The University of Calgary

PERFORMANCE ANALYSIS OF RADIAL DISTRIBUTION SYSTEMS WITH UNBALANCED LOADS

by

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ABSTRACT

A method for the analysis of unsymmetrical radial distribution systems containing unbalanced loads is proposed. Detailed models and appropriate performance equations are developed for loads, distribution lines, single-phase and three-phase transformers, regulating transformers, and capacitor banks. A comparison of the proposed method's results with theoretical results is presented for single-phase and three-phase uniformly loaded lines, and a sample system containing transformers employing a number of different connections is analyzed. A brief discussion of the convergence properties of the proposed technique is presented.

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Variables

A	transformer input admittance matrix
B	transformer current transfer matrix
C	capacitance
C	transformer voltage transfer matrix
D	deviation of the voltage magnitude from 1.0 per unit.
D	transformer output impedance matrix
E	ideal transformer winding voltage
H	transformer constant defined in Equation (6.13)
I, **I**	current scalar, vector
k	load constant (Chapter 4) or eigenvector coefficient (Chapter 9)
l	load constant
L	tap changer raise/lower limit
m	load constant
n	load constant
N	transformer winding turns (Chapter 6) or number of buses (Chapter 9)
P	real power
Q	reactive power
r	transformer secondary-to-primary turns ratio
R	resistance
s	line length
S	complex power

t	number of taps
u	constant defined in Equation (6.75)
$\mathbf{v}$	eigenvector
V, $\mathbf{V}$	voltage scalar, vector
W	regulator voltage bandwidth
x	distance along a line from the receiving end
X	reactance
y_1, y_2	transformer constants defined in Equations (6.70) and (6.71)
Y, $\mathbf{Y}$	admittance scalar, matrix
Z, $\mathbf{Z}$	impedance scalar, matrix
α	damping coefficient
ε	convergence tolerance
ζ	line constant defined in Equation (9.13)
λ	eigenvalue
ω	angular power line frequency

<u>Subscripts</u>

A	actual
B	series branch
C	capacitor
d	desired
E	shunt element
G	grounded-wye
H	transformer high voltage side

L	line
LDC	line drop compensator
ℓ	load segment
M	magnetizing
max	maximum
min	minimum
O	open delta
P	transformer primary
r	rated
R	receiving end
S	sending end
T	total
U	upstream
X	low voltage side
Y	ungrounded-wye
β	usual bus index
Δ	delta
ξ	iteration number
ρ	regulating point

Superscripts

a,b,c	phase designators
g	ground
n	neutral

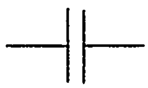
T	transpose
1,2	phase designators (single-phase, two- or three-wire buses)
σ	load segment designator
ϕ	general phase designator
ψ	general phase designator (single-phase, two- or three-wire buses)
*	complex conjugate

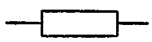
Other Symbols

Im{z}	imaginary part of the complex number z
Re{z}	real part of the complex number z
$\lvert x \rvert$	absolute value of the real number x
$\lVert z \rVert$	magnitude of the complex number z
{I}	set of all system currents
{V}	set of all system voltages

LIST OF GRAPHIC SYMBOLS

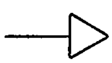
bus number

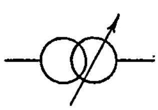

capacitor


general element

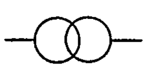
ideal generator

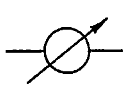

inductance


load


load tap changing transformer


resistance or impedance


transformer


voltage regulator (single-winding autotransformer)

1.0 INTRODUCTION

1.1 General Remarks

The successful development and operation of an electric distribution system requires that, among other things, distribution engineers possess the ability to carry out load flow studies on radial feeders. The information provided by such studies is needed to ensure the adequacy of system equipment, the maintenance of system voltages within specified tolerances, the continued operation of the system under abnormal or emergency conditions, and the minimization of power and energy losses. In order for its outputs to be useful, the load flow method used must recognize that topological and loading-induced unbalances, as well as components and loads with widely varying electrical characteristics, are encountered on distribution systems.

The two methods most frequently used to perform load flow calculations on large transmission and subtransmission networks are the Gauss-Seidel and Newton-Raphson techniques. Both of these have been employed in the analysis of radial distribution systems as well. The Gauss-Seidel method is used by Gregoire, Savulescu, and Trecat [13], Kersting and Seeker [26], and Sun et al. [51]. Given the diversity in impedances encountered in distribution systems, however, this method usually converges slowly, and sometimes the iterations diverge [25, 29]. The Newton-Raphson method, which Fish and Coleman [10], Wortman, Allen, and Grigsby [55], and others have applied to small radial systems, does not normally suffer from numerical instability problems, but it does require a large amount of storage when using a three-phase formulation, and the simplifying assumption of a decoupled Jacobian matrix is not applicable in the distribution case [51]. The storage requirements for both matrix methods can be reduced by employing sparse matrix techniques [21], but in any case the problem size and the difficulty of formulation rise as the size of the system and the sophistication of the component representations increase.

In an attempt to avoid some of the difficulties associated with the methods that use full-system impedance or admittance matrices, several other solution techniques have been developed. Kersting and Mendive [25] suggested a procedure based on ladder network theory, but their method is complicated by the presence of laterals, which require iterations within iterations. A network admittance reduction technique was proposed by Berg, Hawkins, and Pleines [2], but the authors assume the loads to be balanced and to have constant admittance or constant power characteristics. Rustay and Gajjar [44] also use an admittance reduction procedure, but their technique (which is also applicable at frequencies other than the fundamental) allows only constant admittance loads. Several other techniques have also been proposed, including a linearized method used by Juricek, Fukutome, and Chen [24] (for feeder planning rather than detailed load flows), and a voltage iteration method suggested by Koval [29] which assumes a balanced system.

The basic purpose of the work presented herein is the development of a load flow method, specifically for radial distribution feeders, that successfully accommodates the unbalance and load variety mentioned above, and that eliminates some of the difficulties associated with other methods. An attempt has been made to devise a solution technique that combines detailed, accurate component models with a straightforward and readily programmable computational algorithm. The result, for reasons that will become apparent later, is called the Current/Voltage Iteration Method (CVIM).

1.2 Chapter Outlines

The following is a brief summary of the contents of each of the remaining chapters.

Chapter 2: A brief description of electric distribution systems is presented, and the primary feeders and secondary circuits to be studied using the CVIM are described. Some of the terminology and notation used in subsequent chapters is introduced.

Chapter 3: The principles of the CVIM are established by way of a simple single-phase example. The generalization of the method to realistic three-phase systems is discussed.

Chapter 4: The modeling of distribution system loads is discussed, and the concept of a load segment is introduced. Grounded-wye, ungrounded-wye, and delta load segment connections are examined.

Chapter 5: Based on familiar models, CVIM equations are developed for short and intermediate distribution lines. Long lines are discussed briefly.

Chapter 6: Detailed models for single-phase and three-phase distribution transformers are presented. The work in this chapter is required for two reasons: relatively little attention is paid to transformer modeling in most load flow papers, and the transformer modeling that is done (see, for example, Chen and Dillon [7], Dillon and Chen [9], and Roy and Rao [43]) usually results in a formulation of performance equations unlike that required by the CVIM.

Chapter 7: An examination of regulating and load tap changing transformers is presented. Performance equations that are closely related to the transformer performance equations developed in Chapter 6 are derived, and substation voltage regulation is discussed briefly.

Chapter 8: A short, simple development of the equations governing the behaviour of shunt capacitor banks is given.

Chapter 9: Several examples of applications of the CVIM are given, the main objective being to demonstrate the viability of the method. In three cases, a comparison of CVIM results with theoretical results is provided. An example illustrating the effect of transformer connections on load flow is presented, and there is a short discussion of the convergence properties of the CVIM.

Chapter 10: The work of the previous chapters is summarized, and possible directions for future research involving the CVIM are suggested.

1.3 A Note on Units

The per unit system is frequently employed in power system analysis because of the simplifications it introduces. On distribution feeders, however, the advantages of using per unit quantities are not as great due to the necessity of keeping track of the large number of secondary base quantities possible. This, coupled with the fact that turns ratios are maintained explicitly in all transformer equations developed herein, thereby disallowing the often-used simplification of eliminating the ideal windings present in transformer models, suggests the use of physical quantities (volts, amperes, etc.), rather than per unit ones, in performance equations. The per unit system will, however, be used in the presentation of results, since it still has advantages when used for that purpose.

2.0 DISTRIBUTION SYSTEMS

2.1 Introduction

The purpose of this chapter is to provide a general description of electric distribution systems. A brief overview is given in Section 2.2 to illustrate the role played by distribution systems in the overall electric energy supply process, and to introduce the components commonly found on such systems. Primary feeders and secondary circuits are considered in further detail in Section 2.3. In Section 2.4 an individual feeder section is examined; the terminology and notation to be used in subsequent chapters is defined, and the description of the systems to be studied using the CVIM is completed.

2.2 Overview

An electric distribution system is usually defined to be all that part of an electric power system between the bulk power substations and the consumers' service connections. Thus, as illustrated in Figure 2.01, the distribution system forms the final link in the chain of systems that delivers electrical energy from the point of generation to the point of consumption.

Distribution systems are constructed using the following components:

- bulk power substations
- subtransmission circuits
- distribution substations

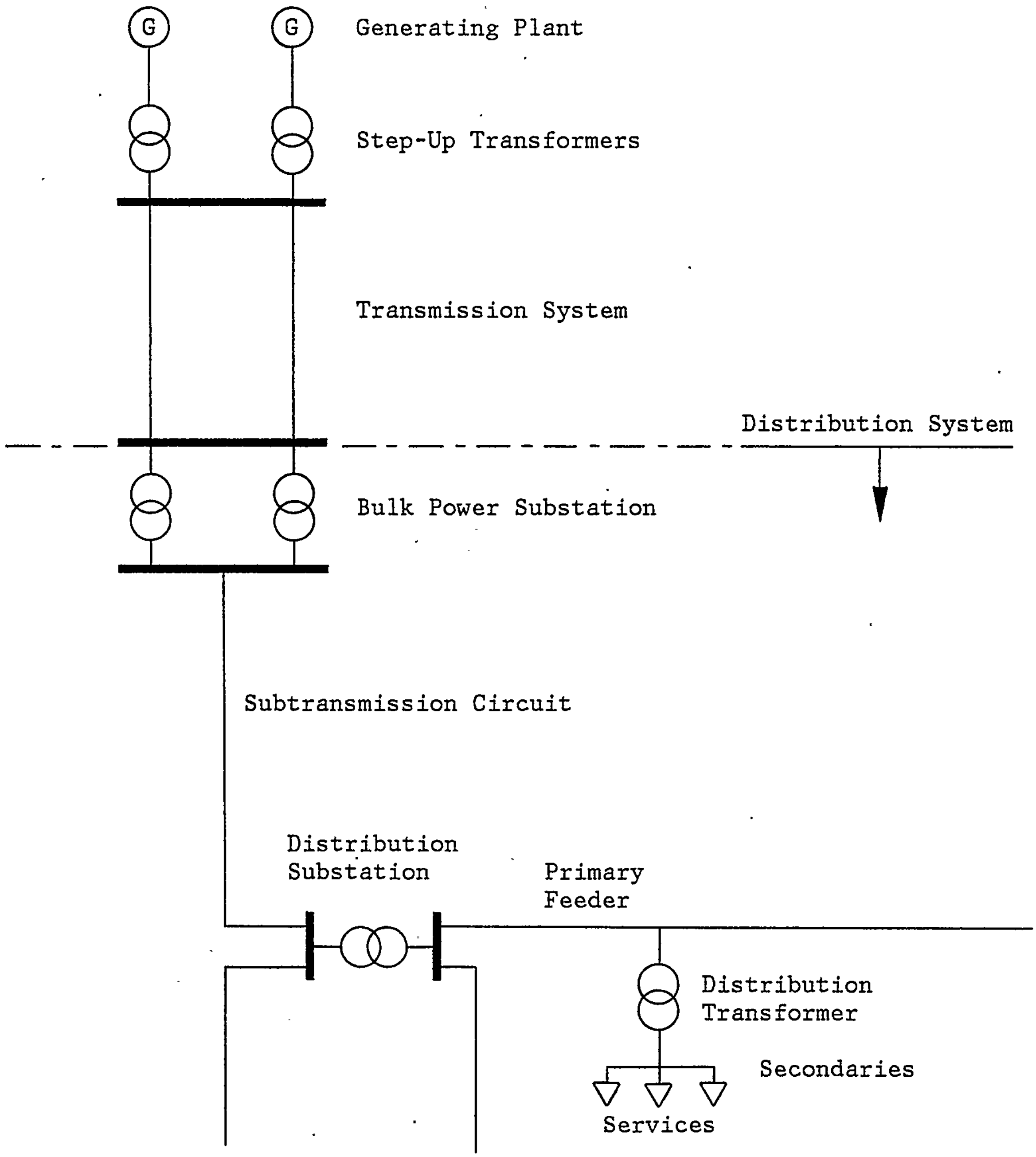


Figure 2.01. The functional components of an electric power system.

- primary feeders
- distribution transformers
- secondaries and services

These components are found on virtually all distribution systems, regardless of classification by the type of load area served (rural, residential, commercial, or industrial), the type of construction employed (underground or overhead), or the design of the components themselves (e.g., radial or non-radial secondary circuits).

The first components encountered on a distribution system are the bulk power substations. They are located in or near the area to be served by the system, and may be supplied either directly by a local generating station, or indirectly by remote generating stations via high-voltage (HV) transmission lines. After the bulk power substations have transformed the voltage from the generation or transmission level to the subtransmission level,[1] subtransmission circuits carry the electrical energy to distribution substations.

Distribution substations serve two principal functions: they receive power from the subtransmission network and distribute it to several primary feeders, and they reduce the voltage from its subtransmission level to its

---

[1]Generation voltages typically lie in the 12 to 30 kV range. Transmission voltages between 220 and 500 kV are common. Subtransmission voltages generally range from 12.47 to 245 kV, with 69, 115, and 138 kV being the most common values.

primary feeder level.[2] Each substation normally serves its own load area, which is a subdivision of the area served by the distribution system as a whole. The equipment used at a distribution substation includes one or more power transformers, buses, and switchgear, as well as voltage-regulating, control, and monitoring devices.

The part of the distribution system of particular interest in this project is that which begins at the low-voltage (LV) bus of the distribution substation, where power flows into primary feeders. Each feeder, which normally consists of a three-phase main feeder, three-phase subfeeders, and single-phase laterals, generally serves an area that is a further subdivision of the area served by its substation.

The last components encountered on the distribution system are the utility customers' service entrances, to which power is delivered by primary feeders via distribution transformers. The transformers supply one or more customers through a secondary circuit after reducing the voltage from its primary feeder level to one of several utilization levels, which generally lie between 120 and 600 V.

2.3 Primary Feeders and Secondary Circuits

Both primary feeders and secondary circuits can be classified as either radial or non-radial circuits. A radial circuit is one that, except for possible multiple current paths within individual devices (such as

---

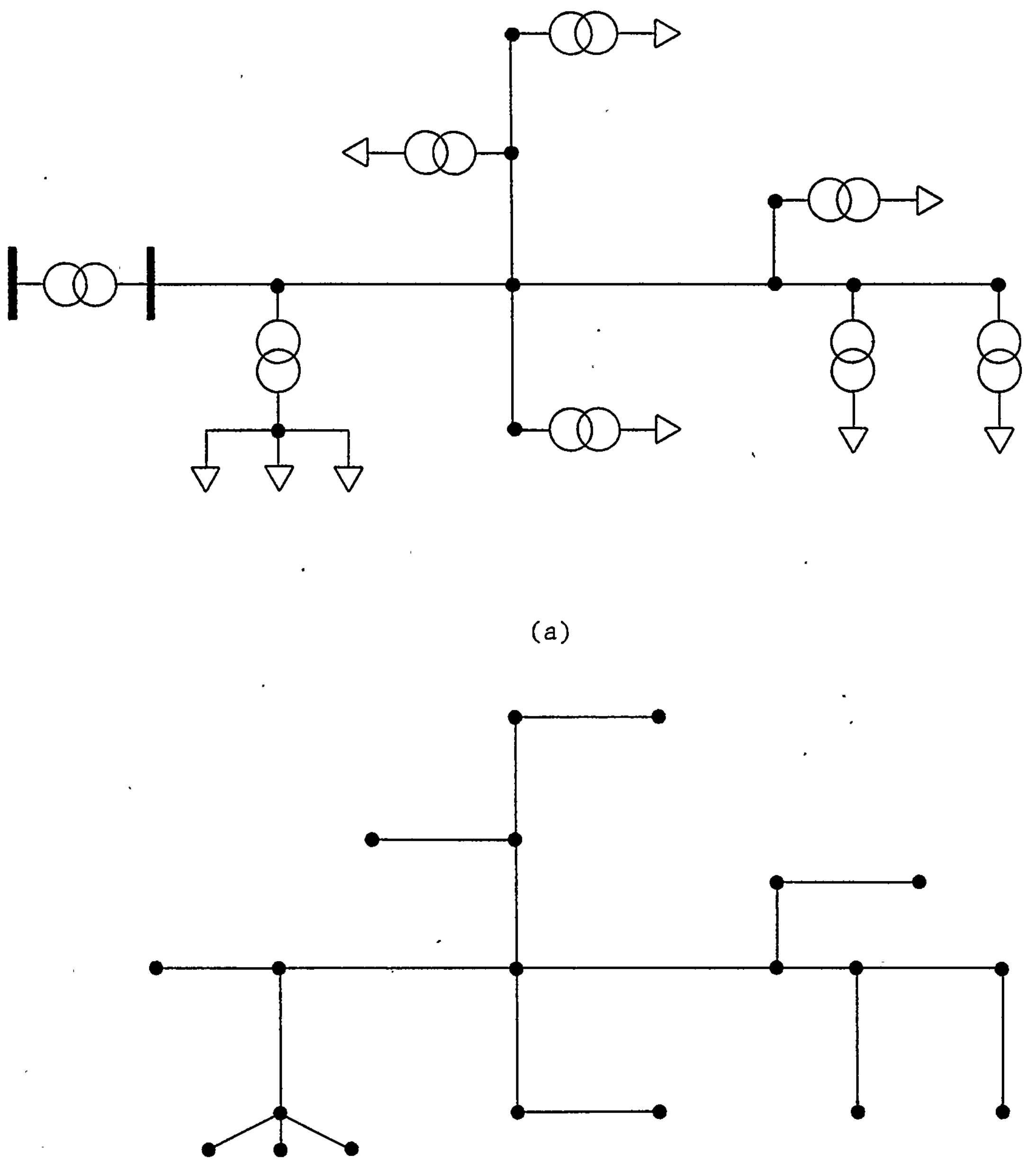
[2]Primary feeder voltages typically lie between 4.16 and 34.5 kV, with voltages in the 15 kV class prevailing.

delta-wye transformers), provides but a single current path between any pair of points Ψ and Π, where Ψ and Π lie anywhere on the circuit other than on a neutral conductor or ground. Stated another way, a circuit is radial if the graph corresponding to its single-line diagram is a tree (see Figure 2.02). A non-radial system provides multiple paths between at least some pairs of circuit points.

In practice, the vast majority of primary feeders are three-phase, four-wire, multi-grounded radial systems. The three-phase sections contain three phase wires and a neutral conductor, while the single-phase laterals are composed of one phase conductor and one neutral conductor. Some sections may carry the neutral wire and two of the three phase wires. The neutral, which is common to both the primary and secondary circuits, is multi-grounded, i.e., it is grounded at each distribution transformer, at frequent intervals where no transformers are present, and to water pipes or driven ground rods at each customer's service entrance.

Not all feeders are of the four-wire multi-grounded type; some use three phase wires only. In these cases, single-phase laterals are composed of two phase wires rather than a phase wire and a neutral wire. Since the equations governing the behaviour of three-wire systems can be derived relatively simply from those presented herein for four-wire systems, the former will not be discussed further.

Whereas primary feeders are almost always radial, there are several secondary circuit arrangements in general use. They are: (1) direct connection; (2) straight radial; (3) secondary banking; and (4) secondary



(a)

(b)

Figure 2.02. A simple radial feeder: (a) the single-line diagram; (b) the corresponding graph.

network. The direct connection system, which is often used in underground residential distribution areas, is illustrated in Figure 2.03a; it is clearly radial because there is only one power flow path from the transformer to the customer's load. The straight radial system, which is very common in residential, light commercial, rural, and industrial areas served by overhead distribution, is illustrated in Figure 2.03b. It is also radial by the definition given above.

A distribution circuit whose primary and secondary systems are both radial will be called "strictly radial." Thus, any circuit using either of the secondary arrangements just discussed, and having a radial primary feeder, is strictly radial. It will be seen in the next chapter that the CVIM can be applied only to strictly radial circuits.

Secondary banking (illustrated in Figure 2.03c) is occasionally used in residential and light commercial areas where there are many services close to each other, where the required spacing between transformers is small, where flicker-producing loads are to be served, or where flexibility is required to accept new loads. Secondary banking differs from low-voltage ac networks (discussed below) in that one primary circuit supplies all transformers where secondaries are banked together, whereas different circuits supply adjacent transformers in a low-voltage ac network. The secondary banking system allows power to flow to a load through two or more transformers, and is therefore not radial. The use of secondary banking is not as common as it once was.

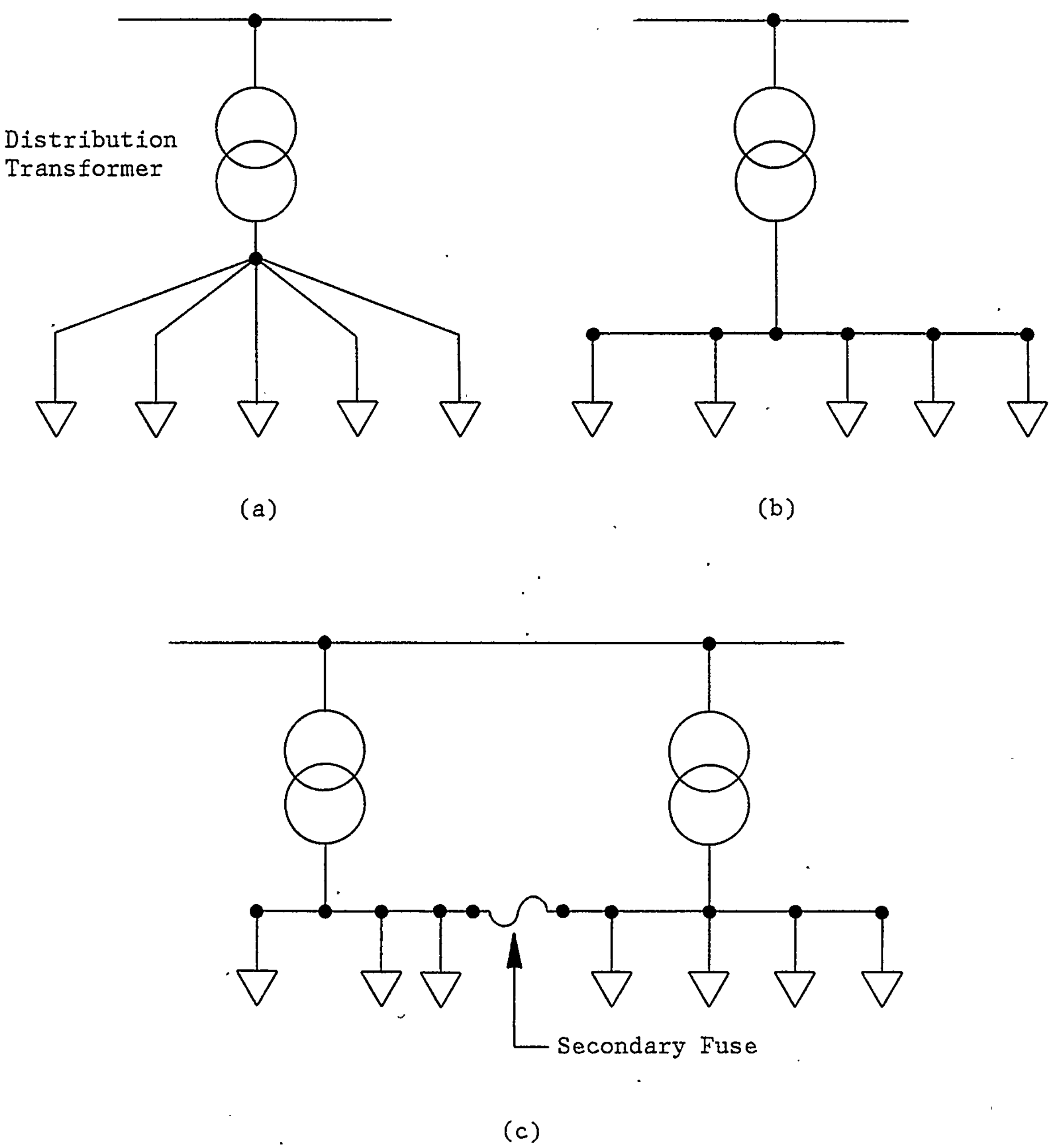


Figure 2.03. The various types of secondary distribution circuits: (a) direct connection; (b) straight radial; (c) secondary banking; (d) network (next page).

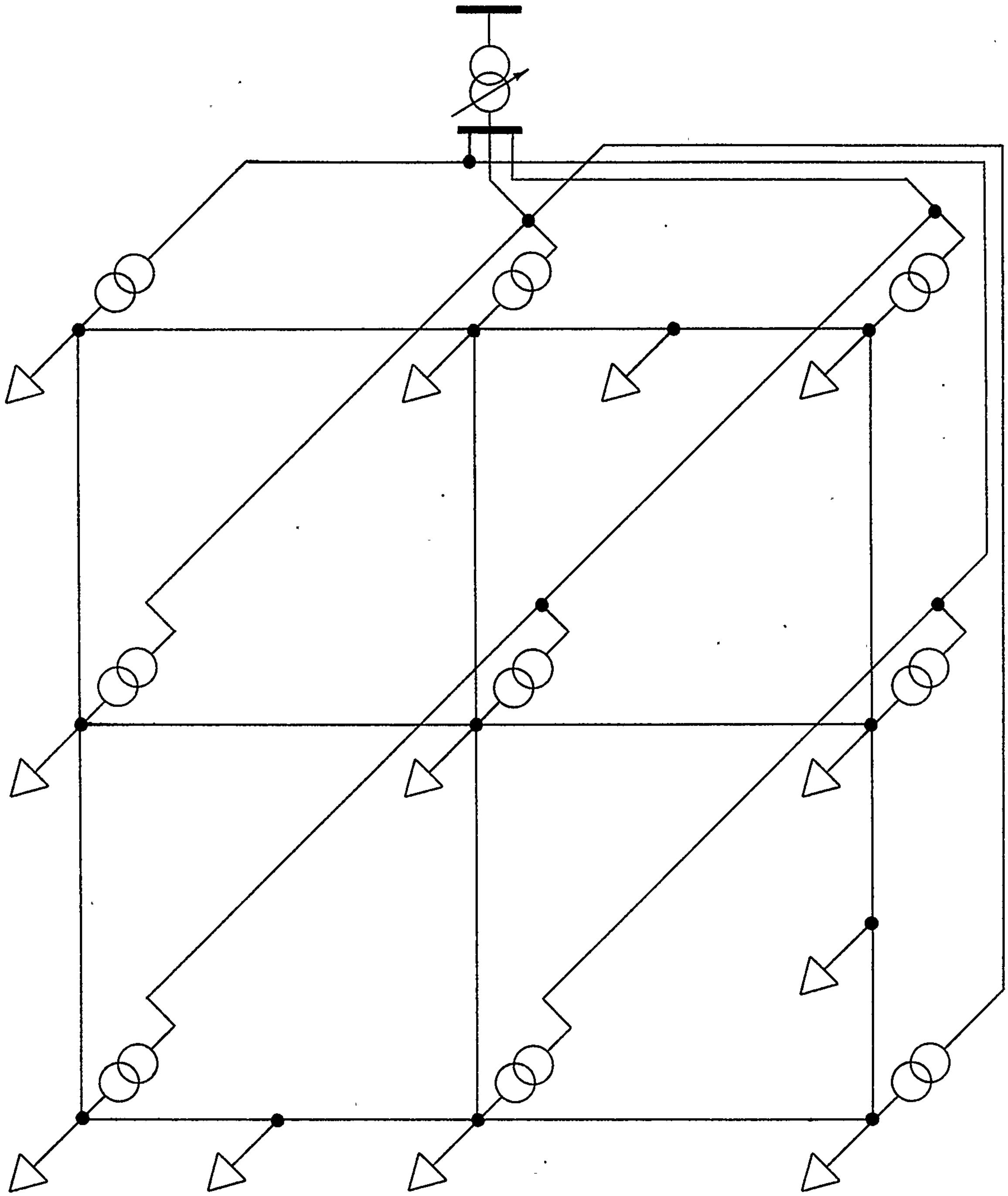


Figure 2.03, part (d).

Secondary network systems are predominant in areas with extremely large load densities, such as the downtown cores of major cities. Such a system is shown in Figure 2.03d. Here, as with secondary banking, there are multiple power flow paths to most loads. Therefore, this system is not radial, and some method other than the CVIM must be used to calculate the power flows.

2.4 Feeder Sections

A strictly radial distribution feeder[3] is shown in Figure 2.04. Details are shown only for the distribution substation (whose LV bus is the feeder source bus) and for a single feeder section consisting of bus β, its associated elements, and bus γ.

On a strictly radial system, bus κ is said to be "upstream" from bus β if any current flowing from the source to β must first pass through κ. Bus β is then "downstream" from bus κ. If bus κ is "immediately upstream" from bus β (a relationship denoted by $\kappa = U\beta$), then κ is upstream from β and is connected directly to it through a single feeder element. On a radial system, there can be only one bus immediately upstream from any other bus; in Figure 2.04, γ is the only bus immediately upstream from β. The number of "immediately downstream" buses is unlimited. The feeder element connecting buses $U\beta$ and β is known as element β.

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[3]Unless specifically noted otherwise, the term "feeder" will henceforth refer to both the primary feeder itself and its associated secondary circuits.

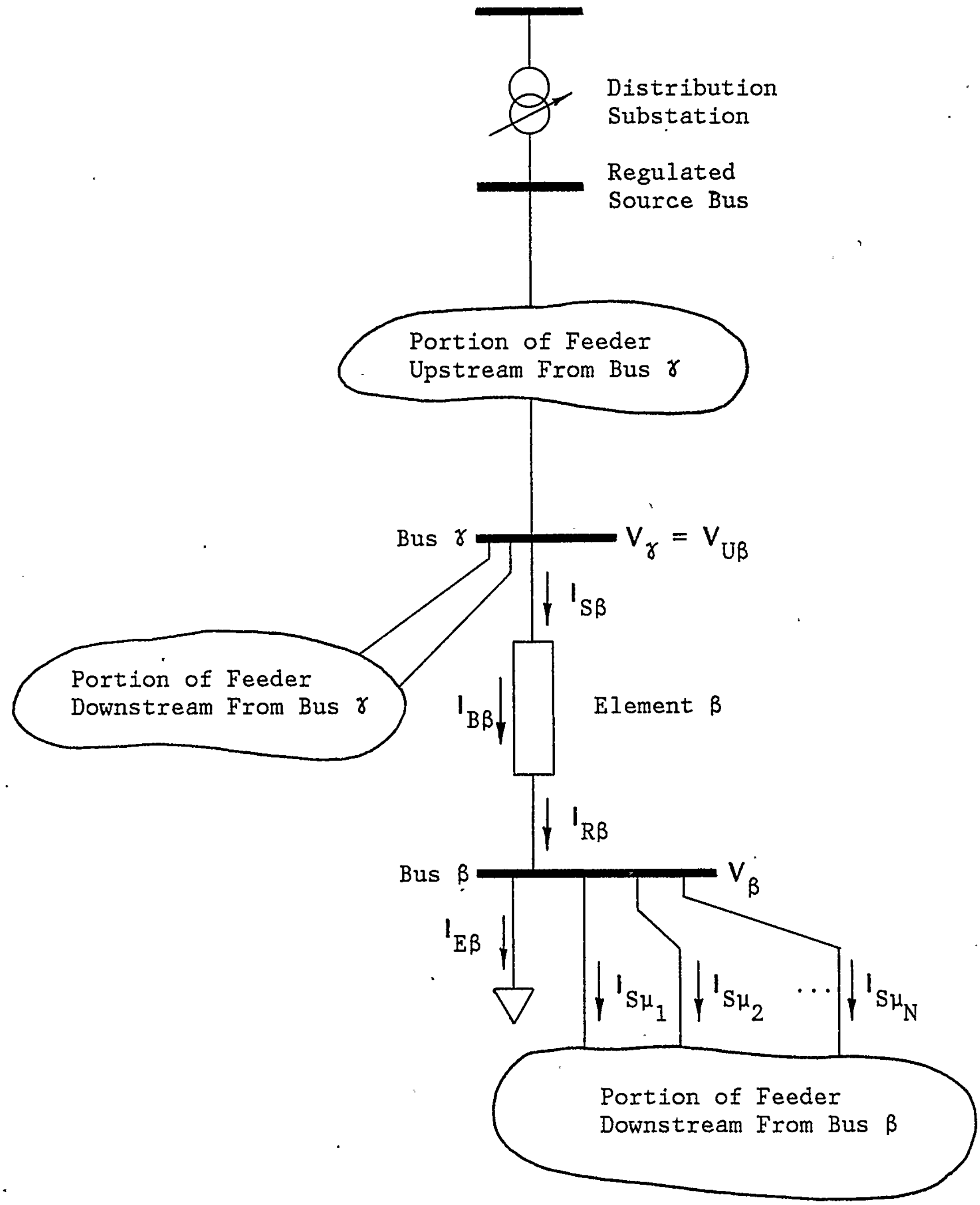

Figure 2.04. A radial feeder with a single section shown in detail.

The currents shown in Figure 2.04 are defined as follows:

$\mathbf{I}_{E\beta}$ = the current flowing into the single-phase or multi-phase load or capacitor bank[4] at bus β

$\mathbf{I}_{R\beta}$ = the current flowing in the receiving (downstream) end of element β

$\mathbf{I}_{B\beta}$ = the current flowing in the series branch of element β

$\mathbf{I}_{S\beta}$ = the current flowing in the sending (upstream) end of element β

Each of these currents is a vector quantity, the form of which can vary. For example, on a three-phase feeder section, $\mathbf{I}_{\chi\beta}$ (χ = E, R, B, S) takes the form

$$(2.01) \qquad \mathbf{I}_{\chi\beta} = \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_{\chi\beta},$$

while on a section fed by a distribution transformer with two secondary windings,

$$(2.02) \qquad \mathbf{I}_{\chi\beta} = \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_{\chi\beta}.$$

[4]If there is more than one shunt element at a bus, a correct analysis can be carried out by creating a fictitious bus for each additional element and connecting these buses to the original through zero-impedance elements.

The form taken by $I_{X\beta}$ at any particular time will be clear from the context in which it is used.

Element β, which is the feeder element connecting buses Uβ and β, will normally be an overhead line, an underground cable, a distribution transformer, or a regulating transformer. The voltage vectors for buses β and Uβ are V_{β} and $V_{U\beta}$, respectively. They are the same as the current vectors $I_{X\beta}$ in the sense that the number of components equals the number of phase circuits.

The vectors in the above equations have components related only to phase quantities. The reason is that one is not normally concerned with neutral and ground currents and voltages. It has been assumed that where required, such as in the evaluation of line impedances, the effect of ground and neutrals is included.

In the interest of enhancing the clarity of the equations presented herein, notation like I_R and V_U will generally be used in place of the more formal $I_{R\beta}$ and $V_{U\beta}$. The subscript β will be used only when an explicit reference to a particular bus or feeder element is desired.

3.0 THE CURRENT/VOLTAGE ITERATION METHOD

3.1 Introduction

It is the purpose of this chapter to establish the principles of the Current/Voltage Iteration Method. An introductory demonstration is given in Section 3.2 by applying the method to the analysis of a very simple feeder. In the section thereafter, the application of the CVIM to more general systems is discussed; several examples of such applications will be given in Chapter 9, following the development of appropriate distribution system element models.

3.2 Application to a Simple Feeder

Figure 3.01 is the single-line diagram of a small single-phase radial distribution system whose voltages and currents are to be determined. The system contains only loads that can be modeled as constant shunt impedances and distribution lines that can be treated as constant series impedances.[1] The circuit shown in Figure 3.02, with the impedance values listed in Table 3.01, is a valid equivalent circuit.

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[1]The restriction to these component types is made in the interest of keeping the example simple.

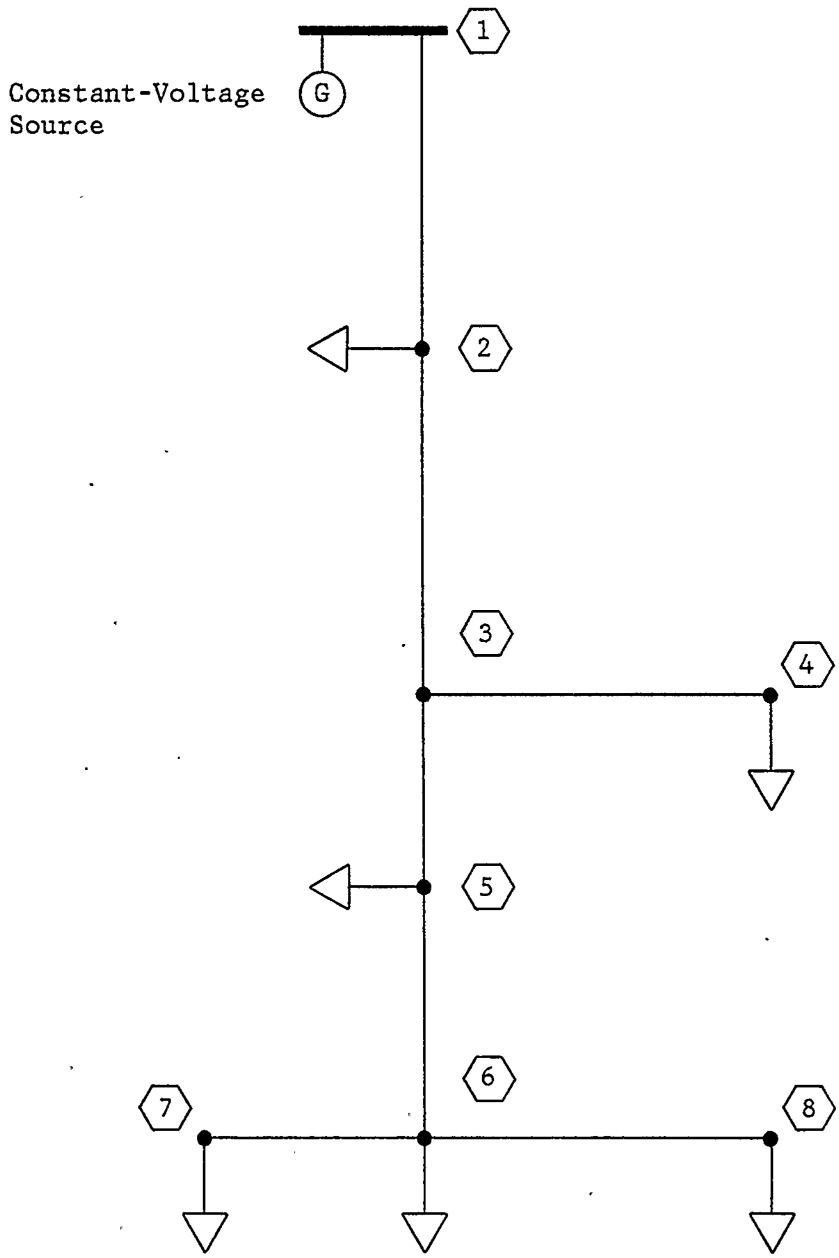

Figure 3.01. The single-line diagram of a simple feeder.

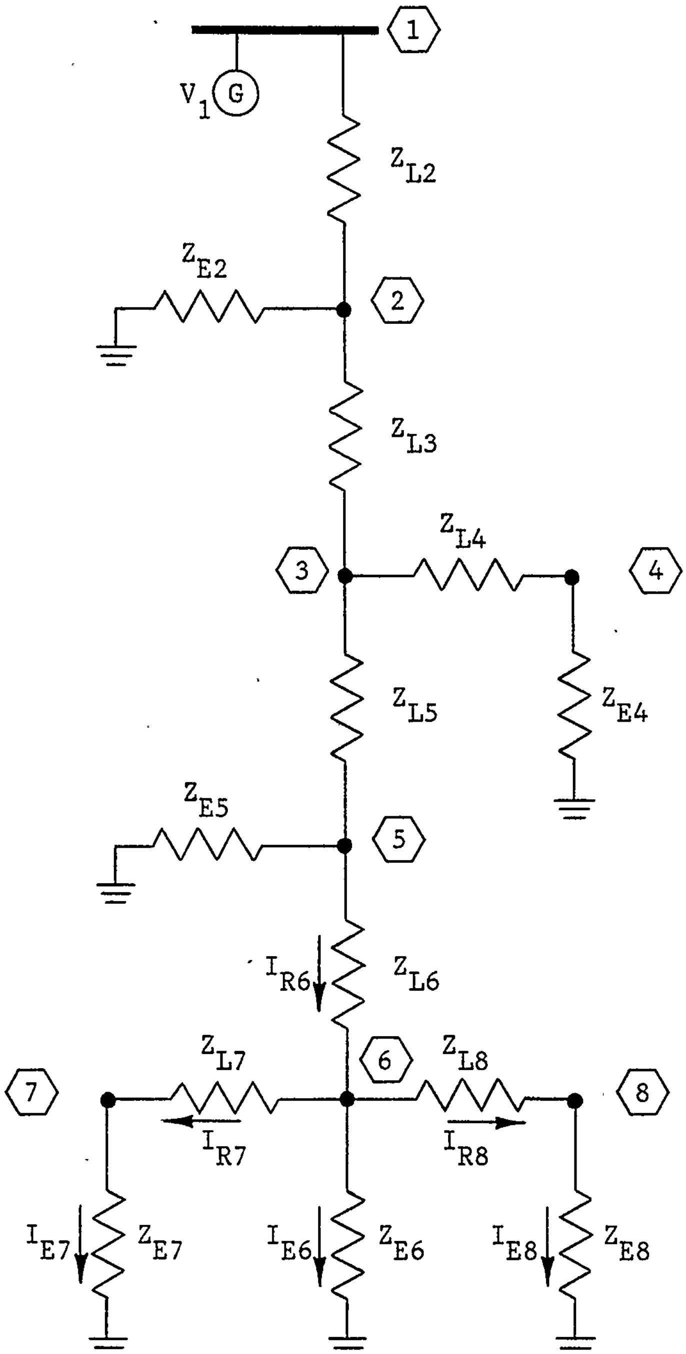

Figure 3.02. The equivalent circuit for the feeder whose single-line diagram is shown in Figure 3.01.

TABLE 3.01
EQUIVALENT CIRCUIT IMPEDANCES

Line	Line Impedance (Z_L)	Bus	Load Impedance (Z_E)
1-2	$(5.098 + j7.647) \times 10^{-3}$	2	1.700 + j1.054
2-3	$(1.274 + j1.912) \times 10^{-3}$	3	
3-4	$(7.853 + j9.300) \times 10^{-3}$	4	1.515 + j1.035
3-5	$(1.722 + j2.583) \times 10^{-3}$	5	9.000 + j4.359
5-6	$(1.309 + j1.963) \times 10^{-3}$	6	1.143 + j0.857
6-7	$(9.162 + j10.850) \times 10^{-3}$	7	0.425 + j0.263
6-8	$(12.827 + j15.190) \times 10^{-3}$	8	0.800 + j0.600

To determine the circuit voltages and currents, begin with the assumption, to be justified in Section 7.4, that bus 1 is a fixed-voltage bus, and take

$$V_1 = 1.050 + j0.000.$$

For the remaining buses (i.e., for $\beta = 2, 3, \ldots, 8$), take

$$V_\beta = 1.000 + j0.000.$$

Call this initial set of bus voltages $\{V\}_0$.

Since the characteristics of all of the circuit's elements are known, then the fact that all bus voltages are "known" means that it is possible to compute the currents. Beginning at bus 8, the load current is found to be

$$\begin{aligned} I_{E8} &= V_8/Z_{E8} \\ &= (1.000 + j0.000)/(0.800 + j0.600) \\ &= 0.800 - j0.600. \end{aligned}$$

There are no buses downstream from 8, and therefore[2]

---

[2]Since $I_R = I_B = I_S$ in a series impedance, the latter two currents can be ignored in this example.

$$I_{R8} = I_{E8} = 0.800 - j0.600.$$

At bus 7, an identical procedure leads to

$$I_{R7} = I_{E7} = 1.700 - j1.054.$$

At bus 6, the load current is

$$I_{E6} = V_6/Z_{E6} = 0.560 - j0.420,$$

and from Figure 3.02,

$$I_{R6} = I_{E6} + I_{R7} + I_{R8},$$

from which

$$I_{R6} = 3.060 - j2.074.$$

At buses 5, 4, 3, and 2, similar calculations can be made; the result is the set $\{I\}_1$ of feeder currents shown in Table 3.02.

The calculation direction, which up to this point has been inward from the electrically remote ends of the feeder, is now reversed. By taking $\{I\}_1$ (and V_1) to be the "known" quantities in place of $\{V\}_0$, a new set of voltages can be calculated. Starting at bus 2, where

TABLE 3.02
FEEDER CURRENTS AFTER THE FIRST CURRENT CALCULATION

Line	Line Current (I_R)	Bus	Load Current (I_E)
1-2	4.025 - j2.688	2	0.425 - j0.263
2-3	3.600 - j2.425	3	
3-4	0.450 - j0.307	4	0.450 - j0.307
3-5	3.150 - j2.117	5	0.090 - j0.044
5-6	3.060 - j2.074	6	0.560 - j0.420
6-7	1.700 - j1.054	7	1.700 - j1.054
6-8	0.800 - j0.600	8	0.800 - j0.600

TABLE 3.03
FEEDER VOLTAGES AFTER THE FIRST VOLTAGE CALCULATION

Bus	Voltage	Bus	Voltage
1	1.050 + j0.000	5	0.989 - j0.025
2	1.009 - j0.017	6	0.981 - j0.029
3	1.000 - j0.021	7	0.954 - j0.037
4	0.993 - j0.023	8	0.961 - j0.033

$$V_2 = V_1 - Z_{L2}I_{R2}$$
$$= (1.050 + j0.000) - (5.098 + j7.647)(10^{-3})(4.025 - j2.688)$$
$$= 1.009 - j0.017,$$

and continuing through to bus 8, gives the set $\{V\}_1$ shown in Table 3.03.

What has been done to this point is shown diagrammatically in Figure 3.03a. The initial set of bus voltages $\{V\}_0$ was used to generate the set $\{I\}_1$ of circuit currents; $\{I\}_1$ was then used to generate a new voltage set $\{V\}_1$.

In order for the pair $[\{V\}_A,\{I\}_A]$ to constitute the solution to the network analysis problem, the situation shown in Figure 3.03b must exist. That diagram indicates that if the voltages $\{V\}_A$ are used to determine the currents $\{I\}_A$, then a subsequent calculation of the voltages based on $\{I\}_A$ must give the set $\{V\}_A$ back again. In practice this criterion must be modified slightly to account for the finite precision available in computations. What should really be expected is shown in Figure 3.03c, where the difference[3] ΔV between the voltage set generated by the currents ($\{V\}_\xi$) and the voltage set that generated the currents ($\{V\}_{\xi-1}$) is less than some acceptable tolerance ε. Since even a subjective comparison of $\{V\}_1$ and $\{V\}_0$ reveals that they are not equal within an acceptable ε, $[\{V\}_1,\{I\}_1]$ is not the correct solution for the problem being considered.

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[3]The difference between two successive voltage sets will be quantified in the next section.

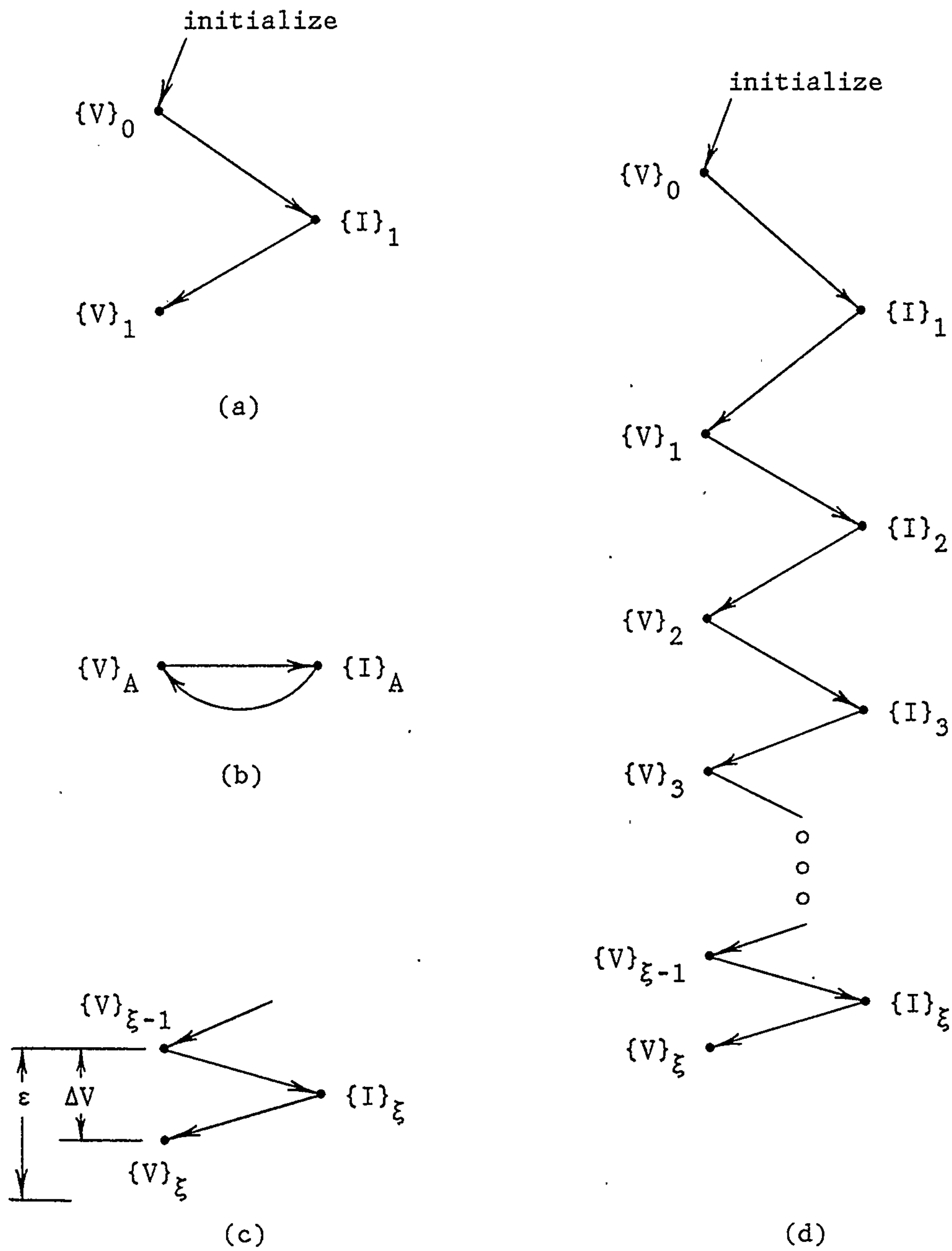

Figure 3.03. A diagrammatic representation of the current/voltage calculation process: (a) the first iteration; (b) the condition for an exact solution; (c) the condition for a practical solution; (d) the complete process.

While $\{V\}_1$ is not the correct voltage set, it might be conjectured that it is closer to the correct set than $\{V\}_0$ is, and that another computation, starting with $\{V\}_1$, might yield a still-closer $\{V\}_2$. This second round of calculations begins, like the first, at bus 8. This time,

$$I_{E8} = V_8/Z_{E8}$$
$$= (0.961 - j0.033)/(0.800 + j0.600)$$
$$= 0.749 - j0.603.$$

Completing the current calculations for the remaining buses yields the set $\{I\}_2$. Subsequent voltage calculations generate $\{V\}_2$, which in turn is used to generate $\{I\}_3$, and so on. As the calculation process is repeated, the difference between successive voltage sets get smaller (see Figure 3.03d). Eventually this difference becomes insignificant (i.e., less than the chosen ε), at which point the currents and voltages are those shown in Tables 3.04 and 3.05. It is fairly simple to verify the correctness of the solution for this example.

3.3 Generalization of the Method

The circuit analysis method used in the last section can be generalized to cope with realistic multi-phase systems in a straightforward way. This generalization is accomplished by writing current and voltage equations applicable to realistic systems in place of the equations used in the example; the principle of calculating alternately the currents and

TABLE 3.04
FINAL FEEDER CURRENTS

Line	Line Current (I_R)	Bus	Load Current (I_E)
1-2	3.836 - j2.725	2	0.425 - j0.272
2-3	3.411 - j2.452	3	N/A
3-4	0.441 - j0.315	4	0.441 - j0.315
3-5	2.970 - j2.137	5	0.088 - j0.045
5-6	2.882 - j2.092	6	0.539 - j0.427
6-7	1.590 - j1.064	7	1.590 - j1.064
6-8	0.753 - j0.602	8	0.753 - j0.602

TABLE 3.05
FINAL FEEDER VOLTAGES

Bus	Voltage	Bus	Voltage
1	1.050 + j0.000	5	0.990 - j0.023
2	1.010 - j0.015	6	0.982 - j0.026
3	1.001 - j0.019	7	0.956 - j0.033
4	0.994 - j0.020	8	0.963 - j0.029

voltages remains, as does the solution criterion. The resulting algorithm (the CVIM) is documented in a flow chart that starts on page 32.

The first step shown in the flow chart is the numbering of the feeder buses. The scheme suggested provides a convenient mechanism for ensuring that the buses are processed in the correct order: current calculations are done in descending bus order, while voltages are calculated in ascending bus order. Any other scheme that provides the same result can be used.

The second step in the algorithm is the initialization of the bus voltages. Because all feeder currents are set to zero, there are no voltage drops in any of the system components, and the bus voltages are therefore initialized to their rated values.[4] Since the substation LV bus voltage is sometimes set a few percent above or below the feeder's nominal voltage (see Section 7.4), its value must be reset once the other bus voltages have been initialized.

Following several "housekeeping" steps, the current and voltage calculation steps are implemented. Except for Equation (3.01), which is simply an appropriate version of Kirchoff's Current Law, the equations to be used in these steps are derived in Chapters 4 through 8, wherein models are developed for the distribution system elements of interest. A slight

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[4]The rated bus voltage magnitudes are needed in load current calculations (see Chapter 4), and are established in this way because the method easily accounts for the phase shifts and magnitude changes induced by distribution transformers. This initialization process was not used in the last section's example because there the initialization was trivial.

modification of the voltage calculation process is sometimes required, as will be discussed in connection with a specific example in Section 9.4.

Once the voltage calculation step has been completed, the difference ΔV between the two latest voltage sets (i.e., between $\{V\}_{\xi}$ and $\{V\}_{\xi-1}$) must be determined to see whether the solution criterion has been met. The chosen definition of ΔV is given in Equation (3.02). If ΔV is greater than the solution tolerance ε then the current and voltage calculations must be repeated; otherwise, the network solution has been found. Several examples of the behaviour of ΔV as a function of the iteration number ξ will be given in Section 9.4.

By now the reason for calling the solution technique the Current/Voltage Iteration Method should be apparent. The first part of each iteration consists of calculating the system currents, beginning at the electrically remote points of the feeder and proceeding (in the "inbound" direction) back toward the source bus. Once the current flowing from the source bus is known, the direction of calculation is reversed; new bus voltages are determined beginning at the bus immediately downstream from the source bus and proceeding (in the "outbound" direction) back toward the load points.

It should be clear from the above that the CVIM cannot be applied to circuits that are not strictly radial; such circuits contain closed loops, and these loops lack the end buses required by the iteration process.

START

1. Number the feeder buses consecutively from 1 through N such that if $\gamma > \kappa$, then bus γ is not upstream from bus κ.

2. Initialize the voltage set $\{V\}_0$ using the following steps:

   (a) Set all feeder currents to zero.

   (b) Set the substation LV bus phase voltages such that each has an appropriate phase angle and a magnitude equal to the system's rated voltage.

   (c) Calculate the voltage vector at all remaining system buses using the same process as employed in Step 6.

   (d) Reset the substation LV bus voltage to its assigned value.

3. Set the solution tolerance ε and the maximum number of iterations ξ_{max}, and initialize the iteration number ξ to zero.

A

A

4. Increment the iteration number ($\xi \leftarrow \xi+1$).

C

5. Compute the current set $\{I\}_\xi$ based on the latest voltage set $\{V\}_{\xi-1}$. Proceeding in descending bus order (i.e., β = N, N-1, ..., 2):

(a) Calculate the current $I_{E\beta}$ drawn by the shunt element at bus β.

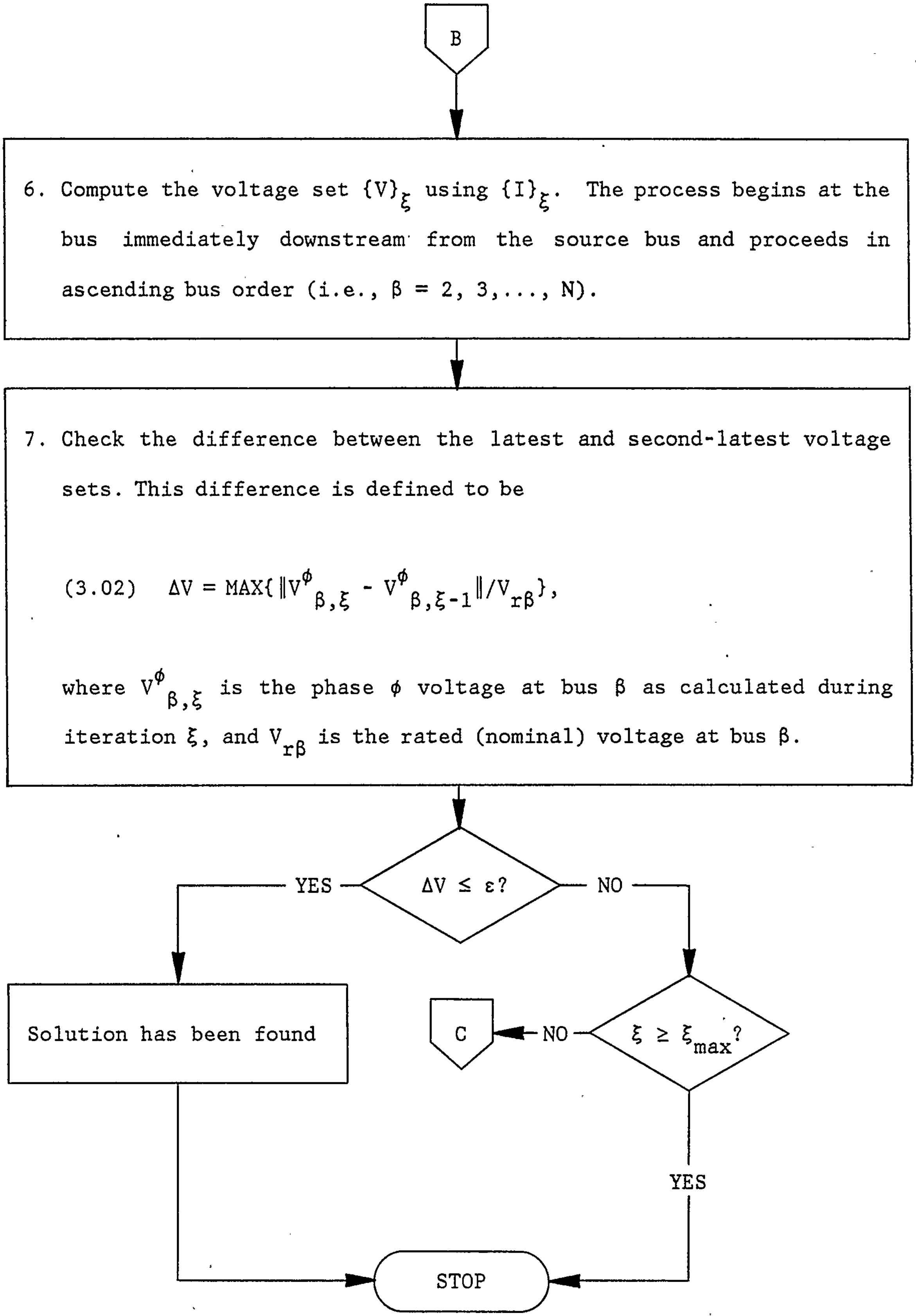
(b) Calculate the current $I_{R\beta}$ in the receiving end of element β using

$$I_{R\beta} = I_{E\beta} + \sum_{\mu} I_{S\mu}. \tag{3.01}$$

The sum is taken over all buses μ that are immediately downstream from bus β. $I_{S\mu}$ is the current flowing into the sending end of element μ.

(c) Calculate the current $I_{S\beta}$ in the sending end of element β. If the element is an intermediate line, then the series branch current $I_{B\beta}$ must be calculated first (see Section 5.3).

B


B
6. Compute the voltage set $\{V\}_\xi$ using $\{I\}_\xi$. The process begins at the bus immediately downstream from the source bus and proceeds in ascending bus order (i.e., $\beta = 2, 3, \ldots, N$).
7. Check the difference between the latest and second-latest voltage sets. This difference is defined to be
(3.02) $\Delta V = \mathrm{MAX}\{\|V^{\phi}_{\beta,\xi} - V^{\phi}_{\beta,\xi-1}\|/V_{r\beta}\}$,
where $V^{\phi}_{\beta,\xi}$ is the phase ϕ voltage at bus β as calculated during iteration ξ, and $V_{r\beta}$ is the rated (nominal) voltage at bus β.
$\Delta V \le \varepsilon$?
YES
NO
Solution has been found
C
NO
$\xi \ge \xi_{max}$?
YES
STOP

4.0 LOADS

4.1 Introduction

Distribution feeders supply a variety of electrical devices and systems, including incandescent, fluorescent, and vapour lamps, various types of heaters, induction and synchronous motors, electronic equipment, and electric transportation systems. Of obvious concern to those responsible for feeder design and operation are the effects that these loads have on voltages and thermal loading. To make an intelligent assessment of these effects, models of the system's loads are required.

The CVIM requires that given the voltage vector $\mathbf{V}$ at a bus, one must be able to calculate the current $\mathbf{I}_E$ flowing into the load at that bus. There are three steps involved:

(1) Determine the load segment[1] voltages from the bus voltages

(2) Calculate the current flowing into each segment

(3) Compute the current vector $\mathbf{I}_E$ from the segment currents

Step (2) is discussed in the next section; Steps (1) and (3) are discussed for single-phase loads in Section 4.3 and for three-phase loads in Section 4.4.

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[1]Load segments are discussed below in connection with single-phase and three-phase loads.

4.2 Determination of Load Segment Currents

The current I^{σ}_{ℓ} flowing into segment σ of a load is related to the voltage across the segment, V^{σ}_{ℓ}, by

$$(4.01)\quad I^{\sigma}_{\ell} = (S^{\sigma}_{\ell})^{*}/(V^{\sigma}_{\ell})^{*}$$

$$= (P^{\sigma}_{\ell} - jQ^{\sigma}_{\ell})/(V^{\sigma}_{\ell})^{*},$$

where

$$(4.02)\quad S^{\sigma}_{\ell} = P^{\sigma}_{\ell} + jQ^{\sigma}_{\ell}$$

is the complex power consumed by the load segment. Thus, equations for the real power P^{σ}_{ℓ} and the reactive power Q^{σ}_{ℓ} are required. There are several forms that these equations can take:[2] two of the most common ones are the exponential form, in which

$$(4.03)\quad P^{\sigma}_{\ell} = P^{\sigma}_{\ell r}\|V^{\sigma}_{\ell}/V^{\sigma}_{\ell r}\|^{k^{\sigma}}$$

and

---

[2]As suggested by the variety of loads encountered, load modeling is a very extensive subject. Because it has been dealt with by a large number of authors (see [6] and the references cited therein), and because it is beyond the scope of this project, the modeling of specific load types is not examined in detail. Only general equations relating real and reactive powers to segment voltages are given.

$$(4.04) \qquad Q^{\sigma}_{\ell} = Q^{\sigma}_{\ell r}\|V^{\sigma}_{\ell}/V^{\sigma}_{\ell r}\|^{l^{\sigma}},$$

and the polynomial form, in which

$$(4.05) \qquad P^{\sigma}_{\ell} = P^{\sigma}_{\ell r}[1 + k^{\sigma}D^{\sigma} + l^{\sigma}(D^{\sigma})^{2}]$$

and

$$(4.06) \qquad Q^{\sigma}_{\ell} = Q^{\sigma}_{\ell r}[1 + m^{\sigma}D^{\sigma} + n^{\sigma}(D^{\sigma})^{2}],$$

where

$$(4.07) \qquad D^{\sigma} = 1 - \|V^{\sigma}_{\ell}/V^{\sigma}_{\ell r}\|.$$

(In Equations (4.03)-(4.07), the subscript r denotes rated, and k^{σ}, l^{σ}, m^{σ}, and n^{σ} are constants.) The exponential load model is useful because constant power, constant current, and constant impedance loads can be created simply by setting the exponents to 0, 1, and 2, respectively. Also, the range of k^{σ} is known (0 to 2), and the model involves only a small number of constants. However, the range of l^{σ} is greater, and (4.04) suffers from an inability to change sign as may be required. The polynomial model is quite versatile, and according to Chen et al. [6], it provides ample accuracy in the modeling of typical loads. This model is also useful in that higher accuracy can be obtained by adding higher-order terms.

4.3 Single-Phase Loads

Most of the smaller loads supplied by distribution feeders, primary examples of which are the loads found in residential areas, are of the single-phase variety. These loads are generally fed through three-winding distribution transformers that provide two secondary voltages. Lights and small appliances can be served at 120 V by connecting them between a phase wire and the neutral wire, while larger appliances (ranges, clothes dryers, etc.) can be served at 240 V by utilizing a connection between the two phase wires.

As shown in Figure 4.01, there are three segments in a single-phase three-wire load. Given the bus voltages, the segment voltages to be substituted into the above equations can be determined from

$$(4.08) \qquad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_\ell = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \end{bmatrix}.$$

Once the segment currents have been calculated based on these voltages, the load currents can be found using

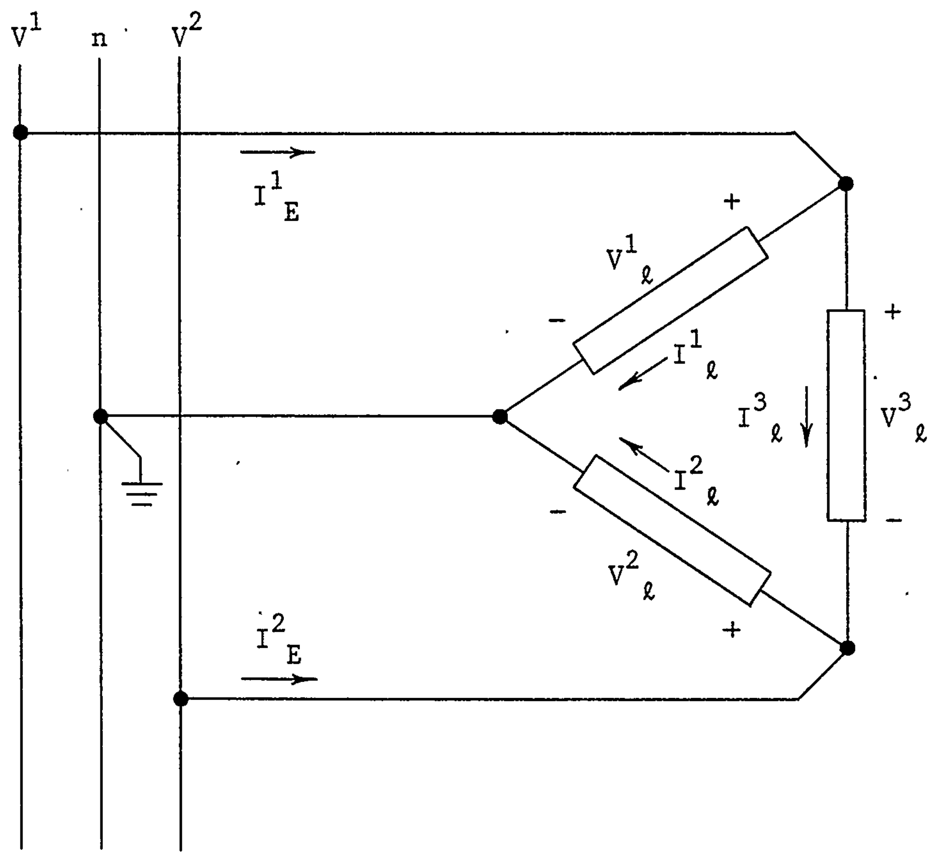

Figure 4.01. A single-phase three-wire load.

$$(4.09)\quad \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_\ell .$$

Not all single-phase loads use three wires; some use only one phase wire and one neutral wire. In this case there is only one segment, and the load current and the segment current are equal, as are the bus voltage and the segment voltage.

4.4 Three-Phase Loads

As load sizes increase, it becomes more economical to provide the customer with a three-phase supply rather than a single-phase one. There are three basic ways in which loads can be connected when a three-phase service is employed. In each case, the calculation of the load currents is slightly different.

4.4.1 Grounded-Wye Loads

One of the ways in which loads can be connected to a three-phase service is shown in Figure 4.02a. Each element of the load is connected between a phase conductor and the grounded neutral conductor, so that

$$(4.10)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_\ell = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} .$$

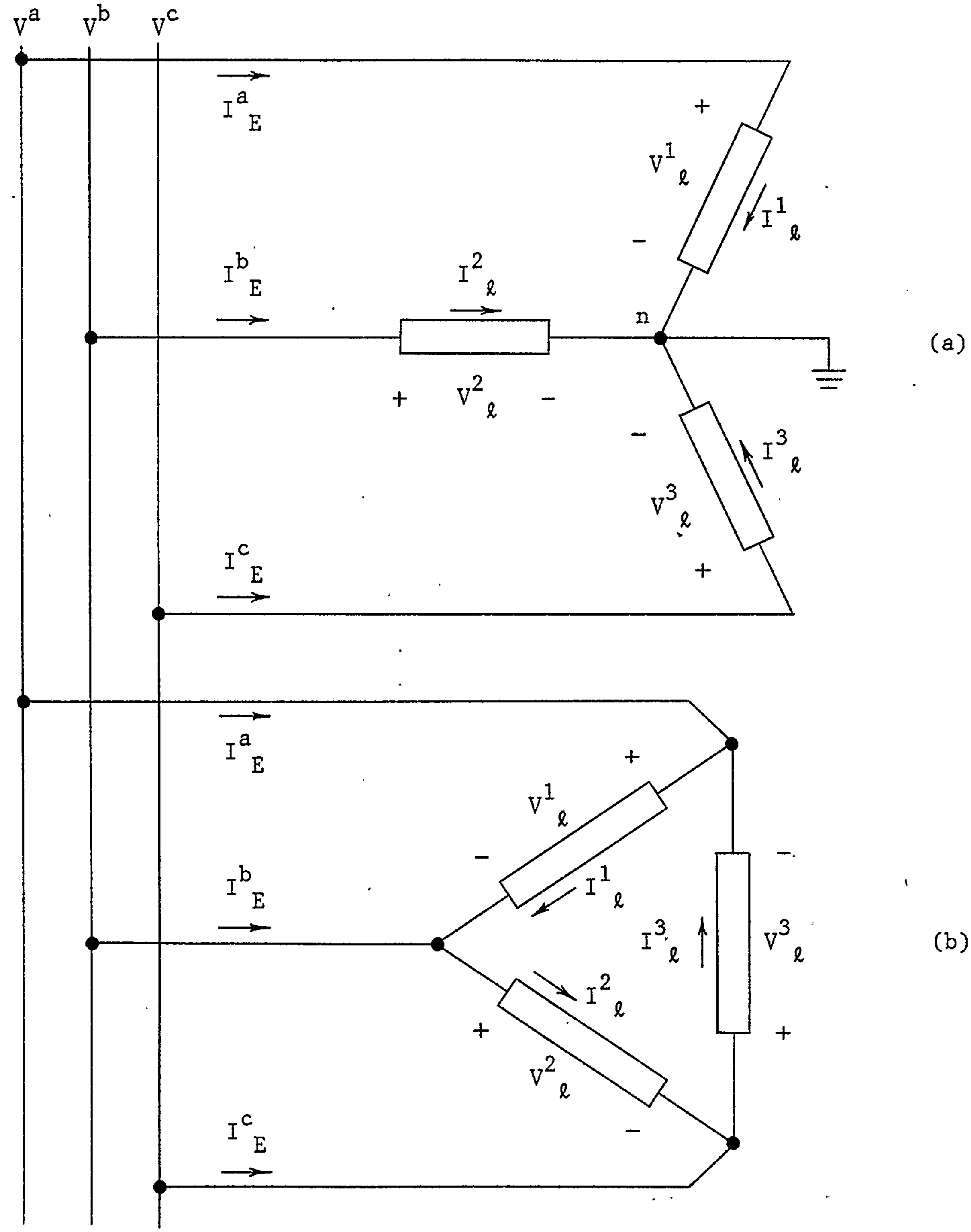

Figure 4.02. Three-phase loads: (a) wye-connected; (b) delta-connected.

Also, it is clear that the load and segment currents are the same, that is,

$$(4.11)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_\ell .$$

4.4.2 <u>Delta Loads</u>

When no neutral conductor is available, when a voltage higher than that available from a phase-to-neutral connection is required, or when certain operational characteristics are desired, a load may be connected in delta. Such a load is illustrated in Figure 4.02b. The segment voltages are given by

$$(4.12)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_\ell = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} .$$

The load currents are related to the segment currents by

$$(4.13)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_\ell .$$

4.4.3 Ungrounded-Wye Loads

Another load connection that may be used under certain conditions (for example, in three-phase induction motors) is the ungrounded-wye. This situation is the same as that illustrated in Figure 4.02a, except that the neutral point n is not grounded. The segment voltages in this case are

$$(4.14)\qquad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_\ell = \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} - V^n{}_\ell \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

$V^n{}_\ell$ is the voltage at the neutral point, and can be determined from the requirement that the total current $I^n{}_\ell$ flowing into the neutral must be zero. After putting

$$(4.15)\qquad V^n{}_\ell = x + jy$$

and

$$(4.16)\qquad \begin{aligned} I^n{}_\ell &= \sum_\sigma I^\sigma{}_\ell \\ &= \sum_\sigma [(P^\sigma{}_\ell - jQ^\sigma{}_\ell)/(V^\sigma{}_\ell)^*] \\ &= f(x,y) + jg(x,y), \end{aligned}$$

some straightforward manipulation shows that finding V^n_ℓ such that $I^n_\ell = 0$ is equivalent to solving the two simultaneous equations

$$(4.17)\qquad f(x,y) = \sum_{\sigma} \frac{P^\sigma_\ell(x,y)[\mathrm{Re}\{V^\sigma_\ell\} - x] + Q^\sigma_\ell(x,y)[\mathrm{Im}\{V^\sigma_\ell\} - y]}{[\mathrm{Re}\{V^\sigma_\ell\} - x]^2 + [\mathrm{Im}\{V^\sigma_\ell\} - y]^2} = 0$$

and

$$(4.18)\qquad g(x,y) = \sum_{\sigma} \frac{P^\sigma_\ell(x,y)[\mathrm{Im}\{V^\sigma_\ell\} - y] - Q^\sigma_\ell(x,y)[\mathrm{Re}\{V^\sigma_\ell\} - x]}{[\mathrm{Re}\{V^\sigma_\ell\} - x]^2 + [\mathrm{Im}\{V^\sigma_\ell\} - y]^2} = 0$$

for x and y. A closed-form solution of Equations (4.17) and (4.18) is very difficult (if not impossible) to obtain, so the CVIM makes use of the Newton-Raphson method [4, 20] to find x and y, and hence, V^n_ℓ.

For the ungrounded-wye load, the load currents are equal to the segment currents:

$$(4.19)\qquad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_\ell .$$

5.0 DISTRIBUTION LINES

5.1 Introduction

Electrical energy is carried from the point of generation to the point of utilization by transmission and subtransmission networks, primary feeders, and secondary systems. The vast majority of transmission and subtransmission lines are constructed using overhead conductors on any of several types of supporting structures, including steel poles and towers, aluminum towers, and wood poles and frames. In areas where it is impractical, unsafe, costly, or environmentally unacceptable to employ overhead construction (such as in heavily urbanized areas, near airports, or across long stretches of water), insulated cable systems are used. In these cases, oil-filled pipe-type cables, extruded dielectric cables, and compressed gas insulated cables may be used.

The construction of distribution feeders, like that of transmission and subtransmission circuits, may be either overhead or underground. Overhead primary feeders and secondary circuits are normally built on wood poles. Underground construction methods employ concentric neutral cables with polyethylene or cross-linked polyethylene insulation; they are being used more and more frequently, especially for sub-feeders and laterals.

Regardless of what type of construction is used, it is important to have accurate models representing the electrical characteristics of the lines. It is the purpose of this chapter to discuss these models, and to derive performance equations appropriate for use with the CVIM.

5.2 Short Lines

A short distribution line is one for which the total capacitive susceptance is so small that it can be neglected. Overhead 60 Hz lines less than about 50 km long are usually treated as short lines. It must be noted, however, that the importance of capacitance and its charging current varies with the characteristics of the line; for this reason, no definite length can be defined as being one below which the short line model will always be applicable.

Figure 5.01 shows the short line model of a distribution feeder section, connecting buses β and Uβ, that has three phase conductors and one neutral conductor. The ground current path is represented by an equivalent conductor whose parameters are functions of the electrical characteristics of the earth. It can be shown that in this case, and in more general cases as well [1, 50], the difference between the phase voltages on the two buses is

$$\begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U - \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} = s \begin{bmatrix} Z^{aa} & Z^{ab} & Z^{ac} \\ Z^{ab} & Z^{bb} & Z^{bc} \\ Z^{ac} & Z^{bc} & Z^{cc} \end{bmatrix}_L \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_B , \tag{5.01}$$

where s is the length of the section and the $Z^{\mu\nu}$ are the conductor self and mutual impedances, per unit length, as adjusted to account for the effects of ground and the neutral.

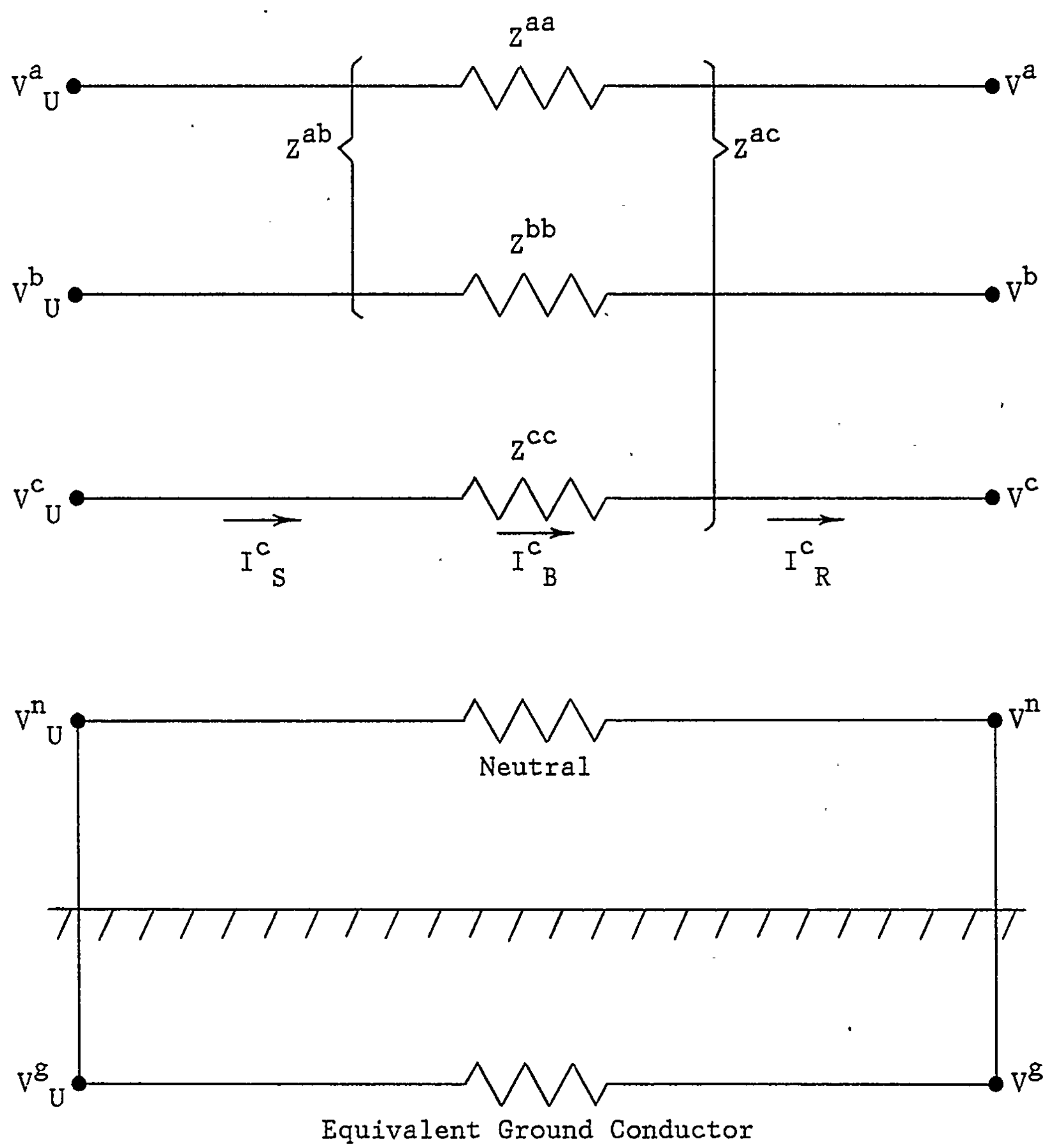

Figure 5.01. The short line model of a three-phase distribution feeder section. The parameters shown have the effects of the ground and the neutral built in. For clarity, only those mutual impedances affecting phase a are shown.

Since it is clear from Figure 5.01 that the sending end, series branch, and receiving end currents are all equal, the equation required on the inbound portion of a CVIM iteration is

$$I_S = I_B = I_R. \tag{5.02}$$

On the outbound portion, V is calculated using

$$V = V_U - sZ_L I_B, \tag{5.03}$$

where Z_L is the line impedance matrix introduced in Equation (5.01). Obviously, on a single-phase line section Z_L is a scalar, while on a two-phase section it is a two-by-two matrix.

5.3 Intermediate Lines

When the capacitance of a line is too large to be neglected completely, the effects of its presence can be modeled by placing half of the shunt capacitance at each end of the line. The resulting equivalent circuit, which is called a nominal π, is shown in Figure 5.02.

The current flowing in the series impedance of conductor a is

$$I^a{}_B = I^a{}_R + I^{ab}{}_R + I^{ac}{}_R + I^{an}{}_R + I^{ag}{}_R. \tag{5.04}$$

Using

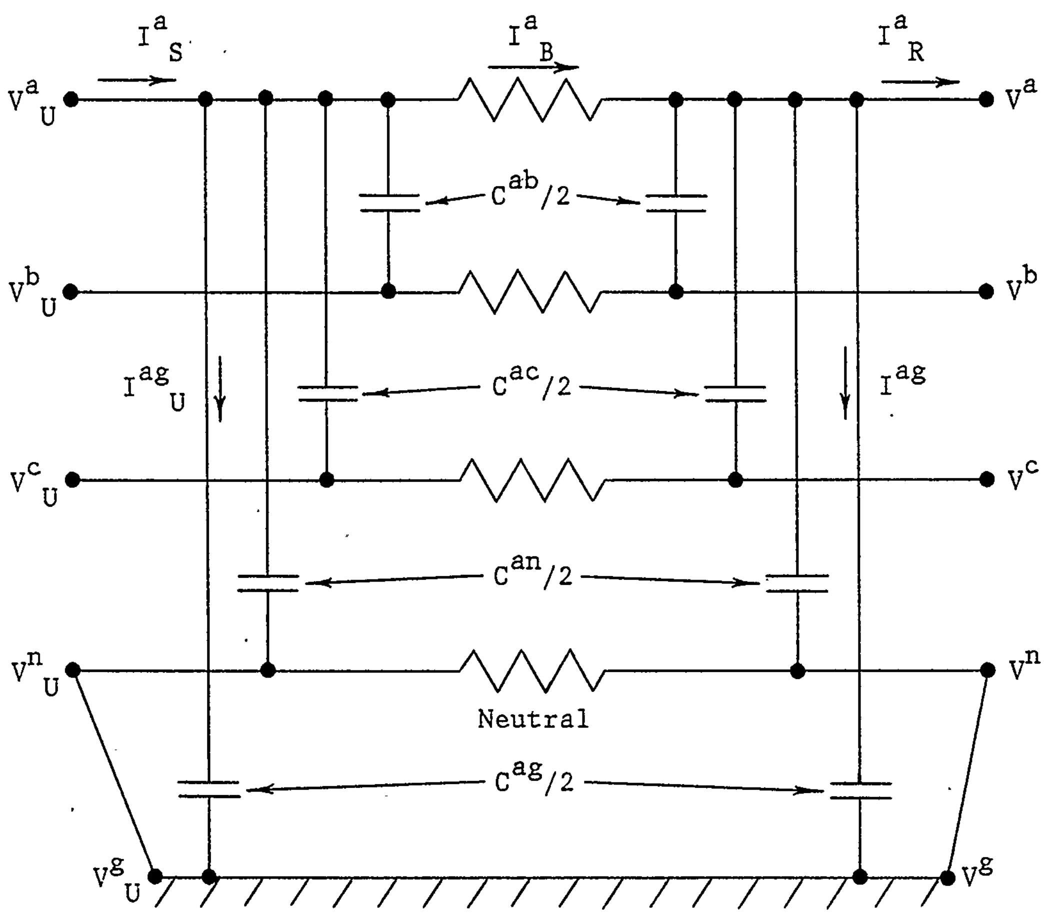

Figure 5.02. The intermediate line model of a three-phase distribution feeder section. For clarity, only the mutual capacitances affecting phase a are shown.

$$(5.05)\quad I^{\nu\tau}{}_{R} = j\omega s(C^{\nu\tau}/2)[V^{\nu} - V^{\tau}],$$

where ω is the angular power-line frequency and $C^{\nu\tau}$ is the capacitance per unit length between conductors ν and τ, Equation (5.04) becomes

$$(5.06)\quad I^{a}{}_{B} = I^{a}{}_{R} + (1/2)j\omega s[C^{ab}(V^{a} - V^{b}) + C^{ac}(V^{a} - V^{c}) + C^{an}(V^{a} - V^{n}) + C^{ag}(V^{a} - V^{g})].$$

Now, when the line capacitances $C^{\nu\tau}$ are being calculated the assumption is generally made that the ground is an infinite, perfectly-conducting plane [1]. As a result,

$$(5.07)\quad V^{n} = V^{g} = 0,$$

and (5.06) simplifies to

$$(5.08)\quad I^{a}{}_{B} = I^{a}{}_{R} + (1/2)j\omega s[C^{aa}V^{a} - C^{ab}V^{b} - C^{ac}V^{c}],$$

where

$$(5.09)\quad C^{aa} = C^{ab} + C^{ac} + C^{an} + C^{ag}.$$

Since equations similar to (5.08) and (5.09) can be derived for phases b and c, the series branch current vector can be calculated using

$$(5.10)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_B = \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R + (s/2)j\omega \begin{bmatrix} C^{aa} & -C^{ab} & -C^{ac} \\ -C^{ab} & C^{bb} & -C^{bc} \\ -C^{ac} & -C^{bc} & C^{cc} \end{bmatrix}_L \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}.$$

This equation can also be written as

$$(5.11)\quad \mathsf{I}_B = \mathsf{I}_R + (s/2)\mathsf{Y}_L\mathsf{V},$$

where the definition of Y_L comes from Equations (5.10) and (5.11).

In exactly the same way that Equation (5.11) was derived, it can be shown that

$$(5.12)\quad \mathsf{I}_S = \mathsf{I}_B + (s/2)\mathsf{Y}_L\mathsf{V}_U.$$

Since Equation (5.03) still applies, all of the equations required to incorporate intermediate line modelling into the CVIM are now known: Equations (5.11) and (5.12) are used on the inbound portion of an iteration to compute the series branch and sending end currents, while on the outbound portion the receiving end voltages are computed using (5.03).

Instead of representing the intermediate line with a nominal π circuit, a nominal T could have been used. The latter circuit is constructed by splitting the series impedance in half and connecting the total shunt admittance between the two halves.

5.4 Long Lines

The nominal π circuit discussed in the last section cannot represent the transmission line exactly because no account is taken of the fact that the parameters of the line are distributed along its length. To include the effects of parameter distribution, it is necessary to compute the voltage and current vectors based on the differential equations governing their distribution along the line. Generalizing the method employed in [50, p.101] to unbalanced three-phase lines, it can be shown that these equations are[1]

$$(5.13)\quad d^2\mathbf{V}(x)/dx^2 = s^2\mathbf{ZYV}(x) = \mathbf{RV}(x)$$

and

$$(5.14)\quad d^2\mathbf{I}(x)/dx^2 = s^2\mathbf{YZI}(x) = \mathbf{R}^T\mathbf{I}(x),$$

where x represents the distance from the receiving end of the line and $\mathbf{R}^T$ is the transpose of $\mathbf{R}$. The solution of these equations requires the determination of the eigenvalues and eigenvectors of $\mathbf{R}$. Since the lines

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[1]The development and solution of an equation like (5.13) is discussed in Section 9.2 in connection with a different but closely-related problem.

and cables found in distribution networks are typically of short or intermediate length, however, the long line model will not significantly improve the accuracy of the circuit analysis, and the use of the additional computational effort associated with the long line model is not justified in this project.

6.0 TRANSFORMERS

6.1 Introduction

The different requirements of the generation, transmission, distribution and utilization components of an electric power system dictate different preferred voltage levels for each of these components. As a result the transformer, whose primary function is the transformation of the voltage from the level used on one part of the system to that used on another, is of major importance to power system operation. Appropriate transformer models are therefore prerequisites to the accurate representation of power systems. It is the purpose of this chapter to develop these models. Before doing so, however, a few remarks will be made about each of the four categories into which the transformers used on electric distribution systems can be placed.

6.1.1 Power Transformers

Power transformers are used in both bulk power substations and distribution substations. In the former they transform the voltage from its generation or transmission level to its subtransmission level; in the latter they reduce the voltage to its primary feeder level from its subtransmission level. In both cases, voltage regulating equipment may be integrated into or associated with the units so that the substation LV bus voltage can be maintained at the desired value. Power transformers are almost always three-phase, consisting of either three-phase units or banks of three single-phase units. They are available with a variety of power and voltage ratings.

Because the assumption has been made that distribution substation LV buses are constant-voltage sources (see Sections 3.2 and 7.4), the modeling of substation power transformers will not be discussed explicitly. However, if the constant voltage assumption is not made, models for the transformers and their associated regulating mechanisms will be needed. They can be derived quite easily from the models for distribution transformers and regulating transformers to be presented below.

6.1.2 Regulating Transformers

Regulating transformers are tapped autotransformers[1] that are used to control the voltages at particular feeder buses, making adjustments in tap settings as required by changes in load conditions. In substations they may exist as separate devices. Often, however, they are incorporated into substation power transformers, forming load tap changing (LTC) transformers. Regulators may also be applied on feeders between substations and loads, and directly at loads. Typically, they can produce output voltages in a range from 10% below to 10% above the input voltage. When not incorporated into power transformers, regulators are usually single-phase units, although a small portion of those in service are

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[1]Since step voltage regulators have almost completely supplanted induction regulators, the latter devices will not be discussed.

three-phase. Regulating transformers, and therefore the LTC capabilities of power transformers, will be discussed in Chapter 7.

6.1.3 Distribution Transformers

By far the most numerous of any of the four transformer types is the distribution transformer, which reduces the voltage from its primary feeder level to the level required by consumers' equipment. Banks of three single-phase transformers, three-phase transformers, and single-phase transformers are all common. In special or emergency cases, a two-transformer bank may be connected using a T or open-delta configuration. Many distribution transformers have a centre-tapped secondary winding so that two secondary voltages (e.g., 120 V and 240 V) can be provided. Generally having ratings below 67 kV and 500 kVA, they may be pole-mounted, platform-mounted, pad-mounted, or placed in subterranean vaults. Some distribution transformers contain off-load, manually-operated tap changers that can be used to compensate for primary voltages that are off-nominal for an extended period. Distribution transformer models will be discussed in some detail later in this chapter.

Network transformers provide a function similar to that provided by distribution transformers, but have several features (e.g., network protectors) not usually found on the latter. Since they are found on secondary networks, and because such networks cannot be analyzed using the CVIM, these devices will not be discussed further.

6.1.4 Instrument Transformers

The control equipment found on distribution systems generally functions based on the values of particular variables. For example, tap changer control circuits must measure regulator output voltages to determine the correct tap position adjustments. Since the control circuits cannot utilize voltages and currents of system magnitude directly, current and potential transformers are used to produce output signals directly proportional to, but much smaller than, the system quantities. Such measurement transformers have an insignificant impact on distribution system behaviour, and will therefore only be mentioned further when regulator control circuits are discussed.

6.2 The Transformer Equivalent Circuit

For the purposes of many power system studies, transformer behaviour is close enough to ideal that the devices can be represented by simple series reactances. In other cases, device imperfections such as core loss and copper loss can be lumped with the loads being supplied. However, a relatively complete transformer analysis is sometimes desired, and in these situations a detailed model is required.

The derivation of equivalent circuits and performance equations for ideal transformers and realistic single-phase transformers is a topic that is covered in a considerable number of references [11, 17, 33, 47-49, 52]. While the circuits developed by the various authors are not identical, virtually all possess the same basic features. Thus, only a very brief sketch of the development of an equivalent circuit will be given.

6.2.1 Ideal Transformers

Consider Figure 6.01, which shows three coils wound around a common core of ferromagnetic material. If it is assumed that the permeability of the magnetic core and the conductivity of the winding conductors are both infinite, then the HV (H) and LV (X) winding voltages are related by

$$(6.01)\quad E^{\psi}_{X} = (N^{\psi}_{X}/N_{H})E_{H} = r^{\psi}E_{H} \qquad (\psi = 1,\ 2).$$

The winding currents obey

$$(6.02)\quad N_{H}I_{H} - (N^{1}_{X})I^{1}_{X} - (N^{2}_{X})I^{2}_{X} = 0,$$

an equation that can also be written

$$(6.03)\quad I_{H} = (r^{1})I^{1}_{X} + (r^{2})I^{2}_{X}.$$

In these equations, the E's are voltages, the I's are currents, the N's are turns of wire, and the r's are the secondary-to-primary turns ratios. If the transformer does not have a tap changer, then the r^{ψ} are fixed. If

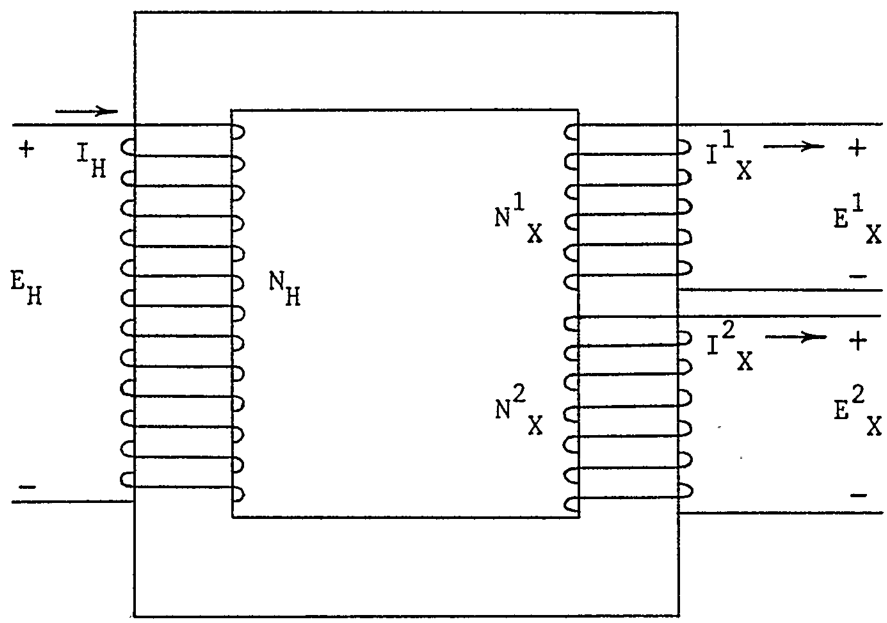

Figure 6.01. The essential elements of a three-winding transformer: the coils and a ferromagnetic core.

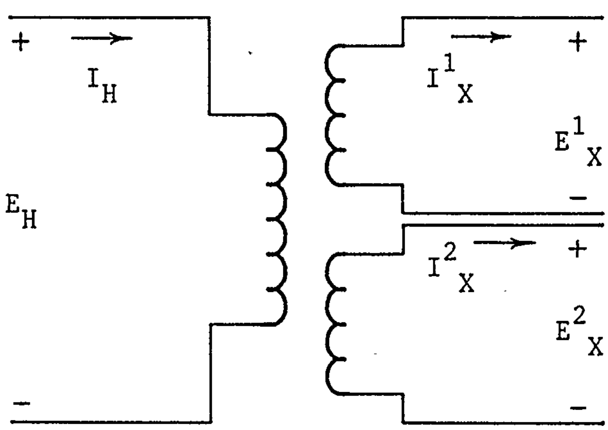

Figure 6.02. The equivalent circuit of an ideal three-winding transformer.

there is a tap changer, then the r^{ψ} vary as the tap position is changed;[2] since distribution transformer tap changing is done manually and off load, however, the r^{ψ} are constants in any load flow study.

The circuit representation of an ideal three-winding transformer is shown in Figure 6.02.

6.2.2 Practical Equivalent Circuits

The equivalent circuit to be used to model real transformers is not that illustrated in Figure 6.02. The reason is that real devices are made of materials with imperfect electromagnetic properties, and they therefore exhibit some deviation from ideal behaviour. To account for this deviation, elements may be added to the ideal equivalent circuit. A configuration that is used quite often is shown in Figure 6.03.

The first material imperfection to be accounted for concerns the winding conductors. Because their conductivity cannot be infinite, the windings must have some resistance. Thus, the resistances R_H, $R^1{}_X$, and $R^2{}_X$ are added to the transformer circuit.

Like the conductivity of the winding material, the permeability of the ferromagnetic core cannot be infinite. Therefore, some of the flux produced by a winding current leaks out of the core material. This time-varying leakage flux induces in each linked winding a voltage whose magnitude is directly proportional to the strength of the flux, and therefore to the

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[2]One common distribution transformer tap changer configuration allows each turns ratio to vary from 95% to 105% of its nominal value in 2.5% steps.

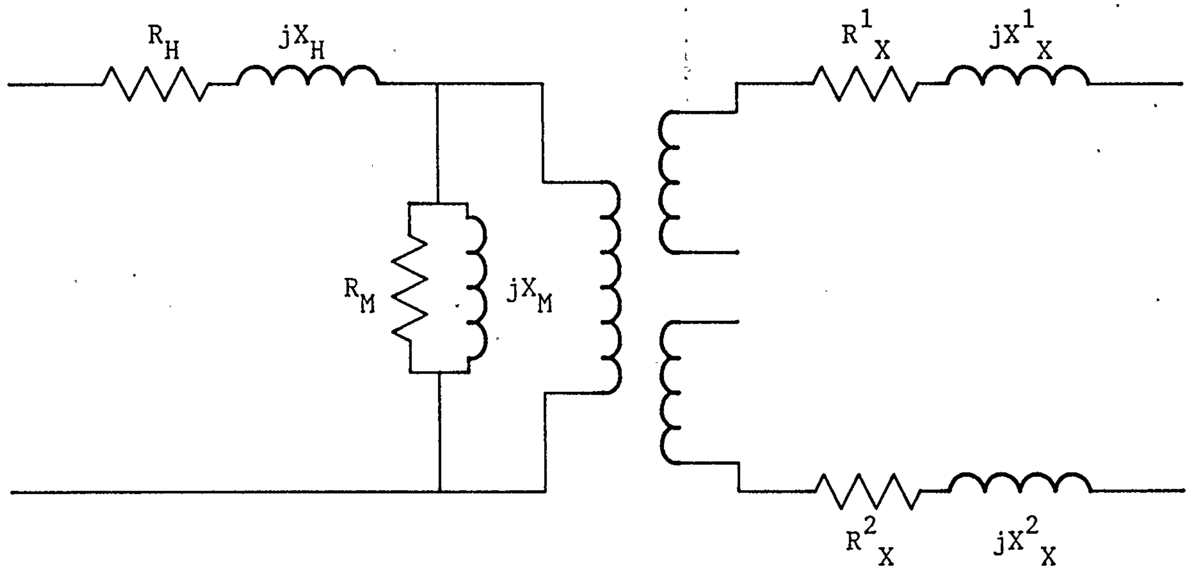

Figure 6.03. The equivalent circuit representation of a realistic three-winding transformer.

magnitude of the winding current. The voltage leads the fundamental harmonic of both the flux and the winding current by $\pi/2$ radians. Since the path of the leakage flux is primarily through magnetically linear media (air and oil), the leakage effect can be modelled by incorporating the linear inductive reactances X_H, $X^1{}_X$, and $X^2{}_X$ into the windings.

The non-infinite permeability of the transformer core is responsible for the presence of another element in the equivalent circuit in addition to the three inductances just mentioned. This element is the magnetizing reactance X_M. In a non-ideal magnetic core, a finite magnetic flux density must produce a non-zero magnetic field intensity. A small magnetizing current must flow to produce this field intensity, and X_M provides the required path. Because of the non-linearity of the core material, X_M is a function of the applied voltage; its magnitude drops rapidly as the core reaches saturation.

The last element to be added to the transformer model is a loss resistance R_M, which is introduced to account for the hysteresis and eddy current losses generated within the magnetic material of the core. Because these losses are proportional to some power of the applied voltage which is less than two, R_M generally tends to rise with increasing voltage.

It should be noted that the equivalent circuit discussed above is a fundamental-frequency circuit only. Because of the non-linearity of the magnetic flux density - magnetic field intensity relationship within the ferromagnetic materials used in transformers, sinusoidal voltages can generate currents that contain both fundamental and odd-harmonic components. In many cases the harmonic current components are ignored because if the voltage is sinusoidal, they deliver no net power. If necessary, the behaviour of the harmonics can be analyzed separately after a first approximation to the solution has been obtained using fundamental frequencies only. This analysis, while similar in principle to the analysis of the fundamental components, is beyond the scope of this project.

The remainder of this chapter will be devoted to developing, based on the transformer model just discussed, equations appropriate for use in the CVIM for single-phase transformers, three-phase banks, and three-phase transformers. In deriving these equations, the two secondary windings will be assumed to be identical, since such is generally the case with distribution transformers. Thus, $r^1 = r^2 = r$ and $Z^1{}_X = Z^2{}_X = Z_X$. Also, the transformer magnetizing admittance,

(6.04) $Y_M = 1/R_M - j1/X_M$,

will be assumed to be constant, since it doesn't vary too much over the voltage range of interest. It should be pointed out, though, that neither of these assumptions are essential to the CVIM.

6.3 Single-Phase Transformers

6.3.1 Transformers Supplying Three-Wire Secondaries

One of the devices most frequently encountered on a distribution system is the three-winding transformer used to supply residential and small commercial consumers. Two secondary windings are provided so that customers have access to two different supply voltages, typically 120 V and 240 V. The former is used for lights and small appliances, while the latter is used for large appliances like ranges and clothes dryers.

The equivalent circuit of a transformer supplying a three-wire secondary is shown in Figure 6.04. From that figure and the ideal transformer equations ((6.01) and (6.03)), the following equations can be derived:

(6.05) $$\begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_X = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_R$$

(6.06) $$I_H = r(I^1{}_X + I^2{}_X)$$

(6.07) $$I_M = Y_M E_H$$

(6.08) $$I^\phi{}_S = I_B = I_M + I_H$$

(6.09) $$E_H = V_H - Z_H I_B = V^\phi{}_U - Z_H I^\phi{}_S$$

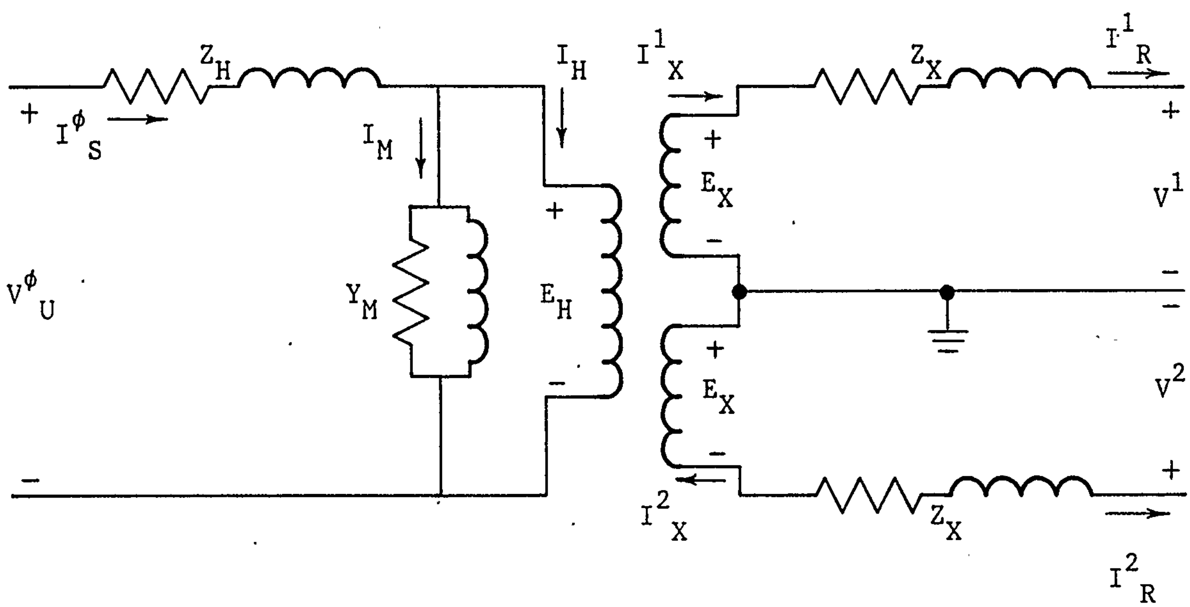

Figure 6.04. A single-phase three-winding transformer used in the supply of three-wire loads.

(6.10) $E_X = rE_H$

(6.11) $$\begin{bmatrix} V^1 \\ V^2 \end{bmatrix}_X = \begin{bmatrix} V^1 \\ V^2 \end{bmatrix} = E_X \begin{bmatrix} 1 \\ -1 \end{bmatrix} - Z_X \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_X$$

In the above equations, V_H and $V^{\psi}{}_X$ are the primary and secondary non-ideal winding voltages. The transformer primary winding is connected between phase ϕ and the system neutral so that $V_H = V^{\phi}{}_U$ and $I_B = I^{\phi}{}_S$, and the secondary winding and secondary bus voltages are one and the same, i.e., $V_X \equiv V$.

An expression for the sending end current $I^{\phi}{}_S$ that depends only on the transformer's parameters and the values of its terminal variables can be derived by using Equations (6.05)-(6.07) and (6.09) in (6.08); the result is

(6.12) $I^{\phi}{}_S = H[Y_M V^{\phi}{}_U + r(I^1{}_R - I^2{}_R)]$,

where

(6.13) $H = 1/(1 + Y_M Z_H)$.

Equation (6.12) is used on the inbound portion of an iteration to calculate the sending end current.

The transformer performance equation to be used on the outbound portion of an iteration can be found by substituting Equations (6.10), (6.05), and (6.09) into (6.11). The downstream voltage vector is thereby shown to be

$$(6.14) \quad \begin{bmatrix} V^1 \\ V^2 \end{bmatrix} = r[V^\phi{}_U - Z_H I^\phi{}_S] \begin{bmatrix} 1 \\ -1 \end{bmatrix} - Z_X \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_R .$$

If $I^\phi{}_S$ is eliminated from this equation using (6.12), the secondary voltage can also be expressed as

$$(6.15) \quad \begin{bmatrix} V^1 \\ V^2 \end{bmatrix} = HrV^\phi{}_U \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \begin{bmatrix} Z_X + Z_P & -Z_P \\ -Z_P & Z_X + Z_P \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_R ,$$

where[3]

$$(6.16) \quad Z_P = Hr^2 Z_H .$$

[3]Note that r^2 means "r squared" from now on, rather than "r two."

6.3.2 Transformers Supplying Two-Wire Secondaries

Three-winding transformers are frequently called upon to supply only one secondary voltage rather than two, especially when they are used as components of a three-phase bank. There are two secondary connections of interest, both of which are shown in Figure 6.05.

For the parallel connection, Equation (6.05) is replaced by

$$(6.17) \quad I^1{}_X + I^2{}_X = I_R,$$

while Equation (6.11) is replaced by

$$(6.18) \quad V_X = V = E_X - Z'_X I^1{}_X = E_X - Z'_X I^2{}_X.$$

Equations (6.06)-(6.10) remain valid except that Z_X is replaced by Z'_X and r is replaced by r'. From Equations (6.17) and (6.18) it can be seen that

$$(6.19) \quad I^1{}_X = I^2{}_X = I_R/2.$$

Combining the last three equations with those that remain valid from the three-wire secondary case, the sending end current equation and the secondary voltage equation are found to be

$$(6.20) \quad I^\phi{}_S = H[Y_M V^\phi{}_U + r' I_R]$$

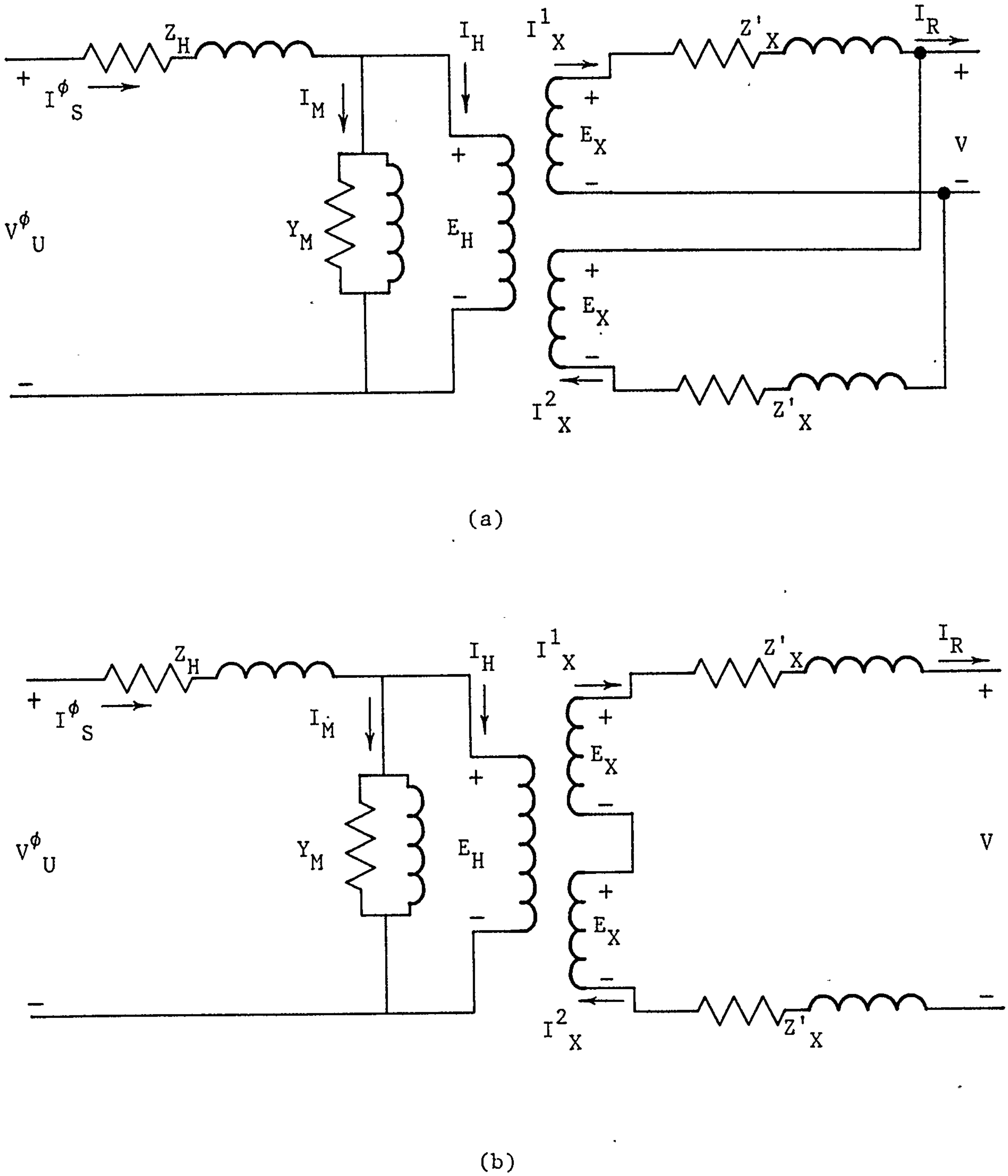

Figure 6.05. Possible two-wire connections for three-winding transformers: (a) parallel; (b) series.

and

(6.21) $V = Hr'V^{\phi}_{U} - [H(r'^2)Z_H + (Z'_X/2)]I_R.$

For the series connection, Equations (6.06)-(6.10) remain valid (with Z_X replaced by Z'_X and r replaced by r'). Equation (6.05) is replaced by

(6.22) $I^1_X = I^2_X = I_R,$

while

(6.23) $V_X = V = 2E_X - Z'_X(I^1_X + I^2_X)$

takes the place of (6.11). The resulting sending end current and secondary voltage equations are

(6.24) $I^{\phi}_S = H[Y_M V^{\phi}_U + (2r')I_R]$

and

(6.25) $V = H(2r')V^{\phi}_U - [H(2r')^2 Z_H + (2Z'_X)]I_R.$

If the definitions $Z_X = Z'_X/2$ and $r = r'$ are used when the secondary windings are connected in parallel, and if $Z_X = 2Z'_X$ and $r = 2r'$ when they are connected in series, then in both cases the sending end current equation is

$$I^{\phi}_{S} = H[Y_M V^{\phi}_{U} + rI_R]. \tag{6.26}$$

Also, in both cases the secondary voltage equation can be written as

$$V = HrV^{\phi}_{U} - Z_T I_R, \tag{6.27}$$

where

$$Z_T = Z_X + Hr^2 Z_H = Z_X + Z_P. \tag{6.28}$$

Further, it is not difficult to show that the last three equations remain valid when the transformer has only one secondary winding. (The turns ratio r would simply be N_X/N_H, while the impedance Z_X would be the impedance of the unit's secondary winding.) Thus, all two-wire secondary cases are governed by the same equations, and they can therefore all be represented by the equivalent circuit of Figure 6.06.

Equations (6.26) and (6.27) can both be written slightly differently, in forms that are useful when dealing with three-phase transformers. Recognizing that $I^{\phi}_{S} = I_B$, $V^{\phi}_{U} = V_H$, $V = V_X$, and $I_R = I_X$ for single-phase two-winding devices, (6.26) becomes

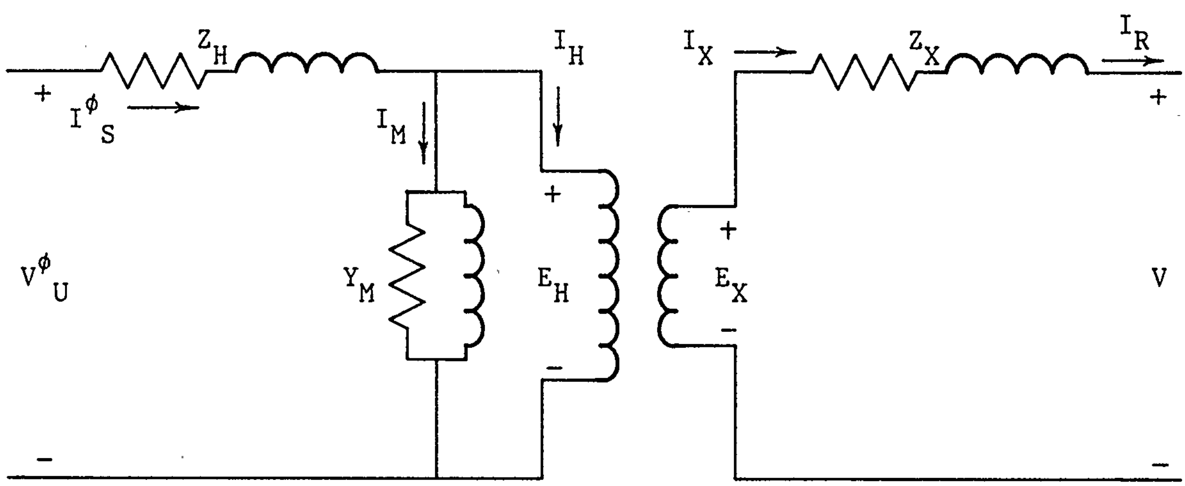

Figure 6.06. The equivalent circuit of a transformer supplying a two-wire secondary.

$$(6.29)\quad I_B = HY_MV_H + HrI_X,$$

while (6.27) becomes

$$(6.30)\quad V_X = HrV_H - Z_TI_X.$$

6.4 Three-Phase Banks of Single-Phase Transformers

6.4.1 Introduction

In order to serve commercial and industrial loads, single-phase transformers are often connected together to form three-phase banks. Given the large number of ways in which the connections can be made, it is impractical to examine them all. However, the method of deriving the appropriate performance equations should be sufficiently well illustrated that those connections not discussed can be modeled.

In all cases presented below it is assumed that: (1) each individual transformer is connected as a two-winding unit having an appropriately adjusted turns ratio and secondary winding impedance; (2) all of the transformers in the bank are the same; (3) the tap setting is the same for all units. These conditions normally exist, but they are not essential to the CVIM.

6.4.2 Common Three-Phase Bank Equations

Consider Figure 6.07, which shows a group of three transformers whose windings have not yet been interconnected in any way. Assuming each unit is represented by the equivalent circuit of Figure 6.06, a straightforward application of Equations (6.29) and (6.30) leads to

$$(6.31)\quad \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_B = HY_M \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_H + Hr \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_X$$

and

$$(6.32)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_X = Hr \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_H - Z_T \begin{bmatrix} I^1 \\ I^2 \\ I^1 \end{bmatrix}_X .$$

These equations are independent of any interconnections, and are therefore valid for all of the three-transformer banks to be discussed below.

Another set of equations to be used throughout this chapter is the following:

$$(6.33)\quad I_S = A_{\pi\sigma} V_U + B_{\pi\sigma} I_R$$

and

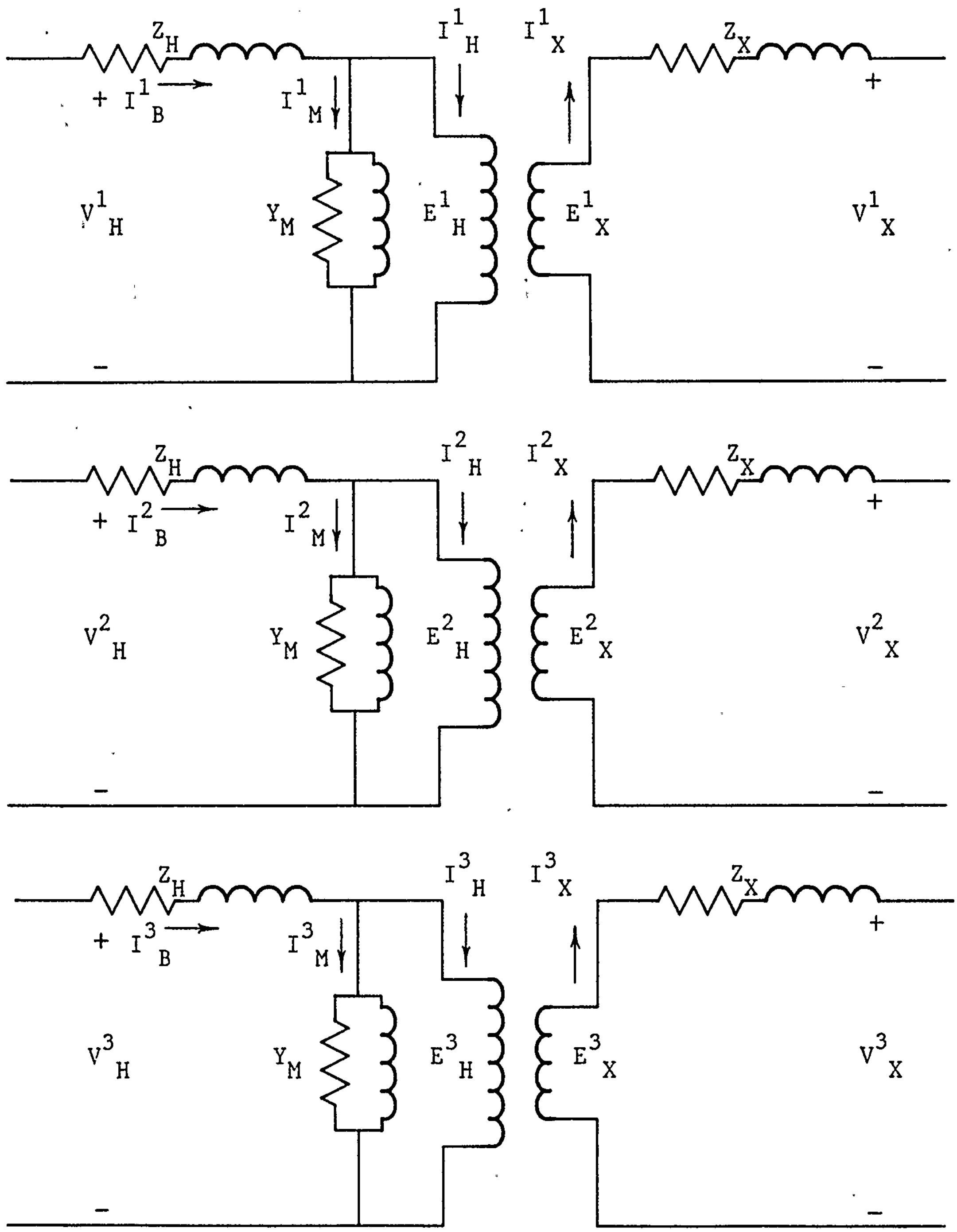

Figure 6.07. A bank of three distribution transformers whose windings have not yet been interconnected.

(6.34) $V = C_{\pi\sigma} V_U - D_{\pi\sigma} I_R.$

It will be shown that (6.33) is the valid inbound equation, and that (6.34) is the valid outbound equation, for all transformer connections considered. The matrices **A**, **B**, **C**, and **D** are functions of the primary and secondary winding connections, which are denoted by the subscripts π and σ, respectively.

6.4.3 Grounded-Wye/Grounded-Wye (GG) Banks

Figure 6.08 shows the equivalent circuit for a three-phase transformer bank in which both the primary and secondary windings are wye-connected and both neutral points are solidly grounded. From that figure it can be seen that:

$$(6.35)\qquad \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_X = \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R$$

$$(6.36)\qquad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_S = \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_B$$

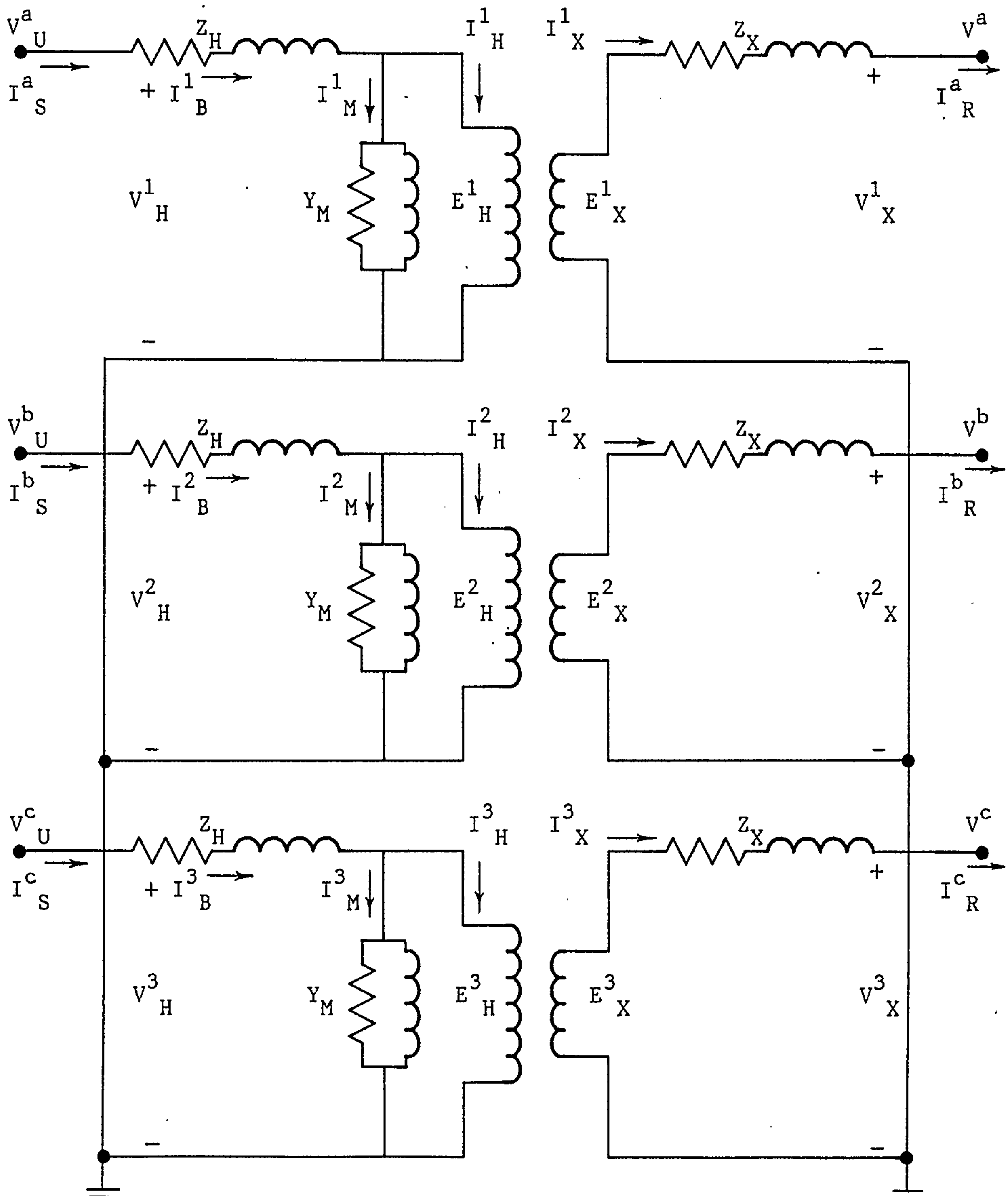

Figure 6.08. The equivalent circuit of a grounded-wye/grounded-wye transformer bank.

(6.37) $$\begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_H = \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U$$

(6.38) $$\begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} = \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_X$$

The sending end current equation for the grounded-wye/grounded-wye transformer can be derived by substituting Equations (6.35)-(6.37) into (6.31). The result, as expected, is simply Equation (6.26) with the voltage and current scalars replaced by vectors:

(6.39) $$\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_S = HY_M \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U + Hr \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R .$$

This equation is the same as Equation (6.33) provided that

(6.40) $A_{GG} = HY_M U$

and

(6.41) $B_{GG} = HrU$,

where $\mathbf{U}$ is the unit matrix.

The receiving end voltage equation can be found by combining Equations (6.35), (6.36), and (6.38) with (6.32). The result is Equation (6.34) with

(6.42) $\mathbf{C}_{GG} = Hr\mathbf{U}$

and

(6.43) $\mathbf{D}_{GG} = Z_T\mathbf{U}.$

6.4.4 Delta/Grounded-Wye (ΔG) Banks

A delta/grounded-wye distribution transformer bank is represented by the equivalent circuit shown in Figure 6.09a. The phases are labelled in such a way that, if the transformers were ideal, the voltages on the HV side would lead their corresponding LV voltages by 30° (see Figure 6.09b).

An examination of the equivalent circuit diagram leads to the following equations:

(6.44) $$\begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_X = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R$$

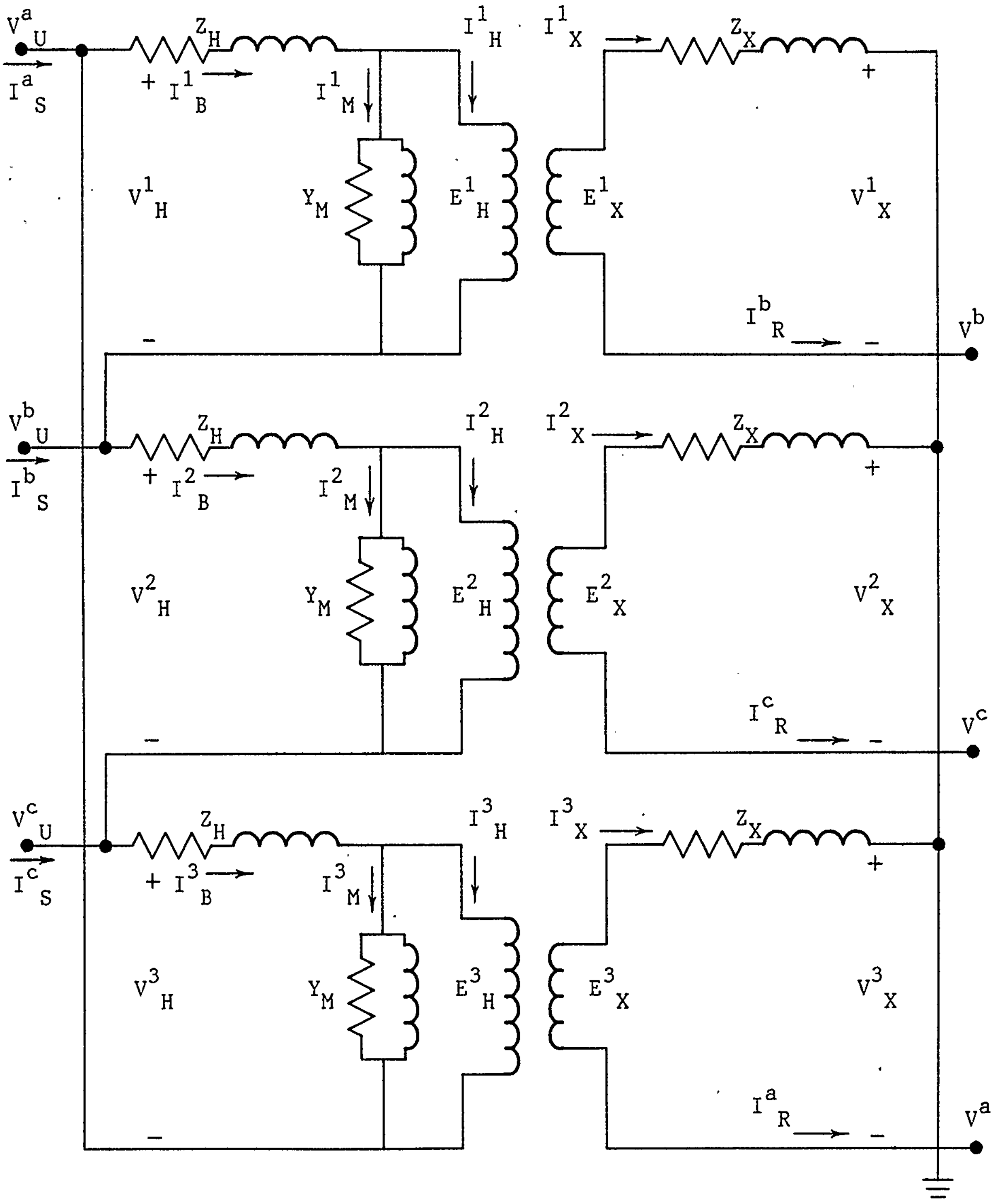

Figure 6.09. A delta/grounded-wye bank: (a) the equivalent circuit; (b) the phasor diagram for an ideal bank (next page).

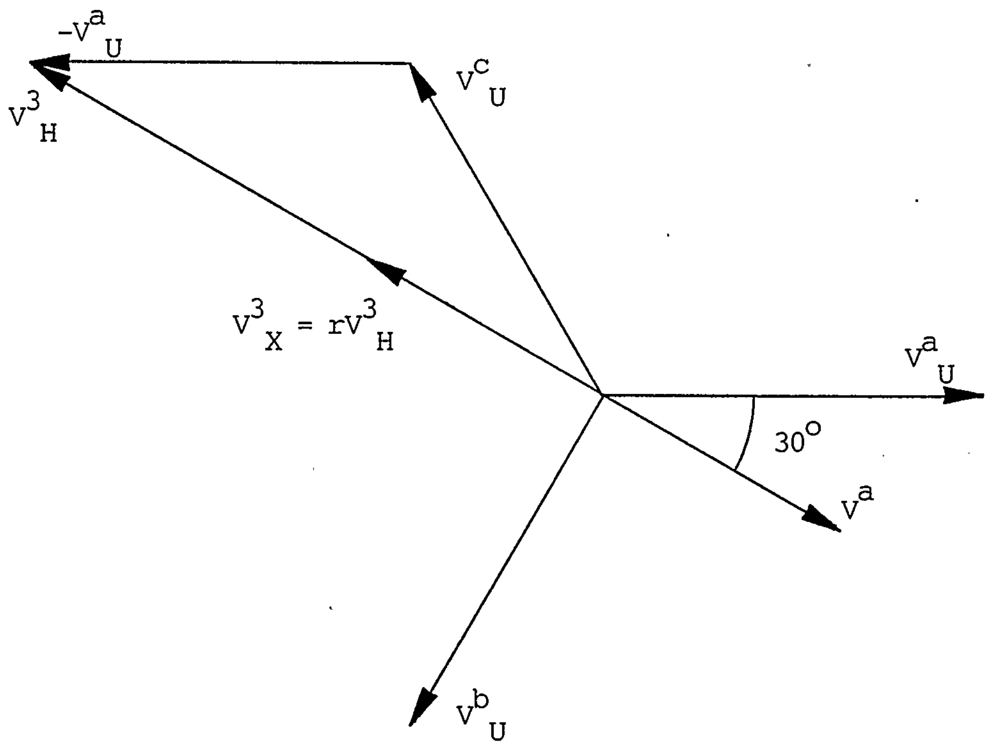

Figure 6.09, Part (b).

$$(6.45)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_S = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_B$$

$$(6.46)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_H = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U$$

$$(6.47)\quad \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_X$$

Premultiplying Equation (6.31) by the 3×3 matrix in (6.45), and then substituting (6.44)-(6.46) into the result, shows that the sending end current matrices are

$$(6.48)\quad A_{\Delta G} = HY_M \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

and

$$B_{\Delta G} = Hr\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}. \tag{6.49}$$

To find the secondary voltage matrices, Equation (6.32) is first premultiplied by the 3×3 matrix in (6.47). Equations (6.46) and (6.44) are then substituted into the result, leading to Equation (6.34) with

$$C_{\Delta G} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{6.50}$$

and

$$D_{\Delta G} = Z_T U. \tag{6.51}$$

6.4.5 <u>Grounded-Wye/Delta (GΔ) Banks</u>

The equivalent circuit for a transformer bank with a grounded-wye primary and a delta-connected secondary is shown in Figure 6.10. The phase labelling convention used for delta/grounded-wye transformers is again followed.

The first step in working out the sending end current equation is the derivation of an equation expressing the secondary winding currents I_X in terms of the transformer's receiving end currents I_R. The derivation of

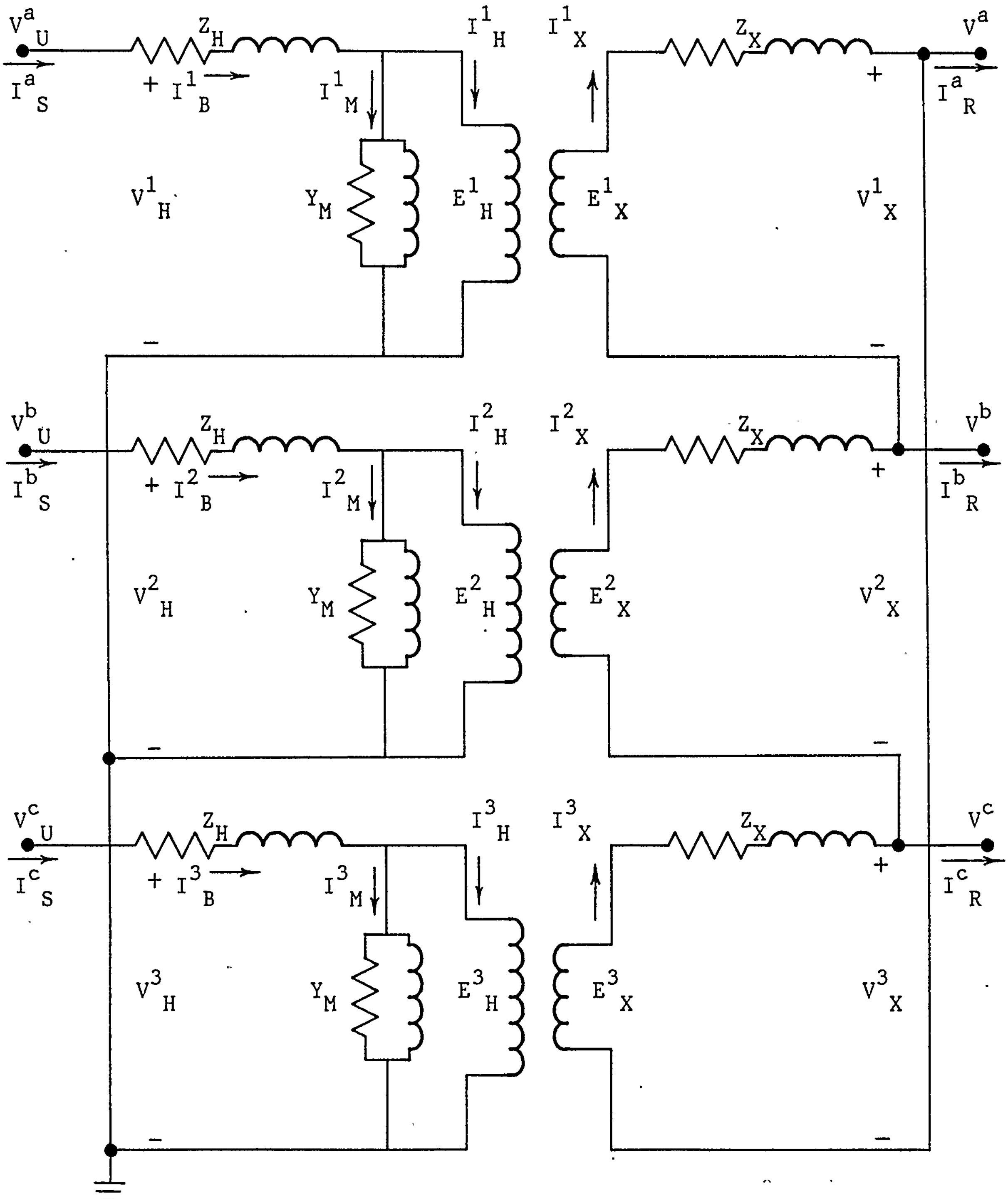

Figure 6.10. The equivalent circuit for a grounded-wye/delta distribution transformer bank.

this equation is not simply a matter of examining the equivalent circuit, as it has been in previous cases, because $\mathbf{I}_X$ cannot be expressed solely in terms of $\mathbf{I}_R$. This can be seen by noting that

$$(6.52) \qquad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_X .$$

The determinant of the 3×3 matrix is zero, so its inverse does not exist.

To find an equation for $\mathbf{I}_X$, use is made of the fact that the sum of the voltage drops around the closed loop formed by the secondary windings must be zero. That is,

$$(6.53) \qquad \sum_{\mu=1}^{3} \{E^{\mu}_X - Z_X I^{\mu}_X\} = 0.$$

Now, Equation (6.52) can be used to eliminate I^2_X and I^3_X from (6.53), with the result that

$$(6.54) \qquad \sum_{\mu=1}^{3} \{E^{\mu}_X\} - Z_X[I^1_X + (I^1_X + I^b_R) + (I^1_X - I^a_R)] = 0.$$

This equation can be solved for I^1_X to give

$$(6.55)\quad I^1{}_X = (1/3Z_X)\sum_{\mu=1}^{3}\{E^\mu{}_X\} + (1/3)(I^a{}_R - I^b{}_R).$$

When a similar procedure is used to determine $I^2{}_X$ and $I^3{}_X$, it is found that

$$(6.56)\quad \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_X = (1/3Z_X)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} E^1 \\ E^2 \\ E^3 \end{bmatrix}_X + 1/3\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R.$$

Substituting for $\mathbf{E}_X$ using

$$(6.57)\quad \mathbf{E}_X = r\mathbf{E}_H = r\mathbf{V}_H - rZ_H\mathbf{I}_B,$$

the secondary winding currents can be expressed as

$$(6.58)\quad \mathbf{I}_X = \mathbf{M}_1\mathbf{V}_H - Z_H\mathbf{M}_1\mathbf{I}_B + \mathbf{M}_2\mathbf{I}_R,$$

where

$$(6.59)\quad \mathbf{M}_1 = (r/3Z_X)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and

$$(6.60) \quad \mathbf{M}_2 = (1/3)\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}.$$

Now that an equation for $\mathbf{I}_X$ is available, the derivation of the sending end current equation proceeds as follows:

$$(6.61) \quad \mathbf{I}_B = HY_M\mathbf{V}_H + Hr\mathbf{I}_X$$

$$= HY_M\mathbf{V}_H + Hr[\mathbf{M}_1\mathbf{V}_H - Z_H\mathbf{M}_1\mathbf{I}_B + \mathbf{M}_2\mathbf{I}_R]$$

$$= H[Y_M\mathbf{U} + r\mathbf{M}_1]\mathbf{V}_H - HrZ_H\mathbf{M}_1\mathbf{I}_B + Hr\mathbf{M}_2\mathbf{I}_R.$$

Since

$$(6.62) \quad \mathbf{V}_H = \mathbf{V}_U$$

and

$$(6.63) \quad \mathbf{I}_S = \mathbf{I}_B$$

for a grounded-wye primary, Equation (6.61) can be rearranged and written as

$$(6.64)\quad M_3 I_S = [Y_M U + rM_1]V_U + rM_2 I_R,$$

or equivalently,

$$(6.65)\quad I_S = (M_3{}^{-1})[Y_M U + rM_1]V_U + r(M_3{}^{-1})M_2 I_R.$$

The matrix M_3 is given by

$$(6.66)\quad M_3 = (1/H)U + rZ_H M_1$$

$$= \begin{bmatrix} m + (1/H) & m & m \\ m & m + (1/H) & m \\ m & m & m + (1/H) \end{bmatrix},$$

where

$$(6.67)\quad m = r^2 Z_H/(3Z_X).$$

Since the inverse of M_3 is given by

$$M_3^{-1} = 1/[3m + 1/H]\begin{bmatrix} 1+2mH & -mH & -mH \\ -mH & 1+2mH & -mH \\ -mH & -mH & 1+2mH \end{bmatrix}, \tag{6.68}$$

the sending end current may be expressed as

$$\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_S = \begin{bmatrix} y_1 & y_2 & y_2 \\ y_2 & y_1 & y_2 \\ y_2 & y_2 & y_1 \end{bmatrix}\begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_U + (Hr/3)\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R, \tag{6.69}$$

where

$$y_1 = [r^2/3Z_X + (1 + 2mH)Y_M]/[3m + 1/H] = Y_M H + (Hr)^2/(3Z_T) \tag{6.70}$$

and

$$y_2 = [r^2/3Z_X - mHY_M]/[3m + 1/H] = (Hr)^2/(3Z_T). \tag{6.71}$$

Thus,

$$A_{G\Delta} = \begin{bmatrix} y_1 & y_2 & y_2 \\ y_2 & y_1 & y_2 \\ y_2 & y_2 & y_1 \end{bmatrix} \tag{6.72}$$

and

$$B_{G\Delta} = (Hr/3)\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}. \tag{6.73}$$

Problems with singular matrices associated with grounded-wye/delta transformer connections are not limited to the derivation of the sending end current equation; they also arise in connection with the development of the secondary voltage equation. Referring to Figure 6.10, it can be seen that

$$\begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_X = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}. \tag{6.74}$$

Since the 3×3 matrix in this equation does not possess an inverse, knowing V_X does not allow the calculation of V. Here, the solution is somewhat different than in the sending end current calculation.

Because none of the wires on the secondary side of a grounded-wye/delta transformer are grounded or electrically connected to the primary, the secondary is an electrically isolated system.[4] Therefore,

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[4]No KVL equation can be written that involves any secondary voltage and a primary or ground voltage.

any of the conductor voltages may be defined to be the secondary reference voltage. Remembering the 30° phase shift convention, let

(6.75) $$V^a \equiv [\exp(-j\pi/6)/\sqrt{3}]V^1{}_X = uV^1{}_X.$$

Given that, referring again to Figure 6.10 shows that

(6.76) $$\begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} = \begin{bmatrix} u & 0 & 0 \\ u-1 & 0 & 0 \\ u & 0 & 1 \end{bmatrix} \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_X.$$

Premultiplying Equation (6.32) by the 3×3 matrix in (6.76) and then using (6.58)-(6.60), (6.62), (6.63), (6.69), and (6.76), the secondary voltage equation is found to be (6.34), where

(6.77) $$\mathbf{C}_{G\Delta} = (Hr/3)\begin{bmatrix} 2u & -u & -u \\ 2u-2 & 1-u & 1-u \\ 2u-1 & -u-1 & 2-u \end{bmatrix}$$

and

$$(6.78) \qquad D_{G\Delta} = (Z_T/3)\begin{bmatrix} u & -u & 0 \\ u-1 & 1-u & 0 \\ u-1 & -u & 1 \end{bmatrix}$$

It should be pointed out that the complete electrical isolation of a grounded-wye/delta transformer's secondary circuit is a consequence of the model chosen to represent the device. This isolation does not occur physically, and it could be avoided if the coupling of the secondary windings to those on the primary through the ambient electric field were to be accounted for by interwinding capacitances in the equivalent circuit. The use of such a detailed model is not warranted, however. At power system frequencies, capacitance effects are very small, and because the secondary load voltages (and therefore the secondary currents) depend only on conductor potential differences, there is no problem in assuming that $V^a = uV^1_X$.

6.4.6 Delta/Delta ($\Delta\Delta$) Banks

The equivalent circuit for a transformer with both primary and secondary windings connected in delta is shown in Figure 6.11. The derivation of its sending end current and secondary voltage equations can make use of much of the work of the previous section.

For a grounded-wye/delta transformer Equations (6.33) and (6.34), by virtue of Equations (6.62) and (6.63), can be written as

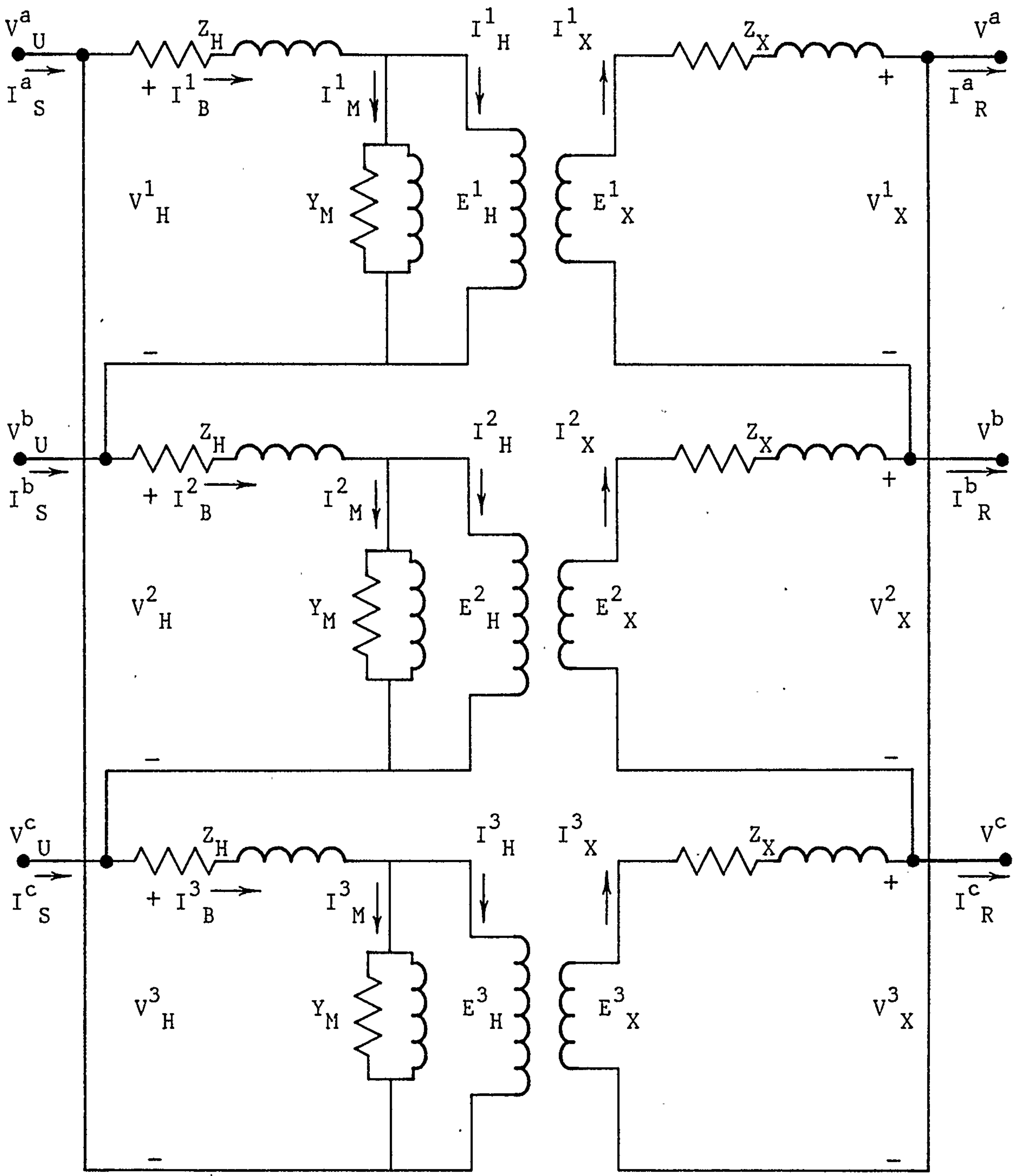


Figure 6.11. A delta/delta transformer bank.

(6.79) $\mathbf{I}_B = \mathbf{A}_{G\Delta}\mathbf{V}_H + \mathbf{B}_{G\Delta}\mathbf{I}_R$

and

(6.80) $\mathbf{V} = \mathbf{C}_{G\Delta}\mathbf{V}_H - \mathbf{D}_{G\Delta}\mathbf{I}_R$.

Since the last two equations are independent of the primary connection (the primary phase quantities $\mathbf{I}_S$ and $\mathbf{V}_U$ are not involved), they are valid for delta/delta transformers as well. Now, from Figure 6.11, it can be seen that

$$(6.81) \quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_H = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U$$

and

$$(6.82) \quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_S = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_B .$$

Premultiplying Equation (6.79) by the 3×3 matrix of (6.82), using the last two equations and the definitions of $\mathbf{A}_{G\Delta}$ and $\mathbf{B}_{G\Delta}$, and noting that

(6.83) $y_1 - y_2 = Y_M H,$

the sending end current matrices are found to be

$$\mathbf{A}_{\Delta\Delta} = HY_M \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (6.84)$$

and

$$\mathbf{B}_{\Delta\Delta} = (Hr/3) \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}. \quad (6.85)$$

Using Equation (6.81) and the definitions of $\mathbf{C}_{G\Delta}$ and $\mathbf{D}_{G\Delta}$ in (6.80), the secondary voltage matrices are found to be

$$\mathbf{C}_{\Delta\Delta} = Hr \begin{bmatrix} u & -u & 0 \\ u-1 & 1-u & 0 \\ u-1 & -u & 1 \end{bmatrix} \quad (6.86)$$

and

$$(6.87)\quad \mathbf{D}_{\Delta\Delta} = (Z_T/3)\begin{bmatrix} u & -u & 0 \\ u-1 & 1-u & 0 \\ u-1 & -u & 1 \end{bmatrix}.$$

6.4.7 Ungrounded-Wye/Ungrounded-Wye (YY) Banks

The equivalent circuit used to model a transformer bank in which both windings are wye-connected and have ungrounded neutral points is shown in Figure 6.12. It can be seen from that figure that:

$$(6.88)\quad \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_X = \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R$$

$$(6.89)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_S = \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_B$$

$$(6.90)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_H = \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U - V^n{}_H\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

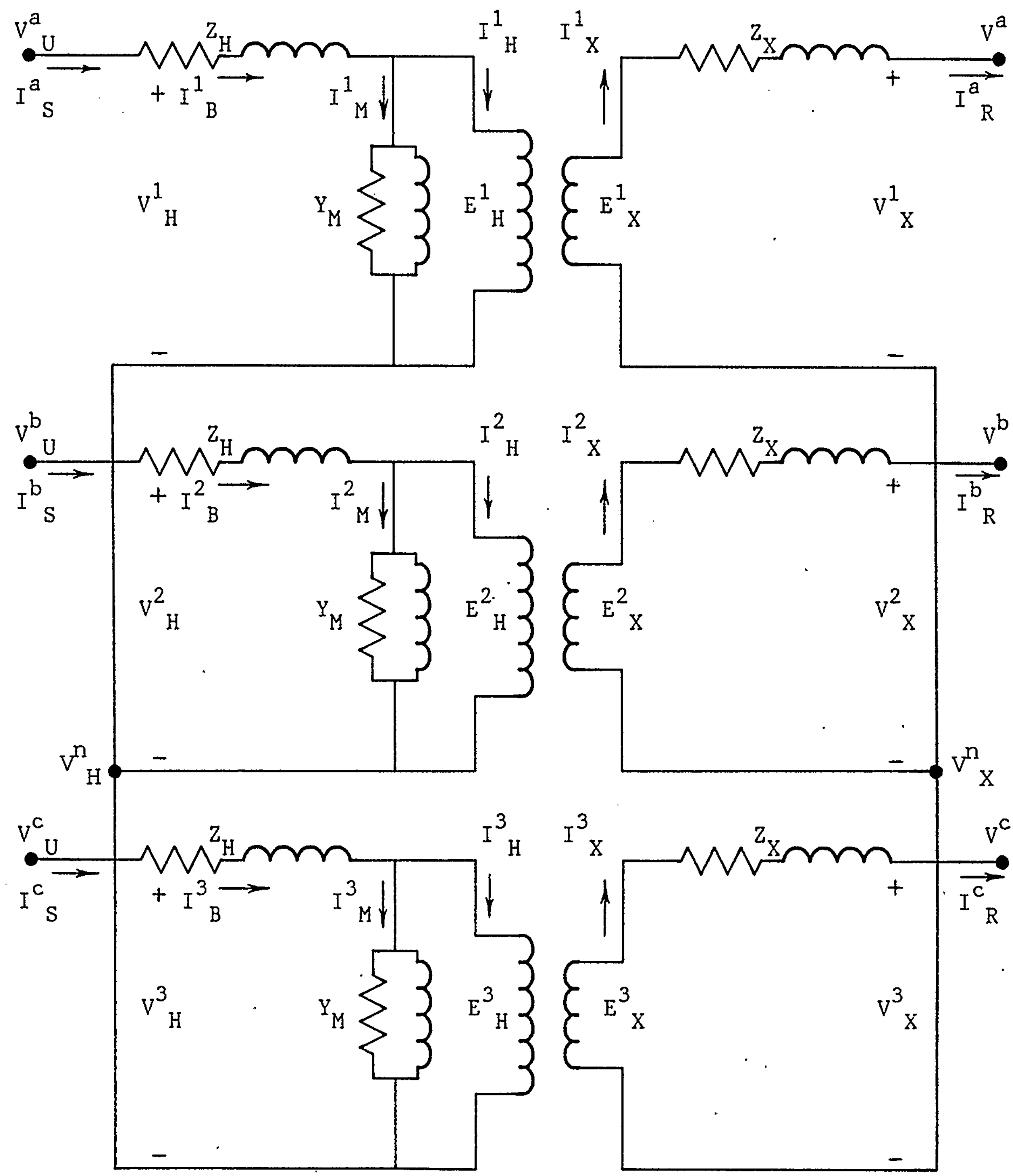


Figure 6.12. An ungrounded-wye/ungrounded-wye transformer bank.

$$(6.91)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_X = \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} - V^n{}_X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

In these equations, $V^n{}_H$ and $V^n{}_X$ are the primary and secondary neutral point voltages.

To derive the sending end current equation, Equations (6.31), (6.88), and (6.91) are first combined to yield

$$(6.92)\quad \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_B = HY_M \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U - HY_M(V^n{}_H) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + Hr \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R .$$

Now, the fact that

$$(6.93)\quad \sum_{\phi=1}^{3} I^\phi{}_B = 0$$

(by KCL at the primary neutral point) can be used to determine $V^n{}_H$. Using Equation (6.92) in (6.93) gives

$$(6.94) \qquad \sum_{\phi=a}^{c} \{HY_M V^{\phi}{}_U - HY_M(V^n{}_H) + Hr(I^{\phi}{}_R)\} = 0.$$

Since

$$(6.95) \qquad \sum_{\phi=a}^{c} I^{\phi}{}_R = 0$$

(by KCL at the secondary neutral point) and

$$(6.96) \qquad \sum_{\phi=a}^{c} V^n{}_H = 3V^n{}_H,$$

the neutral point voltage may be written as

$$(6.97) \qquad V^n{}_H = (1/3)\sum_{\phi=a}^{c} V^{\phi}{}_U.$$

When this equation is combined with (6.90), one finds that

$$(6.98)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_H = (1/3) \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U .$$

Finally, substituting Equations (6.98), (6.89), and (6.88) into Equation (6.31), the sending end current equation is found to be (6.33), with

$$(6.99)\quad \mathbf{A}_{YY} = (HY_M/3) \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

and

$$(6.100)\quad \mathbf{B}_{YY} = Hr\mathbf{U}.$$

As is the case with a delta-connected secondary with no connection to ground, an ungrounded-wye secondary is an isolated electrical system. Since any secondary voltage may therefore be chosen as the reference voltage, let

$$(6.101)\quad V^n{}_X = 0.$$

Using Equations (6.91), (6.101), and (6.98) in Equation (6.32) shows that the secondary voltage matrices are

$$(6.102)\quad \mathbf{C}_{YY} = (Hr/3)\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

and

$$(6.103)\quad \mathbf{D}_{YY} = Z_T\mathbf{U}.$$

6.4.8 Open Delta (OO) Banks

It is not necessary to use three single-phase transformers when a bank must supply three-phase loads. Under certain conditions (for example, in emergency situations when one of the three units of a three-transformer bank has failed), a connection like the open delta shown in Figure 6.13 may be employed. From the equivalent circuit for the case where the c-a transformer is missing:

$$(6.104)\quad \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_R$$

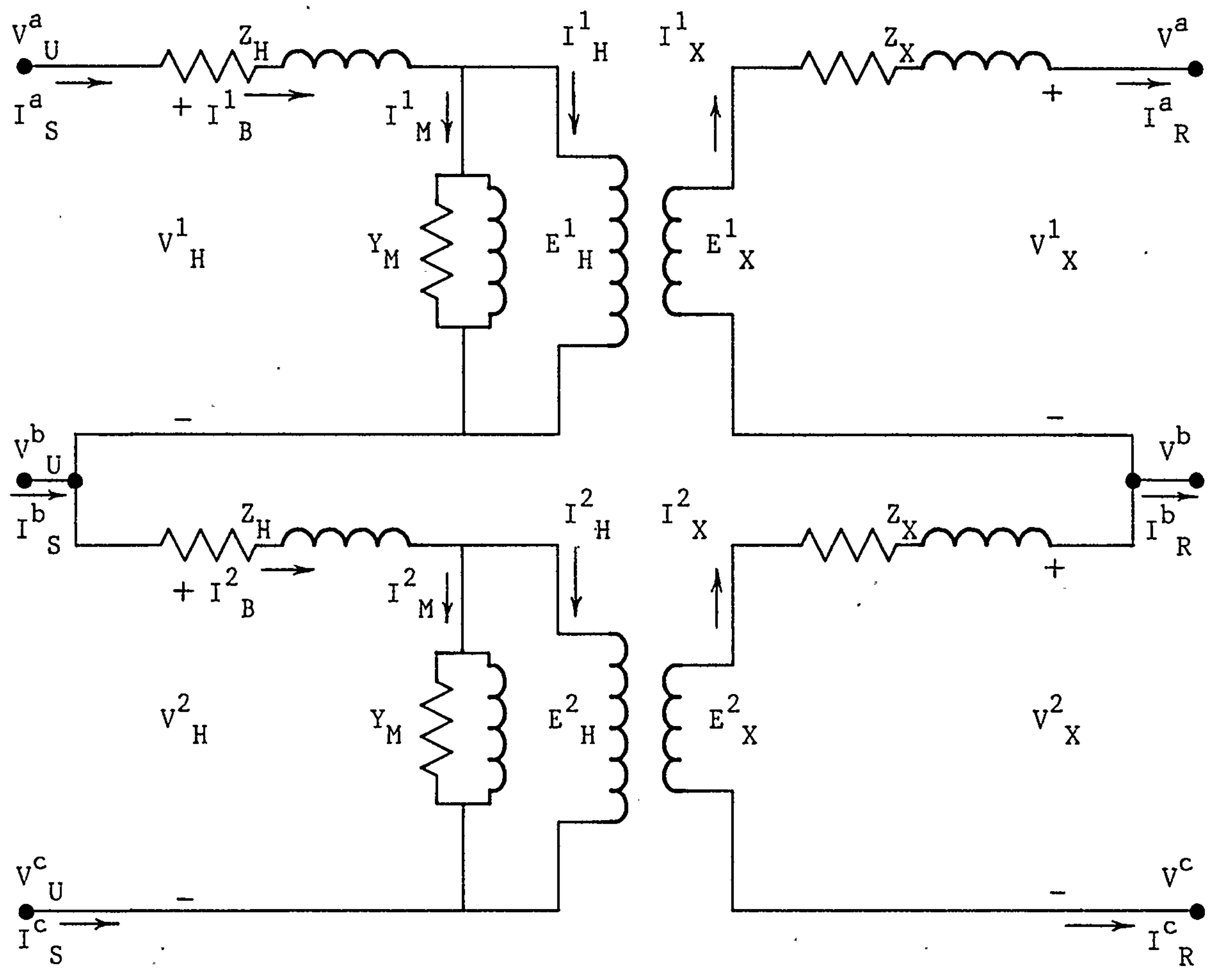


Figure 6.13. The equivalent circuit for an open-delta transformer bank. The c-a transformer is the missing one.

$$(6.105)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_S = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_B$$

$$(6.106)\quad \begin{bmatrix} V^1 \\ V^2 \end{bmatrix}_H = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_U$$

$$(6.107)\quad \begin{bmatrix} V^1 \\ V^2 \end{bmatrix}_X = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}$$

For a two-transformer group, Equation (6.31) is replaced by

$$(6.108)\quad \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_B = HY_M \begin{bmatrix} V^1 \\ V^2 \end{bmatrix}_H + Hr \begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_X.$$

Premultiplying this equation by the 3×2 matrix in (6.105), and then substituting Equations (6.104)-(6.106) into the result, gives Equation (6.33), where

$$(6.109)\quad A_{00} = HY_M \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

and

$$(6.110) \qquad B_{00} = Hr\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

The secondary windings of the open delta bank are, like the secondaries of several other transformer banks, electrically isolated. Therefore, the b-phase secondary voltage can be defined to be

$$(6.111) \qquad V^b = rV^b{}_U.$$

The a- and c-phase voltages can then be expressed as

$$(6.112) \qquad V^a = V^b + V^1{}_X = rV^b{}_U + V^1{}_X$$

and

$$(6.113) \qquad V^c = V^b - V^2{}_X = rV^b{}_U - V^2{}_X.$$

If substitutions for $V^1{}_X$ and $V^2{}_X$ are made from

$$(6.114)\quad \begin{bmatrix} V^1 \\ V^2 \end{bmatrix}_X = Hr\begin{bmatrix} V^1 \\ V^2 \end{bmatrix}_H - Z_T\begin{bmatrix} I^1 \\ I^2 \end{bmatrix}_X$$

(which is the two-transformer analogue of Equation (6.32)), and then (6.104) is used in the resulting equations, it is found that

$$(6.115)\quad \mathbf{C}_{00} = \begin{bmatrix} Hr & r(1-H) & 0 \\ 0 & r & 0 \\ 0 & r(1-H) & Hr \end{bmatrix}$$

and

$$(6.116)\quad \mathbf{D}_{00} = Z_T\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The above equations were all derived assuming that the c-a transformer was the missing one. When the same procedure is followed with the a-b or b-c transformer missing, similar equations result.

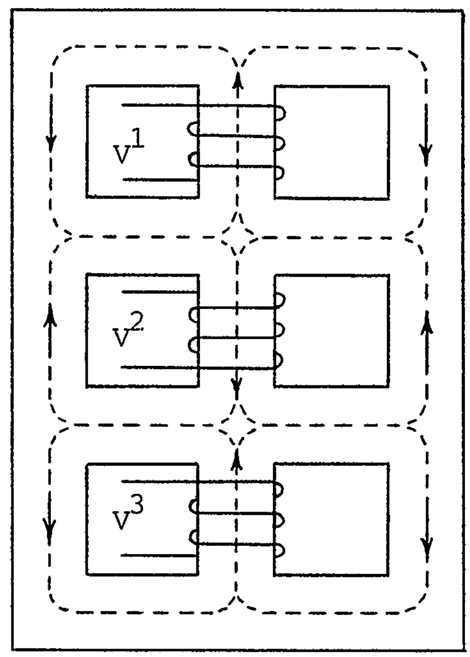
6.5 Three-Phase Transformers

While three-phase loads can be fed adequately from single-phase transformers connected to form three-phase banks, there are conditions (dictated by such things as space restrictions and economics) under which a single three-phase unit is more appropriate. The major performance difference between a three-phase transformer and a bank of single-phase devices arises because in the three-phase unit, all three phases are wound on the same ferromagnetic core. As a result there is coupling between the phases that is not present when separate single-phase transformers are used.

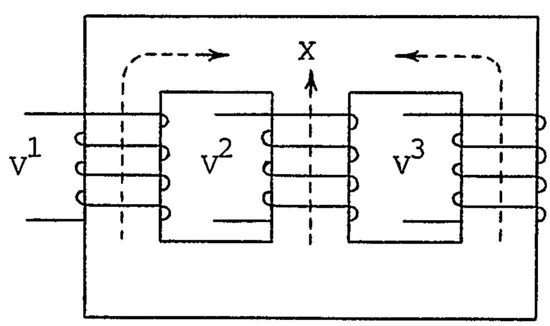
Three-phase transformer cores are built using either shell-form or core-form construction. When shell-form construction is used (Figure 6.14a) the windings are enclosed by the magnetic circuit. In core-form transformers (Figure 6.14b) the windings enclose the core.

Figure 6.14a shows that the magnetic flux produced by each individual winding in the shell-form transformer has available to it a closed magnetic path. This is essentially the same situation encountered when three single-phase transformers are used. Hence, a three-phase shell-form transformer will exhibit the same operational characteristics as a three-phase bank of single-phase units [52], and the models already discussed can be used.

In the core-form transformer, the length of the path taken by the b-phase flux is less than the path lengths of the a- and c-phase fluxes. The resulting unbalance in the corresponding magnetic circuits is, however, slight [48]. This, coupled with the fact that the fluxes must sum to zero (as can be seen by examining point X in Figure 6.14b), implies that the phase voltages must sum essentially to zero. As a result, the equivalent circuit for the three-phase core-form transformer has the same



(a)



(b)

Figure 6.14. Three-phase transformer cores: (a) shell-form; (b) core-form. For clarity, only the primary windings are shown.

form as three single-phase transformers connected delta/delta. Therefore, transformer models that have already been discussed may again be used.

7.0 LTC AND REGULATING TRANSFORMERS

7.1 Introduction

As was noted in Section 6.1.2, regulating transformers are used on distribution systems to control the voltages at particular system buses. They may be used in substations to control feeder source bus voltages, out on feeders to improve circuit voltage profiles, and directly adjacent to loads to control load voltages. The objective of this chapter is to develop a model for regulating transformers that is suitable for incorporation into the CVIM.

7.2 Voltage Regulator Operation and Control

Voltage regulators are actually tapped autotransformers in which one winding (the exciter winding) is common to both the primary and secondary circuits, as shown in Figure 7.01. The exciter winding is coupled to the series winding both electrically and magnetically. A number of taps are present on the series winding, and by changing from one tap position to another, the effective turns ratio of the autotransformer is changed; the regulator output voltage is thereby altered.

Load tap changing (LTC) transformers are used primarily in substations, and are combination transformer/regulators in which the tap changing mechanism is mounted in an oil-filled tank separate from the transformer. If the transformer is a three-phase unit, then tap changing is accomplished by a three-phase gang-operated switch.

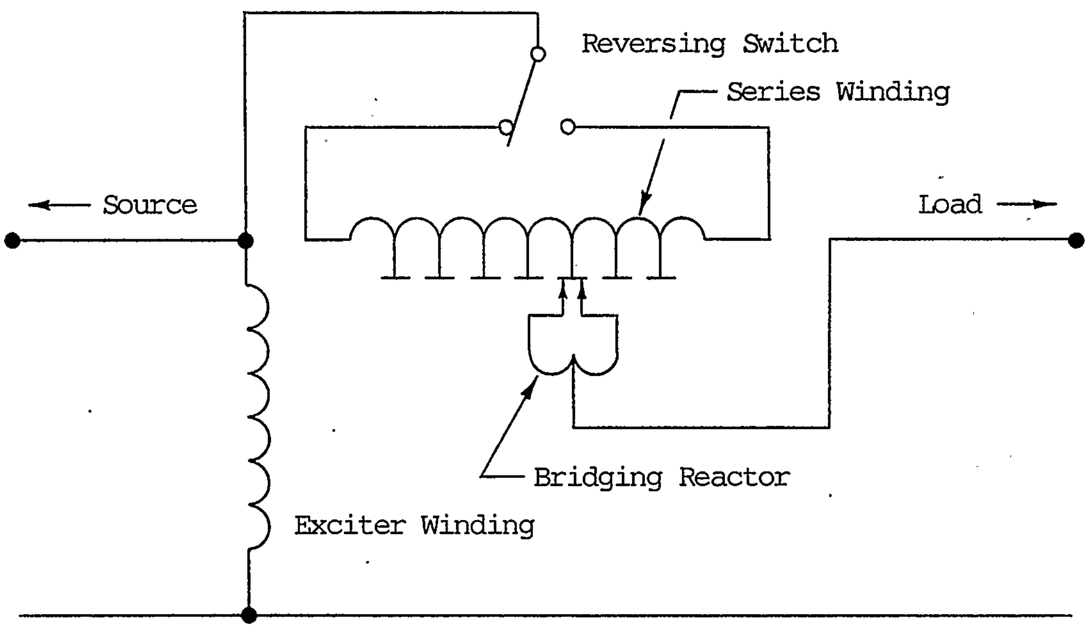
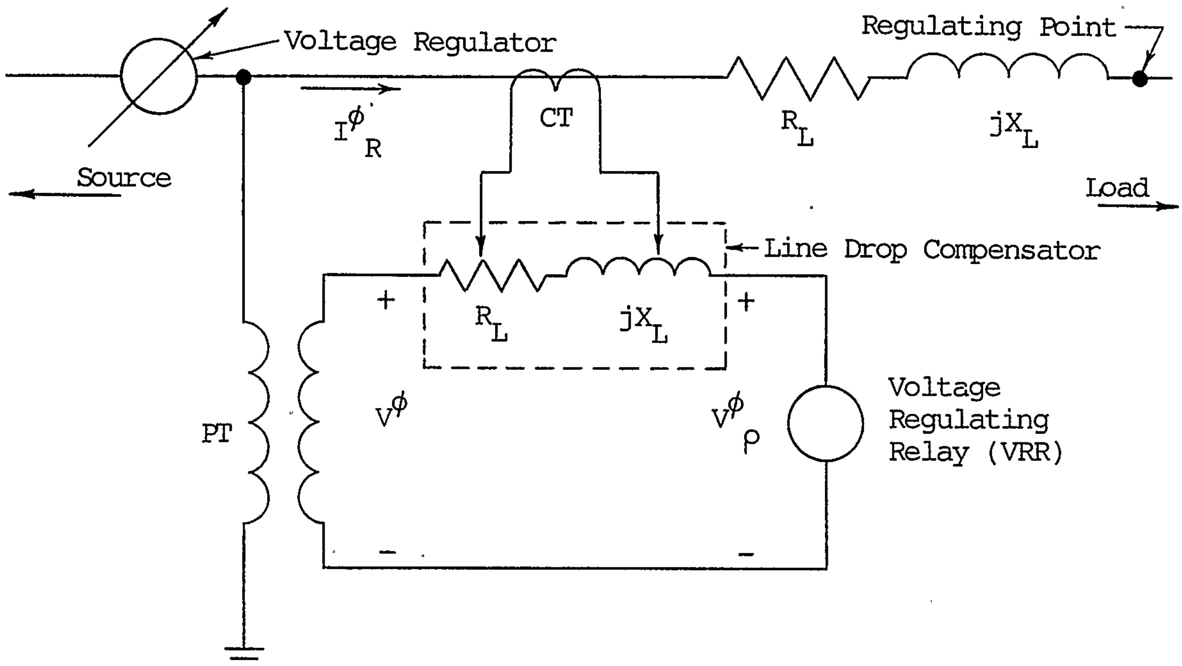


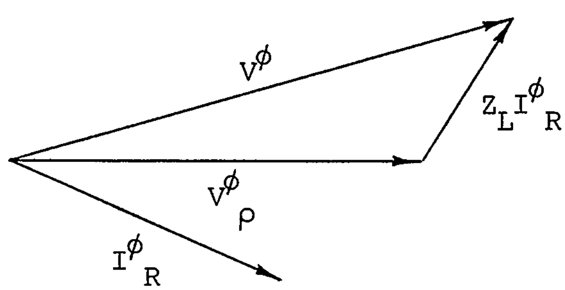
Figure 7.01. The main features of a regulating transformer.

To control the tap setting of a regulator or LTC transformer, a voltage transformer connected between the output and the system neutral provides the regulator's control circuit with a supply voltage proportional to the output voltage. A current transformer supplies the control circuit with a current proportional to the output current. Using these supplies as references, the control circuit determines whether the correct voltage is being supplied. If the voltage lies outside the acceptable range, a motor causes the tap-changing mechanism to change taps by rotating the contact points to the proper position on the dial switch. When a three-phase gang-operated switch is used, either a single phase voltage or the average of two phase voltages is monitored by the control circuit.

When it is desired to have the regulator control the voltage on its downstream bus, it is sufficient simply to measure the output voltage and adjust the tap setting accordingly. However, if it is desired to hold the voltage constant at some point remote from the regulator (the regulating point), a line drop compensator (LDC) is employed. The functioning of the LDC can be described with reference to the simplified schematic diagram and the voltage phasor diagram of Figure 7.02. The resistance in the potential transformer secondary circuit is relatively high compared to the reactance of the circuit, so that the current supplied by the potential transformer is almost in phase with the voltage. The current transformer produces an additional current through the resistance and reactance elements which is directly proportional to, and in phase with, the current flowing in the



(a)



(b)

Figure 7.02. A line drop compensator: (a) the schematic diagram; (b) the voltage phasor diagram.

line. The voltage regulating relay is set such that with zero load current, the regulator output voltage is equal to the voltage to be held at the regulating point. The LDC is then adjusted so that its elements produce a voltage drop proportional to the one that will arise on the feeder itself. If there are no loads, feeder taps, or shunt capacitors between the regulator and the regulating point, the compensator resistance and reactance will be the same (in per unit) as the total resistance and reactance of the feeder segment. If such devices are present, if the effect of the line's own shunt capacitance is not negligible, or if the voltage drop due to mutual coupling is appreciable, then the voltage drop in the LDC can only approximate the actual voltage drop.

If the LDC is set correctly, the regulator will maintain (approximately) the predetermined voltage at the regulating point. This can be seen by examining the phasor diagram of Figure 7.02b. V^{ϕ} represents the feeder voltage at the regulator output terminals as well as the secondary voltage of the potential transformer. Similarly, I^{ϕ}_{R} represents both the feeder line current and the current through the LDC provided by the current transformer, while $Z_{L}I^{\phi}_{R}$ is the voltage drop in the feeder and in the LDC. Finally, V^{ϕ}_{ρ}, which is the voltage at the regulating point, is also the voltage measured by the voltage regulating relay; it is therefore the one that is maintained within the specified tolerances, at least until the limits of adjustment of the regulator itself have been reached.

7.3 Regulator Performance Equations

For the purpose of developing its performance equations, an autotransformer can be considered to be a two-winding transformer with its windings connected in series. A voltage regulator may therefore be represented by the equivalent circuit[1] of Figure 7.03, wherein the switch position marked R is used when raising the voltage, while that marked L is used when lowering it. From the diagram:[2]

(7.01R) $I_X = I^{\phi}_R$

(7.01L) $I_X = -I^{\phi}_R$

(7.02) $I_H = rI_X$

(7.03) $I_M = Y_M E_H$

(7.04) $I_B = I_M + I_H$

(7.05) $I^{\phi}_S = I_B + I^{\phi}_R$

---

[1]A change in tap setting alters the electromagnetic configuration of the regulator, and must therefore alter the device's equivalent circuit parameters as well. The circuit parameters' variation with tap position is moderate, and the parameters will therefore be treated as constants.

[2]An equation with an R or L in its number applies only when the switch is in the R or L position, respectively. If no letter appears, the equation applies in both situations.

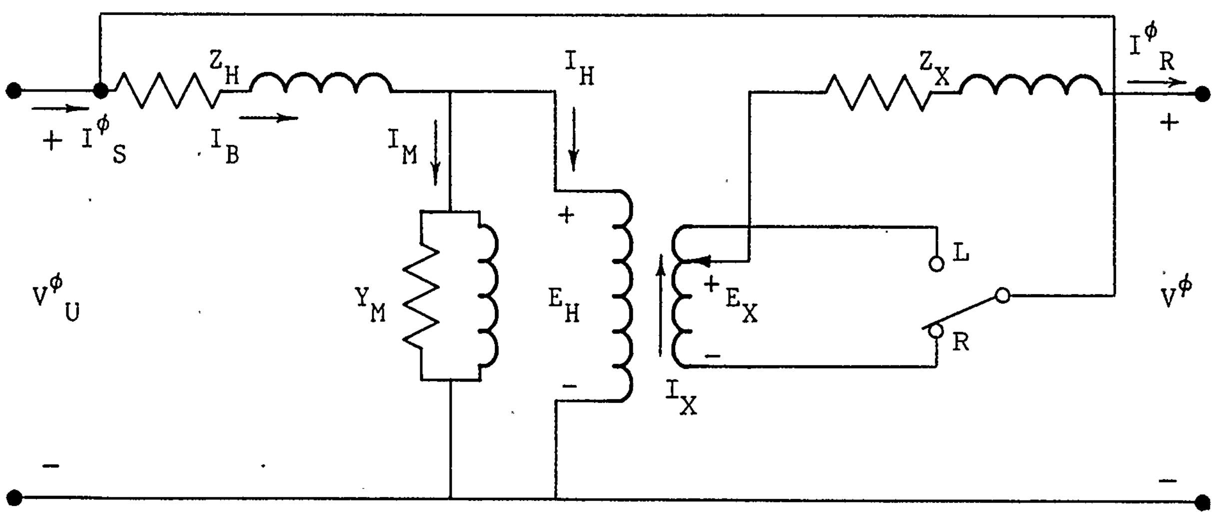


Figure 7.03. An equivalent circuit for a single-phase autotransformer.

$$(7.06)\quad E_H = V^{\phi}_U - Z_H I_B$$

$$(7.07)\quad E_X = rE_H$$

$$(7.08R)\quad V^{\phi} = V^{\phi}_U + E_X - Z_X I^{\phi}_R$$

$$(7.08L)\quad V^{\phi} = V^{\phi}_U - E_X - Z_X I^{\phi}_R$$

When these equations are combined in a manner virtually identical to that used in Section 6.3, the raise and lower mode voltage and current equations turn out to be:

$$(7.09R)\quad I^{\phi}_S = HY_M V^{\phi}_U + (1 + Hr)I^{\phi}_R$$

$$(7.09L)\quad I^{\phi}_S = HY_M V^{\phi}_U + (1 - Hr)I^{\phi}_R$$

$$(7.10R)\quad V^{\phi} = (1 + Hr)V^{\phi}_U - Z_T I^{\phi}_R$$

$$(7.10L)\quad V^{\phi} = (1 - Hr)V^{\phi}_U - Z_T I^{\phi}_R$$

If r is assumed to be the negative of the turns ratio when the reversing switch is in the L position, as will be done henceforth, then regardless of the switch position, Equations (7.09R) and (7.10R) are valid. Note that if the regulator is ideal, then $H = 1$, $Y_M = 0$, and $Z_T = 0$, so that

(7.11) $I^{\phi}{}_S = (1 + r)I^{\phi}{}_R$

and

(7.12) $V^{\phi} = (1 + r)V^{\phi}{}_U$.

If the voltage regulator has a raise/lower limit of L percent, then r can vary from

(7.13) $r_{min} = -L/100$

to

(7.14) $r_{max} = +L/100$

in discrete steps of

(7.15) $\Delta r = 2L/(100t)$,

where t is the number of tap steps provided on the regulator. In one frequently-used configuration, eight 1-1/4% taps, a centre-tapped bridging reactor that divides the steps in half, and a reversing switch combine to provide a total of 32 steps, 16 above and 16 below neutral. This configuration gives values or r from -0.10 to +0.10 in steps of 0.00625 per unit.

The use of the regulator performance equations in distribution system analysis is as follows. On the inbound portion of an iteration, Equation (7.09R) is used to determine the regulator's sending end current. Later, when the outbound portion of that iteration reaches the regulator, the downstream bus voltage V^{ϕ} is calculated using (7.10R) and the appropriate value of the turns ratio r. The voltage at the regulating point ρ is then calculated from

$$(7.16) \quad V^{\phi}_{\rho} = V^{\phi} - Z_{LDC} I^{\phi}_{R}.$$

If V^{ϕ}_{ρ} satisfies the inequality

$$(7.17) \quad [\|V^{\phi}_{d\rho}\| + W/2] \geq \|V^{\phi}_{\rho}\| \geq [\|V^{\phi}_{d\rho}\| - W/2],$$

where $\|V^{\phi}_{d\rho}\|$ is the magnitude of the desired regulating point voltage and W is the voltage bandwidth of the regulator, then the iteration proceeds to the next bus. If the regulating point voltage is too high, then r is decremented by the step size Δr until either (7.17) is true or r_{min} has been reached. Conversely, if $\|V^{\phi}_{\rho}\|$ is too low, then r is incremented until the inequality is satisfied or r_{max} has been reached. In either case the iteration then proceeds to the next bus.

Up to this point only single-phase regulators have been discussed. Since three-phase regulators for four-wire circuits are always wye-connected (and in fact usually consist of three single-phase regulators in a single tank), the equations presented above apply on a phase-by-phase basis. If the tap changer is of the three-phase gang-operated variety, then V^{ϕ}_{ρ} in (7.17) is replaced by either the voltage on the one monitored phase or by the average of the two monitored phase voltages, and the turns ratio is the same in each phase.

7.4 Substation Voltage Regulation

LTC transformers or regulators are frequently employed in distribution substations to provide some degree of compensation for variations in both upstream and downstream conditions. On the upstream side, the HV bus voltage provided by the subtransmission system may deviate from its desired value. As it does so, the tap setting on the regulating device changes, thereby maintaining the LV bus voltage within predetermined limits (provided, of course, that the compensation required is not beyond the capability of the device). On the downstream side, feeder loading varies with time of day and season; the controls on the regulating device can be set so that a high bus voltage is held during peak load periods, and a lower bus voltage is held during light load periods. In this way the voltage spread, which is the difference between the highest and lowest voltages encountered on the primary feeder, can be maintained within required tolerances. (It should be pointed out that bus voltage regulation cannot compensate for low primary voltages at remote points on a feeder that result from excessive feeder voltage drops.)

In view of the voltage control capabilities present in most substations, the author and others [25, 51] feel that the assumption of a constant-voltage feeder source bus is realistic. There are, of course, other assumptions that could be made. For example, the substation LV bus voltage could be taken to be a known function of the total current flowing from the bus. Alternatively, the substation HV bus could be taken to be a constant-voltage source, and the LV bus voltage could be determined based on the feeder currents and a representation of the substation's power transformer. In the former case, however, all feeders fed from the LV bus would have to be analyzed simultaneously, since a value for the total bus current would be required in the calculation of the LV bus voltage. In the latter case a similar statement applies: all feeders fed by the substation transformer would have to be analyzed at the same time, so that the transformer tap positions and voltage drops could be determined. It is unlikely that there would be a sufficient increase in the accuracy of the feeder analysis to justify the additional effort required in either of these cases. Any other assumptions are likely to be even less justifiable, so the constant-voltage LV bus assumption is used. Clearly, however, if the assumption leads to urealistic load flow results (for example, a power output greater than the substation transformer's rating), engineering judgement must be used in determining what assumptions need to be modified.

8.0 CAPACITOR BANKS

8.1 Introduction

In addition to regulating transformers, banks of capacitors are frequently employed to improve voltage and power flow conditions on distribution feeders. The function of shunt capacitors, applied as single units or in groups, is to supply lagging kilovars to the system at the point where they are connected. This reduces the lagging kilovars that must be carried by the feeder between the source and the location of the bank, so that the lagging component of the circuit current is reduced. Among the beneficial effects of this reduction are an increase in voltage level between the source and the bank, a decrease in system losses, improved voltage regulation, and a reduction in kVA loading on generators. Shunt capacitor banks are often switched, so that they can be inserted into or removed from the circuit as load conditions change.

Series capacitors, which are used much less frequently than shunt capacitors, compensate for the inductive reactance of distribution circuits. When a series capacitor is inserted in a feeder, the line inductance, as viewed between two points which include the capacitor, is reduced by the amount of the capacitive reactance. The series capacitor therefore reduces the voltage drop in the line caused by the inductive reactance. Because they are quite rare on distribution systems, series capacitors will not be discussed further.

8.2 Shunt Capacitor Performance Equations

It is assumed that the capacitors used on distribution systems are "perfect", i.e., the capacitance (and therefore the admittance) of each device is constant, and the associated losses are small enough that they can be ignored.

8.2.1 Single-Phase Capacitors

Figure 8.01 shows a single-phase capacitor bank, connected between phase ϕ and the system neutral, whose total rated output is Q_{Cr} VARs. The rated power delivered by the feeder to the capacitor, when the magnitude of the voltage across the latter is equal to its rated voltage V_{Cr}, is

$$(8.01) \quad S_C = V_{Cr}^2 (Y_{Cr})^* = -jQ_{Cr}.$$

(The negative sign appears on the right hand side because the capacitor actually delivers VARs to the system, i.e., it absorbs negative VARs). Since the admittance of the bank is constant, it is equal to the rated admittance Y_{Cr} under all conditions, and

$$(8.02) \quad Y_C = Y_{Cr} = jQ_{Cr}/V_{Cr}^2.$$

The only CVIM calculation involving capacitors occurs during the inbound portion of an iteration, when the current I^{ϕ}_{E} delivered to the single-phase bank is calculated using

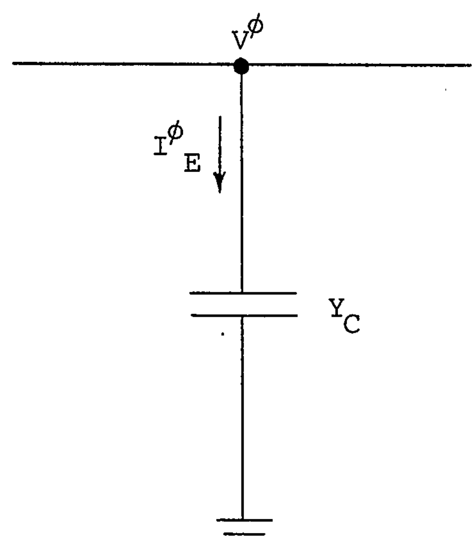


Figure 8.01. A single-phase capacitor.

$$(8.03)\quad I^{\phi}_{E} = Y_C V^{\phi}.$$

8.2.2 Three-Phase Delta-Connected Banks

Figure 8.02a shows a three-phase capacitor bank in which the units are connected in delta. The admittance of each leg, which is related to the capacitive VARs installed in the leg by Equation (8.02), is Y^{μ}_{C} (μ = 1, 2, 3). An examination of the figure shows that the following equations can be written:

$$(8.04)\quad \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_C = \begin{bmatrix} Y^1 & 0 & 0 \\ 0 & Y^2 & 0 \\ 0 & 0 & Y^3 \end{bmatrix}_C \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_C$$

$$(8.05)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \end{bmatrix}_C$$

$$(8.06)\quad \begin{bmatrix} V^1 \\ V^2 \\ V^3 \end{bmatrix}_C = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}$$

Using (8.04) and (8.06) in Equation (8.05) shows that

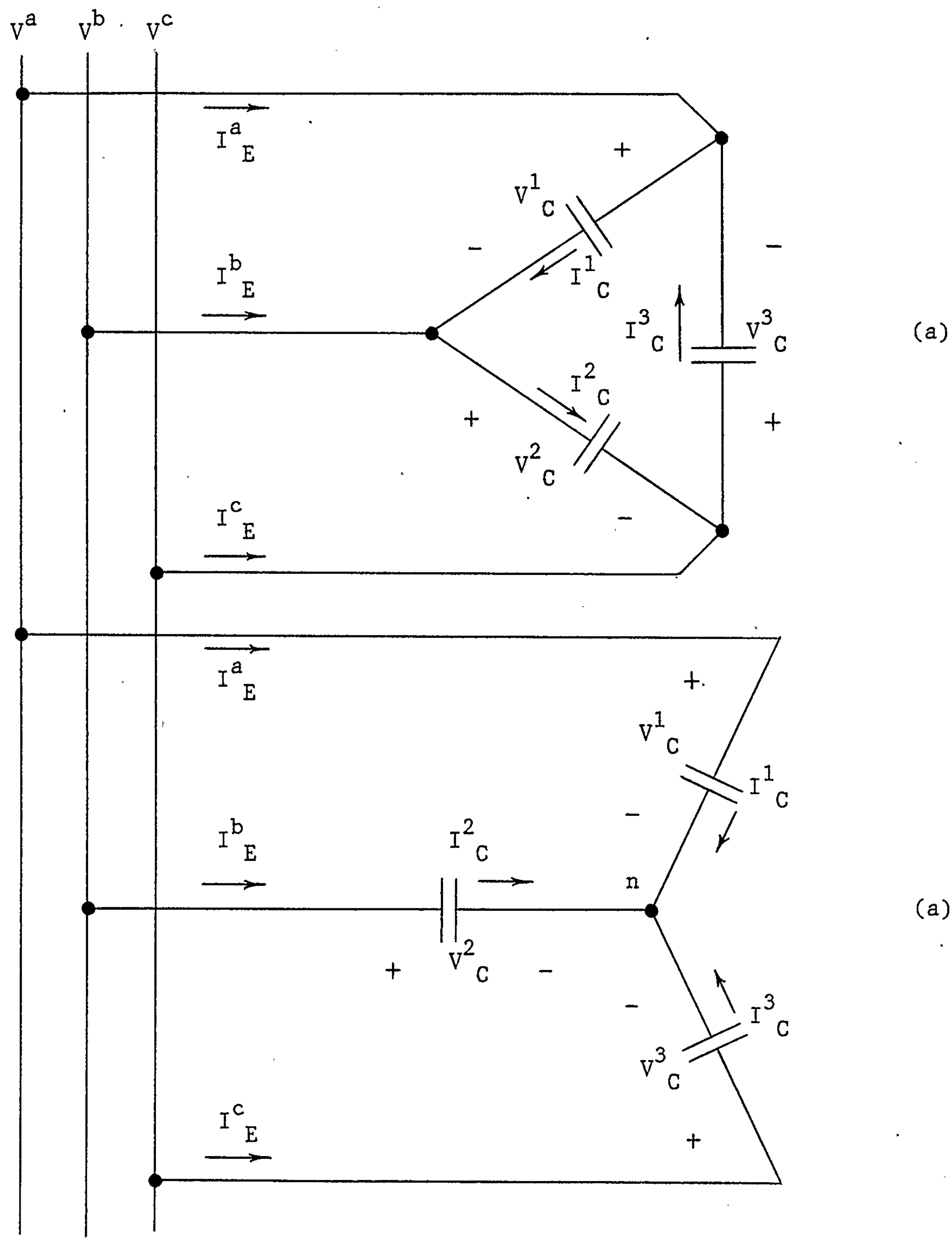


Figure 8.02. Three-phase capacitor banks: (a) delta-connected; (b) wye-connected.

(8.07)
$$\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = \begin{bmatrix} Y^1 + Y^3 & -Y^1 & -Y^3 \\ -Y^1 & Y^1 + Y^2 & -Y^2 \\ -Y^3 & -Y^2 & Y^2 + Y^3 \end{bmatrix}_C \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}.$$

If the bank is balanced, then the admittances are all equal to Y_C, and

(8.08)
$$\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = Y_C \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}.$$

8.2.3 Three-Phase Wye-Connected Banks

Figure 8.02b shows a capacitor bank in which the units are connected in wye. The neutral point may or may not be grounded. The current flowing into the bank is given by

(8.09)
$$\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = \begin{bmatrix} Y^1 & 0 & 0 \\ 0 & Y^2 & 0 \\ 0 & 0 & Y^3 \end{bmatrix}_C \left(\begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} - \begin{bmatrix} V^n \\ V^n \\ V^n \end{bmatrix} \right).$$

If the neutral point is not grounded, then by KCL,

$$(8.10) \qquad \sum_{\phi=a}^{c} I^{\phi}_{E} = 0.$$

Substituting for each I^{ϕ}_{E} from (8.09) leads to an equation for the neutral point voltage:

$$(8.11) \qquad V^{n} = (1/Y_{T})[Y^{1}_{C}V^{a} + Y^{2}_{C}V^{b} + Y^{3}_{C}V^{c}],$$

where

$$(8.12) \qquad Y_{T} = Y^{a}_{C} + Y^{b}_{C} + Y^{c}_{C}.$$

Using Equation (8.11) in (8.09), it is found that the current flowing into an ungrounded-wye capacitor bank is

$$(8.13) \qquad \begin{bmatrix} I^{a} \\ I^{b} \\ I^{c} \end{bmatrix}_{E} = (1/Y_{T}) \begin{bmatrix} Y^{1}(Y^{2} + Y^{3}) & -Y^{1}Y^{2} & -Y^{1}Y^{3} \\ -Y^{1}Y^{2} & Y^{2}(Y^{1} + Y^{3}) & -Y^{2}Y^{3} \\ -Y^{1}Y^{3} & -Y^{2}Y^{3} & Y^{3}(Y^{1} + Y^{2}) \end{bmatrix}_{C} \begin{bmatrix} V^{a} \\ V^{b} \\ V^{c} \end{bmatrix}.$$

If the bank is balanced, then (8.13) simplifies to

$$(8.14)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = (Y_C/3)\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}_C \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}.$$

If the neutral point of the wye-connected capacitor bank is grounded, then $V^n = 0$, and (8.09) simplifies immediately to

$$(8.15)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = \begin{bmatrix} Y^1 & 0 & 0 \\ 0 & Y^2 & 0 \\ 0 & 0 & Y^3 \end{bmatrix}_C \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix};$$

in the balanced case, this becomes

$$(8.16)\quad \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_E = Y_C \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}.$$

9.0 APPLICATIONS AND CONVERGENCE PROPERTIES OF THE CVIM

9.1 Introduction

The purpose of this chapter is to present the results of the CVIM analyses of several systems, and to briefly discuss the convergence properties of the method. In Section 9.2, idealized situations involving uniformly distributed loads on single-phase and three-phase distribution lines are investigated. The primary intent of presenting these situations is the demonstration of the CVIM's viability by comparing its results with those expected from theoretical considerations. In Section 9.3, the effect of transformer connections on system voltages and currents is considered briefly by studying a small, unbalanced three-phase system. Finally, in the last section, a few remarks are made about the convergence properties of the CVIM.

9.2 Uniformly Distributed Loads

9.2.1 Single-Phase Lines

Figure 9.01 shows a single-phase distribution line one unit in length that contains N identical loads, each of which is connected at the receiving end of a line section $\Delta x = 1/N$ units long. The total impedance of the line is Z_L, and the rated total load is S_T. Thus, the impedance of each line section is $Z_L/N = Z_L\Delta x$, and the rated complex power for each load is

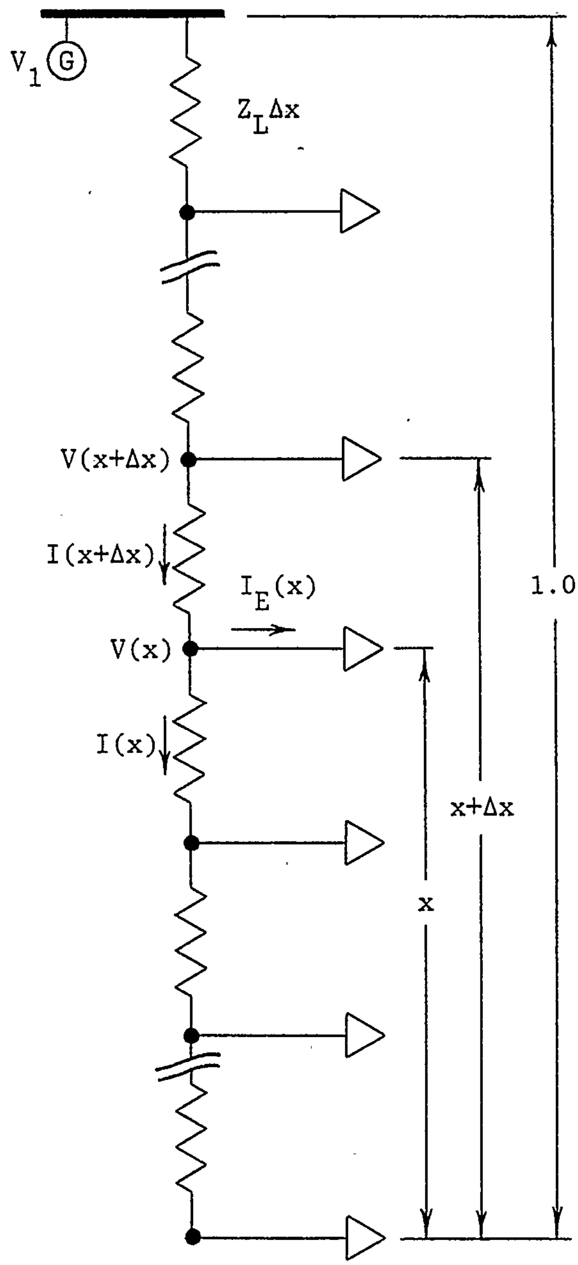


Figure 9.01. A uniformly loaded single-phase distribution line.

$S_r = S_T/N = S_T\Delta x$. The variable x represents the distance from the receiving end of the line.

From Figure 9.01 and KVL,

$$(9.01) \quad V(x+\Delta x) - [Z_L\Delta x]I(x+\Delta x) - V(x) = 0.$$

Rearranging and taking the limit as $\Delta x \to 0$ (i.e., the limit as $N \to \infty$), this equation becomes

$$(9.02) \quad \lim_{\Delta x\to 0}\{[V(x+\Delta x) - V(x)]/\Delta x\} = Z_L \lim_{\Delta x\to 0}\{I(x+\Delta x)\}.$$

Then, using the definition

$$(9.03) \quad dV(x)/dx = \lim_{\Delta x\to 0}\{[V(x+\Delta x) - V(x)]/\Delta x\}$$

and the fact that

$$(9.04) \quad \lim_{\Delta x\to 0}\{I(x+\Delta x)\} = I(x),$$

one finds that

$$(9.05) \quad dV(x)/dx = Z_L I(x).$$

Differentiating (9.05) with respect to x gives

(9.06) $d^2V(x)/dx^2 = Z_L[dI(x)/dx]$.

From Figure 9.01 and KCL,

(9.07) $I(x+\Delta x) - I_E(x) - I(x) = 0$.

Assuming that the system loads are of the constant admittance type, the load current $I_E(x)$ is given by

(9.08) $I_E(x) = Y_r V(x)$,

where Y_r is the load's rated admittance,

(9.09)
$$Y_r = S_r^*/V_r^2$$
$$= S_T^*\Delta x/V_r^2$$
$$= Y_T\Delta x;$$

V_r is the rated line-to-neutral voltage. Substituting Equations (9.08) and (9.09) into (9.07), rearranging the result, and then taking the limit as $\Delta x \rightarrow 0$, gives

(9.10) $dI(x)/dx = Y_T V(x)$.

When this is used in Equation (9.06), the result is a second-order linear differential equation for the line voltage as a function of position:

(9.11) $d^2V(x)/dx^2 = [Z_L Y_T]V(x)$.

The solution of this equation with the boundary conditions $V(1) = V_1$ and $I(0) = 0$ is

(9.12) $V(x) = [V_1/\cosh(\zeta)]\cosh(\zeta x)$,

where

(9.13) $\zeta = \pm\sqrt{Z_L Y_T}$.

The per unit magnitude and the phase angle of V(x) are plotted in Figures 9.02 and 9.03, based on Equation (9.12) and the following parameters:

$V_1 = V_r = V_{base} = 7620$ V

$S_T = 2000$ kVA at 0.85 power factor => $Y_T = 29.278 - j18.145$ mS

$Z_L = 2 + j3\ \Omega$

Also graphed in those figures are the per unit magnitudes and the phase angles of the voltages obtained from CVIM calculations with N = 5, N = 10,

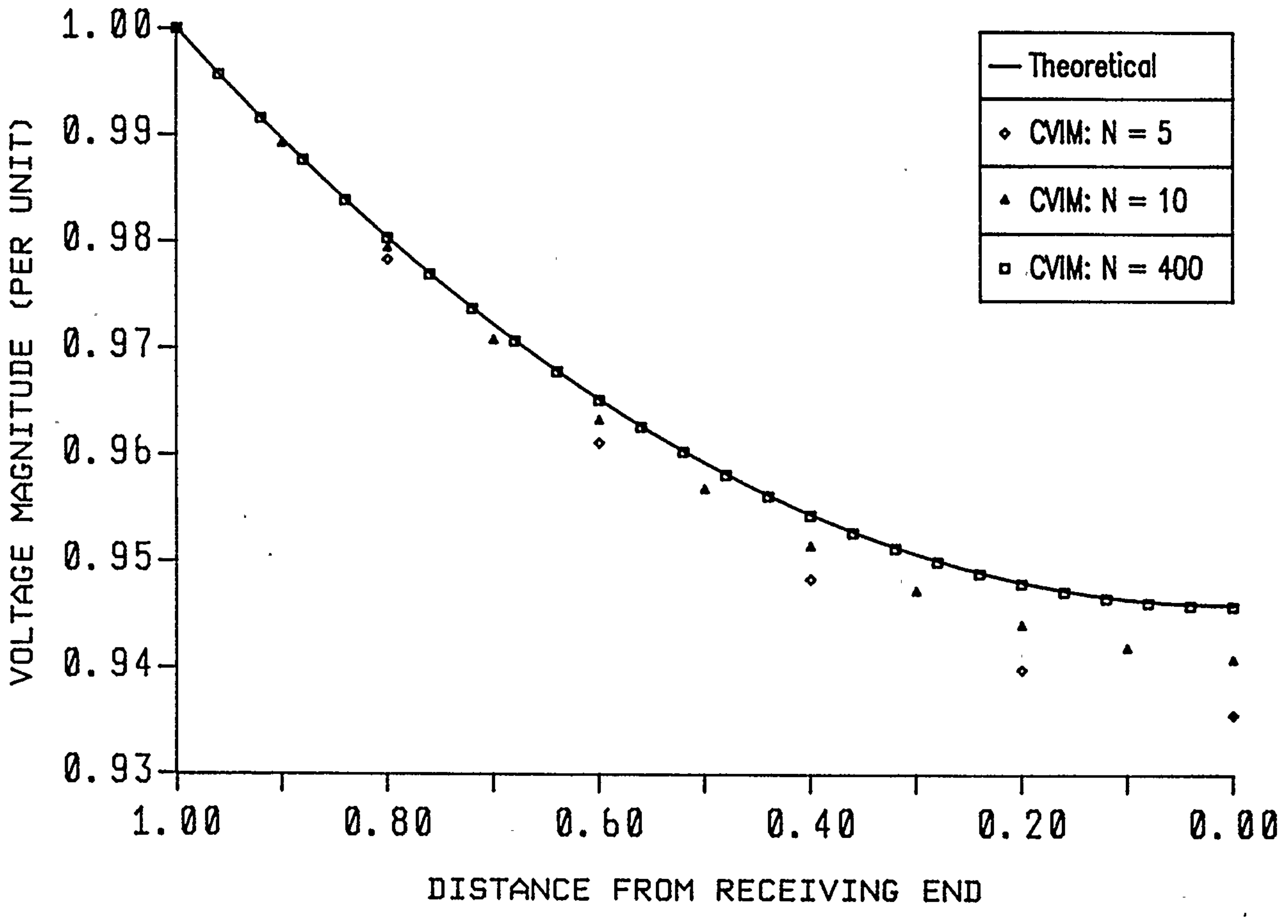


Figure 9.02. The theoretical and CVIM per unit voltage magnitudes on a uniformly loaded single-phase line.

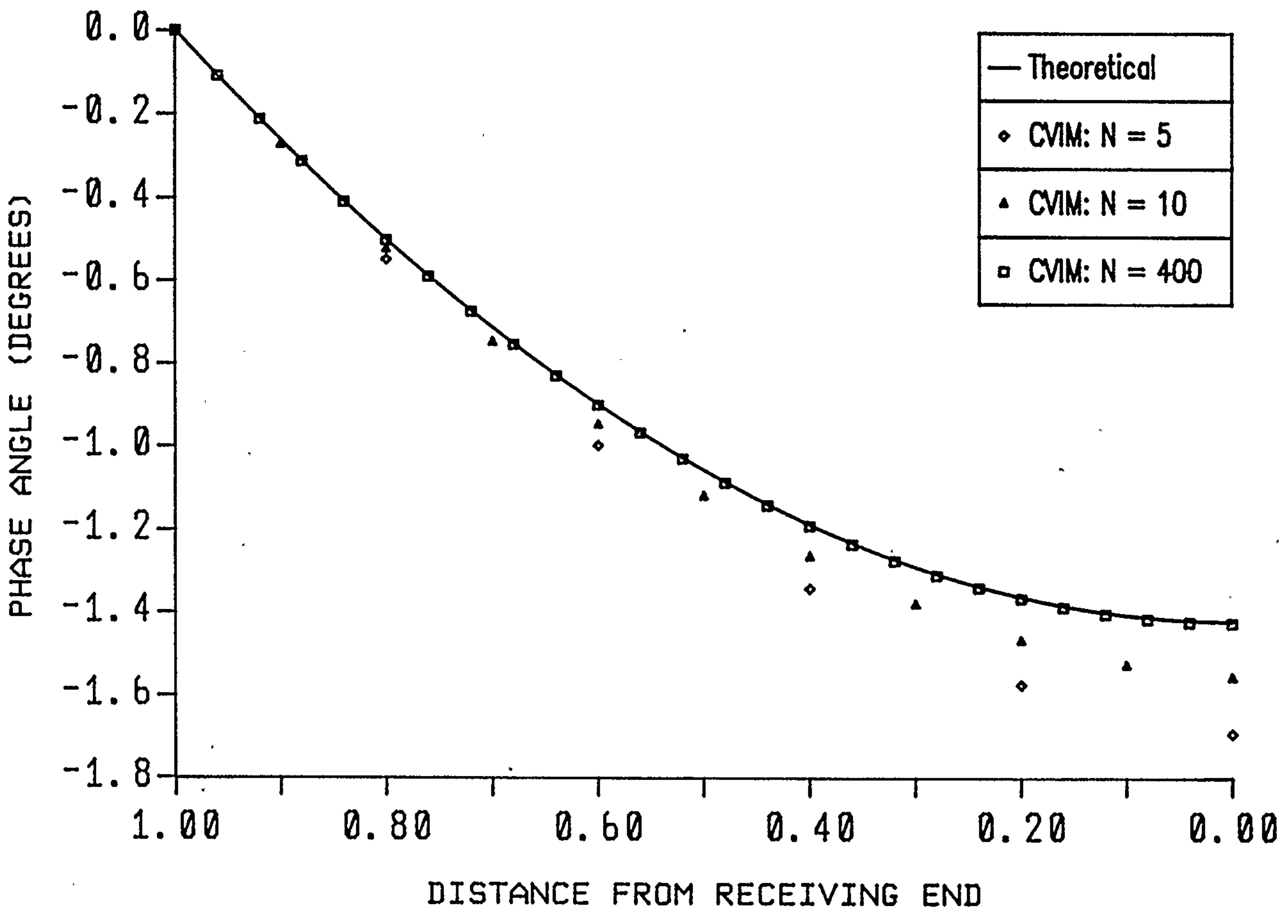


Figure 9.03. The voltage phase angle on a uniformly loaded single-phase line.

and N = 400. As expected, the difference between the theoretical results and the CVIM results is fairly large for the 5-bus and 10-bus approximations. However, using 400 buses, the difference between the theoretical and CVIM receiving end voltage magnitudes is only 0.0001 per unit, while the phase angles differ by only 0.003 degrees. Thus, the agreement between the theoretical results and the CVIM results is excellent, especially considering that some of the difference is due to the fact that $N = \infty$ was approximated by $N = 400$.

9.2.2 Three-Phase Lines

The derivation of the equations governing a uniformly loaded three-phase line proceeds in much the same way as the derivation of the single-phase equations. From Figure 9.04 and KVL, one has

$$(9.14) \qquad V^a(x+\Delta x) - \sum_{\phi=a}^{c}\{(Z^{a\phi}{}_L\Delta x)I^{\phi}(x+\Delta x)\} - V^a(x) = 0.$$

Writing similar equations for phases b and c, and then using the same limit process that was used in the single-phase case, it turns out that

$$(9.15) \qquad d/dx\begin{bmatrix} V^a(x) \\ V^b(x) \\ V^c(x) \end{bmatrix} = \begin{bmatrix} Z^{aa} & Z^{ab} & Z^{ac} \\ Z^{ab} & Z^{bb} & Z^{bc} \\ Z^{ac} & Z^{bc} & Z^{cc} \end{bmatrix}_L \begin{bmatrix} I^a(x) \\ I^b(x) \\ I^c(x) \end{bmatrix}.$$

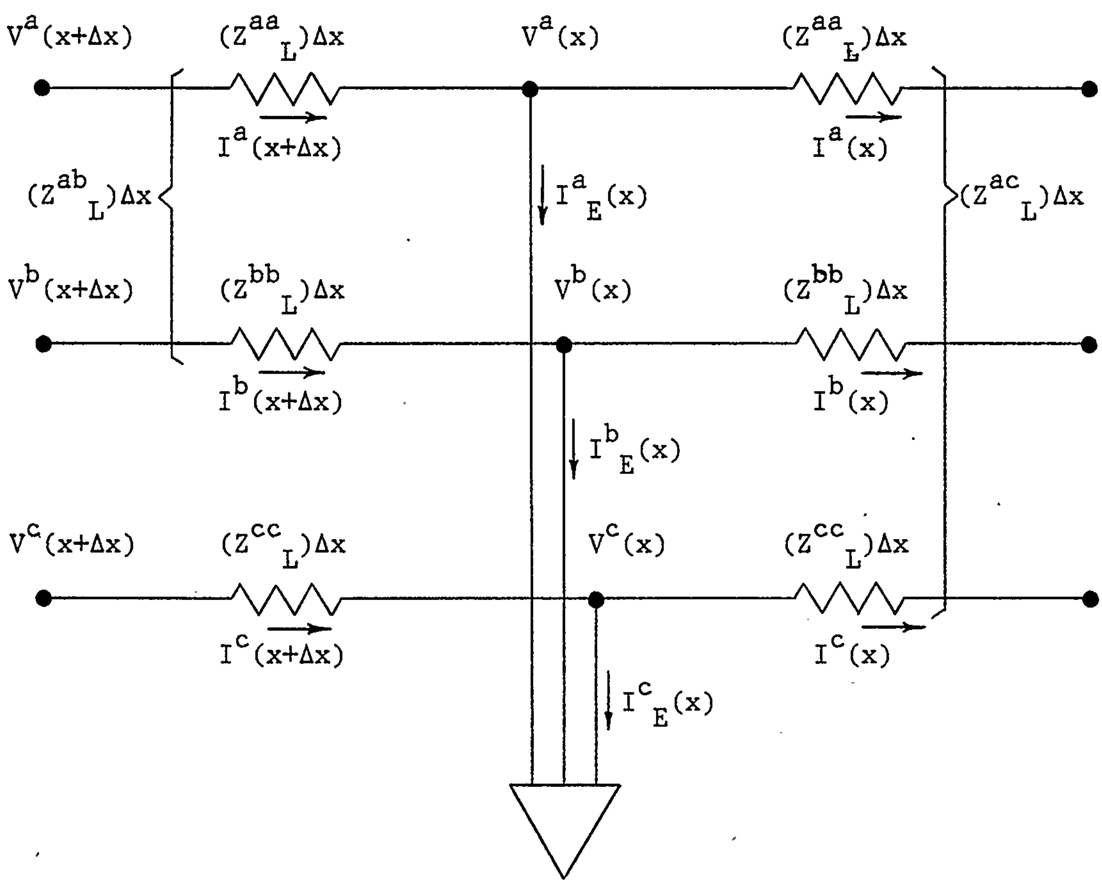


Figure 9.04. Part of a uniformly loaded three-phase distribution line.

This equation can also be written

$$(9.16)\quad dV(x)/dx = Z_L I(x),$$

and can be differentiated with respect to x to give

$$(9.17)\quad d^2V(x)/dx^2 = Z_L[dI(x)/dx].$$

From Figure 9.04 and KCL,

$$(9.18)\quad I^{\phi}(x+\Delta x) - I^{\phi}_E(x) - I^{\phi}(x) = 0$$

for ϕ = a, b, c. In the three-phase case the load currents I^{ϕ}_E depend on both the nature of each load segment and on the connection employed.

If the load segments, which are again assumed to be constant admittances, are connected in a grounded-wye configuration, then

$$(9.19)\quad \begin{bmatrix} I^a(x) \\ I^b(x) \\ I^c(x) \end{bmatrix} = \begin{bmatrix} Y^1 & 0 & 0 \\ 0 & Y^2 & 0 \\ 0 & 0 & Y^3 \end{bmatrix}_r \begin{bmatrix} V^a(x) \\ V^b(x) \\ V^c(x) \end{bmatrix}$$

$$= \Delta x \begin{bmatrix} Y^1 & 0 & 0 \\ 0 & Y^2 & 0 \\ 0 & 0 & Y^3 \end{bmatrix}_T \begin{bmatrix} V^a(x) \\ V^b(x) \\ V^c(x) \end{bmatrix}.$$

Using this equation in (9.18) and taking the limit as $\Delta x \to 0$ leads to

$$(9.20)\qquad d/dx\begin{bmatrix} I^a(x) \\ I^b(x) \\ I^c(x) \end{bmatrix} = \begin{bmatrix} Y^1 & 0 & 0 \\ 0 & Y^2 & 0 \\ 0 & 0 & Y^3 \end{bmatrix}_T \begin{bmatrix} V^a(x) \\ V^b(x) \\ V^c(x) \end{bmatrix} = \mathbf{Y}_{TG}\mathbf{V}(x),$$

which, when used in (9.17), results in the voltage differential equation[1]

$$(9.21)\qquad d^2\mathbf{V}(x)/dx^2 = \mathbf{Z}_L\mathbf{Y}_{TG}\mathbf{V}(x) = \mathbf{R}_G\mathbf{V}(x).$$

The solution of this equation is

$$(9.22)\qquad \mathbf{V}(x) = \sum_{\mu=1}^{3} k_\mu \mathbf{v}_\mu \cosh(\sqrt{\lambda_\mu}\,x),$$

where the λ_μ are the eigenvalues of $\mathbf{R}_G$ and the $\mathbf{v}_\mu$ are the corresponding eigenvectors, normalized such that the component with the largest magnitude is $1 + j0$. At $x = 1$, Equation (9.22) can be written

[1]Note that this equation is identical in form to the equation that governs the voltage distribution on long lines (Equation (5.13)).

$$
(9.23)\quad \begin{bmatrix} v^a{}_1\cosh(\sqrt{\lambda_1}) & v^a{}_2\cosh(\sqrt{\lambda_2}) & v^a{}_3\cosh(\sqrt{\lambda_3}) \\ v^b{}_1\cosh(\sqrt{\lambda_1}) & v^b{}_2\cosh(\sqrt{\lambda_2}) & v^b{}_3\cosh(\sqrt{\lambda_3}) \\ v^c{}_1\cosh(\sqrt{\lambda_1}) & v^c{}_2\cosh(\sqrt{\lambda_2}) & v^c{}_3\cosh(\sqrt{\lambda_3}) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} V^a{}_1 \\ V^b{}_1 \\ V^c{}_1 \end{bmatrix};
$$

this set of three equations is used to determine the coefficients k_μ.

As a specific example, consider a system having the following parameters:

$$
V(1) = 7620 \begin{bmatrix} 1 \\ \exp(-j2\pi/3) \\ \exp(+j2\pi/3) \end{bmatrix} \text{V}
$$

$S^a{}_T$ = 4000 kVA at 0.85 p.f. => $Y^a{}_T$ = 58.556 - j36.290 mS

$S^b{}_T$ = 400 kVA at 0.80 p.f. => $Y^b{}_T$ = 5.511 - j4.133 mS

$S^c{}_T$ = 2000 kVA at 0.95 p.f. => $Y^c{}_T$ = 32.722 - j10.755 mS

$$
Z_L = \begin{bmatrix} 0.858 + j1.768 & 0.070 + j0.634 & 0.065 + j0.503 \\ 0.070 + j0.634 & 0.904 + j1.825 & 0.070 + j0.634 \\ 0.065 + j0.503 & 0.070 + j0.634 & 0.858 + j1.768 \end{bmatrix} \Omega
$$

The eigenvalues, eigenvectors, and coefficients corresponding to these parameters are [19]:

$$\lambda_1 = 0.115 + j0.083$$

$$\lambda_2 = 0.048 + j0.041$$

$$\lambda_3 = 0.011 + j0.004$$

$$[v_1 \quad v_2 \quad v_3] = \begin{bmatrix} 1.000 + j0.000 & -0.198 - j0.155 & -0.027 - j0.005 \\ 0.380 + j0.158 & 0.259 + j0.001 & 1.000 + j0.000 \\ 0.436 + j0.206 & 1.000 + j0.000 & -0.060 + j0.009 \end{bmatrix}$$

$$k_1 = 0.675 - j0.059$$

$$k_2 = -0.829 + j0.670$$

$$k_3 = -0.552 - j1.132$$

The per unit voltage magnitudes and the deviations of the phase angles from their nominal (source bus) values, based on this solution, are shown in Figures 9.05 and 9.06 for phases a, b, and c. The figures also show the CVIM results for N = 400, and as was the case for the single-phase line, the agreement between the CVIM and theory is excellent. It is interesting to note that the unbalanced conditions lead to a voltage rise on phase b.

If the load segments are connected in delta instead of grounded-wye, the voltage differential equation is

$$(9.24) \quad d^2V(x)/dx^2 = Z_L Y_{T\Delta} V(x) = R_\Delta V(x).$$

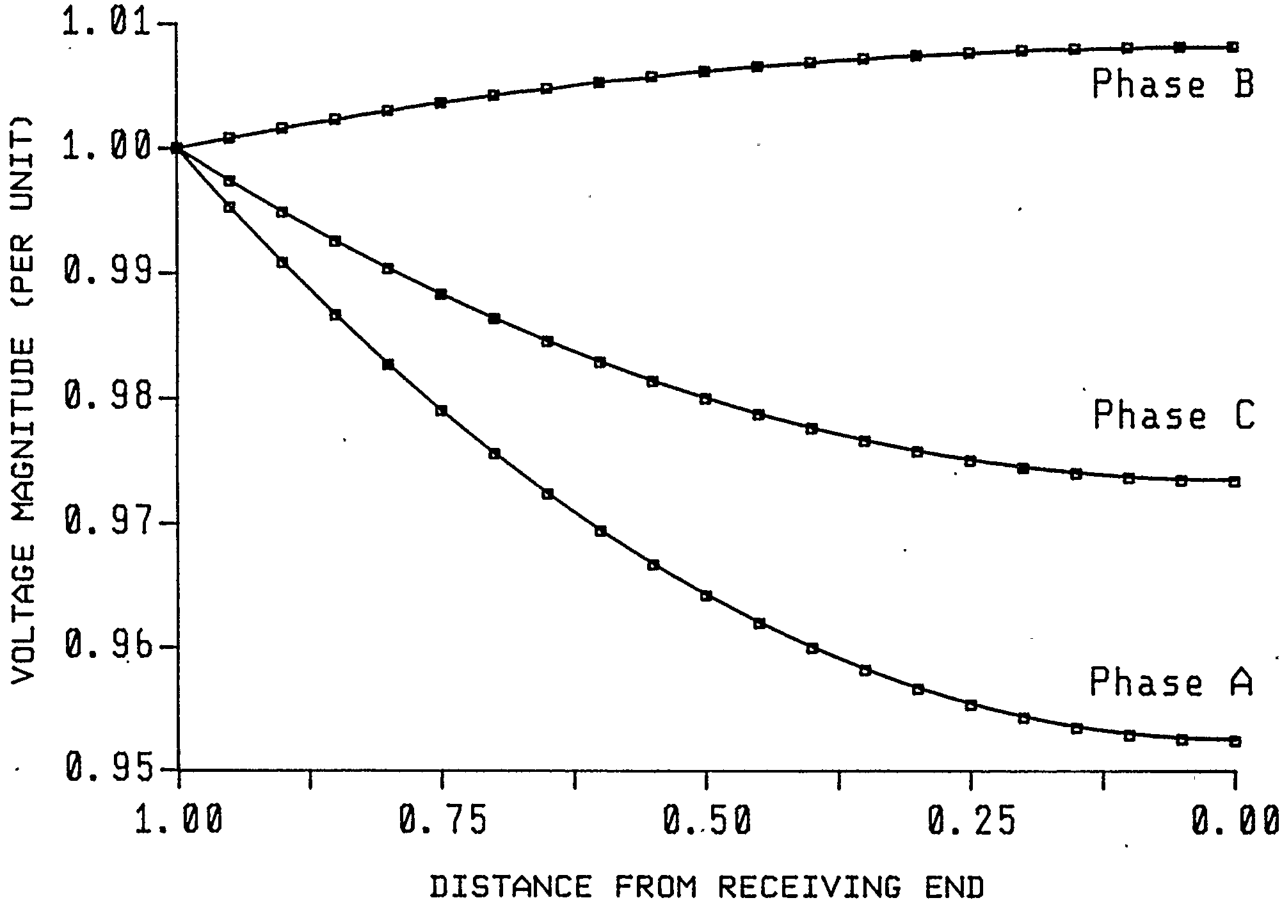


Figure 9.05. The theoretical and CVIM per unit voltage magnitudes on a three-phase line with uniformly distributed grounded-wye loads.

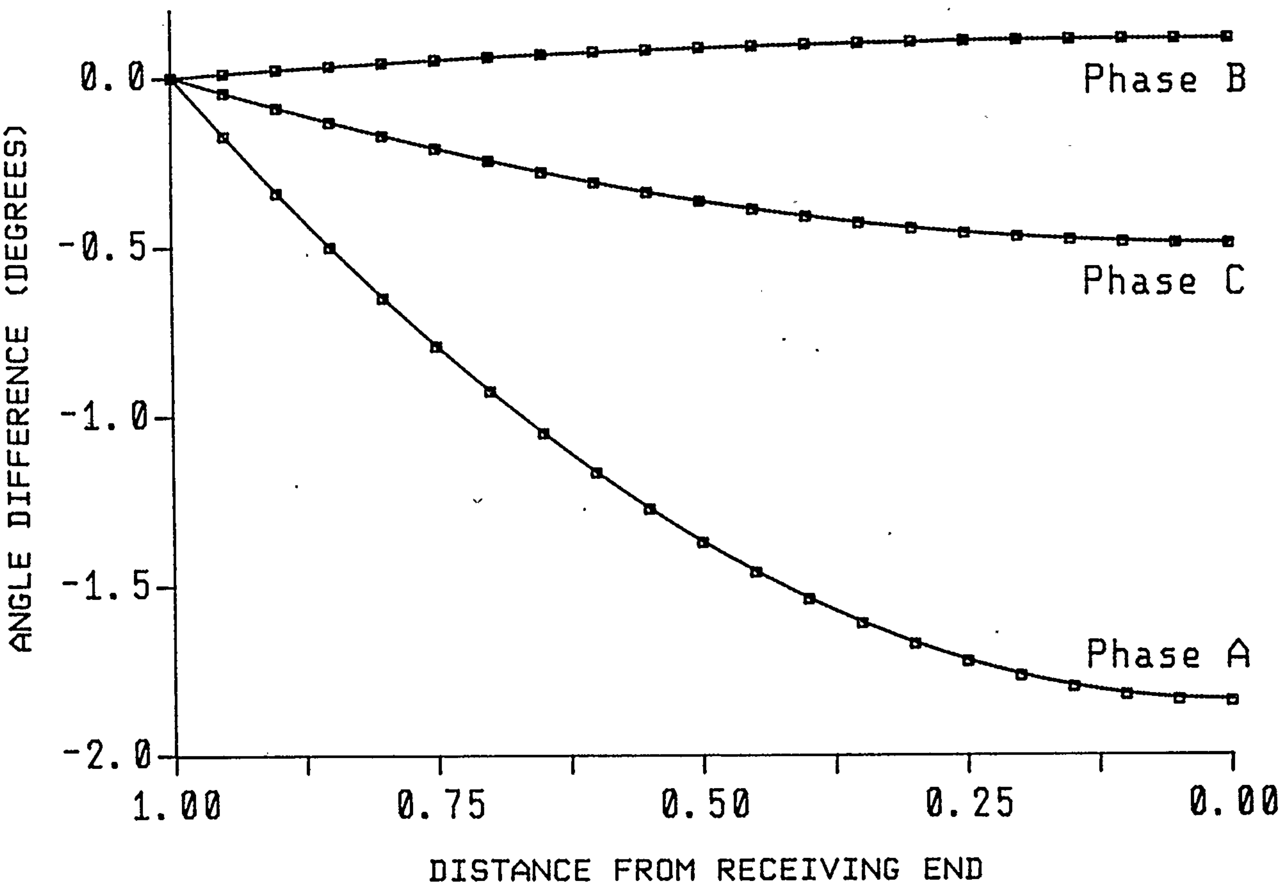


Figure 9.06. The deviations from nominal of the voltage phase angles on a three-phase line with uniformly distributed grounded-wye loads.

In a manner similar to that used in Section 8.2.2, it can be shown that the delta load admittance matrix is

$$(9.25)\qquad Y_{T\Delta} = \begin{bmatrix} Y^1 + Y^3 & -Y^1 & -Y^3 \\ -Y^1 & Y^1 + Y^2 & -Y^2 \\ -Y^3 & -Y^2 & Y^2 + Y^3 \end{bmatrix}_T ,$$

where

$$(9.26)\qquad Y^{\mu}{}_T = (S^{\mu}{}_T)^* / (V_r\sqrt{3})^2$$

is the admittance of load segment μ. For this example, Z_L is the same as in the grounded-wye case, and

$$S^1{}_T = 4000 \text{ kVA at } 0.85 \text{ p.f.} \Rightarrow Y^1{}_T = 19.519 - j12.097 \text{ mS}$$
$$S^2{}_T = 400 \text{ kVA at } 0.80 \text{ p.f.} \Rightarrow Y^2{}_T = 1.837 - j1.378 \text{ mS}$$
$$S^3{}_T = 2000 \text{ kVA at } 0.95 \text{ p.f.} \Rightarrow Y^3{}_T = 10.907 - j3.585 \text{ mS}$$

The eigenvalues and eigenvectors of R_Δ and the resulting eigenvector coefficients are [19]:

$$\lambda_1 = 0.069 + j0.035$$
$$\lambda_2 = 0$$
$$\lambda_3 = 0.023 + j0.014$$

$$[V_1 \quad V_2 \quad V_3] = \begin{bmatrix} 1.000 + j0.000 & 1.000 + j0.000 & -0.333 + j0.072 \\ -0.778 + j0.115 & 1.000 + j0.000 & -0.756 - j0.122 \\ -0.325 - j0.136 & 1.000 + j0.000 & 1.000 + j0.000 \end{bmatrix}$$

$$k_1 = 0.964 + j0.296$$

$$k_2 = 0.008 + j0.046$$

$$k_3 = -0.219 + j1.049$$

The theoretical and CVIM-calculated per unit voltages and phase angle deviations for the delta connection are shown in Figures 9.07 and 9.08. The closer coupling between the phases that results from the delta connection clearly reduces the spread between the phase voltage magnitudes.

9.3 The Effect of Transformer Connections on Load Flow

As a final pair of examples of the application of the CVIM to distribution feeder analysis, two load flow studies will be carried out on the small system shown in Figure 9.09. The only difference between the studies is that in the first the transformer feeding bus 8 has ungrounded-wye connections on both primary and secondary windings, while in the second the transformer is connected in grounded-wye/delta. The objectives of these studies are to illustrate the use of some of the transformer models developed in Chapter 6, and to provide a brief look at the effect that

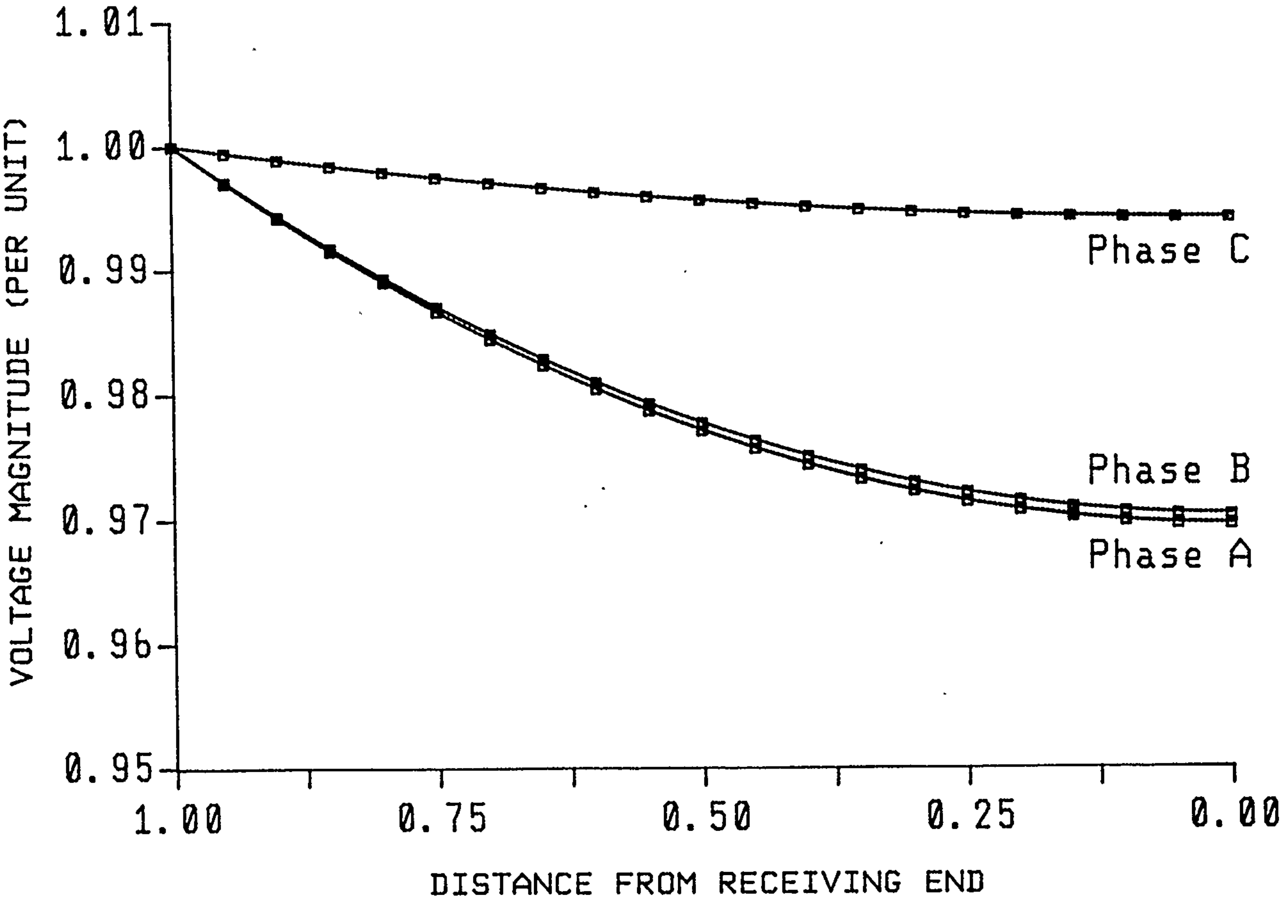


Figure 9.07. The theoretical and CVIM per unit voltage magnitudes on a three-phase line with uniformly distributed delta loads.

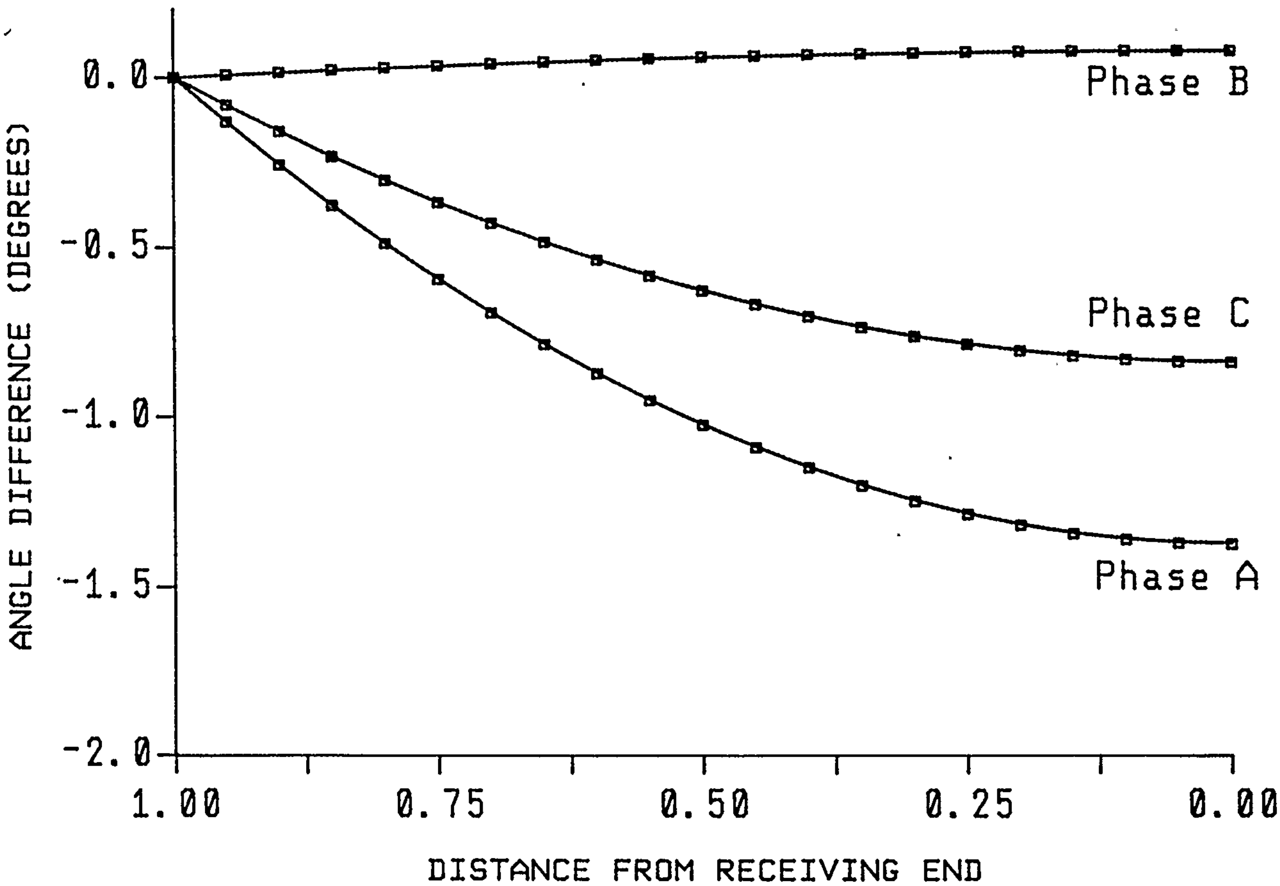


Figure 9.08. The deviations from nominal of the voltage phase angles on a three-phase line with uniformly distributed delta loads.

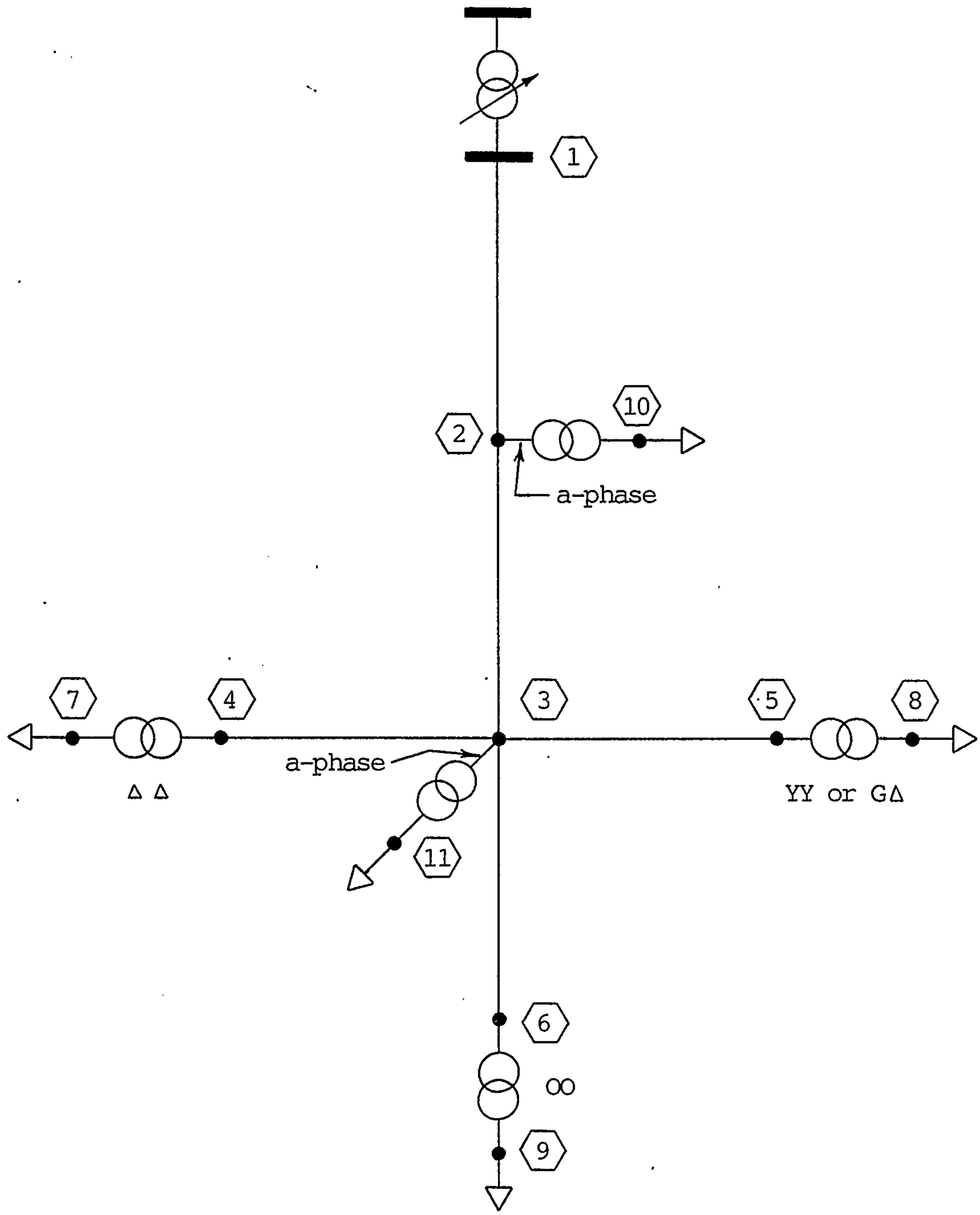


Figure 9.09. The distribution feeder used in the study of the effect of transformer connections on system load flows: (a) single-line diagram; (b) system loads (next page).

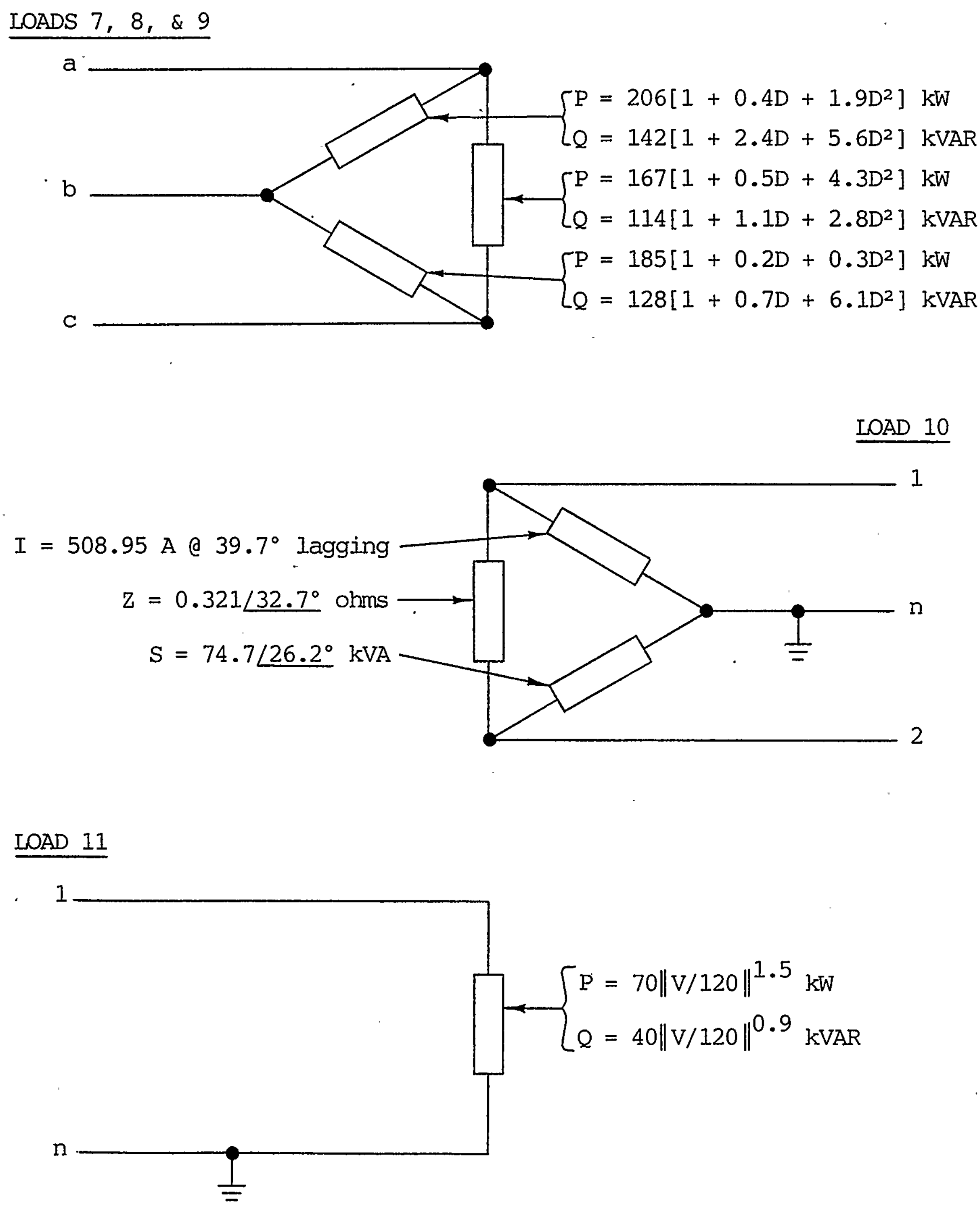


Figure 9.09, part (b).

transformer connections can have on load flow. The system details are as follows:

BASE QUANTITIES:

$S_{Hbase} = 1000$ kVA

$V_{Hbase} = 7620$ V

$S_{Xbase} = 1000$ kVA

$V_{Xbase} = 120/\sqrt{3}$ V on GΔ transformers, 120 V otherwise

LOADS: All three-phase loads are the same; see Figure 9.09b.

TRANSFORMERS: The system's three-phase loads are fed by banks of three single-phase transformers. Every transformer on the system has the same parameters, which are:

$V_{Hr} = 7620$ V

$V_{Xr} = 120$ V

$r = 120/7620$

$Z_H = 0.700 + j3.400\ \Omega$

$Y_M = 4.500 - j25.400\ \mu S$

$Z_X = 0.120 + j0.560\ m\Omega$

The secondary windings on all units, except that feeding the three-wire load at bus 10, are connected in parallel.

LINES: The short line model is used for all lines. The impedance matrices are as follows:

$$Z_{L2} = \begin{bmatrix} 0.800 + j2.400 & 0.060 + j0.800 & 0.040 + j0.700 \\ 0.060 + j0.800 & 0.900 + j2.580 & 0.060 + j0.800 \\ 0.040 + j0.700 & 0.060 + j0.800 & 0.800 + j2.400 \end{bmatrix} \Omega$$

$$Z_{L3} = \begin{bmatrix} 0.200 + j0.600 & 0.015 + j0.200 & 0.010 + j0.170 \\ 0.015 + j0.200 & 0.220 + j0.670 & 0.015 + j0.200 \\ 0.010 + j0.170 & 0.015 + j0.200 & 0.200 + j0.600 \end{bmatrix} \Omega$$

$$Z_{L4} = \begin{bmatrix} 0.260 + j0.750 & 0.022 + j0.240 & 0.022 + j0.240 \\ 0.022 + j0.240 & 0.260 + j0.750 & 0.022 + j0.240 \\ 0.022 + j0.240 & 0.022 + j0.240 & 0.260 + j0.750 \end{bmatrix} \Omega$$

$$Z_{L6} = Z_{L5} = Z_{L4}$$

The results of the load flow analyses are shown in Tables 9.01 through 9.04. While it is difficult to verify the accuracy of the solutions in a simple manner, the results seem plausible given that the line currents flowing into all delta and ungrounded-wye connections sum to zero, the voltage magnitudes look reasonable, and, as expected on distribution systems, the voltage phase angles differ little from their nominal values.

TABLE 9.01
SYSTEM VOLTAGES - UNGROUNDED-WYE/UNGROUNDED-WYE CASE

	V^a or V^1		V^b or V^2		V^c	
Bus	Mag.	Angle	Mag.	Angle	Mag.	Angle
1	1.000	0.0	1.000	-120.0	1.000	120.0
2	0.966	-1.3	0.981	-120.7	0.980	119.6
3	0.960	-1.5	0.976	-120.8	0.975	119.5
4	0.958	-1.6	0.973	-120.9	0.973	119.4
5	0.958	-1.6	0.973	-120.9	0.973	119.4
6	0.958	-1.6	0.973	-120.9	0.973	119.4
7	0.957	-1.3	0.957	-121.3	0.975	118.7
8	0.950	-2.2	0.950	-121.5	0.960	118.2
9	0.944	-2.5	0.973	-120.9	0.959	118.7
10	0.948	-2.3	0.948	177.6		
11	0.956	-1.8				

TABLE 9.02
SYSTEM CURRENTS - UNGROUNDED-WYE/UNGROUNDED-WYE CASE

Bus	I^a_R or I^1_R	I^b_R or I^2_R	I^c_R
2	0.949 - j0.648	-0.731 - j0.254	0.104 + j0.680
3	0.695 - j0.471	-0.731 - j0.254	0.104 + j0.680
4	0.207 - j0.141	-0.242 - j0.086	0.034 + j0.226
5	0.210 - j0.144	-0.245 - j0.084	0.035 + j0.228
6	0.210 - j0.142	-0.244 - j0.084	0.034 + j0.226
7	0.207 - j0.136	-0.238 - j0.087	0.031 + j0.224
8	0.210 - j0.143	-0.244 - j0.084	0.034 + j0.227
9	0.209 - j0.140	-0.240 - j0.086	0.032 + j0.226
10	0.115 - j0.090	-0.139 + j0.087	
11	0.067 - j0.042		

TABLE 9.03
SYSTEM VOLTAGES - GROUNDED-WYE/DELTA CASE

Bus	V^a or V^1		V^b or V^2		V^c	
	Mag.	Angle	Mag.	Angle	Mag.	Angle
1	1.000	0.0	1.000	-120.0	1.000	120.0
2	0.968	-1.2	0.979	-120.6	0.980	119.4
3	0.962	-1.4	0.972	-120.7	0.976	119.2
4	0.960	-1.4	0.970	-120.8	0.974	119.2
5	0.961	-1.4	0.969	-120.7	0.974	119.1
6	0.960	-1.4	0.970	-120.8	0.974	119.2
7	0.957	-1.2	0.957	-121.2	0.976	118.6
8	0.948	-32.1	0.948	-152.1	0.964	88.9
9	0.946	-2.3	0.970	-120.8	0.960	118.5
10	0.950	-2.2	0.950	177.8		
11	0.958	-1.6				

TABLE 9.04
SYSTEM CURRENTS - GROUNDED-WYE/DELTA CASE

Bus	I^a_R or I^1_R	I^b_R or I^2_R	I^c_R
2	0.918 - j0.635	-0.748 - j0.215	0.057 + j0.719
3	0.664 - j0.457	-0.748 - j0.215	0.057 + j0.719
4	0.207 - j0.141	-0.242 - j0.086	0.034 + j0.227
5	0.179 - j0.131	-0.262 - j0.045	0.011 + j0.266
6	0.210 - j0.142	-0.244 - j0.084	0.034 + j0.226
7	0.207 - j0.137	-0.238 - j0.087	0.031 + j0.224
8	0.110 - j0.228	-0.254 + j0.049	0.143 + j0.179
9	0.209 - j0.140	-0.240 - j0.086	0.032 + j0.226
10	0.115 - j0.090	-0.139 + j0.086	
11	0.067 - j0.042		

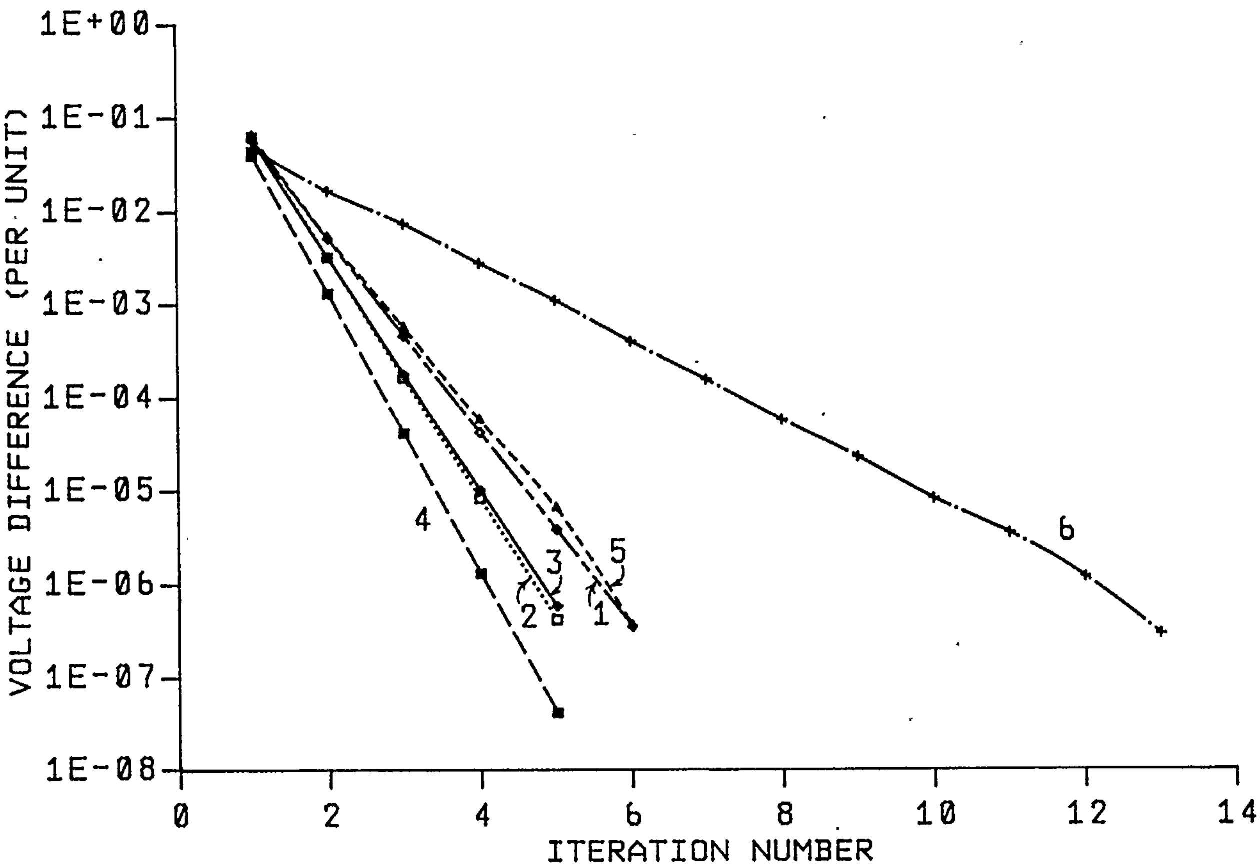
Based on these results, some information on the effect of transformer connections on system behaviour can be obtained.

Since the system under consideration was designed such that the paths 3-4-7, 3-5-8, and 3-6-9 are identical except for the transformer connections used, the differences in voltages and currents along these paths must be due to those connections. In the case of the voltages, the results for the delta/delta, ungrounded-wye/ungrounded-wye, grounded-wye/delta, and open delta transformers are essentially identical. In the case of the currents, however, the grounded-wye/delta transformer exhibits a characteristic somewhat different from the other units; for example, the c-phase current magnitude is about 16 percent higher for this transformer than the others. This current difference is probably due to the presence of the primary neutral connection, which allows the flow of line currents that do not sum to zero.

9.4 Convergence Properties

Of obvious interest to those responsible for distribution system analysis are the convergence properties of the methods employed in that analysis. While no definitive statements can be made about the CVIM given the limited number of trials it has been subjected to so far, some indication of its properties can be given by examining its performance in the cases presented above.

Figure 9.10 is a graph of the difference between successive voltage sets as a function of iteration number for the case studies presented in



1 - Chapter 3 Example

2 - Single-Phase Uniformly Loaded Line

3 - Three-Phase Uniformly Loaded Line, Grounded-Wye Loads

4 - Three-Phase Uniformly Loaded Line, Delta Loads

5 - Transformer System, Ungrounded-Wye/Ungrounded-Wye Case

6 - Transformer System, Grounded-Wye/Delta Case

Figure 9.10. The iteration-to-iteration voltages changes as a function of iteration number for the examples presented in Chapters 3 and 9.

Chapters 3 and 9. Excluding for a moment the case from the last section involving the grounded-wye/delta transformer, the voltage difference is seen to be reduced by a factor of approximately 10 in each iteration. In this respect it is similar to the Newton-Raphson technique, in which the maximum power mismatch drops at a similar rate [28]. There is no sign of the voltage convergence being overwhelmed by numerical effects, but this may be a consequence of the fact that only relatively small systems (the largest having 400 buses) have been studied thus far.

The load flow study involving the grounded-wye/delta transformer is unique among those presented in that convergence was not reached using the CVIM exactly as detailed in Chapter 3. To obtain a solution, it was necessary to damp the iteration-to-iteration voltage changes by introducing the damping coefficient α. For each bus β and iteration ξ, the voltage vector $V_{\beta,\xi}$ was computed using

$$(9.27)\quad V_{\beta,\xi} = V_{\beta,\xi-1} + \alpha[\Delta V_{\beta,\xi}]$$

$$= V_{\beta,\xi-1} + \alpha[V'_{\beta,\xi} - V_{\beta,\xi-1}],$$

where $V'_{\beta,\xi}$ is the voltage vector calculated directly from the currents and the component performance equations. (Note that setting $\alpha = 1$ is equivalent to using the CVIM as exactly as discussed in Chapter 3.) The number of iterations required for convergence to 10^{-6} per unit is shown as a function of α in Figure 9.11; the optimum value of α, which is the one

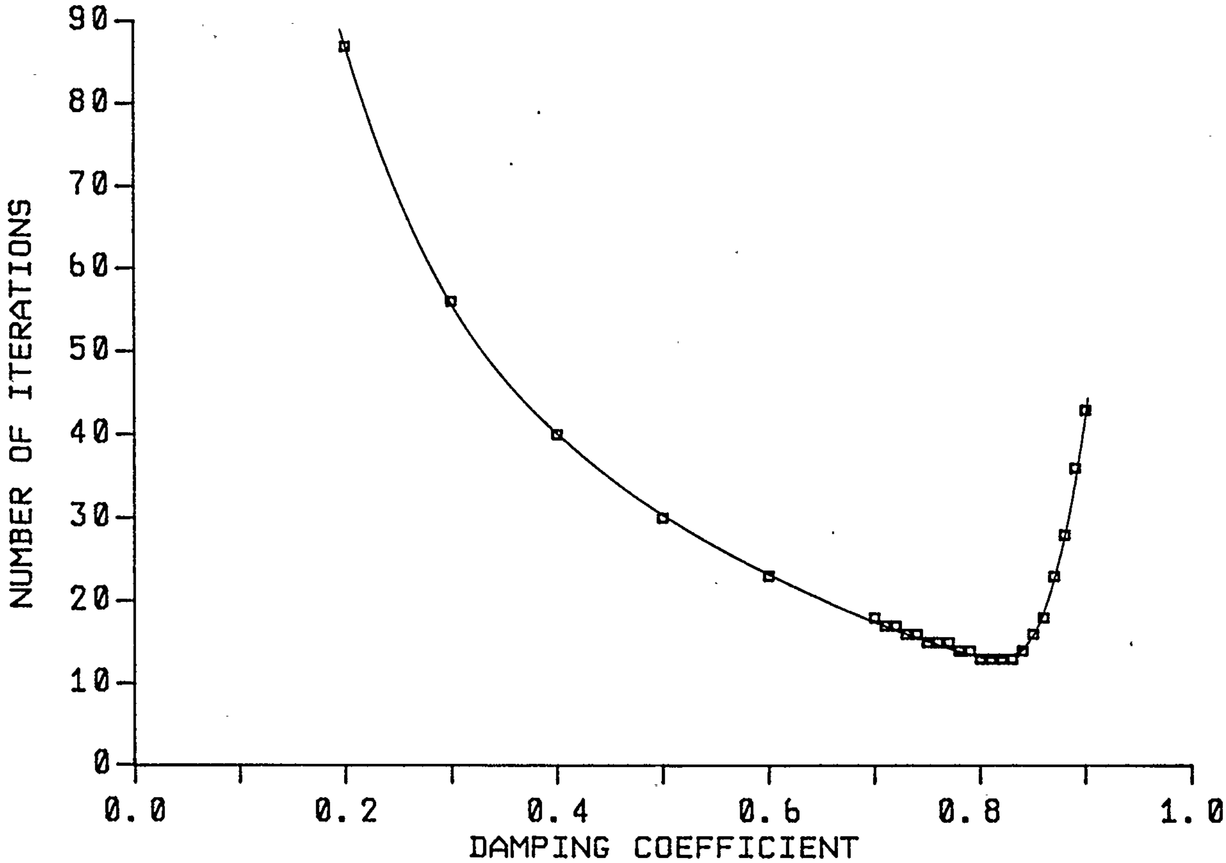


Figure 9.11. The number of iterations required for convergence, for the system containing the grounded-wye/delta transformer, as a function of the damping coefficient.

that resulted in the convergence shown in Figure 9.10, is approximately 0.83 for this particular system.

While it complicates the method slightly, the requirement for voltage damping is not viewed as a defect in the CVIM, since even the well-established Newton-Raphson technique uses damping to avoid divergence under certain conditions [3]. Convergence is slower than in the cases where no damping is used, but some improvement might be made here if a procedure that optimizes α for each iteration is used, instead of treating it as a constant as was done in the work presented above. There is also the possibility that divergence-avoiding mechanisms other than voltage damping can be found, but no investigation of this has been carried out.

10.0 CONCLUDING REMARKS

10.1 Summary

A technique for the solution of load flow problems on unsymmetrical radial electric distribution systems that contain unbalanced loads has been proposed. The viability of the method was demonstrated in Chapter 9, wherein it was shown that the CVIM results matched very closely the results expected from theoretical considerations in a number of situations. In spite of the complexity encountered during the development of some of the equations, especially in Chapter 6, the end results provide sophisticated component models that are nevertheless simple to integrate into the load flow algorithm.

10.2 Possible Future Work

There are several possible avenues for future work either using or directed at the Current/Voltage Iteration Method, including:

1. the comparison of field measurements with CVIM load flow predictions
2. the further investigation of situations in which it is necessary to use a damping coefficient or other divergence-avoiding mechanism in the algorithm, and methods for optimizing those mechanisms

3. the detailed investigation of component performance and interaction (e.g., load/transformer interaction) on unbalanced systems
4. the assessment of the sensitivity of feeder performance to load variation, uncertainty, and unbalance
5. the relaxation of some of the assumptions made in component modeling, particularly through:
   - the incorporation of long lines into the load flow process
   - the use of variable (non-linear) transformer magnetizing admittances
   - the allowance of dissimilar transformers in three-phase banks
   - the recognition of transformer parameter variation with tap position, perhaps along the lines of work done by Kilmer et al. [25]
6. the adaptation of the method to frequencies other than the fundamental for harmonic and carrier signal analysis
7. the extension of the CVIM to allow for cogeneration studies

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HEE: D. G. Fink and H. W. Beaty, eds. Standard Handbook for Electrical Engineers, 11th edition. New York: McGraw-Hill, 1978.

PAS: IEEE Transactions on Power Apparatus and Systems.

WDS: Westinghouse Electric Corporation. Distribution Systems. Electric Utility Engineering Reference Book, vol. 3. East Pittsburgh: Westinghouse, 1959.

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