Report of Working Group D
Rapport du Groupe de travail D

ROBOTICS IN MATHEMATICS EDUCATION

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INTRODUCTION

Robots and robotics have spread out of research laboratories, industrial and commercial settings to a variety of new locations including living rooms and classrooms. This incursion has afforded different learning opportunities for children and adults. In the tradition of Papert (1980), who identified educational robots as ‘objects-to-think-with’, our working group set out to explore some of the potential for using robots to think about mathematics and other powerful ideas through engaging with building, programming, testing, mathematising, playing and discussing emergent ethical issues.

Participants came with a variety of previous experiences and expertise in mathematics education and coding across diverse settings and with different goals. These included making stronger connections between coding and robotics, or computational thinking and mathematical thinking; examining potential for inclusion in courses for pre-service teachers given jurisdictional pushes or emerging curriculum emphases; seeking active hands-on experiences for applied mathematics courses and modelling; and also because of the ‘that’s cool!’ factor.

DAY 1: NECESSARY FOUNDATIONAL EXPERIENCES

Getting started with robots in the classroom for learning requires some necessary foundational experiences to learn how the robot is put together and how the programming works.

BUILDING THE ROBOT

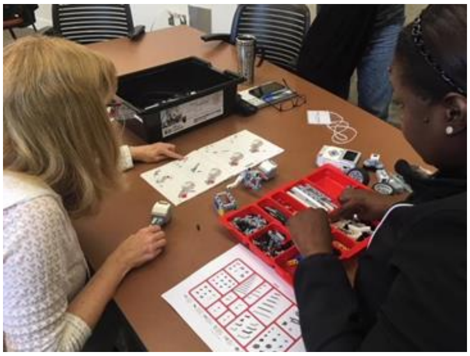


Figure 1. Participants building the EV3 robot.

First we built the robot as per the instructions in the manual provided with the EV3 educational robot kit. The design in the manual provides a solid, stable robot that can easily be adapted to suit a variety of tasks. This robot is suitable for beginners and intermediates. Building a robot from the manual also provides opportunities for using and developing spatial reasoning skills. "[S]patial reasoning is the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects" (Bruce et al., 2017, p. 146). A child's spatial skills at three years of age are strong predictors of how well a child will do in kindergarten. Spatial skills are a better predictor of later mathematics than vocabulary or mathematics (Newcombe, 2010). Spatial ability and mathematics ability are strongly correlated (Mix et al., 2016; Mix & Cheng, 2012). All STEM careers demand strong spatial reasoning (Newcombe & Shipley, 2015; Wai, Lubinski, & Benbow, 2009)—so does architecture, art, drafting, carpentry. Some spatial skills described in Davis et al. (2015) that were engaged when building the robot from the instruction booklet included the following:

- shifting back and forth between dimensions of the 2-D booklet and the 3D emerging robot;
- scaling to match the size of the represented and actual shapes;
- assembling the pieces together requires rotating, arranging, fitting, balancing, comparing, interpreting, visualising, locating and orientating for accurate building.

ROBOT DANCE

Once the robot was built, a robot dance provided opportunities to learn how to instruct the robot to move forward, backwards, and turn repeatedly with a loop. Very quickly, everyone learned how to add programming blocks to their chain, connect their robot to the computer or tablet and download the program to the robot. One group was particularly fast building their robot, so they had already attached many of the sensors. They played with the programming blocks and figured out how to dance using their ultrasonic sensor. The sensor kept them away from walls and other robots. Visit https://vimeo.com/291510578 to see our dance. The dance added an element of fun and set the tone for playful exploration throughout the workshop.

These first tasks established the group dynamics and how the teams worked together. Some groups split roles and responsibilities. For example, while building often one person was in charge of assembling the pieces, while another was in charge of finding pieces for each step. When it came to programming, often one person did most of the on-screen coding. We wondered: How does this splitting of roles build or prevent development or skills? and How could we structure the activities differently to ensure everyone has equal and meaningful opportunities?

RACE TO THE WALL

A race to the wall challenge provided the groups a chance to understand how the power option in the steering block works. The challenge was to program the robot to get as close to the wall as possible without touching it, as quickly as possible, while having a Lego person (mini-figure) remain standing on top of the robot for the entire race. The competitive nature of our working group and this cultural aspect of robotics became apparent in this race. Watch https://vimeo.com/292340498 to see our race.

Figure 2. Ready, set…robot race to the wall.

After the race, we discussed the strategies for how the groups attempted to win. Trial and error was a dominant strategy for approaching this challenge. We discussed how limiting the number of trials might encourage more strategic mathematical thinking with the use of measurement tools. One group found the highest speed that the robot could have without the Lego mini-figure falling off, and then computed the number of rotations for the right distance. Another group found a programming example on the internet that made the robots' speed increase steadily without stopping.

Our discussion of the elements of mathematics in the race to the wall generated the following:

- The feedback from the environment (e.g., the robot and peers) lent itself to developing habits of mind: iterating, adjusting, problem solving, striving for accuracy, flexibility of thinking and innovating.
- Calculating the relationship between wheel circumference and distance travelled required proportional reasoning.
- Programming distances required applying knowledge of integers, decimals, and negative numbers.
- The group that found the fastest speed the robot could go while balancing the Lego mini-figure, used a stepwise linear function for the velocity. We questioned if a constant value for the power translates into a constant value for the speed: if the robot starts at fully stopped, there must be acceleration at first. What function should be used for modelling the velocity of the robot? The group that found a program that used steady acceleration applied continuity and derivatives. These could be explored mathematically.
- Tension between satisfaction and dissatisfaction: there was satisfaction in getting the robot to race to the wall, but there was a dissatisfaction knowing that by fine-tuning the model/program to be more sophisticated, the robot could perform the task better. This created the need for using more variables and/or more advanced mathematics concepts such as continuity.

EMBODIED NOTIONS OF NUMBER

During the working group questions arose around whether activities/tasks like the race to the wall challenge should only be used for knowledge integration and whether new mathematics could be learned with it. In this section we draw on ideas from embodied cognition to address these questions in a more direct way.

Conceptual metaphors are one of the ways we understand mathematics (Lakoff & Núñez, 2000). With regard to the concept of number, Lakoff and Núñez describe four fundamental metaphors of arithmetic: arithmetic as object collection, arithmetic as object construction, arithmetic as measurement, and arithmetic as object along a path (Figures 3-6). The metaphor of arithmetic as an object collection is based on a one-to-one correspondence of numbers to physical objects (see Figure 3). With this metaphor a greater quantity of objects corresponds to a bigger number. For instance, five is greater than two because it is quantitatively more objects. The metaphor of arithmetic as object construction is based on fitting objects/parts and arithmetic operations (see Figure 4). For instance, five is greater than two because an object comprising five units is larger than one comprising two. The measuring stick metaphor maps numbers onto distances, whereby five is greater than two because it is longer (see Figure 5). The metaphor of arithmetic as an object along the path is based on arithmetic as motion by which five is greater than two because it entails moving further from a common starting point (e.g., zero; see Figure 6). Programming robots provides opportunities for illustrating, experiencing and connecting these multiple interpretations of arithmetic metaphors.

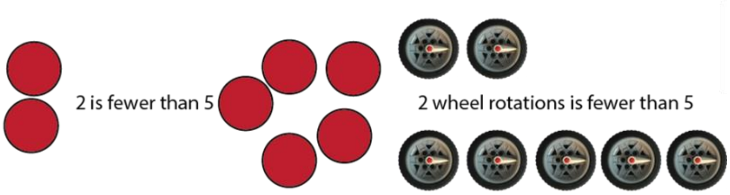


Figure 3. Arithmetic (number) as object collection.

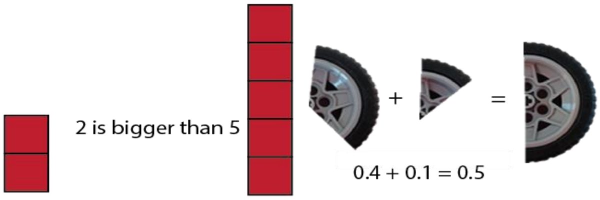


Figure 4. Arithmetic (Number) as object construction.

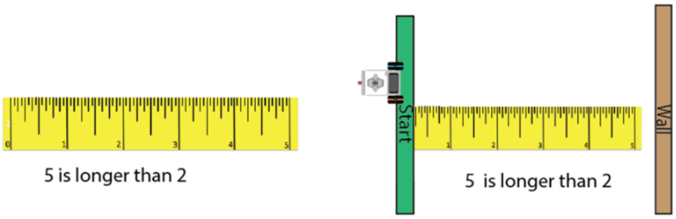


Figure 5. Arithmetic (Number) as Measuring.

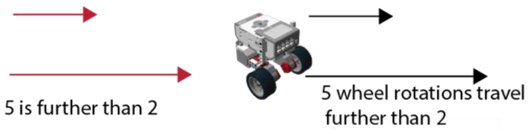


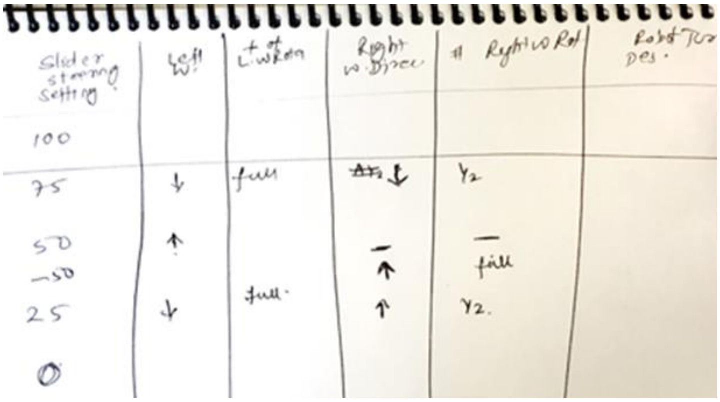
Figure 6. Arithmetic (number) as distance.

DAY 2: UNPACKING THE STEERING BLACK BOX

EXPLORING THE EFFECT OF THE SINGLE PARAMETER FOR STEERING

The activity for understanding the role of the single parameter $(-100 \leq s \leq 100)$ for steering (fixed wheels) revealed the richness of the mathematics that could be done with the integration of robots. As a low floor and high ceiling activity, it opened up many approaches for exploring and modelling.

1. Looking at what each wheel does for different values of s, when the (maximum) number of rotations (r) for the two wheels is set to 1: idle, full rotation, half rotation, etc. This experimental approach prompted a systematic data collection activity in a table of values.



Slider steering Setting	Left W.	# of L.W Rot	Right w. Direc	# Right w Rot	Robot Tur Des.
100					
75	↓	full	↓	½	
50	↑		—	—	
-50			↑	full	
25	↓	full.	↑	½.	
0					

Figure 7. Data collection during an inquiry task.

If one decides to use negative numbers to account for the direction of a wheel (positive as going forward and negative as going backward), such data can be summarised in one or two graphs to get a better feel of the coordination of the two wheels with respect to the steering parameter. This brings an opportunity for meeting and using piecewise linear functions.

Using a reciprocal approach, one may also want to find the function that uses as input the number of rotations of each of the wheels (LR and RR) and computes as output the steering parameter. This would typically invoke inductive reasoning to come up with a rather simple expression such as $s = (LR - RR)/2$.

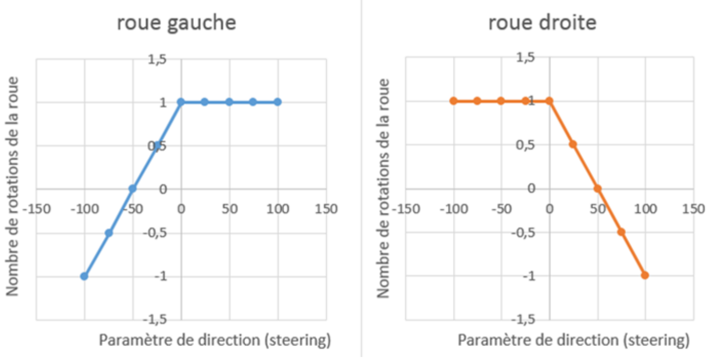


Figure 8. Graph of steering parameter versus number of rotations for each wheel.

The connection between the steering parameter and the number of rotations on each of the wheels was perceived as a true ‘black box’, as it had an *arbitrary* component (how that steering parameter had been defined and programmed in the design of the robot) that only exploration and data collection could manage to unveil.

2. Looking at the trajectory of the robot

After having looked at the connection between the steering parameter and wheel rotations, teams started looking at the trajectory of the robot.

One team mapped that trajectory on paper: for different values of the steering parameter and with one as the maximum number of rotations for the two wheels.

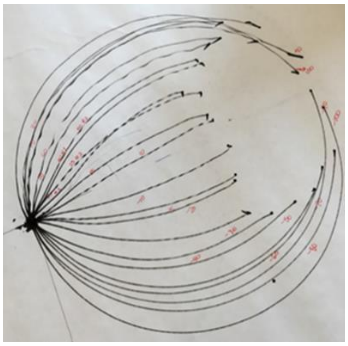


Figure 9. Trajectories of the robot for different steering parameters.

From this empirical investigation, an interesting (and open) question emerged: Could the locus of the end points of the trajectory be an ellipse? And if so, why?

To support their exploration, teams were provided with a mat that showed different concentric circles along which the robot could be made to move (see https://www.ucalgary.ca/IOSTEM/files/IOSTEM/steering-mat.pdf for a downloadable version of the mat). Team members started considering the different arcs of circle that different values of the steering parameter would generate as trajectories for the center of the robot and, eventually, for each of the two wheels.

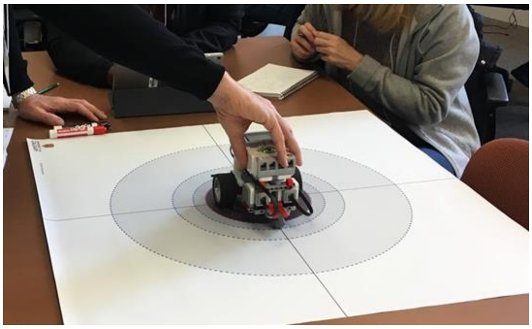


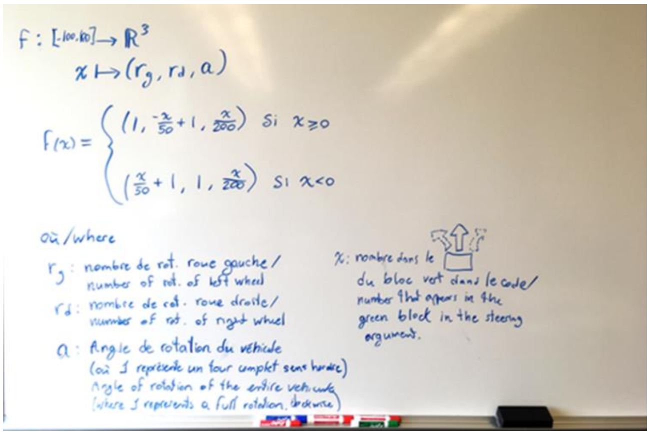
Figure 10. Exploring the trajectory more systematically.

This led to associating the number of rotations performed by each wheel to the distance covered by the wheel and, consequently to

- the corresponding radius for each of the arcs
- the angle of the rotation made by the robot
- the center of that rotation

Contrary to the initial relationship between the steering parameter and the wheel rotation, these relationships did not have an arbitrary aspect but came as consequences of the specific geometry of the system.

A team combined some of these relationships into a single function.



$f : [-100, 100] \to \mathbb{R}^3$

$x \mapsto (r_g, r_d, a)$

$$f(x) = \begin{cases} \left(1, -\frac{x}{50}+1, \frac{x}{200}\right) & \text{si } x \geq 0 \\ \left(\frac{x}{50}+1, 1, \frac{x}{200}\right) & \text{si } x < 0 \end{cases}$$

où/where

r_g : nombre de rot. roue gauche / number of rot. of left wheel

r_d : nombre de rot. roue droite / number of rot. of right wheel

a : Angle de rotation du véhicule (où 1 représente un tour complet sens horaire) Angle of rotation of the entire vehicle (where 1 represents a full rotation, clockwise)

x : nombre dans le du bloc vert dans le code/ number that appears in the green block in the steering argument.

Figure 11. Defining a function for mapping the steering parameter to the movement of the robot.

Their functional model was very close to the geometric model we had designed with GeoGebra and with which participants were later invited to play:

- Steering Wheels Power (https://goo.gl/6e16Py)
- Steering Wheels Rotation (https://goo.gl/VKypNj)
- Steering Wheels Rotation Power (https://goo.gl/CMSYvv)

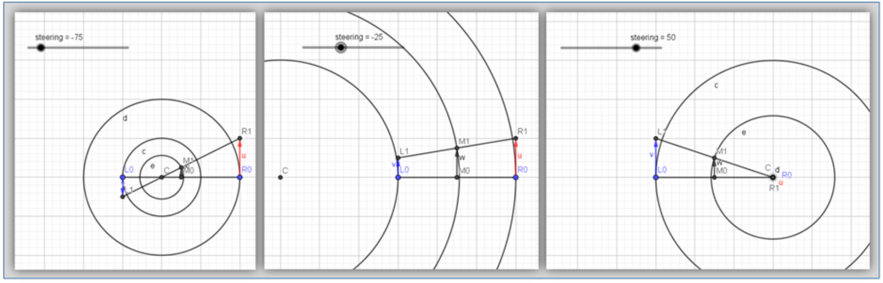


Figure 12. The velocity of each of the wheels modelled as vectors.

Participants spontaneously expressed their appreciation of exploring, all possible angles in the steering of the robot.

- Samuel: *Je suis impressionné et surpris du plaisir qu'on a eu à explorer la boîte noire du robot*.
- James: *It was interesting to see how each team focused on different aspects of the situation, and how the combination of these explorations or models enriched the understanding of all*.
- Frédéric: *Moving from modelling the distance to modelling the velocity is a step up in the abstraction*.

LA TÂCHE DU POLYGONE RÉGULIER

En tirant parti de ce qu'avaient appris les participants avec l'étude du paramètre de direction, la tâche suivante leur demandait de programmer leur robot de façon à ce qu'il se promène le long des côtés d'un polygone régulier. Des tapis avec le dessin d'un tel polygone servaient à la fois à définir la tâche et à fournir un environnement de validation. Il fallait donc construire le modèle mathématique, le traduire en modèle informatique sous forme de programme, l'implémenter dans le robot et en valider l'exécution.

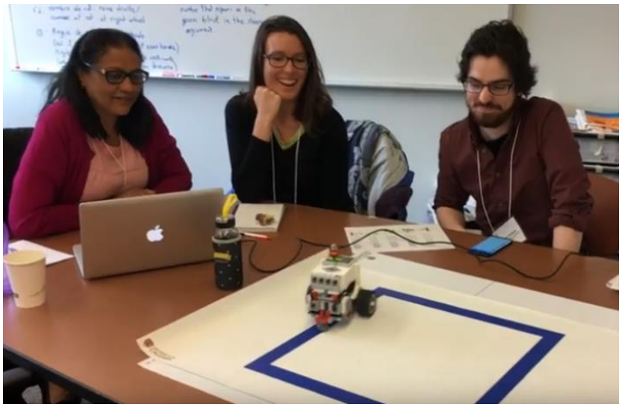


Figure 13. Programmer une trajectoire sur un polygone régulier.

Cette activité donna lieu à deux découvertes surprenantes.

1. On semblait obtenir un meilleur respect du parcours si l'on choisissait de ne pas recourir au paramètre unique pour la direction, et d'y aller plutôt pour un contrôle indépendant de chacune des deux roues, comme le permet aussi le robot utilisé.

2. Si l’activité paraissait *a priori* l’occasion tout indiquée d’utiliser une boucle dans les instructions de déplacement (pour chacun des côtés du polygone), l’utilisation d’une telle boucle semblait créer un moment d’hésitation dans le changement de direction qui se traduisait par un décalage au regard de la trajectoire visée. Le trajet était plus précis si l’on recopiait la séquence des instructions, autant de fois que l’on avait de côtés.

Bien que ces différentes approches soient théoriquement équivalentes et qu’il paraisse légitime de penser *a priori* qu’elles donnent lieu à une même trajectoire, les différences observées constituaient un rappel qu’on était ici avec des objets réels, autant du côté mécanique que du côté informatique, avec la matérialisation de l’information et son traitement (Bertrandias, 1992).

Sur le plan informatique, des hypothèses ont été avancées pour expliquer les écarts observes : erreurs d’arrondi dans la conversion du paramètre unique pour la direction en commandes pour chacune des deux roues; délai lié à la vérification de la condition associée à une boucle.

Sur le plan mécanique, en raison du fait que la direction dépend de la différence entre les vitesses des deux roues, le parcours en ligne droite est extrêmement sensible au moindre écart entre ces deux vitesses. Et quand il s’agit en plus de faire un virage sur une ligne polygonale, les arcs de cercle sur lesquels se réalise un tel virage rendent l’exercice toujours un peu approximatif.

L’importance accordée à la validation, dans cette activité comme celles qui ont précédé, permet d’apprendre à cerner autant les limites que les apports des modèles mathématiques et informatiques qu’on utilise pour atteindre un but au regard d’une situation réelle. Cela peut donner lieu à certains moments de frustration, quand on semble loin du but, mais aussi à de grandes joies quand on y arrive finalement!

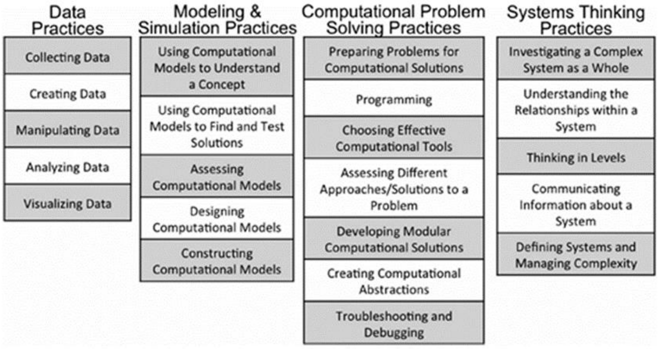
Dans ce travail itératif, le robot participe au milieu didactique (au sens de Brousseau, 1990), dans la mesure où il renvoie l’effet des actions posées, même si cet effet fait parfois intervenir des éléments qui n’étaient pas considérés au départ. Un tel travail nous paraît s’inscrire non seulement dans le développement de compétences de modélisation, mais aussi dans le développement d’une pensée informatique.

DAY 3: LOOKING AT THE BIGGER PICTURE

CONTRIBUTION TO THE DEVELOPMENT OF COMPUTATIONAL THINKING

Weintrop et al. (2016) have proposed a taxonomy aimed at providing a clearer and more operational definition of computational thinking for mathematics and science (Figure 14).

Using this taxonomy, the robot tasks that the group experienced (robot dance, race to the wall, steering & polygon tracing) seem to address mainly Computational Problem Solving Practices, and Modeling & Simulation Practices: participants had to assess, design and construct computational models, which were more or less tied to mathematical models. Most of these computational models were implemented with some amount of programming accompanied with some troubleshooting and debugging.



Data Practices
- Collecting Data
- Creating Data
- Manipulating Data
- Analyzing Data
- Visualizing Data

Modeling & Simulation Practices
- Using Computational Models to Understand a Concept
- Using Computational Models to Find and Test Solutions
- Assessing Computational Models
- Designing Computational Models
- Constructing Computational Models

Computational Problem Solving Practices
- Preparing Problems for Computational Solutions
- Programming
- Choosing Effective Computational Tools
- Assessing Different Approaches/Solutions to a Problem
- Developing Modular Computational Solutions
- Creating Computational Abstractions
- Troubleshooting and Debugging

Systems Thinking Practices
- Investigating a Complex System as a Whole
- Understanding the Relationships within a System
- Thinking in Levels
- Communicating Information about a System
- Defining Systems and Managing Complexity

Figure 14. Weintrop et al. (2016) taxonomy for computational thinking in mathematics and science

Through the different tasks, we also saw participants express the need to collect, organise and analyse data: this was mainly done with paper and pen. A noteworthy exception was when sensors were used by a team to direct their robot eliminating the need to record the specifics of each experiment. This gave rise to a very different approach to the polygon task where the robot was instructed to follow the line as it would 'see' it.

The point could be made that some thinking in levels also occurred when participants had to reconcile the moves a robot makes with the moves its wheels make, and later used the steering box that they had just opened to define a more elaborate trajectory.

ETHICAL AND SOCIOPOLITICAL ISSUES

In our third day, we did not get to spend as much time on the ethical and sociopolitical issues as we had originally planned. Therefore,we feel that the discussion we had should be expanded in the report, recognizing that it is an area that we often do not address when working with robotics in classrooms.

Overview

- Ethical and sociopolitical concerns and themes are woven into human engagement with technologies like robotics that promise large scale social transformation.
- These themes are not often engaged with in mathematics education (school, research).
- Learning mathematics with robots should include such themes and discussions.

Sociopolitical concerns in mathematics education have moved from the margins to the mainstream over the last four decades (see Jurdak & Vithal, 2018). This shift includes a clearer relationship between sociopolitical issues and sociocultural aspects of teaching and learning and a concern with ethics. Political and ethical issues have been woven into human engagement with technologies, including mathematics, across time and space. Technologies like robotics promise rapid large scale social and economic transformation. Such transformations raise important issues of concern to mathematics educators beyond 'the mathematics' such as those related to equity, access, identity development, and social justice. The discussion of ethics and responsibility in mathematics education in the context of learning robotics was framed by D'Ambrosio (1998) is still a pointed reminder to our field that, "particularly in mathematics, there is an acceptance that we are fulfilling our responsibilities if we do our mathematics well…But this is not enough. This must be subordinated to a much broader attitude towards life" (p.71) and his recommendation to engage learners in inquiring how mathematics is implicated in the problems as well as the solutions we face today and tomorrow.

The contest/competition dynamic

Some elements related to this theme emerged from participants' questions and concerns over the three days. For example, questions were raised about the dynamics of the 'contest' format during the 'race to the wall' challenge and possible differential engagement within school-age populations. More explicitly, might the framing as a 'contest' with 'winners' and by extension, 'losers' make the learning, and the learning of mathematics through robotics a more fraught experience for some learners? We note that robotics and informatics contests (e.g., Bebras, https://www.bebras.org/) and competitions are a current part of the modern milieu. This ranges across a variety of school level contests and competitions (e.g., FIRST Robotics Competition, VEX Robotics Competitions, World Robot Olympiad), some of which have been running for over a decade. In a similar vein there currently exist many varieties of school-level mathematics competitions, some of which have been running for over a century, some of which have been the focus of research. The form of engagement in math competitions versus robotics competitions however differs in that the latter is more visible and collaborative (in contrast to individual achievements) and sometimes has a higher cost of entry for participation. That competitions and contests are part of the landscape should prompt reflection and inquiry however as to the influences on robotics education in schools and how this has been and is being shaped. This remains an open question.

Changes to work and workforce

The McKinsey Global Institute Report (2017) on workforce transitions in a time of automation notes that very few occupations, less than 5%, consist of activities that can be fully automated, though in about 60% of occupations at least one-third of activities could be. Those jobs most susceptible to automation are physical ones in predictable (stable) environments and jobs involving the collecting and processing of data. This will likely result in shifts in task allocations as robots and automated processes are not yet better at managing people and social interactions. As many as 800 million people may need to find new jobs by 2030. Though some job displacement may be offset by the creation of new occupational categories that do not currently exist, the challenge will be in ensuring that workers have the right skill-sets to transition quickly. The report also notes that, "We will all need creative visions for how our lives are organized and valued in the future, in a world where the role and meaning of work start to shift" (p. 20).

Given the strong association that has been made between mathematics and labour/workplace knowledge and skills in the political and public spheres, it is relevant to draw learners' attention to the potential disruptive impact of robots on labour and employment.

Robot racism & sexism

Although robots are playing increasingly larger roles in the social sphere with children, the diversity of robots remains limited. Most robots are either stylised as white or metallic (Beswick, 2018). Researchers demonstrated a shooter bias for robots racialised as black. Several examples also demonstrate that some decision algorithms used for artificial intelligence systems produce behaviour that resembles racist and sexist human ones. The biases of the person programming algorithms matter as well as the training data, sets of experiences that one draws upon. There are good analogues for mathematics teaching.

Kyriakidou, Padda, and Parry (2017) note that there do not currently exist guidelines on robot ethics in ethical frameworks and there is limited discussion in research reports. They suggest teachers and researchers have a duty to explain a robot's operational nature to children and to report on this in their methodology. Other issues around preferential attachment for robots over humans and emotional distress due to perceived harm and not understanding the nature of a robot is also an emerging concern. With voice-assistants and A.I., features of social interaction and communicative norms of young children are being re-shaped (Rosenwald, 2017). Papert (1980) too had concerns about the influence of mechanized thinking machines on the development of children's values and self-image. This is a dimension to which we need also attend and draw learners' attention to.

CONCLUSION

We believe we are still at the beginning stages of studying the impacts of robots deployed in mathematics education towards an end of learning mathematics. We acknowledge that there is much still to be learned about teaching and learning with robots, in particular the specific mathematical opportunities that can be afforded. We see one productive avenue as to explicitly create opportunities to open the many black boxes encountered, which involves using mathematical ideas, tools and representations with others. We acknowledge as well that the increasing presence of robots in classrooms, homes and in communities will have transformative effects on human social relationships—a dimension of education we cannot ignore.

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