasks and to the paper-and-pencil test, a two-sample Kolmogorov-Smirnov test was used to determine whether or not there were significant differences between the two ordered distributions of responses. By assessing the absolute differences between the cummulative proportions of responses, the Kolmogorov-Smirnov procedure tests whether or not the two samples have been drawn from populations with the same distribution. For a given sample size, probabilities can be associated with the occurrence of a difference as large as the observed K-S "D". A decision was made to accept the null hypothesis of no significant difference between the two distributions if the probability of observing the calculated K-S D was greater than 0.05. As shown in Figure 2, the probability associated with the observed K-S D of 0.072 was greater than 0.200. Consequently, the null hypothesis was accepted and it was concluded that the two modes of assessment, interview and paper-and-pencil, were consistent and yielded similar cognitive response level distributions.

FIGURE 2

GRADE 3 INTERVIEWS AND GRADE 3 PAPER-AND-PENCIL

ASSESSMENT CONTRAST

Gr. 3 Interview Ratings 60 pupils	PO 26 %	EC 40 %	LC 27 %	EF 7 %	Interview K-S D* Prob. Dec'n
Gr. 3 Paper-and-Pencil Test 112 pupils	PO 24 %	EC 43 %	LC 33 %		Paper 0.072 K-S D* > 0.200 Prob. Acc.

PO - Preoperational

EC - Early Concerete Operational

LC - Late Concrete Operational

EF - Early Formal Operational

*Kolmogorov-Smirnov Goodness of Fit Test: The cognitive response frequency distributions from the Grade 3 Interviews were compared with those from the Grade 3 Paper-and-Pencil assessment. The probability of observing a Kolmogorov-Smirnov D of 0.072 is greater than 0.200 under the null hypothesis that the two distributions are not different (i.e., that they are drawn from the same population).

CHAPTER 5

SUMMARY AND CONCLUSIONS

RESTATEMENT OF THE PROBLEM

Mathematics educators generally agree that teachers should become better informed of their pupils' abilities or modes of cognitive functioning and endeavour to programme their class work both to capitalize on and extend these. One of the difficulties encountered, however, has been the problem of adequately assessing children's thinking processes. This study explored the use of two modes of assessment which offer a promising approach to assessing children's understanding, interviews and a paper-and-pencil test based on insights from Piagetian research.

Specifically, the purposes of the investigation described in this report were:

1. to assess the levels of cognitive response demonstrated in geometry and graphing by Alberta students in Grades 1, 2, and 3, using standardized, Piagetian-based interviews;

2. to construct and administer a Grade 3 paper-and-pencil assessment in geometry and graphing and determine whether such an instrument could serve as an effective alternative to the individual, Piagetian-based interviews for assessing student developmental levels.

DESCRIPTION OF PROCEDURES USED

Interview task assessments were made of the responses from 360 Grade 1 to 3 students who were interviewed by 23 Teacher-Interviewers. Grade 3 student response levels were also assessed by means of a paper-and-pencil test which was administered to 112 students.

Four interview tasks, " Sorting", "Dot", "Loci", and "Beads", were selected from the rich source of cognitive assessment tasks provided by the work of Piaget and his associates. The tasks, were considered to be highly related to key concepts contained in the geometry and graphing strands of the 1982 Alberta Education Elementary Mathematics Curriculum Guide, Grades 1 to 3. They were edited, simplified, and adapted for standard use by 23 Teacher-Interviewers, who participated in workshops to prepare them for conducting the interviews.

Grade 3 student response levels were also assessed by means of a paper-and-pencil test which was administered to 112 students by their regular classroom teachers. The items used in the test were selected according to their degree of difficulty, topic coverage, and cognitive response elicited, and were considered to be compatible with both the interview tasks previously identified and with the objectives specified in the geometry and graphing topic strands of the Alberta Elementary Mathematics Curriculum. The cognitive assessment items were drawn primarily from tests developed by the Australian Council for Educational Research (ACER) (Cornish and Wines, 1978). The reliability of the cognitive items,using Cronbach's Coefficient Alpha, was 0.661.

The data collected from the individual interviews of the students in Grades 1, 2, and 3, and from the paper-and-pencil test administered to Grade 3 students was tabulated and presented in the form of percentage distributions of responses found at each of the cognitive levels. A Kolmogorov-Smirnov goodness-of-fit test (Siegel, 1956) was used to assess the comparability of the distributions of cognitive level ratings of Grade 3 pupil responses to the interview tasks and that produced by the Grade 3 paper-and-pencil test.

PRINCIPAL FINDINGS AND CONCLUSIONS

Cognitive levels of students' responses were observed in interviews in Grades 1, 2, and 3, and in a paper-and-pencil test administered to Grade 3 students. Generally, about three-quarters of all the student responses were at the Concrete Operational level (Early Concrete and Late Concrete levels combined) in Grades 2 and 3. The remaining quarter were primarily at the Preoperational level. In Grade 1, approximately two-thirds of students' responses were at the Concrete Operational level, the other responses being classified as Preoperational. None of the Grade 1 and very few of the Grade 2 students responded above the Late Concrete Operational level, but the percentage of Early Formal Operational responses increased, although almost imperceptibly, from 2% in Grade 2 to 7% in Grade 3. The findings also showed that the levels of responses obtained by administering the Grade 3 paper-and-pencil test closely parallel those obtained by conducting individual interviews. There were differences of only 2% reported at the Preoperational level and 3% at the Early Concrete Operational level. To evaluate the comparability of the cognitive level ratings of the Grade 3 student responses to the Geometry and Graphing interview tasks and to the paper-and-pencil test, a two-sample Kolmogorov-Smirnov test was used to determine whether or not there were significant differences between the two ordered distributions of responses. A decision was made to accept the null hypothesis of no significant difference between the two distributions if the probability of observing the calculated K-S D was greater than 0.05. The probability associated with the observed K-S D of 0.072 was greater than 0.200. Consequently, the null hypothesis was accepted and it was concluded that the two modes of assessment, interview and paper-and-pencil, were consistent and yielded similar cognitive response level distributions.

The results of this study suggest that it is possible to formulate standardized versions of Piagetian tasks that would provide teachers with a means of assessing levels of cognitive response in young children and that would be appropriate for classroom use in terms of ease of administration and modest time commitments. Furthermore, it is possible to construct a paper-and-pencil assessment for Grade 3 students which could serve as an effective alternative to the individual, Piagetian-based interviews for assessing student developmental levels.

IMPLICATIONS FOR FURTHER RESEARCH

Although the topic and grade levels investigated in this study were limited, the findings reported generally reflect and reaffirm the work of Jean Piaget. Many of the young children involved in this study responded to the various tasks and items presented to them at either Preoperational or Concrete Operational levels. This is a clear indication that many primary school children are not ready to "learn" mathematics by sequentially working through a series of textbook exercises.

Too often mathematics educators have neglected to consider the thought processes of children. The role and the mode of thinking in the young child have been extrapolated from those of the adult. Generally, attempts have been made to make the child fit the mathematics rather than the mathematics fit the child. Piaget's work has given strong evidence and direction for making a good match, between content and child. Considering this, it would be valuable to investigate in greater depth selected topics in each strand of the mathematics curriculum to determine students' response levels at each of the grade levels. A study might also be conducted to determine the cognitive levels of curricular demands throughout the mathematics curriculum. The implication of Piaget's method is that the curriculum should be so arranged as to organize information so that it is within the grasp of the child, once the child has had sufficient opportunity to explore and manipulate it.

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APPENDICES

APPENDIX 1

INTERVIEW PROCEDURE/RECORD SHEETS

SORTING RECORD 1 OF 5

REVISED

INTERVIEW RECORD: SORTING (ABOUT 20 MINUTES)

NAME: BOY/GIRL GRADE:

AGE: BIRTHDAY: SCHOOL:

INTERVIEWER: DATE: CASSETTE: COUNTER:

MATERIALS:

GRADE 1

2 blank sheets of paper
A set of geometric solids including: a sphere, a large cube, a smaller cube, a cone, a long narrow cylinder, a shorter wide cylinder, a long rectangular prism, a pyramid, a triangular pyramid, a long triangular prism
Cardboard cutouts of three red squares, two blue squares and three blue circles.

GRADES 2 AND 3

As for Grade 1, but with the addition of the following attribute blocks:
a large, thin, blue circle; a small, thin, yellow circle; small, thin, red square; small, thin, blue triangle; large,thin, blue rectangle; large, thin, red triangle.

PROCEDURE:

GRADE 1

CLASSIFICATION

A. Mix the objects up and place them on a table before the child. Ask the child to put the objects into 2 groups so that the things in each group are alike in some way. All of the objects must be used. Have him place each group of objects onto a sheet of paper. After the child has sorted them all, point to one of the groups and ask, "How are all the objects in this group alike?"

GROUP ONE: ______________________________

____

CORRECT INCORRECT

GROUP TWO: ______________________________

____

CORRECT INCORRECT

SORTING RECORD 2 OF 5

B. Mix the objects again and ask the child to group them into 2 groups, a different way. When he has finished sorting again ask, "How are all the objects in this group alike?" Then repeat the question for the second group.

GROUP ONE: ______________________________

____

CORRECT INCORRECT

GROUP TWO: ______________________________

____

CORRECT INCORRECT

INCLUSION:

Bring out the red and blue cardboard cutouts and arrange them in a line (e.g., blue square, red square, blue circle, red square, blue square, blue circle, red square, blue circle). Then ask: "What colour is this?" (pointing to one of the figures) "What shape is that?" And then....

"Are all the circles blue?"

Successful Unsuccessful

COMMENTS:

"Are all the blue ones circles?" "Why?"

Successful Unsuccessful

COMMENTS:

SORTING RECORD 3 OF 5
GRADES 2 AND 3

As for Grade 1, but repeat the CLASSIFICATION procedure using the set of attribute blocks described under MATERIALS.

CLASSIFICATION (GEOMETRIC SHAPES), A.

GROUP ONE: ______________________________

____
CORRECT INCORRECT

GROUP TWO: ______________________________

____
CORRECT INCORRECT

CLASSIFICATION (GEOMETRIC SHAPES), B.

GROUP ONE: ______________________________

____
CORRECT INCORRECT

GROUP TWO: ______________________________

____
CORRECT INCORRECT

CLASSIFICATION (ATTRIBUTE BLOCKS), A.

GROUP ONE: ______________________________

____
CORRECT INCORRECT

GROUP TWO: ______________________________

____
CORRECT INCORRECT

CLASSIFICATION (ATTRIBUTE BLOCKS), B.

GROUP ONE: ______________________________

____
CORRECT INCORRECT

GROUP TWO: ______________________________

____
CORRECT INCORRECT

SORTING RECORD 4 OF 5

INCLUSION

"Are all the circles blue?"

Successful Unsuccessful

COMMENTS:

"Are all the blue ones circles?" "Why?"

Successful Unsuccessful

COMMENTS:

IDENTIFICATION OF COGNITIVE LEVEL OF RESPONSE

CLASSIFICATION

PREOPERATIONAL.

Able to sort objects but inconsistent in naming an attribute common to all the objects in a group.

Incapable of considering an entire group of objects simultaneously and of naming a single common attribute.

OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL.

Able to group the objects in only one or two ways. When the objects are mixed up again, unable to sort them into 2 different groups, on request.

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL.

The child's thinking is flexible.

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

SORTING RECORD 5 OF 5

INCLUSION

PREOPERATIONAL.

Only knows what is "seen". ".... cannot mentally separate the circles as a class from the whole series. "All" can only mean .... the whole of the graphic collection" (Copeland, 1974, 39).

OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL

Successfully dissociates squares as a class from circles and red from blues, but not yet able to set up classes based on the logic of inclusion.

Response to "Are all the circles blue?": "No, because there are blue squares." Since the child does not yet have the logical structure required to answer the question "Are all of the circles some of the blues?" the question is interpreted "Are all the circles all the blues?"

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL.

Child can establish logical classes of "circles," "squares," and "blues," considering the entire heterogenous grouping "all" of the shapes, and the circles as "some" of the shapes which are blue. (Copeland, 1974, 39, 40)

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

DOT RECORD 1 OF 2

INTERVIEW RECORD: DOT (ABOUT 15 MINUTES)

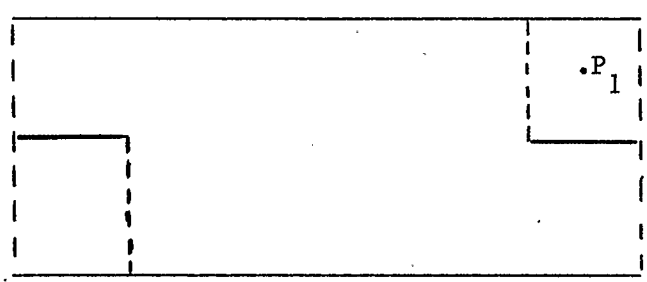
NAME: BOY/GIRL GRADE:

AGE: BIRTHDAY: SCHOOL:

INTERVIEWER: DATE: CASSETTE: COUNTER:

MATERIAL: sheets of plan white rectangular paper, a thirty centimetre ruler (marked in cm only), an unmarked stick, strips of paper, lengths of string.

PROCEDURES: Give the child two identical sheets of plain white rectangular paper, placing them at opposite corners of the table, as shown below. On one of the sheets mark a point, P_1, in red about halfway between the centre of the rectangle and its upper right-hand corner. Ask the child to mark a point on the second sheet in the same position as P_1 has on the first sheet .... so that if the second sheet is placed on top of the first, the two points will be in the same place.



COGNITIVE LEVELS OF RESPONSE

PREOPERATIONAL:

Children make no use whatever of the material provided. Instead of attempting to measure, they place their point by visual estimate.

OTHER PREOPERATIONAL RESPONSE:

DOT RECORD 2 OF 2

EARLY CONCRETE OPERATIONAL:

The point is located visually. Measuring devices are used perceptually and inappropriately.

Beginnings of measurement - however, measurement is one-dimensional. Oblique measurement is common from a corner of the rectangle.

OTHER EARLY CONCRETE OPERATIONAL RESPONSE:

LATE CONCRETE OPERATIONAL:

Empirical discovery of two-dimensional measurement. (Trial-and-error).

OTHER LATE CONCRETE OPERATIONAL RESPONSE:

EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS)

There is no trial-and-error behavior; the child immediately coordinates the two rectangular measurements.

OTHER EARLY FORMAL OPERATIONAL RESPONSE (CONCRETE GENERALIZATION):

LOCI RECORD 1 OF 2

INTERVIEW RECORD: LOCI (ABOUT 15 MINUTES)

NAME: BOY/GIRL GRADE:

AGE: BIRTHDAY: SCHOOL:

INTERVIEWER: DATE: CASSETTE: COUNTER:

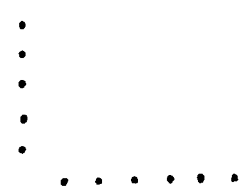
MATERIALS: a set of beads, ruler, pencil, sheets of paper, felt pen

PROCEDURE:

TASK 1: Begin with a blank sheet of paper. As the child watches, mark two points on the sheet, saying, "Let's imagine that this is a tree, and this is another tree." (The points should not be centred on the sheet.) "Where can you stand so as to be the same distance (or 'just as far') from either tree?" (Ask the child to indicate the position with a bead.) "Are there any other points?" After the questions have been answered in this form, remove bead(s) from paper and ask the child to draw all possible positions. If the child says that the beads are on a line, ask whether the points can touch, and how far the line can go.

TASK 2: The problem is extended to cover equidistance from several points A and A^1, B and B^1, etc., where points, A, B, C and D lie in a straight line, and points A^1, B^1, C^1 and D^1 lie at corresponding distances in another line at right angles to it. (The locus of points equidistant from A and A^1, etc. is then the bisector of the angle.)

As the child watches, mark the points on the sheet, saying, "This is a row of trees and here is another row of trees. Where can you stand so as to be the same distance form (or 'just as far') from either row?"



TASK 3: A single dot is marked on a sheet of paper. Ask the child to show where a set of beads should be placed, or where a series of trees should stand, in order to be the same distance, or "just as far", from the dot.

LOCI RECORD 2 OF 2
IDENTIFICATION OF COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL.

Task 1. The child indicates a point at random without regard for distances involved.
Task 2.
Task 3.

COMMENTS:

EARLY CONCRETE OPERATIONAL.

Task 1: The child finds one solution, the midpoint estimated perceptually, but fairly accurately, or a few other points nearby.
Task 2: The child only considers two of the points or produces irregular and random intervals between various points.
Task 3: The child arranges the beads either in a row or else in an irregular ring around the point without any attempt to measure (without discovering for each point the point which is symmetrical to it in relation to the centre).

COMMENTS:

LATE CONCRETE OPERATIONAL.

Task 1. The child shows an inkling of the "locus" but this is achieved by extending the method used in
Task 2. placing the first (central bead) and placing one bead behind another in a continuous line
Task 3. following the same direction. There are occasional errors in equidistance and these due to over emphasis on continuing in a chosen direction, to the neglect of a careful return to the point of departure, i.e., no thought is given to symmetry.

COMMENTS:

EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS).

Task 1. The most important achievement at this stage is reasoning by recurrence. The child determines a few
Task 2. points in the series and immediately concludes that all points on the circle or straight line must
Task 3. have the same property.

COMMENTS:

BEADS RECORD 1 OF 2

INTERVIEW RECORD: BEADS (ABOUT 15 MINUTES)

Linear and Circular Order (Piaget, 1956, 80-103)

NAME: BOY/GIRL GRADE:

AGE: BIRTHDAY: SCHOOL:

INTERVIEWER: DATE: CASSETTE: COUNTER:

MATERIALS: a string of nine vari-coloured beads arranged in a circle
a string of nine vari-coloured beads arranged in a simple linear order
a string of twelve vari-coloured beads arranged in a figure 8 pattern
lengths of string, loose vari-coloured beads

PROCEDURES:

TASK 1: Transposition of circular into simple linear order making the linear order correspond to beads arranged in a circular loop. "What would the necklace look like if it were in a straight line?"

Successful Unsuccessful

COMMENTS:

TASK 2: Establishment of reverse order. Show the child a set of beads arranged in a a row. Ask the child to arrange his own row of beads in the reverse order to that shown. (Can you start from the other end?)

Successful Unsuccessful

COMMENTS:

TASK 3: Transportation of a figure 8 pattern into linear order. Reproducing a string of beads arranged in a figure 8 pattern, in simple linear order.

Successful Unsuccessful

COMMENTS:

BEADS RECORD 2 of 2

COGNITIVE LEVEL OF RESPONSES

PREOPERATIONAL.

Unable to make another row of beads in the same order. May arrange 2 beads in order correctly but unable to coordinate the whole sequence of beads into a given simple linear order.

OTHER PREOPERATIONAL RESPONSE:

EARLY CONCRETE OPERATIONAL.

Unable to transpose the circular order to a linear order and unable to make a row in reverse order. Toward the end of the EARLY CONCRETE OPERATIONAL Period a child may be able to reverse the order, but it is a trial-and-error process (Often loses track after centre of row, making the last half a copy of the model instead of its reverse).

OTHER EARLY CONCRETE OPERATIONAL RESPONSE:

LATE CONCRETE OPERATIONAL.

Solves the problems quickly and with ease. Can reverse the order and correctly consider the intertwining relationship that exists in the figure 8 form.

OTHER LATE CONCRETE OPERATIONAL RESPONSE:

APPENDIX 2

PAPER-AND-PENCIL TEST ITEM CHARACTERISTICS AND SOURCE REFERENCES

PAPER-AND-PENCIL TEST ITEM CHARACTERISTICS

AND SOURCE REFERENCES

TEST ITEM #	SOURCE	MATHEMATICAL CONTEXT	TEST *P value (%) Gr 3	BRYTES	COG-NITIVE LEVEL	ALBERTA EDUCATION P value (%)
1	1,UI,2	Spatial Orientation	58	41.8	LC	
2	1,UI,3	Ordering	62	42.5	LC	
3	1,UI,4	Space Orientation	50	44.0	LC	
4	1,UI,5	Shape Iteration	17	44.1	LC	
5	1,UI,6	R-L Orientation	47	44.7	LC	
6	1,UI,7	Graphing Ordered Pair	58	44.9	LC	
7	1,UI,9	Properties-Shapes	60	45.7	LC	
8	1,UI,15	Reflection-Detail	58	47.6	LC	
9	1,UI,17	Spatial Orientation	15	48.5	LC	
10	1,UI,19	Form Perception	45	49.1	LC	
11	1,UI,22	Form Perception	23	49.0	LC	
12	1,UI,26	Multiple Steps	20	49.8	LC	
13	1,UII,2	Volume-Iteration	33	44.8	LC	
14	1,UII,3	Rotation	33	45.9	LC	
15	1,UII,6	Rotation	12	48.5	LC	
16	1,UII,7	Properties of Shape	49	48.7	LC	
17	1,UII,10	Seriation	58	46.1	LC	
18	1,UII,11	Transformations	40	50.2	EF	
19	1,UII,12	Conservation-Length	26	50.3	EF	
20	1,UII,24	3-D Space Perception	17	53.2	EF	
21	1,UII,31	Space Perception	02	57.3	EF	
22	1,UIII,2	Rotation	20	48.4	LC	
23	1,UIII,3	Volume Iteration	24	48.5	LC	
24	1,UIII,6	Comparison-Area	29	50.1	EF	
25	1,UIV,24	Symmetry	26	59.9	EF	
26	2 20	3-D Space Perception	43			48
27	2 21	Volume-Iteration	44			44
28	2 23	Graphing-Interpretation	76			80
29	2 25	Coordinates-Ordered Pairs	25			62
30	2 24	Coordinates-Ordered Pairs	37			66

1. ACER MPS SPACE
2. Alberta Education Grade 3 Achievement Test

* P value = Probability of Success