2023-12-20

Locating En-Route Charging Stations and Time Points for a Transit Route with Battery- Electric Buses

Esmaeilnejad, Seyedshahab

UNIVERSITY OF CALGARY

Locating En-Route Charging Stations and Time Points for a Transit Route with Battery-Electric Buses

by

Seyedshahab Esmaeilnejad

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

DEGREE OF DOCTOR OF PHILOSOPHY

GRADUATE PROGRAM IN CIVIL ENGINEERING

CALGARY, ALBERTA

DECEMBER, 2023

Abstract

While the acknowledged environmental benefits of battery-electric buses (BEBs) are widely recognized, their distinct differences from diesel buses necessitate adjustments in both route planning (including charging station placement) and operations (comprising schedule management and holding control). Overall, life cycle emissions of BEBs tend to be lower than those of diesel buses. However, the actual environmental impact depends on various factors, including the energy source for electricity generation, battery manufacturing practices, and disposal/recycling methods. Transitioning to electric buses, particularly in regions with a clean energy grid, holds significant potential for reducing greenhouse gas emissions and air pollution. The strategic placement of charging stations, their number, duration of charging, and station types all factor into the comprehensive planning process. These stations can be strategically situated at depots, termini, or even along the route. This study addresses the long-term planning and optimization challenge of revising formulations for dispatch policy, determining optimal en-route charging station locations and corresponding charging durations, and determining the location of the holding point and their slack time. This optimization endeavor aims to enhance passengers' waiting time, operational efficiency, and capital costs, all while mitigating the inherent variability arising from weather-induced ridership fluctuations and battery performance uncertainties intrinsic to BEBs while improving the reliability of the transit service. Two linear deterministic optimization models and a two-stage stochastic programming (SP) optimization process are developed to pursue these goals. These approaches facilitate the strategic placement of BEB charging stations along the route and calculate their associated charging times in addition to the placement of the time points. The application of these models encompasses both one-way and two-way operations. The practicality and efficacy of these methodologies are tested on two high-demand bus routes within Calgary's transit network. Additionally, the study evaluates the potential implications of charging station malfunctions, mainly focusing on scenarios where the maximum charging time is exceeded and its subsequent impact on operational schedules and BEB operation costs. Furthermore, the study explores the solution yielded by the stochastic model, using the expected value of perfect information as an evaluative metric.

Acknowledgments

Looking back at my first email correspondence with Dr. Lina Kattan where we discussed the possibility of beginning my Ph.D. journey, I still remember the excitement and uncertainty about what was ahead. Today, I feel pride and profound gratitude for the trust that Dr. Kattan put in me and every step of the journey it took for me to get to this point. While the path was not straightforward, Dr. Kattan's invaluable leadership, oversight, and mentorship gave me the confidence to push forward and complete this work in a way that I am incredibly proud of our contributions. The opportunity to glean insights from her profound expertise is a privilege for which I am profoundly fortunate, and forever grateful. Aside from the invaluable academic insights and guidance, my family and I have deeply appreciated the heartwarming empathy, generosity, and care that Dr. Kattan shows to all her students. Every single day, she reminds us of the value of solid faith, work-life balance, and how much more efficient one can be with these despite numerous challenges, such as the unprecedented pandemic.

An equally remarkable privilege of this journey has been working with my co-supervisor, Dr. Chan Wirasinghe, whose leadership, mentorship, and attention to the finest details have been invaluable to my Ph.D. and how I now critically think in tackling and approaching new challenges. In the meetings with Dr. Wirasinghe, his momentary silence is followed by feedback, comment, or question that has opened my eyes to a level of thinking that one can only experience in his presence. He has also set an example of the value of celebrating differences in culture and opinion, extending friendship, and learning from each other.

I worked and collaborated with numerous friends and colleagues at the University of Calgary, and I am deeply grateful for their contributions. I want to thank Dr. Mohammad Ansari Esfeh, Dr.

Saeid Saidi, Rouzbeh Azargoshasbi, Amirhossein Saeidmehr, and Dr. Benjamin Forrest for their invaluable support on both intellectual and technical fronts. Their friendly guidance and expertise have played a significant role in shaping my research trajectory. Their collaboration and input have been truly invaluable.

Furthermore, I would like to thank Calgary Transit for providing the essential data that made this thesis possible. I am deeply grateful to Mr. Asim Arsalan for his extensive support and assistance in the data acquisition process, which has been instrumental in completing my work. This work is financially supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery Grant (DG) and Discovery Accelerator Supplement (DAS) Grant on Urban Transportation Operation and Management in a Connected and Autonomous vehicle, Alberta Innovate – Strategic Research Projects on Integrated Urban Mobility through Emerging Transportation Technologies, Canada Research Chair in Integrative Transportation Systems through Automation and Connectivity and a NSERC-CREATE grant on Integrated Infrastructure for Sustainable Cities.

Foremost, I could not have completed this work without endless love, patience, and support from my amazing wife and best friend, Seti. There are not enough words to express how profoundly grateful I am to have you in my life and for the two wonderful children we are raising together. To our exceptional children, Sam and Aria, you have contributed to my Ph.D. more than I can describe. Because of your adorable and endless curiosity and love for learning, I have become a better learner and researcher. Your determination and persistence from learning to crawl, walk, and feeding yourselves, to learning and teaching me the names of all the animals that have ever existed – you have helped me look at the world in a whole new way. You inspired me to keep on working and learning. And my hope is that this work will help your generation have a more sustainable future.

Last, I wish to circle back and thank Drs. Kattan and Wirasinghe again. While I may be one of many Ph.D. students you have supervised, your kind and supportive approach to supervision really stands out. It is not lost on me just how lucky I have been to work with you both, my remarkable mentors and co-supervisors.

Dedication

To my children, Sam and Aria - may this work

help your generation have a more sustainable future.

And to Seti, your honorary Ph.D. is in my heart.

Table of Contents

List of Tables

List of Figures and Illustrations

List of Acronyms

μ_s	Auxiliary variable for the decision tree
γ_{B}	Cost of an extra battery
γ_{BEB}	Cost of a BEB
γ_{r}	Value of passenger in-vehicle travel time
$\bar{p}$	Average rate of arrival of the transit passengers per unit time
$\bar{p}$'	Induced demand
x_k	Relative distance of a bus stop from the origin terminal
α_s	Auxiliary parameter
γ_w	Cost of the waiting time per unit time per person
γ_{w1}	value of the waiting-time of passengers
γ_{w3}	In-vehicle value of time of passengers
γ_{wo}	Cost per unit time of the regular transit service
λ_D	Cost of the dispatch of a regular diesel bus
λ_E	Operator cost for dispatching one BEB for a route
a_E	Acceleration rate of a BEB
APC	Automated passenger count
A_s	Number of alighting passengers at station "s"
B	Minimum allowed percent-charge level of the battery
b_E	Deceleration rate of a BEB
BEB	Battery electric buses

BEV	Battery electric vehicle
B_s	Number of boarding passengers at station “s”
C_D	Discharge rate
c_E	Cost of electricity
C_{ES}	Cost of an early bus
C_F	Charging rate
C_L	Cost of a late bus
c_O	Operator cost
CS_{min}	Minimum number of required en-route charging stations
C_T	Cost of through passengers
D	Dwell time
D_s	Planned departure time
E	Extra required batteries
EVPI	Expected value of perfect information
EVS	Expected value solution
FCEB	Fuel cell electric buses
H	Uniform transit headway
h*	Optimal headway
h_c	Capacity headway
HEB	Hybrid electric buses
HVAC	Heating, ventilation, and air conditioning
L	Length of the bus route in each direction

L_{Max}	Maximum running range
LRT	Light rail transit
N	Number of bus stops
N_C	minimum Minimum required capacity of the charging facility
n_E	Required BEB for one bus route
N_R	The number of round trips that a BEB can finish with the initial charge
OF	Objective function
PEV	Plug-in electric vehicle
PHEV	Plug-in hybrid electric vehicle
R	Required total charge
ST	Slack time
T	Required charging time
T_C	Required charging time at a terminal
T_{C1}	Charging time in terminal 1
T_{C2}	Charging time in terminal 2
T_{COE}	Total required charging time with plug-in charging stations at both ends
TCO	Total cost of ownership
T_P	Round-trip time
TP	Time point
TRV	Average travel time between two stops
Ts	Earliest possible departure time
V	Average cruising speed

VSS	Value of stochastic Solution
W	Charge deficit
Z	Total cost per round trip
θ	Transit ridership

CHAPTER 1: INTRODUCTION

The recent advancements in transit electrification mark the start of a new era, fundamentally transforming our cities' design, planning, operation, and maintenance of public transport systems. Not only must the battery electric bus (BEB) planning problem adhere to the schedule, but it must also consider battery range limitations and the fleet's recharging plans. BEBs have zero on-road emissions, low noise levels, and better acceleration compared to diesel buses (Hall et al., 2019). The use of motor vehicles in transportation is continuously growing globally (4%), while the transportation sector is ranked second in CO_2 emission (23% of emissions in Canada) (Lamb et al., 2021, Ansari Esfeh, 2020). The electrification of transit fleets can decrease the reliance on oil and help reduce the emission level (Zheng et al., 2021). In tandem with the general benefits of public transit, electric buses can contribute to sustainable transportation systems while addressing concerns such as climate change and reduced inequality. Compared to earlier electric-operated transit vehicles such as trolleybuses, BEBs are more flexible and offer higher levels of adaptability in deviating from their original routes. Electric buses, in their manufacturing, entail the mining and processing of materials like lithium, cobalt, and nickel, with potential environmental impacts, yet their emissions are generally lower compared to diesel buses (Velandia et al., 2019). During operation, electric buses produce zero tailpipe emissions, decreasing local air pollution, and greenhouse gases, particularly in cleaner energy grid regions (Hensher et al., 2022). However, BEBs' environmental impact hinges on electricity sources. Diesel buses emit pollutants during combustion, contributing to air pollution (Rupp et al., 2019). In maintenance, electric buses need less upkeep due to fewer moving parts but face concerns with lithium-ion battery disposal. Meanwhile, diesel buses require regular engine maintenance, generating waste. Despite the lower emissions during operation, the overall impact depends on various factors, urging the transition to electric buses, especially in areas with clean energy, for a significant reduction in greenhouse gas emissions and air pollution.

One of the challenges of this developing technology is that BEBs have a limited operational range compared to diesel buses, traveling relatively short distances before needing to recharge. This is primarily a concern in cold weather conditions, which expedites the BEB battery drainage rate. The operational range of BEBs on a route is variant. It is a function of topography, driving style, bus size, weather, load, battery capacity, battery age, and traffic conditions (Wang et al., 2020). The BEB's battery needs to be charged or swapped during the service depending on route lengths and characteristics. Electric buses can be charged/refueled in different ways: at a depot, at the termini of a route, at en-route plug-in charging stations, via en-route lane charging while in motion, and with battery swapping (Panda et al., 2023; Chen et al. 2018). Typically, regular charging stations are placed/installed at bus terminals or depots. Buses that can operate for an entire day on a single charge can take advantage of low-cost overnight charging. The primary advantage of overnight charging is that it promotes electricity use during off-peak hours. Otherwise, daytime electricity recharge is required. For short routes, a bus with a close-to-drained battery will return to its assigned charging station. For long routes, additional charging stations should be located along the bus routes. Alternatively, additional BEBs should be added to the fleet to avoid using en-route charging, which still would not be a viable solution for long routes. There are several major decisions required about charging facilities on a route served by BEBs: number, location, charging duration, and type of charging station assigned to each charging site. Using predetermined charging station locations, some studies considered charging rules and scheduling along a route or throughout a complete transit network. Currently, the charging facilities can be categorized into three groups: battery swapping, plug-in charging (Verbrugge et al., 2021), and wireless charging (Panda et al., 2023). In addition, while BEB must stop to charge at a charging station, they create a de facto time points at selected bus stops along the route. Time points are part of a common system of holding control, which increases transit reliability by holding early buses.

1.1 Objectives of Study

The primary objectives of this research are to design effective strategies for optimizing the placement of en-route charging stations, determine their charging duration, and redefine the optimal time point locations for BEB transit routes. This research considers the long-term planning and optimization problem of en-route charging station locations, charging durations, and the location of time points considering passengers' waiting time, operating costs, and capital costs. For the first time, this research effort addresses the demand's stochastic behavior and the battery performance of the new bus fleets affected by cold weather conditions, which is a serious concern in Canada.

By addressing these objectives, this study seeks to provide valuable insights into the practical transition to battery electric buses and implementing this new technology within public transportation networks.

1.2 Proposed Methodology and Contributions

A comprehensive approach combining mathematical optimization techniques and transportation planning principles is employed to achieve the objectives mentioned above. The proposed methodologies involve formulating deterministic and stochastic models to optimize the locations of en-route charging stations and time points for battery-electric bus routes. The contributions of this research lie in developing novel linear optimization models for charging station locations and the integration of stochastic programming (SP) to account for uncertainties in demand and supply.

The work conducted in this study can be categorized into three interconnected phases: 1. enhancing the cost function for dispatch policy, 2. determining the optimal placement of en-route charging stations, and 3. attaining the optimal arrangement of time points for a battery-electric bus (BEB) route. In the first phase, this study addresses diverse charging station types and refines the equations to derive optimal parameters facilitating the transition of bus routes from diesel-powered to BEBs. During the second phase, the study introduces optimization models (both deterministic and stochastic) applicable to one-way and two-way bus routes. These models are developed for identifying suitable locations for en-route charging stations. Notably, the innovative incorporation of a two-stage stochastic optimization approach represents the first attempt to effectively account for weather-induced uncertainties impacting battery-electric bus routes' demand and supply dynamics. Lastly, the third phase revolves around enhancing the reliability of electric bus route services. This is achieved by formulating the problem of determining the optimal time points for a bus route as a linear model.

1.3 Organization of Thesis

This thesis comprises seven chapters, organized as follows:

Chapter Two presents a literature review on Battery Electric Buses (BEBs), focusing on previous research on dispatch policies, the placement of en-route charging stations, and determining time points for battery-electric bus routes.

Chapter Three addresses the enhancement of the cost function for deriving optimal dispatch parameters, including headway and the required number of buses, for transit routes transitioning from conventional buses to battery-electric buses.

Chapter Four outlines two deterministic optimization models designed to position en-route charging stations for one-way bus routes. Real-world data from Route 3 in Calgary is employed for evaluation. The same chapter adapts the model to accommodate two-way bus routes, demonstrated through an evaluation using real-world data from Route 303.

Chapter Five introduces a two-stage stochastic model built upon the deterministic optimization models established in Chapter Four. This model addresses uncertainties stemming from weather-induced variations in demand and supply.

Chapter Six devises a deterministic optimization model to determine the optimal arrangement of time points for one-way bus routes. The model leverages outputs from the two-stage Stochastic Programming model introduced earlier.

Chapter seven summarizes the research findings and provides concluding thoughts on the study. It explains the contributions of the research, discusses its limitations, explores potential real-world applications, and suggests directions for future research.

This thesis aims to contribute to advancing sustainable public transportation by optimizing the integration of battery-electric buses and en-route charging stations. This research endeavors to facilitate the widespread adoption of environmentally friendly transit solutions by addressing various operational and planning challenges.

CHAPTER 2: LITERATURE REVIEW

Within the transportation sector, transit buses present an excellent business case for electrification. Transit electrification is expected to significantly reduce energy consumption and greenhouse gas (GHG) emissions in urban networks (Garcia et al., 2022). Studies such as Al-Ogaili et al. (2021) have provided detailed analyses of the energy performance of battery-electric buses. The transition from conventional diesel to battery-operated electric buses will inevitably introduce changes to public transit planning, maintenance, and operations, therefore requiring revisions to the formulation of the headway, stop spacing, reliability, fleet size, and possibly even bus routing.

Electric buses operate with different types of electrification technologies that can be categorized into Hybrid Electric Buses (HEB), Fuel Cell Electric Buses (FCEB), and Battery Electric Buses (BEB) (Al-Ogaili et al., 2021). All these technologies are equipped with an electric engine, but the main difference is the power source. The market share of electric buses of all types has featured steady growth in recent years. HEBs use both an internal combustion engine and an electric motor to provide traction power. HEBs are, in turn, segmented into two categories: series and parallel. In the first category, the internal combustion engine, referred to as a generator, constantly charges the electric motor, while in the second, both engines take care of the required traction power. In some of the HEBs, there is a plug-in configuration that charges the onboard battery with an external source. In the FECBs, fossil fuel is burned to generate electricity through an electrochemical process (Chan 2007, Zivanovic and Nikolic 2012). The main focus of this thesis is on the third category of electric buses, BEBs.

BEBs are often described as pure electric buses powered by electricity stored/saved in an onboard battery. BEBs have various operational features that are different from conventional buses (Al-Ogaili et al., 2021). These features can be categorized as economic aspects (capital cost, infrastructure investments, maintenance, and operational costs), operational aspects (range, acceleration, charging time, availability, and infrastructure), environmental aspects (GHG emissions, noise, and air quality), and energy aspects (energy source, energy consumption, and fuel efficiency) (Blynn and Attanucci, 2019). Despite all the advantages that BEBs have, some factors are holding back this industry from faster growth, including 1. high upfront capital costs, 2. scalability, 3. flexibility and operational experience, 4. decline of technology cost over time, 5. electricity charges or grid issues, and 6. lack of charging infrastructure standardization. Notably, the total cost of ownership (TCO) of the BEBs is one of the main barriers to widely implementing them in public transit fleets (Blynn and Attanucci, 2019). TCO depends on many constantly changing factors, such as taxation policies and energy costs. Canada has recently launched several projects to move towards the operation of zero-emission buses. BEBs are being implemented in Toronto, Montreal, Edmonton, St. Albert, and Vancouver (Global mass transit report, July 2017). While implementing battery-electric buses (BEBs) in the public transit system is a breakthrough in the electrification of the transportation system, BEBs may need frequent refueling/recharging compared to conventional buses. The wide variety of BEBs and the alternatives for recharging are considered for planning and scheduling the electrified bus routes.

The literature for this thesis is divided into three different sections. Section 2.1 covers the literature review for Chapter 3 of this thesis, revisiting dispatch policy. Sections 2.2 and 2.3

respectively cover the literature for locating en-route charging stations (Chapters 4 and 5) and arranging the location of holding points (Chapter 6) for battery-electric bus routes.

2.1 Scheduling and Planning

Enabling the seamless integration of battery electric buses (BEBs) into existing public transportation networks is a pivotal step toward achieving sustainable urban mobility. As the global shift towards cleaner and more efficient transit systems gains momentum, the electrification of buses stands out as a promising solution. This section delves into the literature surrounding optimizing dispatch policies for battery electric buses, emphasizing the vital role of dispatch strategies in maximizing the operational efficiency of electric bus fleets. By revisiting dispatch policies and considering factors such as route optimization and charging infrastructure management, this section aims to shed light on the multifaceted challenges and innovative solutions that underlie the successful implementation of battery electric buses in urban transit networks.

The initial phase of supply optimization necessitates formulating an objective function, serving as the focal point for the planners' intent to amplify certain aspects. The majority of research in our assessment gravitates toward two predominant objectives: enhancing social welfare or maximizing profit. Additionally, exploring the second-best scenario, wherein social welfare optimization is pursued under a predefined budgetary constraint, is a recurrent theme. These preferences align with the established paradigms of welfare economics, whereby the concepts of societal financial gains are firmly entrenched, particularly within transport economics. An inclination arises that such objectives could limit applicability within the real-world industry. Notably, the policy difficulties of transportation economics research frequently encounter obstacles due to the nuanced understanding and acceptance of social welfare within the policy domain. Hence, the academic community wants to demonstrate the benefits of focusing on the well-being of everyone by using more straightforward ways to measure how well things are working. Alternatively, researchers seek to propose alternative objectives that maintain a measure of welfare maximization yet are more understandable to a broader audience. This pursuit aligns with the aspiration to bridge the gap between academic research and practical implementation. The literature review for revising the dispatch policy in this section traverses beyond the conventional approaches of welfare and profit maximization, venturing into these potential alternatives that better resonate with the intricate interplay of theoretical underpinnings and pragmatic applicability. Social surplus is about adding up the good and bad things that people, suppliers, and others in society feel when using public transport. This is a big concept in studies about how to plan public transport. Social welfare is about adding up how much good the users and suppliers get and considering how it affects the whole society. How we determine the users' benefits depends on how we describe the demand for public transport. Social welfare becomes about making a difference when we have a simple way of expressing demand, such as using numbers.

Glaister (1974) introduced the concept of generalized consumer surplus, which means how much people are willing to pay, including different things like time and inconvenience. When we understand demand using a model with a typical consumer, the total consumer surplus is just the result of how valuable things are for the consumer when everything is balanced and how many of these ordinary consumers there are. This understanding will allow for significant insight into consumers’ WTP for the travel time savings (Weiss et al., 2023), which is an important step to implementing BEBs.

In the case of the logit model, consumer surplus is shown as the expected usefulness in making choices, measured using the logsum formula. Small and Rosen (1981), adjusted for how much each extra unit of money matters. In other words, this means that the approach of random choices can be implemented to determine how well policies about our services work economically. Nash (1978, 1982) considered an alternative approach by looking into increasing the number of miles passengers travel and the number of miles vehicles travel, but within a budget limit. These options were considered more often because they make sense in the real-world where public transport operates. Trying to maximize vehicle miles could be seen as offering as much capacity as possible within the budget while maximizing demand aims to make the service appealing to as many people as possible. In the context of public transport, achieving higher demand and increasing the number of vehicle miles traveled can also be pursued with a focus on making a profit. This is especially true if a single provider (monopolist) gets financial support (subsidy) for each service. It's been demonstrated that this subsidy corresponds to the inverse of a specific factor used in budget considerations. This factor comes from mathematical optimization methods when maximizing passenger or vehicle miles. In a practical case study, it was observed that prioritizing the maximum number of riders while ensuring no profits are made is slightly less effective than aiming for welfare maximization while staying within the same financial constraints.

Glaister and Collings (1978) examined how to maximize ridership while considering different ways of giving importance to various aspects. They explored the idea of assigning specific values to different individuals or groups of users in the goal of their study. They concluded that maximizing the miles traveled by passengers could lead to the best possible outcome for overall well-being by using a well-designed set of these values. Over forty years ago, Nash (1978) recognized that aiming for Pareto-style social welfare maximization was "too complex to be widely understood and implemented" by those involved in the industry. This insight remains relevant today. Thus, exploring different ways of setting goals for the public good and looking at how they work in different policy situations can provide researchers with insights into why the best decisions are made and help them evaluate the economic outcomes of these decisions. The body of research in public transport economics has made significant progress in understanding the best ways to plan public transport services and determine appropriate capacity levels. This progress is primarily attributed to the development of microeconomic models, which, although somewhat simplified, offer valuable insights by identifying the most effective values for various decision factors. These factors include how often services should run, the size of vehicles, the density of stations or stops, and the overall structure of the transportation network. Despite the complexities inherent in advanced capacity models, the core economic principles guiding optimal service provision have remained steadfast. The general guideline remains that increasing capacity is beneficial as long as the added benefits outweigh the extra costs. This principle holds regardless of the mode of transport or the specific capacity-related aspects being considered. Passengers generally experience higher capacity advantages, including improved consumer satisfaction and reduced user expenses.

On the other hand, increasing the service capacity naturally leads to higher overall operating costs for transportation operators. This balances the positives and negatives, forming the core challenge of the optimization problem addressed in microeconomic capacity models. The evolution of research typically corresponds to introducing new factors related to capacity, advancements in understanding demand, or the imposition of constraints in the optimization problem, often in financial or technological limitations. The literature on service frequency originates primarily from Mohring (1972). In his study, Mohring aimed to determine the most favorable service frequency for an individual bus route. Central to Mohring's model is the delicate trade-off between the cost of passengers' waiting time and the operational cost incurred by adjusting the service frequency. The combined minimization of these two cost factors occurs when the additional benefit in waiting time reduction, resulting from frequency adjustments, is equivalent to the corresponding increase in operational costs. This fundamental optimization strategy has since been expanded to incorporate various additional impacts that changes in frequency could exert on the technological process or the overall user encounter. Jansson's study in 1980 proposed that if the time between cycles relies on the demand per vehicle due to boarding and alighting durations, then the frequency essentially governs the duration of time spent within the vehicle. Subsequently, Kraus (1991), Jara-Díaz and Gschwender (2005), and Tirachini et al. (2014) introduced the concept of passenger discomfort due to crowding, which augments the importance of time saved within the vehicle.

In addition to the frequency of public transportation services, the specific timing of each departure holds significance for consumers, giving rise to what is termed the timetabling problem. Newell (1971) investigated methods to minimize waiting times by optimally dispatching departures to ensure a smooth distribution of external passenger arrival rates across time. However, it is more realistic to acknowledge that passengers might have preferred departure and/or arrival times, thus perceiving costs associated with deviations from the schedule.

The confluence of scheduling preferences and the timetabling conundrum draws parallels with classical models of spatial economic location decisions and, more broadly, optimal supply featuring product differentiation. When preferred travel times are distributed within a fixed time span (e.g., a day) without substitution between days, the timetabling challenge mirrors Hotelling's line model. Conversely, Salop's circle model comes into play when preferences fluctuate throughout the day, and the transportation provider sets departure times at hourly intervals, allowing travelers to reschedule beyond a one-hour window. Alfa and Chen (1995) examined user equilibrium properties within the line model, accounting for bus capacity limitations and instances of failed boarding. De Palma and Lindsey (2001), assuming a uniform distribution of desired arrival times, demonstrated that an optimal timetable, with no capacity constraints, results in consistent headways within the line and circle models.

Further analytical insights are derived when considering variable schedule delay costs. Within the Hotelling model, this circumstance leads to more widely spaced departures, signifying earlier and later departures for the first and last buses. This differentiation aims to accommodate passengers highly averse to schedule deviations. As highlighted by de Palma and Lindsey (2001), neglecting this heterogeneity results in suboptimal timetables and elevated average user costs. Subsequently, de Palma et al. (2015) introduced factors of crowding costs and varied pricing into the problem. They determined that crowding does not impact the optimal timetable if scheduling preferences are consistent.

Nonetheless, with a peak in the preferred arrival time pattern, optimal train departures cluster closely around the peak. Furthermore, differentiated pricing enhances efficiency by diverting demand from the most congested services. In Section 3.3, we revisit the dynamic trip scheduling framework, delving into pricing studies where timetables are no longer endogenous. Vehicle size optimization, intricately intertwined with frequency optimization, emerged as a significant focal point within public transport economics during the 1980s. Various studies contributed to the discourse known as the "minibus debate," which was sparked by Walters (1982) proposition. Walters asserted that the density economies featured in Mohring's (1972), and Nash's (1978) work largely diminish when accounting for endogenous bus size. Notably, Walters demonstrated that high-frequency minibus services might outperform their counterparts operated with standard 55-seat urban buses. This led to the disappearance of the gap between marginal and average social costs in the former scenario, consequently questioning the rationale for subsidies.

In response, Mohring (1983) acknowledged the possibility of milder density economies for minibus services. However, he highlighted that Walters' findings heavily relied on the assumption that minibus drivers' wages could be equivalent to taxi drivers, which was approximately one-third of the average bus driver's salary. Mohring (1983) established that even if this cost difference were feasible in a competitive minibus market, the minimum average social cost only minimally deviates from standard bus operations. Gwilliam et al. (1985) took a more critical stance, asserting that maintaining a constant average waiting time with standard buses, irrespective of demand, was a "fundamental error in Walters' model." After rectifying this assumption by equating the waiting time to half the headway, they concluded that the optimal bus size increases with demand, contrary to Walters' findings. Their numerical optimization under the correct model using the parameters from Walters' (1982) work led to a configuration resembling a standard single-decker bus used in the United States.

These deliberations on optimal bus size assumed great policy significance in Britain, particularly within the context of bus deregulation. The expectation was that private operators, under such deregulation, would run smaller buses more frequently (Glaister, 1986).

In the most rudimentary static models discussed thus far, insights into optimal vehicle size can be gleaned by introducing a capacity constraint to the model. This entails assuming a constant ratio of ridership to optimal frequency. Jansson (1980) pioneered the consideration of separate peak and off-peak operations, where the vehicle's capacity constraint is primarily binding during peak times. Through analytical exploration, he demonstrated that as fewer passengers are transported outside critical sections and periods of service provision due to uneven demand patterns, the relative importance of frequency and waiting time diminishes concerning bus size. An uneven demand distribution points to a larger optimal bus size.

Oldfield and Bly (1988) revisited the single-period static model of a representative origin-destination pair, making advancements in modeling average waiting time. Their innovations included potential failed boarding instances when buses are undersized and demand varies. Three waiting time assumptions were examined:

1. A constant average load factor assumption establishes a deterministic link between demand, frequency, and vehicle size.
2. Incorporating a load factor-dependent multiplier in the waiting time function reflects the impact of both service frequency and occupancy rate, given that the cost of failed boarding increases with vehicle occupancy.
3. Adding a load factor-dependent cost to the standard headway-dependent waiting time cost.

Oldfield and Bly (1988) introduced a notable innovation by calibrating their model to accommodate elastic demand. This calibration found that assumption one leads to the highest optimal vehicle size, while assumption yields the lowest. The introduction of external subsidies positively influences vehicle size across all three scenarios. The existing literature has generally assumed that user costs remain unaffected by occupancy rates, implying that operators lack motivation to increase vehicle size unless capacity constraints are binding. An empirical analysis of train lengths in the Netherlands by Rietveld et al. (2002) revealed consistently suboptimal occupancy rates, prompting a call for theoretical investigations into the underlying causes. This coincided with the work of Jara-Díaz and Gschwender (2003), who extended Jansson's (1980) model to incorporate the impact of in-vehicle travel time valuation on occupancy rates. This development corresponded with an increasing body of empirical evidence on the discomfort of crowding, often quantified as a multiplier on the value of time. Jara-Díaz and Gschwender (2003) demonstrated through numerical exercises that this extension significantly enhances the optimal frequency compared to the original Mohring and Jansson models. Pels and Verhoef (2007) extended the theoretical framework to encompass a broader demand system with imperfect modal substitution across multiple origin-destination pairs. Basso et al. (2011) introduced a feature relevant to bus services, considering the maneuverability of buses concerning their size and factoring this into the time in motion. Within the realm of buses, Tirachini et al. (2014) explored a discrete approach, where vehicle size as a decision variable is limited to four commercially available configurations: mini, standard, rigid long, and articulated. Jara-Díaz et al. (2020) proposed a dual-fleet approach, suggesting using two vehicle sizes to address demand fluctuations between peak and off-peak periods. Tirachini et al. (2014) introduced the number of seats as an additional decision variable to allow greater flexibility in vehicle capacity. Their numerical calibration with Sydney data revealed that maximizing seat supply is optimal when crowding is a concern. Hörcher et al. (2018) derived an analytical guideline for optimal seat provision, suggesting that lower than maximal seat supply might be optimal under certain conditions. In the context of railways, de Palma et al. (2015) maintained a constant vehicle size but allowed for variable seat provision. In a dynamic peak-period setup, they demonstrated how regulating seat availability on the busiest trains could mitigate crowding discomfort, emphasizing the potential benefits of reallocating space between seated and standing passengers. De Palma et al. (2017) later explored the reverse scenario, maintaining a fixed seat layout while allowing train length to be endogenous. Through various pricing scenarios in the dynamic peak setup, they revealed that, in this context, capacity investments and efficient pricing do not substitute for congestion relief, implying that expanding capacity yields more incredible benefits when aligned with other socially optimal supply variables.

2.2 En-route charging stations

With the increasing integration of battery electric buses (BEBs) into urban transit systems, the optimization of en-route charging station locations has emerged as a crucial challenge for ensuring the practicality and efficiency of electric public transportation. As cities worldwide strive to embrace sustainable mobility solutions, the strategic placement of charging infrastructure plays a pivotal role in shaping the viability and effectiveness of battery electric bus routes and networks. This section reaches into the literature focused on the optimization of en-route charging station locations, exploring various methodologies, algorithms, and factors that influence the decision-making process. By analyzing the insights gleaned from past studies and real-world implementations, this section aims to provide a comprehensive overview of the advancements and considerations driving the optimization of en-route charging station locations for battery electric buses. In most research, the electric charging station location is analyzed at the network level, where the intersections of routes are considered the only feasible locations. In addition, most charging station locations are determined with the main purpose of optimizing operational costs. For example, Sweda et al. (2017) developed an EV recharging policy, considering the operating costs associated with battery longevity and avoiding charging the battery to its maximum/full capacity. Yao et al. (2020) and Wang et al. (2017) created optimal BEB charge schedule models to lower annual scheduled charging costs for a transportation network. Other studies attempted to optimize BEB charging station locations exclusively by considering candidate locations at the endpoints of routes (Rogge et al., 2015; Liu and Ceder, 2020). Lin et al. (2019) created a two-stage spatial-temporal model for the infrastructure planning of a bus network to optimize the total cost of transportation and the load on the power grid.

While a transit network perspective is vital in a grid network, looking at the transition problem to BEB on a route-by-route basis is increasingly attracting research attention (Sebastiani et al., 2016; Zhu et al., 2016; Kunith A., 2017; Xylia et al., 2017; Liu et al. 2018; Wei et al. 2018; Hu et al. 2022; Wang et al. 2023). Locating fast battery charging facilities at the bus stop takes advantage of the fact that the transit bus is already stopping at the transit stops to board and alight passengers; at the same time, opportunistically charge the BEBs (Hu et al. 2022). In addition, consideration of the route level is essential as cities are likely to transition to BEB by piloting it on a route-by-route basis. This is especially important in North American cities that are characterized by low transit line densities, necessitating en-route charging stations at transit stops. Recently, Vancouver and Chicago have been piloting a complete transition for selected routes to BEB with charging stations at transit stops (Translink 2023; City News 2023).

Kuby and Lim (2005) examined generic refueling stations on a network, demonstrating that the network's nodes are not a finite domain set; including candidate sites at midpoints between nodes could provide better round-trip coverage. Kuby and Lim (2007) and later Lim and Kuby (2010) augmented the set of nodes in the network to include other midpoint candidate sites. In closely related studies, Xylia et al. (2017), Kunith et al. (2017), and Zhu et al. (2016) developed a mixed-integer linear program to optimize the location of the charging facilities where every transit stop was considered as a candidate charging station location and to minimize the operator's costs. Successful planning and operations of a transit system require that the cost to passengers as well as the cost to operators are estimated properly (Ansari Esfeh et al., 2021, 2022). Numerous studies have explored the implementation of various charging station types (Luo et al., 2018; Wang and Lin, 2013; Cui et al., 2018). However, this research has primarily focused on battery electric vehicles (BEVs). Introducing a fresh perspective, Liu and Wang (2017) proposed an innovative framework to strategically position diverse BEV charging stations. Their model accounts for advancements in wireless static and dynamic charging, encompassing BEV type selection and user routing preferences. To solve this intricate model, they employed an efficient algorithm based on a stochastic radial basis function response surface. Lee et al. (2020) examine a week-long survey of Plug-in Electric Vehicles (PEVs) charging behaviors, revealing diverse patterns. Analyzing various charger levels and locations uncovers distinct weekday and weekend charging habits among Battery Electric Vehicle (BEV) and Plug-in Hybrid Electric Vehicle (PHEV) owners.

Wei et al. (2018) created a mixed-integer programming model to tackle the challenge of arranging charging stations along a route where diesel buses are being phased out partially during a transition phase in favor of BEBs. Esmaeilnejad et al. (2022) developed a two-stage deterministic optimization framework for locating the en-route charging stations and their charging time in the first stage, followed by locating the optimal arrangement of the optimal holding points.

So far, only a few BEB studies have taken a stochastic modeling approach that reflects the uncertainty of key input transit parameters. Tang et al. (2019) used robust optimization to tackle the problem of fluctuation of BEB charging durations that result from the changes in bus travel time caused by prevailing traffic conditions. Using stochastic optimization, An (2022) examined the problem of locating normal plug-in chargers at the network level with time-varying electricity prices. Panah et al. (2021) used a stochastic programming (SP) technique to incorporate the fluctuation of energy cost that varies with the source of electricity generation (i.e., solar, thermal, or wind). Using two-stage stochastic programming, Zhou et al. (2023) examined the electric transit facility planning for a fleet of heterogeneous electric buses considering the uncertainties in BEB travel time and its battery degradation. Solving SP models is notorious for being computationally complicated and expensive when compared to the deterministic models for solving the same problems. Many recommend solving the same problem by replacing all the random variables with their expected values or their means. The optimal solution in this approach is called the "expected value solution" (EVS) (Birge and Louveaux, 2011). This EVS approach is beneficial in the absence of enough information to model the total randomness of the input variable(s). The difference between EVS and the solution of the optimal objective value of the recursive problem (SP model in the case of this paper) is called the "value of stochastic solution" (VSS). In other words, VSS is the cost of ignoring the uncertainty in making the final decision. Madansky (1960) established that the value of the recursive problem is always smaller or equal to the EVS. In a recent paper, Hu et al. (2022) examined the problem of the optimal location of the charging stations and their charging schedule while also considering both passengers' and operators' costs under time-variant electricity pricing. A robust optimization approach was adopted mainly to address the uncertainties associated with bus travel time and ridership loading and unloading profile in determining the optimal en-route battery charging station locations and their opportunistic charging schedule. Hu et al. (2022) separately tackled the problem of optimal location and optimal charging time of the en-route charging stations. However, the problem of the location of the charging stations cannot be separated from that of charging scheduling as they mutually affect each other. In other words, fixing the location of the charging stations dictates their charging duration and vice versa, so if both problems are solved separately, a sub-optimal solution (local optimum) can result. Ideally, the problem should be tackled simultaneously. In addition, as the study mainly focused on an Australian case study, little attention was paid to the uncertainty concerns of the BEB performance caused by the extreme cold weather conditions.

Two significant challenges exist in solving the BEB charging location and duration problem with uncertain weather conditions. While the first challenge involves the stochasticity of the demand pattern, the second challenge is induced by BEB technology and deals with the fluctuation of the battery discharging rate. Considering such stochasticity in both BEB demand and operation is paramount when transitioning into a BEB route. These weather-related considerations are especially a concern in cold countries like Canada, where the temperature fluctuates considerably from season to season.

Liu et al. (2018) developed a robust optimization model implementing mixed integer programming for planning an electric bus route economically and efficiently. The optimization model is designed to minimize the operation cost. Besides, uncertainty in the energy consumption of electric buses caused by instability in traffic conditions and travel demand is addressed in this model. Implementing this model on real-world data demonstrates that the model can provide an optimal plan for a BEB system with fast-charging stations along the route. In addition, the trade-off between the robustness and cost of the system is addressed.

Lin et al. (2019) propose an optimization model for planning optimum charging infrastructure for BEBs considering both the transportation network and the power grid. The designed model is a three-tier assignment problem which is solved using MIP. The evaluation of their model is performed by a case study of Shenzhen (the first city with an all-BEB transit network).

Li et al. (2018) proposed a new routing and scheduling for bus routes equipped with BEBs. The model in this study suggests equations to calculate the battery swap station (BSS) scales and spacing. This study is evaluated by implementing a numerical simulation performed on the data of a BSS, Xuejiadao, in Qingdao, China.

Abdelati et al. (2023) worked on the optimization of battery electric bus (BEB) systems in transit networks with a focus on their robustness during charging disruptions. Unlike traditional cost-centric approaches, their study employs complex network theory to assess BEB system resilience. The research finds that timely resolution of disruptions maintains system robustness, but multi-charger stations have adverse effects. Key factors affecting robustness include bus charging frequency and event duration. This study quantifies BEB system robustness using a service frequency indicator under disruptions and suggests minimizing charging events and spreading infrastructure to enhance it.

Mohamed et al. (2017) presented a twofold investigation into the integration of Battery Electric Buses (BEBs) within a full transit network. It employs real-time simulation models to assess operational feasibility, energy demands, and infrastructure requirements of different BEB configurations (flash, opportunity, and overnight charging). The study highlights the feasibility of flash and opportunity BEBs despite intermittent power demands. Subsequently, the paper evaluates the impact of BEBs on utility distribution grids, emphasizing transformer sizing, load profiles, and grid stability. Contradictory recommendations between operational feasibility and grid impact assessments underscore the need for holistic decision-making. The research contributes insights for optimizing BEB implementation, accounting for operational and utility constraints.

2.3 Time Points

The efficient operation of battery electric buses (BEBs) within urban transit networks hinges on their environmental benefits and ability to offer passengers reliable and punctual services. Achieving optimal schedules and time point locations for battery electric buses is a comprehensive challenge that requires careful consideration of various parameters such as passenger preferences, operational costs, and charging infrastructure. This section delves into the literature surrounding determining optimal time point locations for transit routes with en-route charging stations, focusing on the intricate interplay between scheduling efficiency and passenger satisfaction. By exploring existing research, methodologies, and key findings, this section aims to comprehensively understand the strategies and considerations contributing to successfully integrating battery electric buses with well-designed time point schedules.

Decisions regarding the operations of transit services are made at various planning stages: the strategic planning stage, the tactical planning stage, and the operational stage (Gkiotsalitis and Van Berkum, 2020). The focus of the last section of this study is the operational stage of the transit routes, where dynamic control approaches, like holding, are employed to improve the reliability of the transit service. This contains significant relevance, especially for public transport services that operate in mixed traffic, as they encounter heightened levels of uncertainty. Bus services, in particular, frequently experience recurring disturbances attributed to the inherently unpredictable nature of the operational environment. Fluctuations in system operations commonly arise from interactions with mixed traffic, traffic signals, constrained stop capacity, variability in passenger demand, and driver behavior (Hans et al., 2015).

Corrective actions are implemented daily, including stop-skipping, vehicle holding at specific stops, and rescheduling. Stop-skipping strategies have been explored in works by Sun and Hickman (2005), Chen et al. (2015c), Yu et al. (2015), and Liu et al. (2013). Vehicle holding at specific stops has been studied by Newell (1974), Berrebi et al. (2015), Hernández et al. (2015), and Wu et al. (2017). Rescheduling methods have been proposed by Adamski and Turnau (1998) and Strathman et al. (1999). Some researchers have attempted to combine stop-skipping and vehicle-holding approaches, as seen in the works of Eberlein (1995), Cortés et al. (2010), and Sáez et al. (2012). Gkiotsalitis et al. (2019) introduced the concept of virtual lines to combine short-turning and interlining options. Munõz et al. (2013) integrated speed control with vehicle holding, while Cortés et al. (2011) explored the integration of short turning and deadheading. Despite the efforts to optimize operational control strategies, the computational complexity of each problem and the need to compute optimal control strategies in quasi-real-time often lead to separate applications of different operational control approaches. In addition to holding, rescheduling, and stop-skipping, inter-station control strategies can also be considered. Common and standard inter-station control methods include traffic signal priority, as examined by Skabardonis (2000), Liu et al. (2003), Koehler and Kraus (2010), and van Oort et al. (2012), and speed control, explored by Daganzo and Pilachowski (2011), Wang et al. (2014), and Ampountolas and Kring (2015).

Research on vehicle holding techniques has been under investigation since the early 1970s, as evident in studies by Osuna and Newell (1972) and Newell (1974). Turnquist (1981) categorized vehicle holding strategies into two distinct classifications. The first is schedule-based holding, which establishes checkpoints along the bus route to manage vehicle operations. However, this approach primarily addresses headway deviations due to buses running ahead of schedule, often incorporating additional time in the published schedule to mitigate bunching effects and enhance system stability at the expense of passenger travel times (Petit et al., 2018; Newell, 1974; Osuna and Newell, 1972). In contrast, the second type, headway-based holding, is centered around maintaining a consistent time interval between consecutive vehicles. Implementing this strategy necessitates real-time data on bus operations, including their positions or, at the very least, their headways (Bie et al., 2019).

Despite this longstanding interest, the holding problem remains a significant research subject due to its inherent complexity. Newell (1974) focused on a single time point for intentionally delaying vehicles and proposed a strategy to minimize passengers' average waiting time. In the literature, time points are defined as stops where vehicles arriving before the scheduled departure time must wait to depart on time, as Klumpenhouwer and Wirasinghe (2018) discussed. The thorough investigation of time point identification has received limited attention thus far. In a study conducted by Lesley (1975), a recommendation was made to designate time points in instances where the coefficient of variation of headways exceeds twice the average value across all stops. This criterion specifically targets situations characterized by unusually high unpredictability of bus arrival times. Likewise, in their study, Abkowitz and Engelstein (1986) reached a similar conclusion regarding the placement of time points. They suggested that time points should be strategically located based on the highest product obtained from multiplying the standard deviation of travel time by the ratio of affected passengers to the total passenger demand. This approach effectively accounted for the impact of unreliability on passengers as well as the varying demand observed on most routes. Numerous analytical and computational investigations have explored the concept of holding control in a broad sense. Seneviratne (1990) utilized a cost-based approach and a simulation model to examine the problem of locating time points and allocating slack time along a route. The study concluded that there is a threshold number of time points on a route, beyond which travel time increases unnecessarily. Wirasinghe and Liu (1995a) focus on determining the optimal number and placements of time points in the design of transit schedules for a single run. It addresses the challenge of balancing operational efficiency and passenger satisfaction by considering factors such as passenger demand, route characteristics, and operational constraints. Through the use of simulation models and optimization techniques, the study explores different scenarios to quantify the impacts of time point configurations on travel time deviations and overall system performance. The findings try to provide insights and recommendations for transit authorities and operators to optimize their schedules and improve operational efficiency while enhancing passenger satisfaction. Daganzo (2009) created a model with a single control point on a loop route, demonstrating that a controlled headway approach required less slack time than scheduled service. Daganzo and Pilachowski (2011) expanded this approach to enable real-time control at any point on the route, allowing for infinite control points. Bartholdi and Eisenstein (2012) proposed that scheduled service exhibits inherent instability. Xuan et al. (2011) adapted Daganzo's work (2009) to explore holding strategies incorporating schedule adherence. Their mathematical model encompassed three control methods: traditional schedule-based control, forward headway control (Daganzo, 2009), and two-way headway control (Daganzo and Pilachowski, 2011). In all cases, the developed control methods ensured buses maintained regular headways and adhered to a schedule. However, the paper did not address the central research question of this thesis pertaining to the strategic placement of timing points along the route. Osuna and Newell (1972) introduced the initial concept of a dynamic holding system, which involved re-dispatching buses at a designated control point. Newell (1974) extended this idea to two vehicles and one control point to minimize passenger waiting time. Other works, such as Eberlein et al. (2001) and Hickman (2001), also focused on reducing headway variance. Daganzo (2009) incorporated real-time information and prioritized minimizing overall slack time while maintaining a satisfactory level of headway variance. This approach was further expanded by Daganzo and Pilachowski (2011) to accommodate a more dynamic situation. Berrebi et al. (2015) advanced the real-time dispatching methodology to encompass subsequent vehicles in their analysis. Fu and Yang (2002) conducted multiple simulations involving various time point stops and demonstrated that holding is more effective when applied at time points with higher boarding demand. Additionally, they examined the influence of the number of time point stops by testing the one-stop control, two-stop control, and all-stop control. From their simulations, they concluded that a targeted two-stop control represents a viable and satisfactory option. Furthermore, Cats et al. (2014) determined the optimal number and ideal locations of time point stops for holding, evaluating their effects on transit performance through simulations. Hickman (2001) employed Marguier's stochastic model (Marguier, 1985) to calculate the vehicle trajectories on a single route. Using this model, Hickman (2001) devised a holding algorithm applied whenever a vehicle reaches a time point stop. For this purpose, Marguier's model was used to approximate the trajectories of all preceding vehicles. The optimal holding time was determined using a line search solution method since obtaining an analytical solution proved challenging due to the complexity involved in deriving the first-order conditions of the optimization problem. Recently, holding control has been combined with transit signal priority (TSP) through the application of rule-based holding criteria (Laskaris et al., 2020).

With the rise of extensive data sets in transportation, research on transit reliability has similarly shifted toward data analysis and specialized modeling. This data-driven approach aims to address the unique challenges of transit systems, focusing on specific datasets to improve reliability.

Earlier works and studies extended the body of existing literature by incorporating emergent technologies such as real-time control. While providing valuable theoretical insights into the principles underlying the holding strategy, these studies tend to overlook the practical reality wherein numerous transit agencies mostly operate within fixed schedules and experience variable headways during peak periods. Additionally, the intricate dynamics of time point placement along a route, encompassing factors such as quantity, location, and scheduling, have not been thoroughly examined in existing research. This specific aspect remains relatively underexplored within the scholarly discourse, and the formulation of a systematic and empirically grounded methodology for determining the optimal number and strategic placement of time points along a given bus route is yet to be established.

This research diverges from the recent trend of using datasets to generate regression models. Instead, this thesis presents a linear optimization model, building upon the work of Klumpenhouwer and Wirasinghe (2018), which encompasses some influential factors on bus reliability. By utilizing an optimization model based on the physical behavior of buses, we can directly modify various aspects of the model using theory or calibrated data sets and observe the model's response. The level of incorporated data can be adjusted according to preference, and the data requirements for a calibrated model are minimal and readily available in most transit centers, such as the City of Calgary. Although this approach may result in less detailed or specific outputs, it enhances the transferability of results and allows the model to be adaptable across diverse scenarios.

In conclusion, the literature on optimizing dispatch policies, en-route charging station locations, and time point schedules for battery electric buses underscores the complex nature of transitioning to sustainable urban transit systems. Insights from diverse studies emphasize the need for flexible strategies that encompass operational efficiency, passenger preferences, and technological progress. This chapter delves into these complexities and innovative solutions, highlighting that successfully integrating battery-electric buses hinges on understanding these interrelated components. The subsequent chapters contribute by proposing novel methodologies and models to enhance the efficacy and viability of battery electric buses in modern urban transportation.

CHAPTER 3: REVISITING THE DISPATCH POLICY FOR BATTERY ELECTRIC BUSES

In transitioning the bus fleets in a transit network to BEBs, the strategy for upgrading and implementing it plays an important role. One of the possible strategies is upgrading all the buses, one route at a time. In other words, the routes can be ranked/prioritized to be electrified based on several factors, such as the number of diesel buses, fleet size, length of the route, bus frequency, characteristics of the transit stops, and the demand profile of the route. Thus, all the buses in a given selected route can be replaced at once with new BEBs, and in doing so, transit operators might reassign the regular buses of that route to other routes to replace the ones that are ready to be phased out. Many factors should be considered while replacing the buses on a route with BEBs. Instances of these factors include the location of charging stations and their charging capacities, the number of required BEBs and their configuration, and the resulting bus dispatch rate.

This chapter delves into revisiting and refining the dispatch policy for BEBs. We explore potential updates in dispatch rates, reassess the number of BEBs required on specific routes, and re-evaluate the charging and refueling facilities' configurations. The objective is to formulate an efficient and practical dispatch policy that ensures seamless BEB operations while maximizing the benefits they offer.

3.1 Updating Dispatch Rate in a Route

The first step to transition into an entirely BEB-operated route is obtaining the dispatch rate or headway and, consequently, the BEB fleet size for that route. The related total cost (Z) is defined as the summation of the cost of dispatching BEBs (operation cost) and the waiting time of the passengers (users' cost) as a function of headway. Assuming a route where the average rate of arrival of the transit passengers per unit time is $\bar{p}$, the uniform transit headway is h, and the cost incurred by the operator for dispatching one BEB for a route is λ_E (including labor cost, the wages of the crew, and the capital BEB cost per run). Therefore, the total dispatching cost per unit of time is $\frac{\lambda_E}{h}$. Having the headway (h) and assuming a random arrival of passengers, the average waiting time per passenger can be approximated to be $\frac{h}{2}$. So, the total cost of the waiting time is $(\gamma_w \bar{p} \frac{h}{2})$ where γ_w is the cost of the waiting time per unit time per person. The total cost (dispatching cost and waiting time cost) as a function of headway can be obtained using Equation (1) (Wirasinghe, 1990).

$$Z = \gamma_w . \bar{p} . \frac{h}{2} + \frac{\lambda_E}{h} \quad (1)$$

The optimum headway of the route (h^*) can be obtained by setting the first derivative of Equation (1) with respect to h to zero:

$$\frac{dz}{dh} = 0 \rightarrow \quad h^* = \sqrt{\frac{2\lambda_E}{\gamma_w . \bar{p}}} \quad (2)$$

The value of the second derivative of the total cost function (Z") is positive at h^*. Therefore, it can be inferred that h^* is the optimum headway for minimizing the cost of the objective function (Equation (1)).

While crucial in dispatch rate analysis, the value of the waiting time of the passengers (γ_w) does not change when transitioning from regular bus to BEB. The new dispatch rate for the upgraded bus fleet changes as $\lambda_E \neq \lambda_D$, where λ_D is the cost of the dispatch of a regular diesel bus.

In the first stage of planning, the travel demand (ridership) is assumed to be unchanged when replacing the diesel buses with BEBs. According to Aber (2016), the dispatching cost of a bus in a round trip of a BEB is expected to be less than that of regular buses ($\lambda_E < \lambda_D$). Based on Equation (2), the optimum headway when implementing the BEBs will decrease (${h^*}_E < h^*$), which is an important advantage of this transition, as earlier studies have shown that decreasing the headway results in increasing the reliability of public transit (Chen et al., 2009). Consequently, with the increase in reliability along with shorter waiting times, the demand for public transit in that specific route will increase in the longer term. In turn, increased transit ridership ($\bar{p}$). The increase in the ridership due to alterations in the service is addressed as induced demand in the literature. By transition to BEBs, the induced demand ($\bar{p}$') should be implemented to obtain the optimized headway (see Equation (3)).

$$h^* = \sqrt{\frac{2\lambda_E}{\gamma_W.\overline{p'}}} \tag{3}$$

Having the optimum headway, the minimum cost of dispatching the BEBs can be obtained by plugging in the h^* in the cost function (Equation (1)).

$$Z^* = \gamma_W.\overline{p}'.\frac{\sqrt{\frac{2\lambda_E}{\gamma_W.\overline{p'}}}}{2} + \frac{\lambda_E}{\sqrt{\frac{2\lambda_E}{\gamma_W.\overline{p'}}}} = \frac{\sqrt{2\lambda_E\gamma_W.\overline{p'}}}{2} + \frac{\sqrt{\lambda_E\gamma_W.\overline{p'}}}{\sqrt{2}} = \sqrt[2]{2\,\gamma_\mathrm{w}\,\overline{p'}\lambda_\mathrm{E}} \tag{4}$$

Equation (4) states that the minimum cost is a function of the passengers' waiting-time value, updated ridership, and the dispatch cost of the BEBs. The value of waiting time does not change within this upgrade. The ridership ($\overline{p'}$) is expected to be increased, and the dispatch cost (λ_E), as mentioned earlier, is expected to be decreased. Therefore, the product of $\overline{p'}$ and λ_E determines how the optimum (minimum) cost of the transit service may change by upgrading the fleet to BEBs.

One of the critical factors in obtaining the headway of a route is the capacity (C) of the BEBs, which is not initially considered in obtaining the optimum headway in Equation (3). Notably, the capacity of a BEB may change, considering that the level of service (LOS) determines all or some of the standees considered for the capacity (Das and Pandit, 2015). The level of service (LOS) has a direct impact on the capacity of a BEB. This is because the LOS is a crucial factor in determining the number of passengers that can be accommodated in a BEB. The capacity of a BEB may change based on the LOS for several reasons, such as standees consideration, seating arrangement, and comfort and passenger experience. By considering the capacity constrain C, a bus should be dispatched based on the minimum of either h^* or the headway capacity is fulfilled, whichever comes first. To meet the capacity constraints, a full bus should be dispatched with the capacity headway of $h_c = \frac{C}{P'}$.

$$h_E^* = \min\{h^*, h_c\} \tag{5}$$

Therefore, schedule headway h_E^* can still be used if it is smaller than h_c. Otherwise if ($h_c < h^*$), the h_c should be used as the operational headway (see Equation (5)).

3.2 Updating the Number of BEBs in a route (n_E)

The number of required buses for a given route (i.e., fleet size) depends on the headway and the time it takes for a bus to finish a round trip and get ready to start a new round trip. With transit electrification, round trip time (T) is expected to change with the change in bus acceleration and deceleration and also change in the time required for BEB charging. Thus, these two factors should be considered in updating the round trip time that can be given by equation (6)

$$T = 2\frac{\mathrm{L}}{\mathrm{V}} + \mathrm{N}\left(\frac{\mathrm{V}\left(\frac{1}{\mathrm{a}_E} + \frac{1}{\mathrm{b}_E}\right)}{2}\right) + \mathrm{T_c} + \Omega \tag{6}$$

In Equation (6), “2L” is the length of the round trip of the bus route, V is the mean cruising speed, N is the number of bus stops, “a_E” and “b_E” are acceleration and deceleration rates of the BEB, and the T_c is the total electric battery charging duration time required in one round trip; thus, T_c includes the charging times and/or battery swapping time in the bus terminals, charging duration at the en-route charging stations, and Ω is the total loss time (in minutes) elapsed along the route for one round trip for boarding and alighting the passengers and crew rest time. In order to obtain the number of required BEBs for a route (n_E), we should divide the round-trip time (T_p) by the designated headway of the route for the BEBs ($h^*{}_E$):

$$\mathrm{n_E} = \frac{\mathrm{T_p}}{\mathrm{h^*{}_E}} \tag{7}$$

3.3 Updating Charging/refueling Facilities

Various factors may reduce the operational range of the BEBs, including air conditioning, bus passenger loading, route characteristics (e.g., vertical alignment), driving behavior, and battery life and age. At the current state of the battery technology, the BEBs on most routes are incapable of finishing a whole day’s work without battery recharging (Schoenberg et al., 2021), which is an issue since the charging time of the BEBs is longer compared to the refueling of regular buses. On average, the fuel efficiency (electric power efficiency) of the BEBs is obtained to be 0.74 km/kWh based on the experimented and simulation studies that considered random conditions and situations (Ma et al., 2021). Many efforts have been devoted to locating the charging stations along predefined bus routes (Kunith et al., 2017; Wei et al., 2018). Different types of techniques can be implemented to charge the batteries:

• Plug-in charging

• Wireless charging (conductive charging)

• Battery swapping

The mentioned types of charging facilities differ significantly in terms of charging time, efficiency, size, and location requirements. Besides, this variety in the type of charging solutions increases the flexibility while planning and scheduling the BEBs. In plug-in charging, the BEB physically connects to a power source using a cable/wire connection. The plug-in charging technique is the most efficient; however, it requires extra steps to connect the charging cable to the BEB. In wireless (conducive) charging, the BEB can be charged without any physical connection with the source of power.

The wireless charging stations are mainly designed for en-route charging opportunities. This technique is less efficient compared to plug-in charging (Jiang et al., 2020). Using a wireless charging facility does not require an operator to install the charger to the BEB and make the charging procedure start/stop. Therefore, wireless charging is a better option when the charging time opportunity/requirement is short or the bus has to be charged while moving. But there are also some other disadvantages to consider such as cost, limited speed, and maintenance and reliability (Panda et al., 2023).

The battery swapping requires replacing the depleted battery with a fully charged one. Battery-swapping is a new alternative strategy for eliminating most of the existing barriers in BEB operation, such as the necessity of fast recharging. The feature of this strategy is that it can be performed in the termini, at both or one end of a bus round trip, in 5-10 minutes (Liang et al., 2017). This strategy is keenly explored and adopted, especially in China, where 12,000 were planned to be constructed and installed in Nanjing and other pilot cities (Zheng et al., 2014).

The optimal location and type of the charging stations chosen for the bus route can be altered based on the length of the route and the type of the BEBs. Consider the length of the route in each direction to be L, the minimum allowed percent-charge level of the battery to be B, and let C_D be the discharge rate per unit of length (%/km) of the battery. Different parameters, as mentioned earlier, affect the operational range of a BEB, such as demand (load), weather, and route topology; instead, a standard parameter, discharge rate "C_D" is considered as a known quantity that incorporates those factors (Li, 2016). This parameter is unique for each BEB for each route, which can be obtained by data collection and drivetrain simulation (Su et al., 2016).

The location and selection of the charging stations for a bus route Is a problem which can be divided into three classes corresponding to ranges of (100 – B):

1. A charging station is only required at one end of the route, with no en-route charging required:

$$2C_D L < 100 - B \tag{8}$$

2. Charging stations are required at both ends of the route with no en-route charging required:

$$C_D L < 100 - B < 2C_D L \tag{9}$$

3. Charging stations are placed at both ends of the route, and en-route charging station(s) is (are) required:

$$100 - B < C_D L \tag{10}$$

In the case of having a charging station at one end of a bus round trip, a plug-in charging station or a swapping battery facility can be used, as shown in Figure 1. Figure 1(a) and Figure 1(b), respectively, represent having one plug-in charging station and one battery-swapping facility in a bus route.

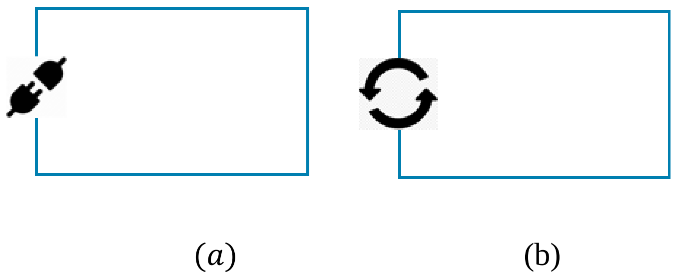

Figure 1 – The alternatives of having a charging station at one end of a bus round trip

In the case of needing a charging facility at both ends of the bus route, three different arrangements can be applied as shown in Figure 2: (a) plug-in charging stations at both ends, (b) battery swapping facilities at both ends and (c) plug-in charging at one end and battery swapping at the other end.

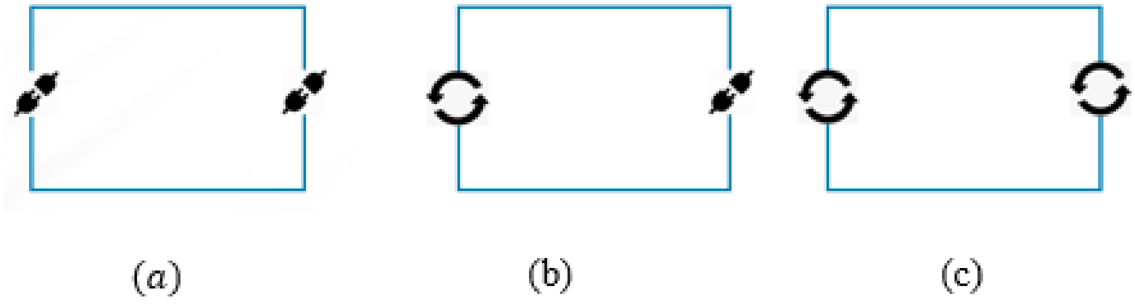

Figure 2 – The alternatives of having charging stations at both ends of a bus round trip

If the charging stations on both ends of the bus route do not satisfy the need for the charge of the BEB to finish the route, en-route charging stations are required. Therefore, the charging stations' location and the charging time assignment are the variables to be determined. For en-route charging stations, the charging duration is critical as there are through-passengers in the vehicle. Therefore, as shown in Figure 3, fast-charging plug-in or wireless charging facilities are required alternatives.

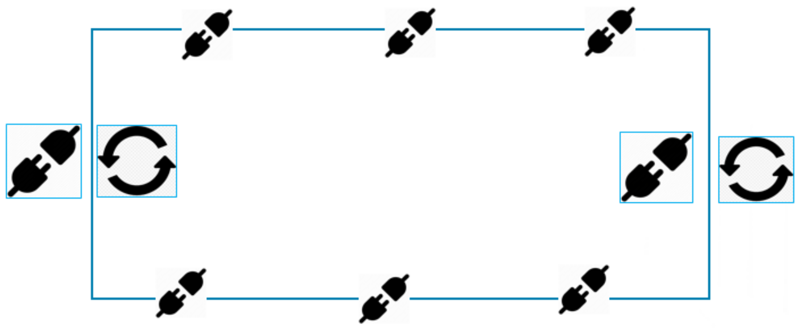

Figure 3 – En-route charging stations are required

However, the allocation of the en-route charging stations, if needed, and their charging times is the second step of the problem of the location of the charging stations and assigning the charging times in each location. Therefore, this study focuses on discussing Cases 1 and 2, i.e., locating charging facilities at one or both ends of a bus route.

3.3.1 Plug-in charging facility at one end of the bus route

There is only one charging facility in the bus route in this layout. All the layouts assume that the BEB fleet starts the day from the starting terminal, fully charged. The charging status of the BEBs should be checked to ensure they can finish the bus round trip and return to the charging facility. In the case that the BEB can finish the bus round trip, the percentage of the required total charge, R, for finishing the bus round trip is given by

$$R = 2\,L\,C_D \tag{11}$$

It is assumed that a BEB can finish the round trip at least once. The number of times (N_R) that the BEB can finish the bus round trip without any additional charge can be obtained using Equation (12):

$$N_R = \left\lfloor \frac{100 - B}{R} \right\rfloor \tag{12}$$

Hence, the BEB needs to get fully charged only after traversing the bus round trip N times. The required charging time (T_C) at a charging station with a charging rate C_F (%/unit time) can be obtained using Equation (13):

$$T_C = \frac{N_R(2\ L\ C_D) - (100 - B)}{C_F} \tag{13}$$

In the case that T_C is greater than the headway, it is required to increase the number of charging plug-ins (or charging bays) at the charging stations. Otherwise, bus-bunching will happen in the terminal. The minimum required capacity of the charging facility (N_C), i.e., the number of fast chargers, to avoid bus bunching, can be obtained using Equation (14):

$$N_C = \left\lceil \frac{T_C}{h^*_E} \right\rceil \tag{14}$$

So, the average charging time as part of the bus round trip time is $\frac{T_C}{N}$. Consequently, the round trip time (T_{CE}) for this charging layout can be obtained using Equation (15), where Ω is the total loss time elapsed along a route for one cycle for boarding and alighting the passengers in the stations, and T_a is the elapsed time for accessing the charging station and connecting the BEB to the charging facility.

$$T_{CE} = 2\frac{L}{V} + N\left(\frac{V\left(\frac{1}{a_E} + \frac{1}{b_E}\right)}{2}\right) + \frac{T_C}{N_R} + \Omega \tag{15}$$

As mentioned earlier, the value of λ changes with the electrification of the bus routes ($\lambda_D \neq \lambda_E$). The extra cost of owning the BEBs, longer cycle times of electrified bus routes, and the decrease in the cost of energy for traversing the BEBs are all included in the λ_E.

3.3.2 Battery swapping facility in origin terminal

The second alternative for having one charging facility in a bus route is having a battery swapping facility at the origin terminal or a depot close to this terminal. In choosing the BEBs and their battery size, it should be assured that the BEBs will be able to traverse the bus route reliably with no need for an extra charge, as shown in Equation (8). The number of round trips that the BEBs can traverse prior to accessing the battery swapping facility can be obtained using Equation (12). Considering the time for accessing the battery swapping facility and having the battery swapped is T_s, the average battery swapping time as part of the duration of the cycle time is $\frac{T_s}{N}$. Therefore, the cycle time of the bus route can be obtained using Equation (16):

$$T_{SO} = 2\frac{L}{V} + N\left(\frac{V\left(\frac{1}{a_E} + \frac{1}{b_E}\right)}{2}\right) + \frac{T_s}{N_R} + \Omega \tag{16}$$

The assumption is the replaced depleted batteries will be fully charged overnight; thus, each replaced battery will be used only once daily. Therefore, the required number of extra batteriesI) needed in a battery swapping facility for a bus route per round trip can be obtained using Equation (17).

$$E = \frac{T_{SO}}{N_R \ h} \tag{17}$$

It can be inferred that by replacing the plug-in charging with a battery swapping facility, there is an extra cost for purchasing extra batteries. However, the number of required BEBs decreases because, with the current state of battery technology, the charging time of a depleted battery (T_C) takes more time compared to swapping the same battery (T_s).

The minimum required number of bays, i.e., the capacity of the battery swapping facility, is $\left\lceil \frac{T_s}{h_E^*} \right\rceil$ to avoid bus bunching. Similar to the previous section, the total cost (Z), optimum headway, and optimum total cost can be obtained using Equations (1), (2), and (3). In this setup, the cost of purchasing and maintenance of the extra batteries should be considered in the dispatch cost of each BEB (λ_E).

3.3.3 Plug-in Charging Stations in both terminals

Two charging facilities at both termini of a bus route are required in case a fully charged BEB may not manage to traverse the bus round trip, but it can reliably manage to finish one direction of the bus route, as shown in Equation (9). The first alternative is locating plug-in charging stations at both terminals of a bus route. Both charging stations are assumed to have the same charging rate (C_F). Besides, it is assumed that the BEBs always leave the origin terminal fully charged, while in the turning terminal, the BEBs charge to a specific percentage to get back to the origin terminal reliably. Therefore, the total charging time in both of the terminals in one cycle can be obtained using Equation (18):

$$T_{COE} = \frac{2 \ L \ C_D}{C_F} \tag{18}$$

However, the charging times in the terminals are not necessarily equal ($T_{C1} \neq T_{C2}$). The charging station duration in the destination terminal should be long enough to charge the BEB. Hence, its battery level is at or above B when it returns to the origin terminal. The required charging duration for returning to the starting terminal can be obtained using Equation (19), and the minimum required charging time in the second terminal can be obtained using Equation (20).

$$2\ L\ C_D - (100 - B) \tag{19}$$

$$T_{C2} = \frac{2\ L\ C_D - (100 - B)}{C_F} \tag{20}$$

Consequently, the charging time in the first terminal at the end of each bus round trip can be obtained using Equation (21):

$$T_{C1} = T_{COE} - T_{C2} \tag{21}$$

The round-trip time (T), the total cost, the optimum headway, and the optimum cost are similar to section 3.3.1. However, the N_R will be eliminated as in this section the initial assumption is that the BEBs cannot finish a round trip without being charged in the destination terminal. Similar to the previous section, the cost of installation and maintenance of an extra charging facility, i.e., extra costs due to electrification of the bus route, should be included in the value of the (λ_E).

3.3.4 Battery swapping in both terminals

When we need charging facilities at both ends of a bus route, the second alternative is having battery-swapping facilities at both terminals. As the batteries cannot manage the BEB to traverse the route, the BEBs' batteries should be swapped every time they reach a terminal. Therefore, the cycle time of the bus route (T_{CS}) can be obtained using Equation (22):

$$T_{CS} = 2\frac{L}{V} + N\left(\frac{V\left(\frac{1}{a_E} + \frac{1}{b_E}\right)}{2}\right) + 2T_s + \Omega \tag{22}$$

The main advantage of this method is that all the batteries owned by the operator are always charged by the plug-in overnight technique. Therefore, the batteries will last longer (compared to fast charging facilities), which is financially and environmentally beneficial. However, in this setup, i.e., having two battery swapping facilities on the route, the operator may purchase a larger number of extra batteries (in addition to the ones that come with the BEB). The number of extra batteries to be purchased can be obtained using Equation (23):

$$E = 2\left(\frac{T_{CS}}{h^*{}_E}\right) \tag{23}$$

Additionally, assuming that the battery swapping time in one cycle is less than the total charging time of a cycle ($2T_s < T_c$), it can be inferred that the operator saves the round trip time in this setup, which helps the operator provide fewer BEBs. The reduced number of BEBs to be provided can be obtained using Equation (24):

$$N_B = \frac{T_c - 2T_s}{h^*{}_E} \tag{24}$$

Having the price of an extra battery (γ_B) and a BEB (γ_{BEB}), Equation (25) lets the operator figure out how much extra or less should be provided using this setup:

$$A = B\gamma_B - N_B\,\gamma_{BEB} \tag{25}$$

where A is the saving in the initial investment. In the case that A>0, having two battery swapping facilities is financially a better alternative compared to the first alternative.

3.3.5 Combination of battery swapping and charging stations in terminals

The third alternative for charging facilities at both ends of a bus route is having one battery swapping facility at one end and one plug-in charging facility at the other. It is reasonable to have the battery swapping facility at the starting terminal where we would like the BEBs to leave the terminal fully charged. The round-trip time of the route can be obtained using Equation (26):

$$T_{\text{Comb}} = 2\frac{L}{V} + N\left(\frac{V\left(\frac{1}{a_E} + \frac{1}{b_E}\right)}{2}\right) + T_{c2} + T_s + \Omega \tag{26}$$

where T_{C2} is the minimum required charging time to ensure that the BEB can return to the starting terminal without an extra en-route charge (see Equation (20)). Assuming that the swapping time (Ts) is less than the charging time in the starting terminal ($T_S < T_{C1}$), the round-trip time of this alternative is less than the first one. However, it depends on the length of the route and the amount of charge needed in the finishing terminal (T_{C2}); it can be less, equal, or greater. So, it can only be inferred that the number of BEBs needed in the third alternative is less than in the first. However, it can be easily determined that the extra batteries needed for the third alternative are half the second. Therefore, the optimum layout for having charging facilities at both ends of the bus route is a function of the length and discharging rate (C_D) of the BEBs.

3.4 Numerical Example

Consider a bus route with a round trip length of 20 km where the away and return roads have the same length. The mean discharge rate (C_D) is 5 (%/km), a round trip time of 30 min (excluding the charging times), and the headway is 2 min. Assuming the charging rate (C_F) is 6 (%/min), the battery swapping time is 3 min, and the minimum allowed battery status is 20% (B=20); compare the number of required BEBs and extra batteries for different possible charging layouts where the price of a new BEB is \$785,000, and the price of each spare battery for the battery swapping facility is \$50,000.

$$\mathrm{C_D L} = 50\%, 2\mathrm{C_D L} = 100\%, 100 - \mathrm{B} = 80\% \ \rightarrow \ \mathrm{C_D L} < 100 - \mathrm{B} < 2\mathrm{C_D L}$$

Therefore, the route needs charging stations in both of the terminals. In this case, there are three alternatives for charging facility arrangements at the terminals:

1. Plug-in charging stations in both terminals

The total required charging times (in both terminals in aggregate):

$$\mathrm{T_{COE}} = \frac{2\ \mathrm{L}\ \mathrm{C_D}}{\mathrm{C_F}} = \frac{100}{6} = 16.66\ min \cong 17\ min$$

The charging time in the turning (destination) terminal:

$$\mathrm{T_{C2}} = \frac{2\ \mathrm{L}\ \mathrm{C_D} - \ (100 - \mathrm{B})}{\mathrm{C_F}} = \frac{200 - \ (100 - 20)}{6} = 3.33\ \mathrm{min} \cong 4\ min$$

The charging time in the first point terminal or starting:

$$\mathrm{T_{C1}} = \mathrm{T_{COE}} - \mathrm{T_{C2}} = 17 - 4 = 13\ min$$

Round trip time = 30 + 17 = 47 min

Number of BEBs = $\frac{47}{2}$ =23.5 → 24 BEBs are required

Initial cost for owning the BEBs = 24×\$785,000 = **\$18,840,000**

2. Battery swapping facility in both terminals

Round trip time = 30 + 2×3 = 36 min

Number of BEBs = $\frac{36}{2}$=18 BEBs are required

Number of extra batteries = $\mathrm{E} = 2\left(\frac{\mathrm{T}}{\mathrm{h^*}_{\mathrm{E}}}\right)$ = 36 extra batteries are required (18 in each terminal)

The initial cost for owning the BEBs and the extra batteries= 18×$785,000 + 36×$50,000= **$15,930,000**

3. Combination of battery swapping and charging stations in terminals (Battery swapping in the start terminal)

   The charging time in the turning (destination) terminal:

$$T_{C2} = \frac{2\ L\ C_D - (100 - B)}{C_F} = \frac{200 - (100 - 20)}{6} = 3.33\ \text{min} \cong 4\ min$$

Round trip time = 30 + 3 + 4 = 37 min

Number of BEBs = $\frac{37}{2}$ =18.5 → 19 BEBs are required

Number of extra batteries = $E = \left(\frac{T}{h^*_E}\right)$ = 18.5 → 19 extra batteries are required

The initial cost

Owning the BEBs and the extra batteries= 19×$785,000 + 19×$50,000= **$15,865,000**

Based on the initial cost, it can be inferred that having a combination of a battery-swapping facility and a plug-in charging station offers a better setup financially.

CHAPTER 4: LOCATING OPTIMUM EN-ROUTE CHARGING STATIONS: ONE-WAY AND TWO-WAY BUS ROUTES (DETERMINISTIC MODEL) [1]

4.1 Introduction

While the environmental advantages of battery-electric buses (BEBs) are well-known, their significant differences from diesel buses require alterations to both route design (i.e., charging station locations) and operations (i.e., schedule management and holding control). With that in mind, integrating battery electric buses (BEBs) into transit systems requires meticulous infrastructure planning and optimization. It is crucial to design appropriate charging schemes for each bus route to ensure that BEBs adhere to their operational schedules. Nevertheless, before transitioning from traditional fuel-powered buses, decision-makers and transit authorities should thoroughly grasp the potential impact of a fully electric bus fleet on the energy infrastructure, as highlighted in the existing literature.

The location, number, duration, and types of charging stations must be considered as part of the planning process. Charging stations can be located at depots, termini, or en-route. This chapter explains the long-term planning and optimization problem of en-route charging station locations and charging duration to optimize passengers' waiting time, operation, and capital costs. A linear

[1] Esmaeilnejad, S. Klumpenhouwer, W. Wirasinghe, S. C. and Kattan, L. (July 2022). Integrating En-Route Charging of Electric Bus with Holding Control on an Urban Route with Time-Points. Paper presented at and published in the proceedings of the International Symposium on Transportation and Traffic Theory (ISTTT), Beijing, China, July 24-26, 2022.

deterministic optimization model is developed to optimally place BEB charging stations along the route and estimate their assigned charging time for both one-way and two-way routes. The developed approaches are tested on two high-demand bus routes in Calgary (Route 3 and Route 303). The main contributions of this work are thus twofold: 1) the development of a passenger-centric optimization approach to simultaneously determine charging station locations and durations, and 2) the consideration of weather-induced stochasticity of both transit ridership and operation in solving the problem. This study considers two transit routes with en-route and terminal charging stations as numerical examples. The first route is a regular one-way bus route, and the second is a two-way bus route sharing stops in both directions.

4.2 Methodology

This section considers the development of the non-linear deterministic model and the linearizing of the deterministic model for locating the en-route charging stations. Table 1 shows the model parameters and typical values adopted in the numerical example for model evaluation (Wirasinghe and Liu, 1995; Luethi et al., 2007). The variable B is the minimum allowed battery charge level in percentage (%) to preserve the life of the BEB lithium battery and, more importantly, the reliability of the service (decreasing the chance of having a dead battery during service hours). For instance, battery management systems are typically kept between 20% and 80% of their maximum rated charge in electric cars. In this study, the batteries' charge level is kept between 20% and 100% (fully charged). The discharge rate of the battery "C_D" is in percentage per unit of length. Instead of considering different parameters that affect the operational range of a BEB, such as demand (load), weather, and route topology, a standard parameter, discharge rate (C_D), is considered as a known quantity that incorporates those factors (Li, 2016). In this study, the unit of C_D is in the percentage of the battery discharge rate per kilometer (%/km). The charging rate at a charging station (C_F) is assumed as a known quantity. The unit of C_F is in battery percentage per unit time (%/min) and states how much charge, in percentage, can be restored to the battery per unit time of charging. The charging rate is a function of the output power of a charging facility and the size of the battery of a BEB (You et al., 2015). The parameters C_D and C_F are unique for each BEB operating on different days of the year for each route and thus should be obtained by experimenting and analyzing collected data (Su, 2016). To address this case study and effectively resolve the issue, the chosen battery size for the BEB is 60 kWh, providing an approximate range of 16.6 km. Notably, this specific battery capacity aligns with the best-selling BEBs of its size in certain regions (Horrox and Casale, 2019). L is the route length that the BEB is covering in one direction. The variable C is the ten years amortized installation cost of the charging station. In other words, the cost of installing the charging stations is divided by the number of times the charging stations are used, which is a function of the headway and working hours of the route. In our model, the procurement and installation costs are $65,984 (USD) for a DC fast charger with a power level of 350 kW, with one charger per site. (Nicholas, 2019); this number is converted to Canadian dollars (CAD). Dividing the installation cost by the number of times each charging station is used gives us an estimation of the cost for each time a BEB uses the charging stations. In this model, it is assumed that the charging stations used are the same. H is the headway of the route, which is equal on all days. Based on the battery size of the BEBs to be used on the route, their range, and the output capacity of the en-route fast chargers, the values of C_D and C_F are given in Table 1. The headway is the upper boundary for the charging time to avoid bus bunching at charging stations. The variable γ_r is the value of passenger in-vehicle travel time.

Hossain et al. (2015) estimates were used for the waiting and in-vehicle time values in this study. γ_r was calculated as the midpoint between out-of-vehicle waiting time and in-vehicle riding time. Hossain et al. (2015) acknowledge that waiting on a bus offers improved comfort and certainty but is not considered as productive as actual riding time. This concept is reflected in the R parameters calculation, which provides a framework for estimating public transport user time value and informing service planning decisions. For ease of notation, we introduce a binary state vector ψ, taking the value of zeros and ones ($\psi_{1\times n} \in [0,1]$), to denote the location of charging stations with the dimension of the [1×n−1]. The binary state vector value ψ_i indicates the state of the i^{th} stop, with one indicating a transit station at the i^{th} stop is instrumented with a charging station, and zero otherwise. For example, the state vector of a bus route with six stops and charging stations at stops 2 and 5 (i.e., $\psi_2 = 1$ and $\psi_5 = 1$) would read as $\psi = [010010]$.

Table 1- Model parameters and inputs

Symbol	Meaning	Typical Value
B	Minimum allowed percent-charge level of the battery (%)	20%
C_D	Discharge rate of battery (%/unit of length)	6%/km
C_F	Charging rate at stations (%/unit of time)	10%/min
L	Length of the route (unit of length)	10 (km)
C	Cost of charging station ($/each time of usage of a charging station)	$0.731/bus/trip (Nicholas, 2019)
H	Route Headway (unit of time)	10 (min)
C_E	Cost of electricity ($/kWh)	$0.10/kWh
C_O	Operator cost ($/unit of time)	$0.50/min
γ_r	Value of passenger in-vehicle travel time $/person/unit of time)	$5.1/passenger/hr (Hossain et al., 2015)

4.2.1 Deterministic equivalency of the charging station location problem

Location and type of charging stations chosen for a bus route depend on the route length, its characteristics, and the type of BEBs. Consider the route length to be L; assuming charging stations are required at both ends and along the route, the following inequality holds:

$$100 - \mathrm{B} < \mathrm{C_D L} \tag{27}$$

For example, the BEB cannot travel between two termini without the battery charge going below B, assuming the BEBs leave the terminal fully charged. Infrastructure and capital investment are required to install en-route charging stations, which should be fixed for many years. In other words, the nature of the problem of the location of charging stations is a long-term planning problem. In contrast, the operational effects, weather conditions, and potential scheduling changes, such as charging times, can be adjusted in response to this change in infrastructure (along with other exogenous effects, such as changes in demand and travel patterns). Therefore, the charging time at a charging station is considered a short-term planning problem that can be updated regularly. The charge deficit (W) across the route is given by:

$$W = \mathrm{C_D L} - (100 - \mathrm{B}) \tag{28}$$

En-route charging stations must accommodate this charge deficit. So, for the routes with $W > 0$, at least one en-route charging station is required. The charging station location and duration should be designed to ensure that the battery of the BEB is drained enough to accommodate the assigned charge at the time of charging. The minimum sum of required en-route charging durations (T) that is required along the route is given by:

$$T = \frac{W}{\mathrm{C_F}} \tag{29}$$

The minimum required charging time 'T', must be distributed among the charging station(s) placed along the route. Let T_i be the charging time assigned to the station at the i^{th} stop. For example, suppose T is six minutes; the scheduler can assign only one en-route charging station for six minutes or multiple charging stations with equal or unequal charging time that sum to a total of six minutes. In our optimization model, the two decision variables are the locations of the charging stations and the charging duration at each station.

Assume an existing route with regular buses planned to be converted to be served by BEBs. Also, assume that the bus stops along the route do not change and are known. A subset of these stops are suitable for installing a charging station. Thus, each stop is a candidate location for installing the en-route charging stations. It is further assumed that the BEBs serving this route are all identical, which results in equal battery size, discharge rate ($\mathrm{C_D}$), and the charging rate ($\mathrm{C_F}$); and that all the BEBs leave the terminals at either end of the route fully charged.

We considered a case where a transit operator's objective is to minimize the extra costs due to the necessary modifications to transition into a BEB-based route while maintaining the current transit schedule. The endured cost is associated with the installation of charging stations, extra driver cost, electricity cost, and the passengers' in-vehicle delays induced by the time needed to charge the BEBs at en-route charging stations.

Figure 4 is a schematic of a bus route with "n" suitable bus stops, including the destination terminal. For each stop, we know the distance in km from the origin stop O (x_i), and the number of boarding and alighting passengers per trip at each stop. Therefore, we can obtain the number of through-passengers (θ_i) at each stop.

Figure 4 – A bus route with n stations

As all the suitable bus stops are candidate locations for the placement of a charging station, there are 2^{n-1} possible solutions for charging stations on a route with "n-1" suitable bus stops, given that the destination terminal is already instrumented with a charging station.

Equation (30) below represents a minimization problem that focuses on minimizing the additional costs incurred to both the operators and transit users due to converting to BEBs. The first term indicates the cost of extra in-vehicle waiting time for through-passengers due to placing charging stations for BEBs. In other words, we consider a wait cost penalty for the charging BEB delay imposed on the through-passengers who are waiting on the bus while the BEB is getting charged. To obtain this cost, the charging time "T" is deducted from the dwell time "D". The second term is additional driver cost due to waiting while charging and the cost of electricity. The third term indicates the operator cost that captures the additional cost per one-way trip carried by the operator due to the installation and maintenance of the charging facilities. The objective is, therefore:

$$\min \sum_{i=1}^{n-1} (\psi_i (T_i - D_i)(\theta_i \gamma_r) + \psi_i T_i (C_E + C_O) + \psi_i C) \tag{30}$$

We can express the maximum running range L_{Max} of a fully charged BEB before needing a charge as:

$$L_{Max} = \frac{(100 - \mathrm{B})}{\mathrm{C_D}} \tag{31}$$

Obtaining the L_{Max} enables finding the minimum number of required en-route charging stations (CS_{min}):

$$CS_{Min} = \left\lceil \frac{L}{\mathrm{L_{Max}}} \right\rceil \tag{32}$$

Fixing the minimum number of required en-route charging stations limits the solution search space by narrowing down the possible arrangements, thereby expediting the solution computation time. Four other constraints should be satisfied:

1. The total required charging time in % needed across the route to reach the destination terminal:

$$\sum_{i=1}^{n-1} T_i C_F \psi_i + (100 - B) \geq C_D L \tag{33}$$

This constraint ensures that the charging deficit W in Equation (28) will be offset with enough charging time distributed among the en-route charging stations to reach the destination stop.

2. Each charging station has a relative distance of x_k from the origin must be reached before the BEB runs below a predefined minimum charge level B:

$$C_D x_k - \sum_{i=1}^{k} T_{i-1} C_F \psi_{i-1} \leq (100 - B); \quad \forall k \tag{34}$$

This restriction is defined for each suitable stop along the route ($\forall k$). For example, this restriction is regenerated in a route with twenty such stops (k=20) and should be satisfied 20 times (for each suitable stop). This constraint ensures that the charging stations are located so that BEBs can reach them before their battery level goes below the permitted minimum charging level B.

3. Each charging station has a maximum restriction for the charging level of the battery that it can reach:

$$\sum_{i=1}^{k} T_{i-1} C_F \psi_{i-1} \leq C_D x_k \, ; \;\; \forall k \tag{35}$$

This restriction ensures that the battery is drained enough for the assigned charging time at the charging station. For example, when a BEB arrives at a charging station with an 80% charge and the $\mathrm{C_F} = 10\ (\frac{\%}{min})$, that station's maximum assigned charging time can be two minutes. Similar to the previous restriction, this restriction should be regenerated and satisfied for each suitable stop along the route ($\forall k$).

4. The charging duration is a value greater than the dwell time and cannot exceed the scheduled route headway "H", to limit the possibility of bus bunching:

$$(\text{dwell time})_k \leq T_k \leq H, and\ T_k \in R^+ \tag{36}$$

This constraint sets a lower and upper bound on the charging time at any station. In this case, H represents the route headway; however, this parameter can be fine-tuned in response to specific route characteristics. If the bus arrival is entirely random, it may be best to use a smaller value (i.e., H/2), spreading the charging time over the route to avoid buses meeting at charging stations. This constraint will impact the model's flexibility to concentrate on assigning charging times at locations where costs are lower. The upper bound for charging time is set to be 6 minutes based on Alamatsaz et al. (2022). In addition, the lower limit of the number of required charging stations is set to CS_{Min} using Equation (37) to save the computational work of the solver:

$$\sum_{i=1}^{n-1} \psi_i \geq CS_{Min} \tag{37}$$

4.2.2 Linearization of the optimization problem

The two decision variables in this study are T_i and ψ_{1xn}. These two decision variables are multiplied by each other in both the objective function and the constraints (Equations (30), (33),(34), and (35)), thereby making this model non-linear. Having a linear optimization model is one of the main requirements for the reformulation of a model to a solvable two-stage SP problem. The M technique is implemented to linearize the model (Asghari et al., 2022). This technique is helpful when a binary decision variable is multiplied by another decision variable (either discrete or continuous). The M technique eliminates the terms of binary decision variables multiplied by the other decision variables in the objective function and in all the restrictions. Instead, it adds the restriction $T_k \leq \mathrm{M}\psi_k$ that confines the non-binary decision variable by multiplying the binary decision variable by a large number (usually addressed as "M"). This technique is explained more explicitly while reformulating the original non-linear model.

Implementing this technique converts the objective function and the constraints of our model as follows:

$$\min\left\{\sum_{i=1}^{n-1}((T_i - D_i)(\theta_i\gamma_r) + T_i(C_E + C_O) + \psi_i C)\right\} \tag{38}$$

$$\begin{gathered} s.t. \quad \sum_{i=1}^{n-1} T_i C_F + (100 - B) \geq C_D L \\ \sum_{i=1}^{k} T_{i-1} C_F + (100 - B) \geq C_D x_k; \quad \forall k \\ \sum_{i=1}^{k} T_{i-1} C_F \leq C_D x_k; \quad \forall k \\ \sum_{i=1}^{n-1} \psi_i \geq CS_{Min} \end{gathered} \tag{39}$$

$$(\mathrm{D})_k \leq T_k \leq 6$$

$$T_k \leq \mathrm{M}\psi_k$$

$$\psi_{1xn} \in [0,1], T_k \geq 0, and\ T_k \in R^+$$

where Equation (38) is the reformulated objective function and Equation (39) is the reformulated constraints of the original deterministic model. The outcome of applying the M technique is that the ψ_i is eliminated from the first term of the Equation (38) (the objective function) and from all the constraints (Equation (39)) in which ψ_k and T_k were multiplied with each other. Instead, a new constraint is added to confine T_k ($T_k \leq \mathrm{M}\psi_k$). By adding the new restriction, when a charging station is not installed at a given suitable stop (i.e. $\psi_k = 0$), the associated charging duration is eliminated (i.e., $T_k = 0$), but when $\psi_k = 1$, T_k can take any non-negative value bounded by the value of M (usually a large number considering the nature of the model). In this problem, T_k is bounded by H (to avoid bus bunching at the charging stations); hence, we set the value of M equal to H. Therefore, the model is linearized without compromising the objective and constraints of our initial optimization model. Also, by setting the "M" equal to "H", the earlier constraint on T_k ($T_k \leq$ H) can be eliminated. It is worth mentioning that implementing the M technique does not change the nature of the decision variables and the model is still a mixed-integer program as the location of the charging stations (ψ_i) is a binary decision variable while the charging durations are continuous variables.

4.2.3 Extending the model for two-way routes

The first model was developed for the en-route charging stations' location and charging time for one BEB route direction. The assumption was that the optimal location of the charging stations for each direction, away and return, would be obtained independently while the BEBs leave the termini fully charged. It can be perfect for a downtown bus route that does not have common stops/sections in the away and return directions of a route; the BEBs are planned to leave the turning terminal fully charged.

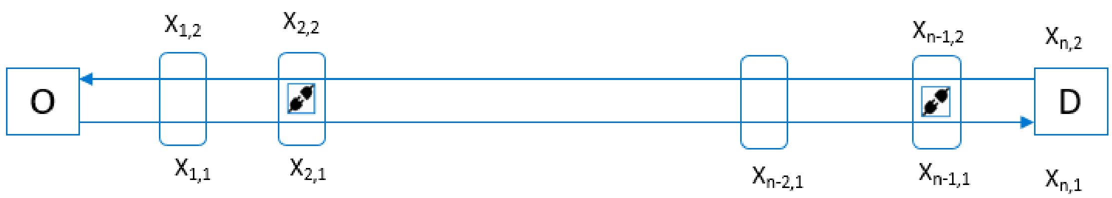

Figure 5 – Bus routes with completely common stops and route sections

However, the one-way model is not financially optimal for two-way routes that share sections and stops in opposite directions, such as some “bus rapid transit” (BRT) routes (Deng and Nelson, 2011). Another challenge is that the previous model may not provide the best arrangement for the charging stations in the case that BEBs do not leave the turning station fully charged. In addition, the problem becomes more complicated as the stop location for a one-directional route seems to meet its demand profile, which is often reversed for the route in the other direction. Thus, to be optimized, the locations of the charging stations should simultaneously meet the fluctuation of the demand in both directions while optimizing the costs. Therefore, as a special use of the one-way mode, the original model is extended and modified for a two-way bus route such as a BRT system sharing bus stops. Hence, all route sections and stops are assumed to be fully common in both directions, as shown in Figure 5. Locating the optimal location of the charging stations and the charging duration of a two-way bus route is a particular case of a one-way model developed in this study. Both the away and return routes share “n” bus stops, and all are suitable for locating battery charging stations. As shown in Figure 5, the stops are shown as $x_{i,j}$ where “i” is the stop number (including the destination) and "j" is the index for direction (j=1 for away and j=2 for return), and $x_{i,j}$ represents the relative distance of the i^{th} stop in the "j" direction from the origin; $T_{i,j}$ represents the assigned charging time at the charging station located at Ihe i^{th} stop in the "j" direction. Each bus stop may have a charging facility for the BEBs serving both directions or neither. However, the charging time of a charging station for opposite sides is not required to be equal and may differ ($T_{i,1} \neq= T_{i,2}$). For example, the BEBs in one direction may charge for two minutes while the BEBs in the return direction in the same charging station may charge for 3 minutes. It is assumed that the BEBs leave the origin stop fully charged, while the charge status for the BEBs leaving the turning point (destination stop D) is relaxed for the optimization model to be decided. In other words, the BEBs may or may not leave terminal D fully charged.

The objective function and the constraints of the deterministic model explained in the previous section (Equations (38) and (39)) are thus modified first to formulate a deterministic optimization model for the location and charging time of the charging stations in a two-way bus route.

$$\min\left\{\sum_{i=1}^{n}\left(\psi_i C + \sum_{j=1}^{2}((T_{i,j} - D_{i,j})(\theta_{i,j}\gamma_r + \ C_O) + T_{i,j}(C_E))\right)\right\} \tag{40}$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^{n}\left(T_{i,1}C_F\right) + \sum_{i=1}^{n}\left(T_{i,2}C_F\right) + (100 - B) \geq C_D(2L) \\ & \sum_{i=1}^{k} T_{i-1,1}C_F + (100 - B) \geq C_D x_{k,1}; \quad \forall k \end{aligned} \tag{41}$$

$$\sum_{i=1}^{n} T_{i-1,1}C_F + \sum_{i=1}^{k}(T_{n+1-i,2}C_F) + (100 - B) \geq C_D(L + x_{n-k,2}); \quad \forall k$$

$$\sum_{i=1}^{k} T_{i-1,1}C_F \leq C_D x_{k,1}; \quad \forall k$$

$$\sum_{i=1}^{n} T_{i-1,1}C_F + \sum_{i=1}^{k}(T_{n+1-i,2}C_F) \leq C_D(L + x_{n-k,2}); \quad \forall k$$

$$T_{n,1} = T_{n,2}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{2} \psi_{ij} \geq CS_{Min}$$

$$T_{i,j} \leq \mathrm{M}\psi_i; \ \forall i,j$$

$$\psi_{1xn} \in [0,1], T_k \geq 0, and\ T_k \in R^+$$

The objective function is updated by an additional index of “j” for considering the charging times ($T_{i,j}$) and demand ($\theta_{i,j}$) for both directions separately. In the constraints of the extension of the model (see Equation (45)), the second and third constraints of the original deterministic model (developed for the one-way route) are written twice (once for the away direction and once for the return direction). In addition, there is a new constraint ($T_{n,1} = T_{n,2}$) that defines the turning terminal for the model. So, half of the assigned charge at the turning stop can be assigned to one direction as $T_{n,1}$ and the other half is assigned to the other direction as $T_{n,2}$. In other words, when the model assigns $T_{n,1} = T_{n,2} = 3\ min$, a total of six minutes of charging time is assigned at the turning stop.

4.3 Results: Applying the deterministic models on two bus routes in Calgary (Route 3 and Route 303)

The developed models are applied and analyzed on two different routes in Calgary. The first is a one-way route, Route 3 (see Figure 6), and the second is a two-way route, Route 303 (see Figure 7). The main difference between the mentioned routes is in their layout. While Route 3 is more of a radial route that passes through the central business district (CBD) with a high directional ridership profile, Route 303 is an East-West BRT line that mainly covers the inner city (avoiding downtown/CBD) and has a relatively more uniform ridership profile without a pronounced directional peak (see Figure 8 and Figure 9).

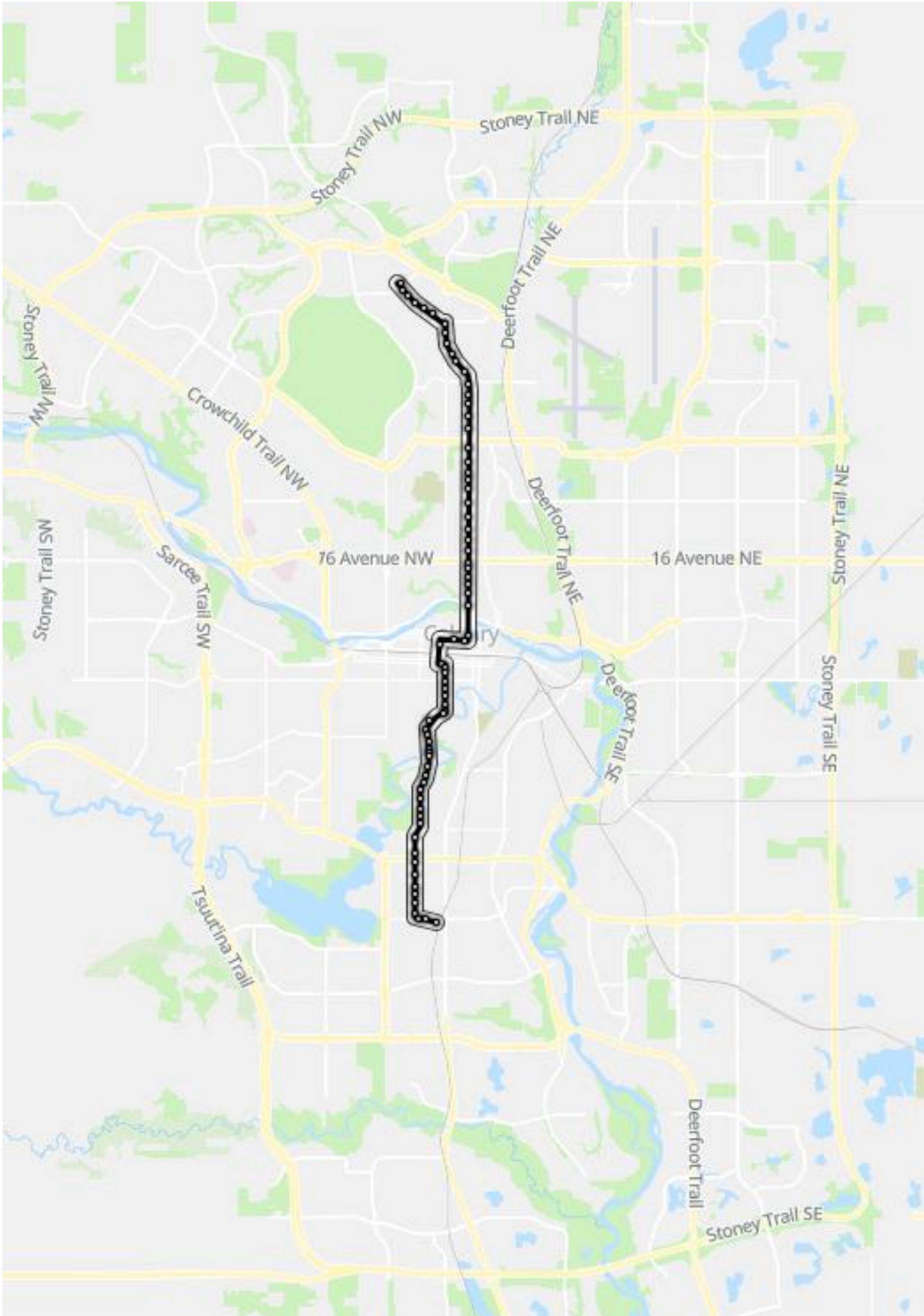

Figure 6 – Route 3: Sandstone/Elbow Drive SW (Moovit website – Calgary Transit Map)

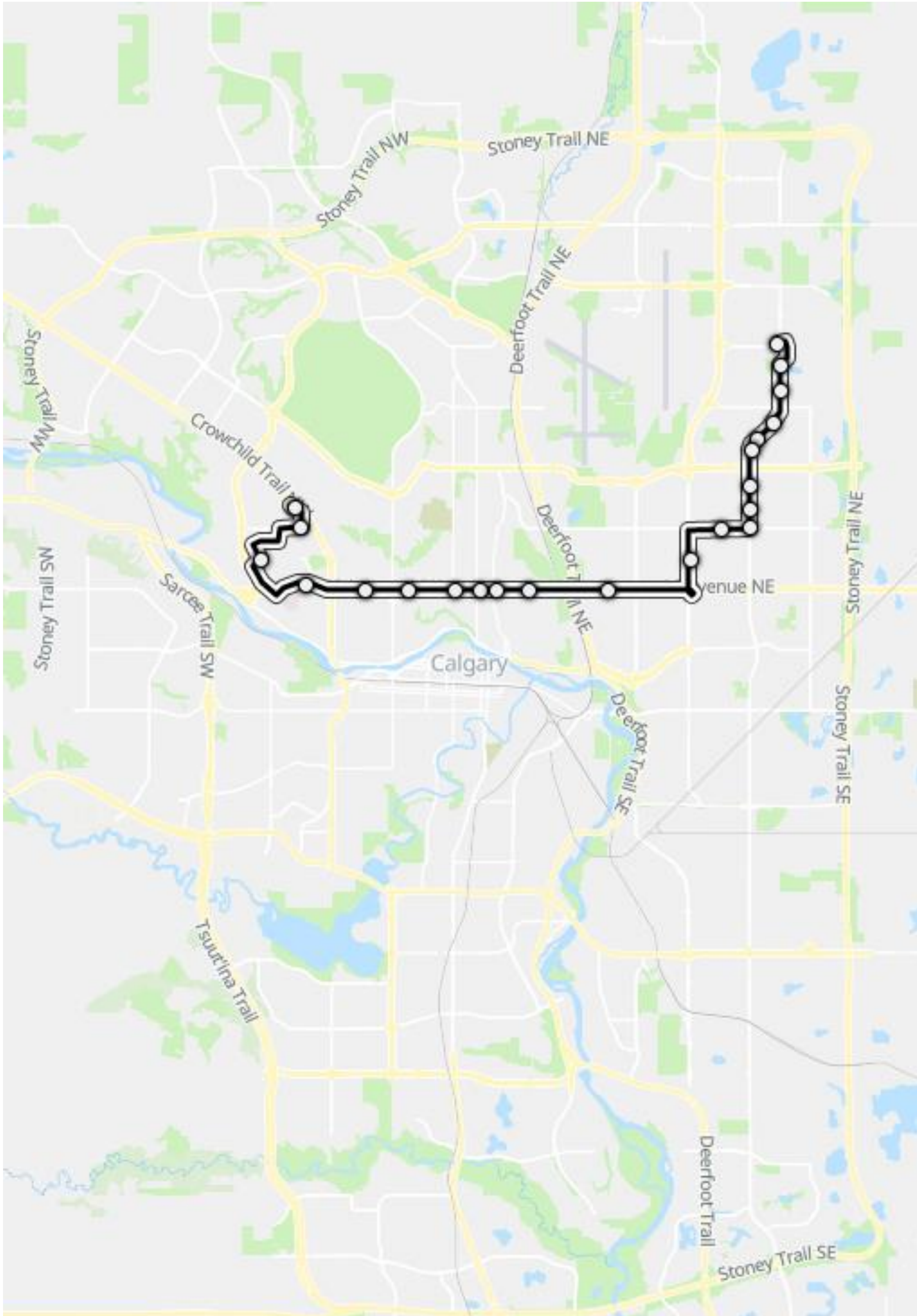

Figure 7 – Route 303: Max Orange Saddletowne (Moovit website – Calgary Transit Map)

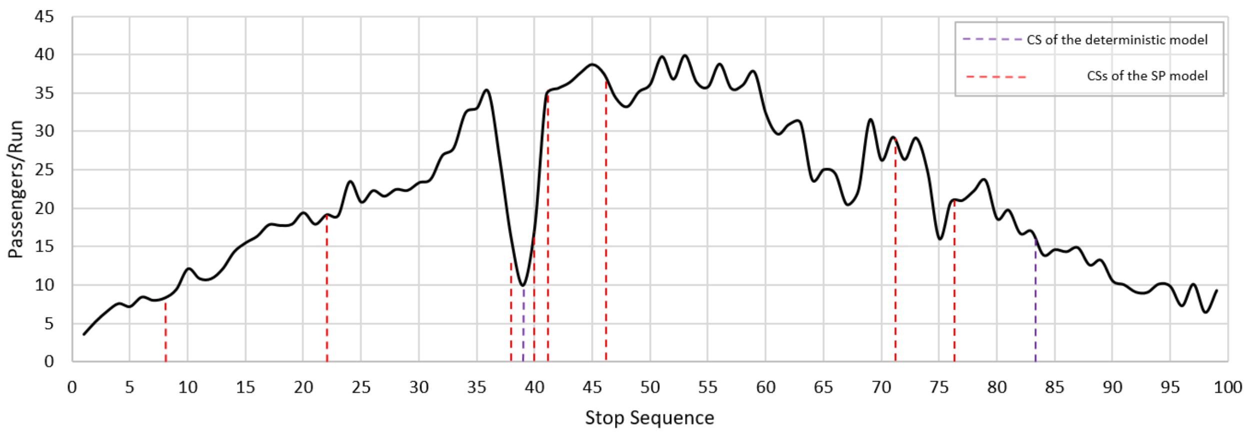

Figure 8 – Through passenger demand, Route 3 in Calgary

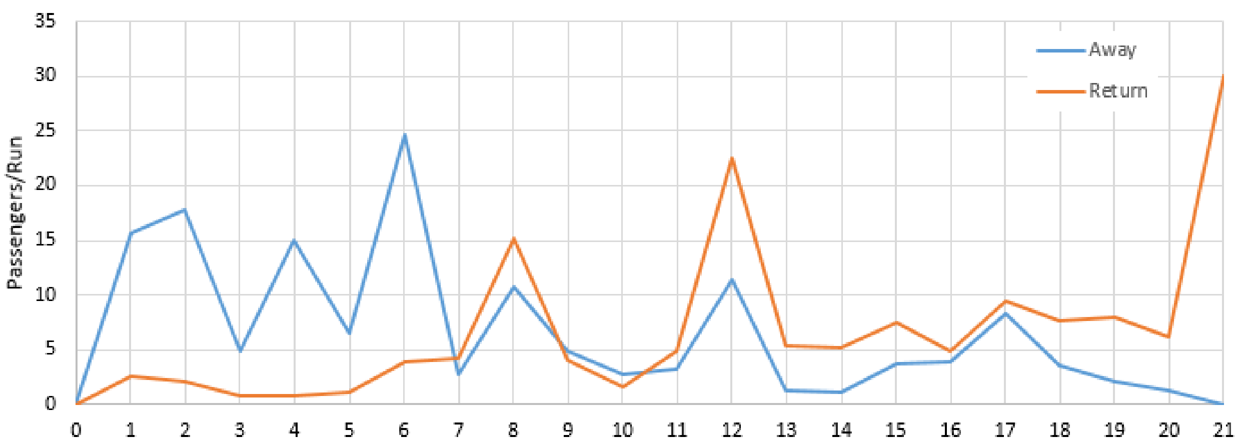

Figure 9 – Through-passenger demand (away and return), Route 303 in Calgary

4.3.1 One-way bus route: Route 3 in Calgary (deterministic model)

The developed linear deterministic model for a one-way bus route was first applied to Route 3. As mentioned earlier, Route 3 is a long radial bus route of high demand that connects the north and south of Calgary through its CBD. Thus, during morning peak hours, the first part of the route (i.e., origin station to CBD) has a high passenger load, which decreases once the bus exits downtown (see Figure 8). Route 3 has one hundred bus stops (including the terminals). Automated passenger count (APC) data of all the stops of Route 3 were obtained from Calgary Transit and used to supply the models with passenger demand and travel times between the bus stops. Similar data is also used to obtain the average number of boarding and alighting passengers for each station of the route to calculate the dwell time (Rahman, 2011).

For the deterministic model optimization, the 95th percentile of passengers' load during the evening peak hours is used in the optimization model to accommodate the case of extreme load where the battery charge drains faster. The optimum deterministic model solution for the location of the charging stations and the charging time is recorded in Table 2. The provided solution states that the route needs three charging stations located at stops #38 (a central transit station downtown), #83, and #91 (outside downtown) with a total charging time of 10.44 minutes (i.e., 4.94, 3.06, and 2.43 minutes, respectively). The model locates the first charging station at stop #38, where many passengers alight, leaving the bus with a low through-passenger load. This allocation choice is directed by the objective function formulation that assures that fewer passengers are affected by the extra wait-time penalty resulting from battery charging activity and getting the advantage of the higher dwell time value.

Similarly, as the BEB needs to charge in the latter half of the route, the algorithm chooses stops #83 and #91, which also have a relatively low through-passengers load to stop for a lower charging duration so the bus can reach the nearby final terminus with the minimum 20% battery charging level requirement. The ridership of buses in stations #38, #83, and #91 are shown using purple dashed lines in Figure 8. The resulting total extra cost per trip due to the electrification of the route is $20.36/trip.

Table 2 - Result of the deterministic optimization model for the one-way route

Stop Number	En-route Charging Time (min)
Stop #38	4.94
Stop #83	3.06
Stop #91	2.43
Total en-route charging time per run (min)	**10.44**
Total cost per run ($)	**$ 20.36**

4.3.2 Two-way route (BRT route) in Calgary (Route 303) (deterministic model)

Route 303, known as "Max Orange Brentwood", is a BRT route that has 23 stops that connect Saddletowne Light Rail Transit (LRT) Station (located in the NE of Calgary) to the Brentwood LRT Station (located in the NW of Calgary). The City of Calgary provides the ridership data of this route with the exact same specifications. This route is around 55 km (27.6 km in each direction). The main specification of this BRT route is that it shares the route and stops in both the away and return directions. To deploy this route with BEBs, it is assumed that the BEBs leave the start terminal fully charged.

Table 3 - Results of the deterministic optimization model for the two-way bus route

Stop Number		En-route Charging Time (min)
Stop #3	Away	2.57
Stop #3	Return	3.48
Stop #5	Away	2.10
Stop #5	Return	3.48
Stop #15	Away	4.14
Stop #15	Return	3.59
Stop #21	Destination Stop (Turning Terminal)	11.31
Total en-route charging time (min)		**30.67**
Total model cost ($/each round-trip)		**$52.74**

Table 3 shows the outcome of the deterministic model under the average daily demand/ridership profile. The demand of the two-way route is chosen differently from the one-way route since the peak demand in the opposite directions does not coincide. In other words, an extreme case scenario may not occur during the peak hours in either direction (see Figure 9). Interestingly, although the model does not force the BEB to be charged at the terminal station, the algorithm chooses to do so to exploit the presence of a low-demand profile at the terminal where most passengers have alighted. Based on Table 3 and the assumption of a charging station in the origin terminal, the route needs three en-route charging stations and one in the turning/destination terminal. The resulting total en-route charging time is 30.67 minutes, with almost one-third assigned to the charging station in the turning terminal. The extra cost of electrification of this route for each round-trip is $52.74. Looking at the ridership profile of Route 303 (Figure 9), it can be inferred that the model chooses the stops where at least the ridership of one of the away or return directions is very low compared to the neighbor stops. In other words, the summation of the ridership in the away and return directions can be considered to approximate the location of the charging stations.

CHAPTER 5: LOCATING OPTI[2]MUM EN-ROUTE CHARGING STATIONS (STOCHASTIC MODELS)

5.1 Introduction

The deterministic model developed for locating the optimal charging stations is not able to address the uncertainty in the demand and the performance of the batteries sourced by the weather conditions. Weather and road conditions (i.e., snowy roads) are key contributing factors that disrupt public transit ridership and operation (Hofmann and O'Mahony, 2005). Additionally, extreme environment and weather condition results in the change in driving and transit usage behavior (Ansari Esfeh, 2020, 2022). Among weather parameters such as wind, snow, or rain, the temperature is the most influential factor affecting the ridership of public transit (Shih and Nicholls, 2011; Diab et al., 2020). De Palma and Rochat (1999) and Cools et al. (2010) confirmed that extreme weather conditions might induce a change in departure time and transit routes, shift of the transport mode, or trip cancelation. In more recent research, Martin (2018) and Ngo (2019) found that extreme weather conditions cause an increase in transit ridership. In addition to the induced fluctuation of transit ridership, weather and ambient temperature significantly affect transit travel time and BEBs operation. While there are different types of available batteries, the

[2] Esmaeilnejad, S., Kattan, L., and Wirasinghe, C. Optimal Charging Station Locations and Durations for a Transit Route with Battery-Electric Buses: A Two-Stage Stochastic Programming Approach with Consideration of Weather Conditions. Transportation Research Part C: Emerging Technologies. Volume 156, November 2023, 104327: https://doi.org/10.1016/j.trc.2023.104327.

electric vehicle industry's current focus is on using lithium battery technologies (Sun, 2020). At cold temperatures, the energy pack in the battery decreases due to an increase in internal resistance (Chan and Sutanto, 2000). A change in the temperature from 20℃ to -20℃ decreases the range of a BEB to 80% of the nominal capacity and to 30% for a temperature around -40℃ (Wang et al., 2020, Xie et al. 2021). BEBs' driving range falls with increased cabin heating, ventilation, and air conditioning (HVAC). Another concern related to lithium batteries in cold temperatures is the charging rate/current, which is restricted to avoid adverse impacts on the battery life cycle (Budde-Meiwes et al., 2013). Reyes et al. (2016) found a reverse relationship between the ambient temperature and the range of the BEBs. Their results depicted that the range decreases approximately linearly to 30% of the nominal range when the temperature decreases from 30℃ to -30℃. In other words, the battery drain rate almost increases by 300% with the decrease in the temperature within the mentioned range. Thus, the charging frequencies and durations are expected to increase significantly in cold ambient temperatures. Failure to capture the stochasticity of battery drainage rate during cold weather conditions might lead to an interruption of transit service, thus severely compromising the reliability of BEB operation. This chapter addresses the mentioned challenges by incorporating the weather stochasticity of the ridership and the associated BEB discharging behavior into the BEB charging location and duration problem formulation. The main contribution of this work is thus simultaneously determining charging station locations and their corresponding charging schedule considering the weather-induced uncertainties of battery depletion rate and ridership profile using a passenger-centric optimization approach.

A two-stage stochastic optimization problem is formulated to simultaneously locate en-route charging stations and their charging duration while minimizing the additional charging stations' capital and operating costs and the induced passenger in-vehicle delay costs. In the first stage, the optimal location of charging stations is decided, followed, in the second stage, by their corresponding charging duration. Unlike a deterministic approach that takes a point estimate of the model parameters, the stochastic optimization approach, as developed in this paper, assigns a probabilistic distribution to the battery depletion rate and ridership. Thus, many candidate scenarios created by varying these parameters are examined, and the charging locations and schedules are optimized accordingly. Therefore, instead of an average number of charging stations and durations, stochastic programming has the potential to model several candidate future scenarios to help transit planners make more informed decisions to improve the reliability of BEB operation under cold weather conditions.

The model's output can be used as a decision support tool to identify the infrastructure needed for a reliable transit service under an extremely cold weather scenario. Today, municipalities are still learning what are the needs for transit electrification. Chicago Transit is facing severe challenges in running its BEBs when the temperature drops, stating that "cold weather is the biggest Chicago Transit challenge" (City News, 2023). This chapter considers two transit routes with en-route and terminal charging stations as numerical examples. The first route is a regular one-way bus route, and the second is a two-way bus route sharing stops in both directions (similar to those used in the previous chapter). Comparing the results with the deterministic problem formulation indicates the importance of incorporating the stochasticity of the weather-induced demand and supply variables in the problem formulation.

Figure 10 illustrates a high-level schematic of the modeling approach implemented in this study (in both Chapters 3 and 4). A non-linear deterministic optimization model is developed first to formulate the long-term location of en-route charging stations and the corresponding charging duration for one direction of a regular bus route converting to BEB use (see section 4.2.1). In the next step, the non-linear deterministic problem is linearized using the "M technique" (Asghari et al., 2022) (see section 4.2.2). In addition, the linearized deterministic model for the one-way bus route is extended to a deterministic model for the two-way bus route (see section 4.2.3). Linearizing the deterministic problem is a requirement to reformulate to a two-stage SP model that optimizes the location and charging time of the en-route charging stations while addressing the uncertainties in both the ridership demand and charging drain rate of batteries. This chapter modifies and extends the deterministic models for the one-way and two-way bus routes to an SP model.

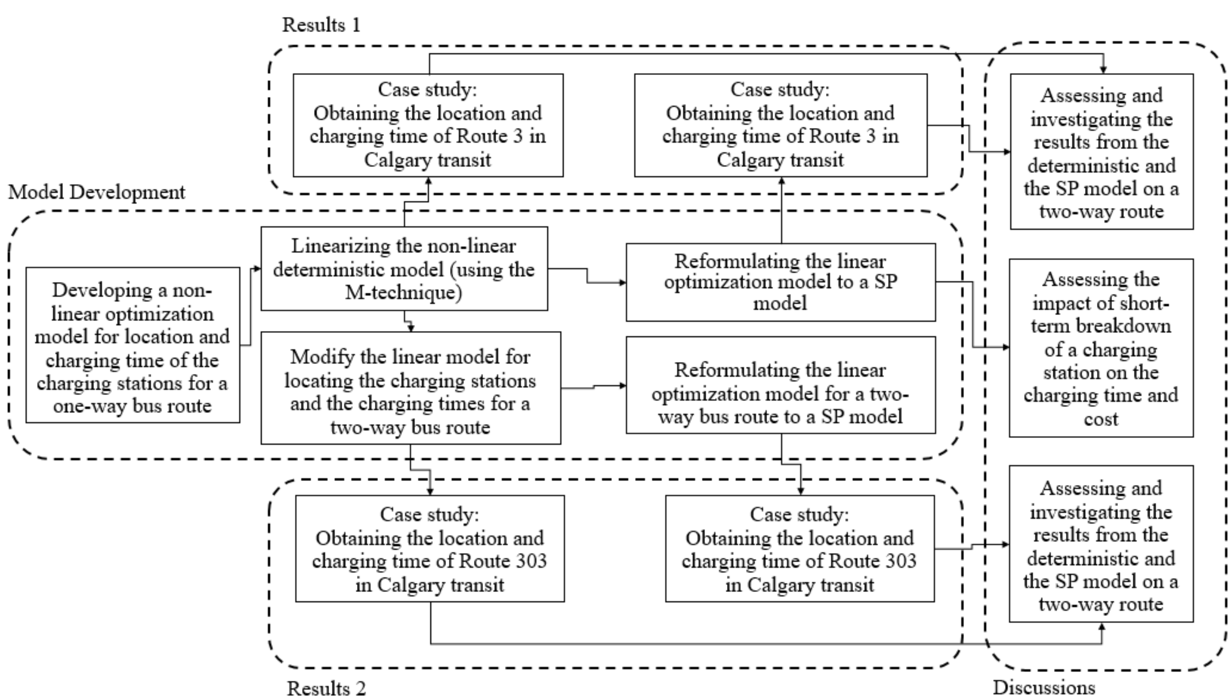

Figure 10 – High–level schematic structure of this chapter

For numerical analysis, the SP models are applied to bus routes in the City of Calgary (the similar routes in the previous chapter). Further, the impact of the breakdown of the charging stations using the SP model is assessed. The results from the optimization models (deterministic and SP model) are used to evaluate and assess the performance and value of the proposed two-stage SP model.

5.2 Two-stage stochastic programming (SP) model

In modeling many optimization problems, an inevitable uncertainty level affects the model's outcome. In deterministic models, the mean or expected value or worst-case scenario can be used for the parameter(s) source of uncertainty. In our model, transit ridership (θ_i), and battery drain rate (C_D) are the parameters/inputs that bring a level of uncertainty caused by weather conditions and thus may change the optimum solution of the model. The average annual value of the mentioned inputs was used in the deterministic model. In cases where the uncertainty of the parameters is independent of the decision variables, Stochastic Programming (SP) is an appropriate mathematical framework (Birge and Louveaux, 2011).

The general form of the objective function (Equation (42)) and constraints (Equation (43)) of a two-stage SP is as follows:

$$\min Z = c^T x + E_{\varepsilon}[\min q(\omega)^T y(\omega)] \tag{42}$$

$$\begin{aligned} s.t. \qquad & Ax = b \\ & T(\omega)x + Wy(\omega) = h(\omega), \\ & x \geq 0, y(\omega) \geq 0 \end{aligned} \tag{43}$$

In the general format of the SP, the first-stage decision variables are represented by vector x; the second-stage decision variables are represented by the vector y(ω), where (ω) is an indicator, a model input, or a parameter. The parameter ω can also reflect uncertainty in the decision variable. The parameters c and q(ω) are constants with known values obtained from the first and the second stage, respectively. "A" and "b" are two known matrices/vectors for defining the linear and deterministic constraints of the SP, including the first-stage decision variable x. T(ω), W, and h(ω) are the parameters for the second group of constraints, including the first and second-stage variables. In the general form, the constraint equations are equality constraints. In addition, the first and second-stage variables are assumed to be non-negative (Shapiro et al., 2021).

5.2.1 Developing the two-stage stochastic model: one-way route

To develop the SP model, we need to reformulate the objective function (i.e., Equation (38)) and all the constraints of the deterministic model (i.e., Equation (39)) so as to follow the two-stage SP format. In the SP model, the vector ψ is the binary variable that addresses the location of the charging stations in the first stage (long-term variable), and the vector T states the corresponding charging duration (in min) of the second-stage variable of the model (medium-term operation variable). As vector T is a second-stage variable, it is used as $\mathrm{T}(\omega)$ in the SP model. Transit ridership (θ_i), and battery drain rate (C_D) are the inputs that bring uncertainty, which are addressed as $\theta_i(\omega)$ and $C_D(\omega)$ in the SP model, respectively. The rest of the variables (C, γ_r, L, C_F, x_k, and CS_{Min}) are fixed values and do not change compared to the deterministic model. In reformulating the deterministic model to the SP model, all the constraints should be changed to equality constraints. Therefore, we use the slack variables (S_i) to change the inequality sign of the constraints to equal signs. It is important to mention that in our SP model, the slack variables are always positive and do not affect the model's outcome. Therefore, the objective function and the constraints of the linear deterministic model (Equation (38) and (39)) can be rewritten as follows to be reformulated to a standard SP model (see Equation (44) and Equation (45)):

$$\min \mathrm{Z} = \mathrm{C}^T\psi_i + E_\varepsilon[\min(T(\omega) - D(\omega))(\theta_i(\omega)^T\gamma_r) + T(\omega)(C_E + C_O)] \tag{44}$$

$$s.t. \quad C_F\sum_{i=1}^{n-1} T_i(\omega) = C_D(\omega)L - (100 - B) + S_1$$

$$\sum_{i=1}^{k} C_F T_{i-1}(\omega) = C_D(\omega)x_k - (100 - B) + S_2; \quad \forall k$$

$$\sum_{i=1}^{k} C_F T_{i-1}(\omega) + S_3 = C_D(\omega)x_k\,; \;\; \forall k \tag{45}$$

$$\sum_{i=1}^{n-1} \psi_i = CS_{Min} + S_4$$

$$T_k + S_5 = \mathrm{M}\psi_k$$

$$\psi_{1xn} \in [0,1], T_k \geq 0, S_i \geq 0$$

In modeling a problem in SP format, a decision tree and the probability of each branch are required. As illustrated in Figure 13, the decision tree of our model has six branches. Schematically, Figure 11 illustrates the structure and steps of developing and solving the two-stage SP model of this study. To shed more light, the SP model requires two sets of fixed and uncertain inputs. The SP model uses a decision tree based on the probability of occurrence of the branches based on the ambient temperature (each branch is considered as a scenario “s”). As the solution, the model delivers two sets of outputs, the first stage and the second stage. The first stage output is independent of the scenario (location of the charging stations, which has a long-term nature), and the second stage output is dependent on the scenario (charging time of the charging stations, which is a function of the ambient weather condition and may change daily).

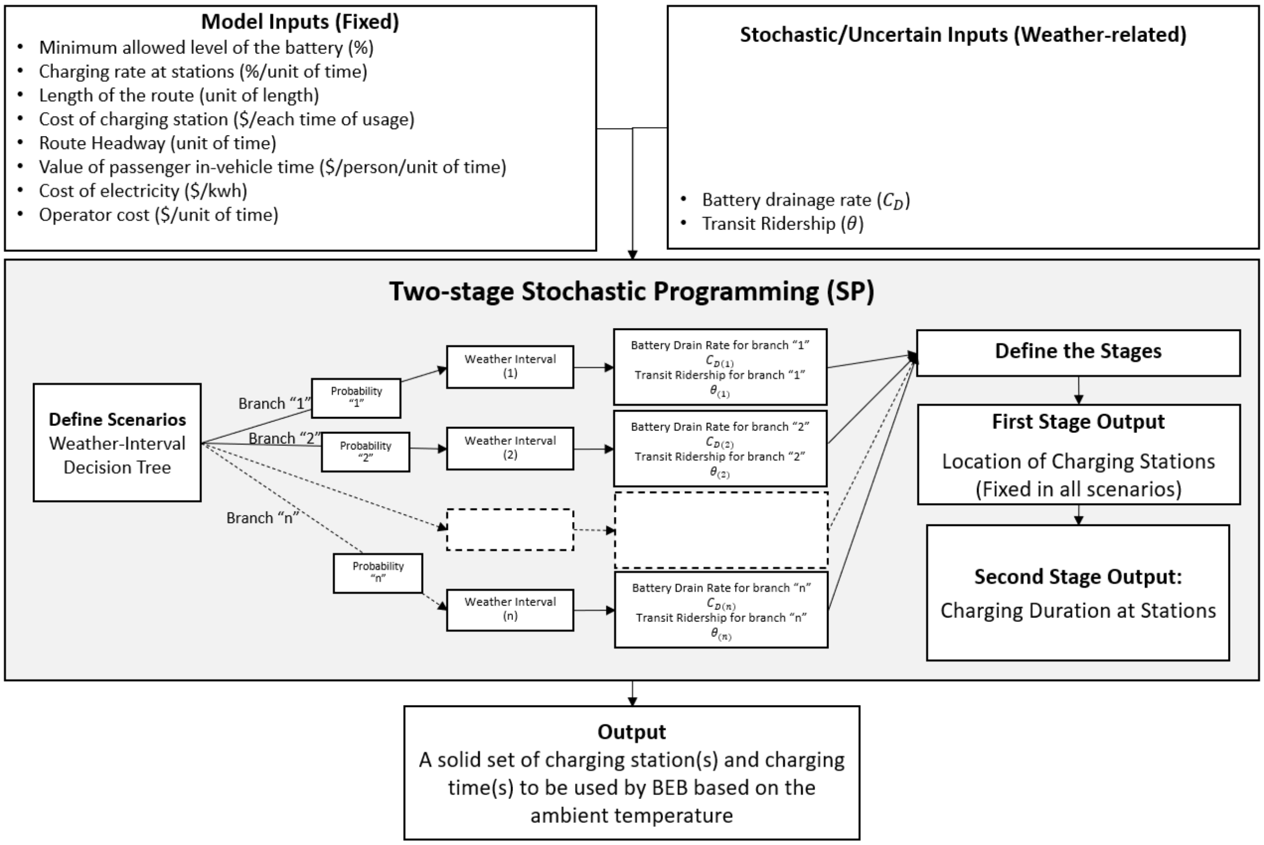

Figure 11 – Steps of developing the stochastic optimization model

To obtain the probability of occurrence of each branch in percentage, the ratio of the days that the average data belongs to each interval is obtained (see Table 4).

Table 4 - Probability of temperature intervals in the City of Calgary

Temperature Intervals (°C)	-30 to -20	-20 to -10	-10 to 0	0 to 10	10 to 20	20 to 30
The probability of temperature occurring within the specified range	3.56%	7.95%	21.64%	31.51%	33.42%	1.92%
Battery Drain Rate (C_D) (%/km)	10	8.4	6.8	5.2	3.6	2
% Change in Transit Ridership	5%	3%	1%	-1%	-3%	-5%

The historical weather data of Calgary (Figure 12), acquired from the official website of the Government of Canada, is used to obtain the range and the probability of the daily temperature (Climate Weather Canada). This probability is used later as the occurrence probability in the branches of the decision tree in the SP model. The temperature intervals are shown at the end of the decision tree branches, and the probability of occurrence of that temperature is written on the branch line (see Figure 13). For example, based on the weather data, the average temperature in Calgary was between -30°C and +30°C. This sixty-degree interval (-30 to 30) is divided into six groups.

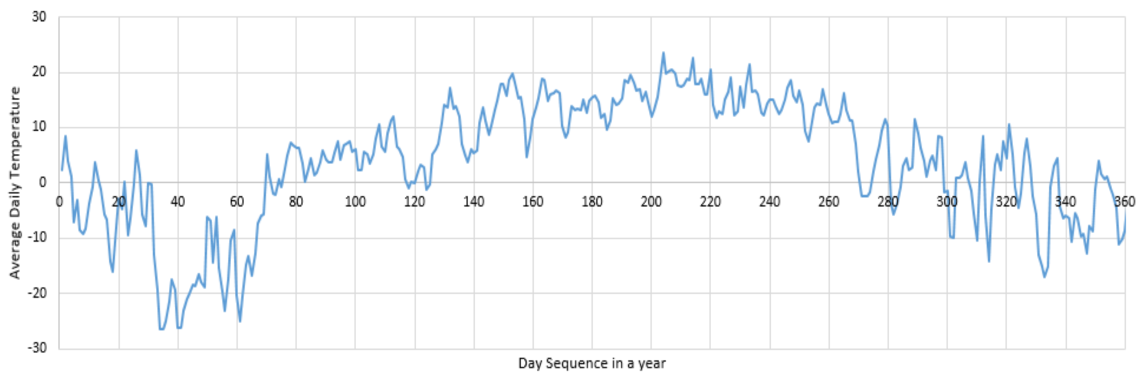

Figure 12 – Historical data of average daily temperature in the City of Calgary

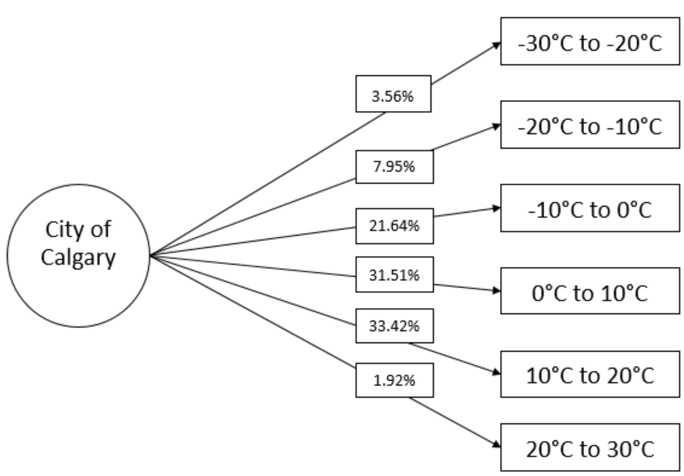

Figure 13 – Decision tree of the probability of daily weather temperature in Calgary

Amongst the 365 data for the average daily temperature of Calgary, in 122 days, it was between 10°C to 20°C, which means a 33.42% probability for the fifth branch of the decision tree (see Figure 13).

In our problem, both transit ridership $\theta_i(\omega)$ and battery drain rate $C_D(\omega)$ are uncertain variables that are a function of the ambient temperature and fluctuate with the changes in weather conditions. The actual transit ridership of 2019 of the two transit routes in Calgary, provided by the City of Calgary, is used.

As discussed earlier, the temperature impacts the battery's performance of the BEBs. In this study, this impact is addressed via the drain rate "C_D" of the batteries using linear interpolation. Based on the work done in the literature by Ngo (2019), the impact of the temperature on ridership is estimated as follows: For each 10°C decrease in the ambient temperature, the transit ridership will increase by approximately 2%. Therefore, linear interpolation approximates the transit ridership in each weather interval (Lindgren et al., 2016). The calculation is similar to what is performed to obtain the drain rate "C_D" in different weather conditions. For example, for the temperature of -30°C to -20°C, the demand is expected to be 5% more than the ridership used in the deterministic model. Due to a lack of experimental data for the battery performance in Calgary, the reported data from the cities with similar weather conditions are used. The C_D is obtained to be 10%/km for the temperatures in the first group (-30°C to -20°C) and 2%/km in the sixth group (20°C to 30°C) based on the 6% drain rate used in the deterministic model. The battery drain rate is linearly interpolated for the other groups of temperatures.

5.2.2 *Developing the two-stage stochastic mode: two-way route*

Using the same method explained in the previous section, in the second step, the modified/linearized deterministic model (Equations (40) and (41)) is reformulated to an SP model (see Equations (46) and (47)) to incorporate the stochasticity of the ridership demand and battery draining behavior as affected by ambient temperature.

$$\min \mathrm{Z} = \mathrm{C}^T\psi_i + E_\varepsilon[\min\,(\mathrm{T}(\omega) - \mathrm{D}(\omega))(\theta_{i,j}(\omega)^T\gamma_r + C_O) + \mathrm{T}(\omega)(C_E)] \tag{46}$$

$$\begin{aligned}
s.t.\quad & C_F\left(\sum_{i=1}^{n} T_{i,1}(\omega) + \sum_{i=1}^{n} T_{i,2}(\omega)\right) = C_D(\omega)L - (100 - B) + S_1 \\
& \sum_{i=1}^{k} C_F T_{i-1,1}(\omega) = C_D(\omega)x_k - (100 - B) + S_2;\ \forall k \\
& C_F\left(\sum_{i=1}^{n} T_{i-1,1}(\omega) + \sum_{i=1}^{k} T_{n+1-i,2}(\omega)\right) = C_D(\omega)x_k - (100 - B) + S_3;\ \forall k \\
& \sum_{i=1}^{k} C_F T_{i-1,1}(\omega) + S_4 = C_D(\omega)x_k;\quad \forall k \\
& C_F\left(\sum_{i=1}^{n} T_{i-1,1}(\omega) + \sum_{i=1}^{k} T_{n+1-i,2}(\omega)\right) + S_5 = C_D(\omega)x_k;\ \forall k \\
& T_{n,1} = T_{n,2} \\
& T_{i,j} + S_6 = \mathrm{M}\psi_i\ \forall i,j; \\
& \sum_{i=1}^{n}\sum_{j=1}^{2} \psi_{ij} = CS_{Min} + S_7 \\
& \psi_{1xn} \in [0,1], T_k \geq 0, S_i \geq 0
\end{aligned} \tag{47}$$

While the developed SP models are linear and discrete, they have a decision tree for addressing the uncertainty (see Figure 13). Therefore, Progressive Hedging (PH) is chosen as the solving algorithm or method based on scenario decomposition renowned for solving multistage stochastic optimization problems (Bareilles et al., 2020). Python's PySP optimization repository (Hart et al., 2011) solves PH algorithms for our SP models. In PySP, we get the advantage of Pyomo, an algebraic modeling language (AML), an advanced environment for modeling and solving deterministic mathematical problems to specify our deterministic base and the scenario tree models.

5.3 Results: Applying the two-stage SP models on two bus routes (Route 3 and Route 303)

Thus, since the above-mentioned results do not consider the impact of weather conditions, the optimum solution to the same problem is conducted with the two-stage SP. The solution using two-stage SP is stated in Table 5. The significant difference is in the input of the models where the impact of the ambient temperature (weather condition) on the battery drain rate (C_D) and the ridership demand is incorporated in the SP model for six different groups with different temperature ranges.

5.3.1 One-way route in Calgary (Route 3)

The number and location of the charging stations (first-stage decision variables) are optimized and obtained for all weather conditions, and the results are reported in Table 5. Based on the solution results, the route needs five charging stations at stops #22, #40, #42, #71, and #76. The stops for both models, deterministic and SP model, are brought in Figure 14. The ridership in the mentioned stations is shown with red dashed lines in Figure 8.

Table 5 - Result of the SP model with no restriction

	En-route Charging Time (min)					
Stop Number	-30°C to -20°C	-20°C to -10°C	-10°C to 0°C	0°C to 10°C	10°C to 20°C	20°C to 30°C
Stop #22	3.23	2.21	2.01	1	1	0
Stop #40	5.37	4.31	3.38	2.02	1.07	0
Stop #42	4.99	2.3	2.5	2.01	1	0
Stop #71	4.09	3.01	2.01	1	0	0
Stop #76	5.06	5.77	3.01	1.95	0	0
Total en-route charging time (min)	22.74	17.6	12.91	7.98	3.07	0
First-stage cost ($)	3.65	3.65	3.65	3.65	3.65	3.65
Second-stage cost ($)	52.74	38.27	26.90	16.27	6.87	0
Total model cost ($)	56.40	41.93	30.56	19.93	10.53	3.66
Expected total charging time (min)	**8.53 (min)**					
Expected total cost ($)/one-way trip	**$21.82**					

The optimization model's choice of the optimum charging locations at stops reflects the bus stations with minimum ridership in their proximity (local minimums) or the stops that are close to the local minimums and have a higher dwell time value when an extra charge is needed. It is noticeable that the chosen stations are either local minimums or neighbors to the local minimums in the demand trajectory. In the solution obtained by SP, the assigned charging time is changed based on the ambient weather temperature. While the results show that no charging is required on a typical summer day (i.e., the temperature in the range of 20°C to 30°C), if the day average temperature is seven degrees, the BEBs will be charged in all the suggested stations. With the temperature dropping below 10°C, the algorithm consistently chooses all the stops while increasing the charging time.

The expected total charging time in Table 5 is obtained by summing up the product of the charging time of each column and its probability. The expected total charging time can be interpreted as the average charging time during the peak hours in a year. The expected total cost reflects the same concept as the expected total charging time. To incorporate the impact of the fluctuation in the weather conditions, the optimum solution obtained by SP requires two extra charging stations compared to the deterministic model. However, the expected total charging time using SP is 8.53 minutes, which is 1.91 min less than the total charging time of the deterministic model. Less charging time results in a shorter cycle, making a route more attractive for the riders. In addition, the shorter cycle time is more desirable for the operators as the route needs a smaller fleet size. Looking at the en-route charging time of each group, for the days with favorable and positive temperatures (with the aggregate probability of almost 65 percent), the charging is less than the outcome of the deterministic model. Thus, the optimum solution provided by the deterministic model can be interpreted as a conservative schedule when the temperature is above 0°C. However, the solution becomes less reliable when the BEB operates in harsher weather conditions, often for six months in Calgary. In cities where the seasonal temperature is less volatile, the schedule of the bus routes can be updated on a seasonal basis. The expected total cost when using the SP model is $21.82, which is $1.46 more expensive per run than the deterministic model. This extra cost can be addressed as the cost paid to decrease the annual expected travel time as the expected charging time in the SP model is decreased compared to the deterministic model. In addition, this $1.46 extra cost can be addressed as the expected value of perfect information (EVPI), which is the cost of not having certain/accurate information in this problem. It can be paraphrased as how much the operator may consider investing for acquiring certain and accurate input data/information for planning and scheduling. It is noteworthy that the value of the VSS is a sensitive function of the volatility of the temperature. For example, in the City of Calgary, only 3.5% of the days were below -20°C. If the probability of the days with extreme weather conditions were 20%, the value of using the SP model would be less based on the values that EVPI and VSS would recommend.

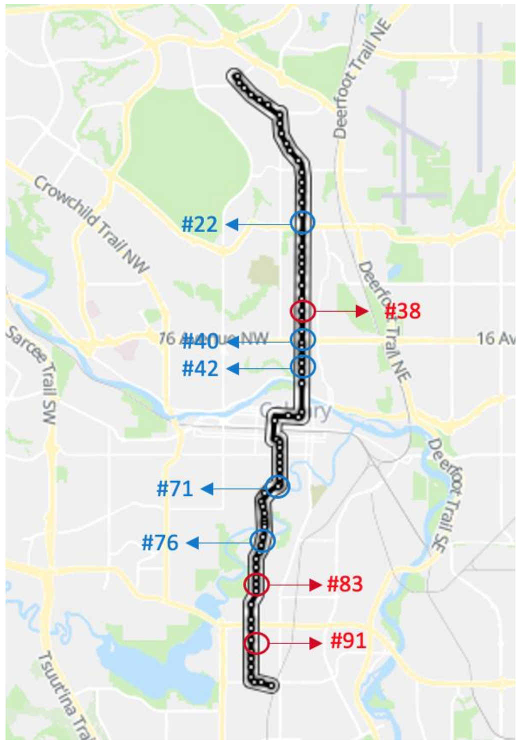

Figure 14 – Stops of the deterministic model (Red) and SP model (Blue) for Route 3

5.3.2 Studying the impact of the failure of a charging station

The impact of the failure of a charging station can be associated with one of the most critical concerns regarding the electrification of the bus routes, which is how this upgrade may affect the reliability of the route. Charging stations may bring new reliability issues to the route operation and, thus, schedule. For routes with multiple en-route charging stations, the breakdown of a charging station may affect the charging times (the plan of the route) and the cost of each trip. As in the sample calculation, the charging station with the maximum share in the total annual charging time (Stop #40) in Route 3 (one-way trip) is chosen to see how the breakdown of this charging station may affect the route. Depending on the weather temperature, the impact could be different. The impact of the breakdown of Stop #40 in each weather condition is stated in Table 6, which shows the updated total charging time and updated total cost (in $ and %) and compares the cost difference before and after the breakdown. The breakdown does not affect the schedule for the days that the temperature is above 20°C, as no charging time is needed. In addition, as expected, the summation of the updated charging time must be equal to the total before the breakdown. This summation can be used to check if the model is working correctly by adding the new constraint for forcing the breakdown of a charging station. However, the breakdown of a charging station may increase the cost of a route up to 43.57%, while the percentage of impact on the expected total cost is 31.57%. So, having a backup charging station for some stations could be financially reasonable in the long run. Such analysis can reveal to decision-makers the ability to avoid choosing nearby stops for installing the charging stations that do not have applications in temporary schedules and at the same time, have minimal application in the original schedule. For example, stop #42 has minimal usage in the original scheduling but can help the charging need while stop #40 is broken, making the charging station at stop #42 a great backup for the charging station at stop #40. This insight is attained after running the model using the following setup, where the condition of the route is assumed after a breakdown at the other charging stations. The same calculation is repeated more times by forcing the other charging stations to become out of service. The result of each calculation generates a table similar to Table 5. For the reader's convenience, the last row of the generated tables (including the last row of Table 6) is also used in Table 7.

It can be inferred that the breakdown of four out of five charging stations (located at stops #22, #40, #42, #71, and #76) in the extreme weather conditions (-30°C to -20°C) will result in a halt of service for the bus route as the model cannot find a feasible solution/arrangement for assigning charging times to the other available four stations, which is shown as “No Feasible Solution” (NFS) in the table. The zero percent or no impact in Table 7 means that a charging station did not have an assigned charging time in the corresponding group.

Table 6 - Assessing the impact of the breakdown of charging station #40

	En-route Charging Time (–in) - After the breakdown of Stop #40					
Station Number	-30°C to -20°C	-20°C to -10°C	-10°C to 0°C	0°C to 10°C	10°C to 20°C	20°C to 30°C
Stop #22	6.67	5.51	4.54	3.47	1.4	0.00
Stop #40	Broken	Broken	Broken	Broken	Broken	Broken
Stop #42	7.64	5.14	2.64	1.13	0.00	0.00
Stop #71	2.03	1.70	1.38	1.05	0.00	0.00
Stop #76	6.40	5.37	4.35	2.32	1.66	0.00
Total en-route charging time (min)	22.74	17.82	12.90	7.99	3.07	0.00
Total model cost ($/each run) (After the breakdown)	77.37	60.20	43.49	27.25	12.49	3.66
The difference in cost due to breakdown ($/each run)	20.98	18.27	12.93	7.32	1.97	0.00
Impact of breakdown on the cost (%)	37.20%	43.57%	42.30%	36.73%	18.67%	0%

Table 7 - *Assessing* the impact of the breakdown of each of the charging stations

Impact of the Breakdown on the Cost (%)							
Station Number	-30°C to -20°C	-20°C to -10°C	-10°C to 0°C	0°C to 10°C	10°C to 20°C	20°C to 30°C	Impact on the Expected Total Cost (%)
Stop #22	NFS*	NFS	27.44	26.73	25.61	0	NFS
Stop #40	37.20	43.57	42.30	36.73	18.67	0	31.75
Stop #42	NFS	33.62	34.91	31.47	17.53	0	NFS
Stop #71	NFS	32.13	30.12	27.19	17.40	0	NFS
Stop #76	NFS	37.21	39.35	29.92	20.46	0	NFS

*NFS: No Feasible Solution

It is important to make sure that under any circumstances, the transit service is restored and passengers are not stranded. Therefore, the NFS status is not acceptable as part of the planning. There are a few options to show some flexibility with the constraints of the model to resolve the above-listed NFS situations. These modifications can be related to the policy and planning part of the model or to the specification of the equipment that are considered in the model, such as that of BEBs. The first option is to serve passengers with BEBs equipped with larger batteries. This modification needs additional capital investment and can be less interesting from the operator's point of view. Using BEBs with larger batteries can be modeled in our model by reducing the value of the charging draining rate per unit of distance “C_D” and increasing the route capital cost. The second option is exceptionally lower than the minimum allowed charge level of the battery “B” in case of station breakdowns. The third option can be changing the headway “H” of the service to give a larger span of charging time to the BEBs for en-route charging stations. The last two suggested modifications are related to the policy of scheduling and planning. We tried to increase the headway as an exercise, but it did not help as it was not a constraint limiting the charging times. In addition, changing the value of battery charge level B was found to be only helpful in resolving the NFS caused by the breakdown of stations #71 and #76. The value of B should be decreased to 11.5% and 19.25% to resolve the NFS of stations #71 and #76, respectively. However, decreasing the value of B to zero does not resolve the NFS for stop #22. To resolve the NFS of station #22, the battery size of the BEB should be increased by 35%. This value, change in the capacity of the battery, for stations #71 and #76 is 15% and 1%, respectively. Under such a disruption scenario, the transit agency can also consider a plan B of battery-swapping option to minimize the service disruption.

5.3.3 Two-way route (BRT route) in Calgary (Route 303)

Based on the SP model, the route only needs three en-route charging stations (in addition to the one in the origin terminal), which is the same as the output of the deterministic model (see

Table 3). The stops for both models, deterministic and SP model, are brought in Figure 15. Similar to the one-route model, while no charging duration other than in the terminals is required when the temperature is above 20°C, the BEB starts requiring more frequency and duration of charging once the temperature drops below that level. For temperatures below 20°C, the model distributes the required charging time between all the charging stations to take advantage of cost reduction due to the dwell time.

Table 8 - Result of the SP model for the two-way bus route

Stop Number		En-route Charging Time (min) -30°C to -20°C	-20°C to -10°C	-10°C to 0°C	0°C to 10°C	10°C to 20°C	20°C to 30°C
Stop #4	Away	5.99	4.66	2.32	1.00	1.44	0.00
	Return	5.54	4.77	4.69	2.59	1.43	0.00
Stop #10	Away	5.99	4.63	2.32	2.27	1.00	0.00
	Return	5.52	4.30	3.31	3.48	1.00	0.00
Stop #14	Away	5.59	4.99	5.57	3.92	1.23	0.00
	Return	5.52	5.71	2.38	2.37	1.00	0.00
Stop #21	Destination Stop (Turning Terminal)	12.48	9.16	8.94	5.07	4.83	3.04
Total en-route charging time (min)		46.64	38.22	29.53	20.70	11.93	3.04
First-stage cost ($/each round-trip)		4.65	4.65	4.65	4.65	4.65	4.65
Second-stage cost ($/each round-trip)		253.94	118.67	76.68	41.74	13.89	2.74
Total model cost ($/each round-trip)		**258.59**	**123.32**	**81.33**	**46.39**	**18.54**	**7.39**
Expected total charging time (min)		**21.66 (min)**					
Expected total cost ($)/round-trip		**$57.57**					

The expected total charging time is 21.67 minutes, almost 30% less than the total charging time from the deterministic model. It is worth noting that for almost 67% of the times (for the times that the temperature is above zero degrees), the charging time obtained by the SP model is less than the result of the deterministic model. In contrast with the result of the previous SP model that serves a pronounced one-direction demand profile, when the ridership has no pronounced directional peak, all the charging stations are used under the worst weather conditions. This efficiency can be related to the optimum design of the BRT routes in that the variance of the demand along the route is lower than that of regular bus routes (see Figure 8 and Figure 9).

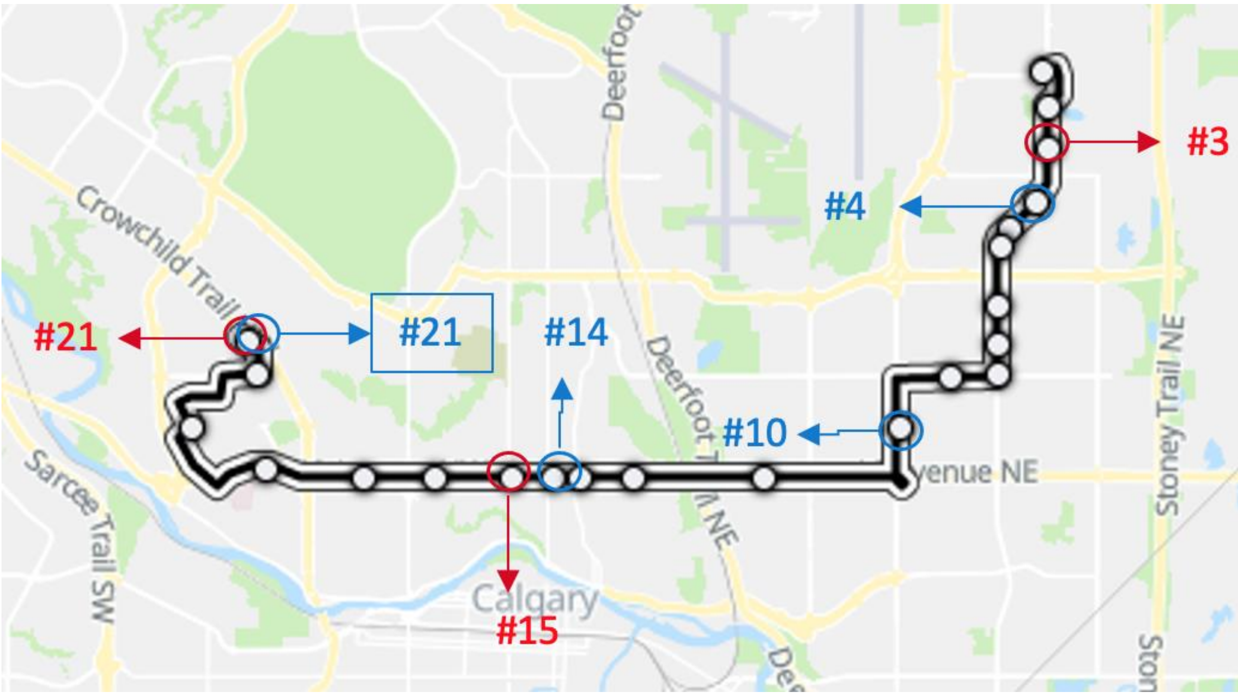

Figure 15 - Stops of the deterministic model (Red) and SP model (Blue) for Route 303

SP models are formulated and applied on a one-way and a two-way bus route in Calgary using real data. The results of the numerical analysis of the one-way route state that implementing the SP model requires the operator to construct more charging stations, resulting in a higher cost for initiating an electric bus route. However, using the SP model's outcome helps decrease the expected travel time compared to the deterministic model. The models are extended and reformulated for two-way bus routes that share stops in the away and return directions. The evaluations of the extension models (deterministic and SP) are performed on a BRT route in Calgary using the data provided by the City of Calgary.

5.3.4 *Studying the impact of using BEBs by larger battery size*

The battery size of the Battery Electric Buses (BEBs) used on a bus route may impact the charging times at the charging stations. In this model, altering the battery size leads to changes in the battery discharge rate (C_D) and the charging rate at the stations (C_F) when using the same charging stations with similar power (kW). To evaluate the significance of this parameter, the SP model for Route 3 is replicated with an increased battery capacity, and the results are presented in Table 9.

Table 9 - Result of the SP model on Route 3 with 10% larger battery

	En-route Charging Time (min)					
Stop Number	-30°C to -20°C	-20°C to -10°C	-10°C to 0°C	0°C to 10°C	10°C to 20°C	20°C to 30°C
Stop #22	3.24	2.21	2.01	1	0	0
Stop #40	5.99	4.78	2.01	2.01	1.18	0
Stop #42	5.15	4.32	2.5	1	1	0
Stop #71	4.10	3.80	2.01	1	0	0
Stop #76	2.40	2.10	2.01	1.95	0	0
Total en-route charging time (min)	20.89	17.21	10.54	6.96	2.18	0
First-stage cost ($)	3.65	3.65	3.65	3.65	3.65	3.65
Second-stage cost ($)	44.12	31.84	20.60	11.57	4.87	0
Total model cost ($)	47.78	35.50	24.26	15.23	8.53	3.65
Expected total charging time (min)	**7.28 (min)**					
Expected total cost ($)/one-way trip	**$17.49**					

Comparing the findings from two tables (Table 5 and Table 9) reveals that increasing the battery capacity does not influence the number and location of stops chosen for charging stations. When the battery capacity is raised by 10%, the expected total charging time decreases by 14%, and the total cost decreases by 20%. It is important to note that augmenting the battery size results in a heavier permanent load. This change in the battery's weight may impact both the battery discharge rate and the capacity of the BEBs for passengers. This model is repeated by increasing the battery capacity of the BEBs by 20% compared to the base model. The result shows that paying extra to increase the battery size helps save the total cost by 23% and decreases the expected total charging time by 27%. The charging times are assigned to five charging stations with one difference compared to the two previous experiments.

CHAPTER 6: OPTIMAL TIME POINT LOCATIONS FOR A TRANSIT ROUTE WITH EN-ROUTE CHARGING STATIONS

6.1 Introduction

The emergence and widespread adoption of electric buses have brought about significant advances in urban transportation. As cities strive to transition to more sustainable and environmentally friendly modes of public transit, electric buses have gained considerable attention due to their reduced carbon emissions and improved energy efficiency. However, optimizing the operational efficiency of electric bus networks remains a key challenge for transit authorities and operators. One crucial aspect of achieving efficiency in electric bus operations is the reliability of the service provided by this new technology. The reliability and interdependability of public transportation services carry significant significance for daily commuters and are linked with the sustainable development of transportation networks. Unreliability is a significant source of inconvenience for passengers, frequently ranking at the forefront of factors impacting the quality of service on a transit route (Eboli and Mazzulla, 2008). The effectiveness of bus operations occurring within the context of shared roadways is notably influenced by a range of unpredictable and uncertain factors present in diverse mixed traffic scenarios. These elements encompass aspects like traffic congestion, unpredictable passenger requirements, varying passenger demands, and inclement weather conditions (Huang et al., 2020, Medina et al., 2017). To tackle the mentioned unreliability, one of the techniques that transit services frequently employ is holding control or time points, focusing on ensuring bus adherence to predetermined schedules. This operational approach entails designating specific stops as time points, requiring buses to reach these stops ahead of the scheduled departure time to wait briefly before departing as per the timetable. In other words, time points are designated locations where buses are scheduled to arrive and depart at specific times, facilitating synchronized operations and increasing reliable passenger service. Integrated with the challenge of determining time point locations is the determination of the optimum slack time to allocate at each time point. Slack time refers to the additional time intentionally built into a bus schedule at time points. This extra time allows a bus to wait briefly before departing to maintain better adherence to the timetable. Adjusting the slack time is a strategic decision to enhance the reliability and consistency of transit operations. Adjusting the slack time directly affects the reliability of the bus route. Ideally, optimizing slack time should be carried out in conjunction with the placement of time points, recognizing their mutual impact on route reliability.

The findings of this research will contribute to the growing body of knowledge on electric bus operations and provide practical recommendations for transit authorities and operators to enhance the efficiency and reliability of their electric bus networks. To achieve that, a linear optimization model is developed in this study. The cost function is formulated to consider passengers' time valuation, encompassing both waiting time due to bus lateness or earliness, as well as time lost at time points. Moreover, operating costs, reflective of enhanced travel time expenses from increased control measures, are integrated into this model. The optimization model employs data from automated vehicle location (AVL) and passenger count (APC) systems. This methodology is applied to optimize an operational route within Calgary, Canada, utilizing the devised model and cost function.

6.2 Methodology

This section considers the development of a linear optimization model for locating the optimum arrangement of the location of the holding points or time points along a bus route. This model uses the output of the two-stage stochastic model (elaborated in Chapter 5) for the location of the charging stations. The linear model is solved by Mixed Integer Programming (MIP).

6.2.1 Decision variable ($\boldsymbol{TP_s}$):

TP_s is a binary value for stating whether a time point is assigned to bus stop “s”. The value of TP_s is one when a time point is assigned to bus stop “s”. Otherwise, it will be zero (see Equation (48)). TP_s is the decision variable(s) for this model. So, the number of decision variables would be the number of stops. It is assumed that the bus stops with charging stations are time points ($TP_s = 1$). Other than the stops with the charging stations, the model solves for the optimum arrangement of the time points.

$$TP_s = \begin{cases} 1 & \leftarrow \text{Time Point (TP) exists at stat“o”s "s"} \\ 0 & \leftarrow \text{Otherwise} \end{cases} \tag{48}$$

6.2.2 Development of the Cost function

Obtaining the cost in this study is an extension of the work done earlier by Wirasinghe and Liu (1995) and Klumpenhouwer and Wirasinghe (2018). This study's total cost accounts for the operational and users’ costs. The cost is broken down into four different categories:

1) The cost of an early bus (C_{Es}),
2) The cost of a late bus (C_L),
3) The cost of through-passengers (C_T), and
4) The operation cost (C_O).

In the following subsections, the additional costs of running a bus due to deviation from the planned schedule (earlier or later) are formulated. In other words, the summation of the additional costs at each bus stop is in addition to the original cost obtained in the case of an ideal, perfect, and on-time bus.

6.2.2.1 Cost of an Early Bus (C_{Es})

An early bus will miss some transit passengers who would be at the bus stop before the planned departure time (D_s) but not early enough to catch the early bus. Therefore, they should wait for an entire additional headway (H) for the next bus. Therefore, the cost of an early bus at the station "s" (C_{Es}) can be obtained using Equation (49).

$$C_{Es} = \gamma_{w1} \times P_s \times (D_s - T_s) \times H \tag{49}$$

where P_s is arrival rate of Passengers to station "s", γ_{w1} is the value of the waiting-time of passengers, T_s is the earliest possible departure time, and $(D_s - T_s)$ states how early buses arrive at the station "s" which helps to obtain the number of passengers who should wait in the station due to an early bus.

6.2.2.2 Cost of a Late Bus (C_L)

A late bus results in additional waiting time for the passengers waiting for a bus at stop "s" and extra travel time for the passengers who are alighting at stop s. The cost of a late bus at station "s" (C_{Ls}) can be obtained using Equation (50).

$$C_{Ls} = \gamma_{w1} \times A_s \times (T_s - D_s) + \gamma_{w1} \times B_s \times (T_s - D_s) = \gamma_{w1}(A_s + B_s)(T_s - D_s) \tag{50}$$

where A_s is the number of alighting passengers at station "s" and B_s is the number of boarding passengers at station "s".

6.2.2.3 Cost of the Through-Passengers (C_T)

For through-passengers who are not planning to alight at the stops that have time points, every pause in the stops due to the holding points is considered a delay. Therefore, the cost of through-passengers at stop “s” (C_{Ts}) due to waiting in a time point can be obtained using Equation (51).

$$C_{Ts} = \gamma_{w3} \times \theta_s \times (D_s - T_s + ST) \tag{51}$$

where θ_s is the number of the through-passengers at stop “s”, γ_{w3} is in-vehicle value of time of passengers and $(D_s - T_s + ST)$ obtains the duration of waiting time at a stop due to a time point.

6.2.2.4 Operation Cost (C_O)

Placement of time points and holding the buses at the holding control points result in longer travel time for buses. An increase in the travel time will increase the operation cost (e.g., driver wage, energy consumption). The operation cost due to the use of a time point at stop “s” (C_{Os}) can be obtained using Equation (52):

$$C_{Os} = \gamma_{wo}(D_s - T_s + ST) \tag{52}$$

where γ_{wo} is the cost per unit time of the regular transit service.

6.2.3 Development of the Decision tree

Implementing the concept of having or not having a time point at stop “s” makes the possibility of five different branches in the decision tree of each stop. Figure 16 illustrates that three independent scenarios can happen if a time point exists, and two independent scenarios may occur if there is no time point at stop s, respectively. These scenarios are named K_{1s} to K_{5s}. $K_{\mathrm{i}s}$s (i=1, …, 5) are binary variables indicating which scenario happened for a bus at stop “s”. In other words, there is exactly only one of K_{is}s is taking value one, and the other four are equal to zero (see Equation (53)).

$$K_{1s} + K_{2s} + K_{3s} + K_{4s} + K_{5s} = 1 \rightarrow \sum_i K_{is} = 1 \; ; (K_{is} = 0, 1) \tag{53}$$

For example, K_{2s} occurs when a bus arrives late within the slack time at stop s, where stop “s” is a time point in the bus route. Figure 17 is a layout of the decision tree (illustrated in Figure 16) on a time axis coordination to address how slack time may vary in the management of transit fleets based on their punctuality. μ_s and α_s (used in both in Figure 16 and Figure 17) similar to K_{is} are auxiliary binary variables that are defined to define the decision tree mathematically. In section 6.2.4, it is explained on how μ_s, α_s , and K_{is} are used to model the decision tree.

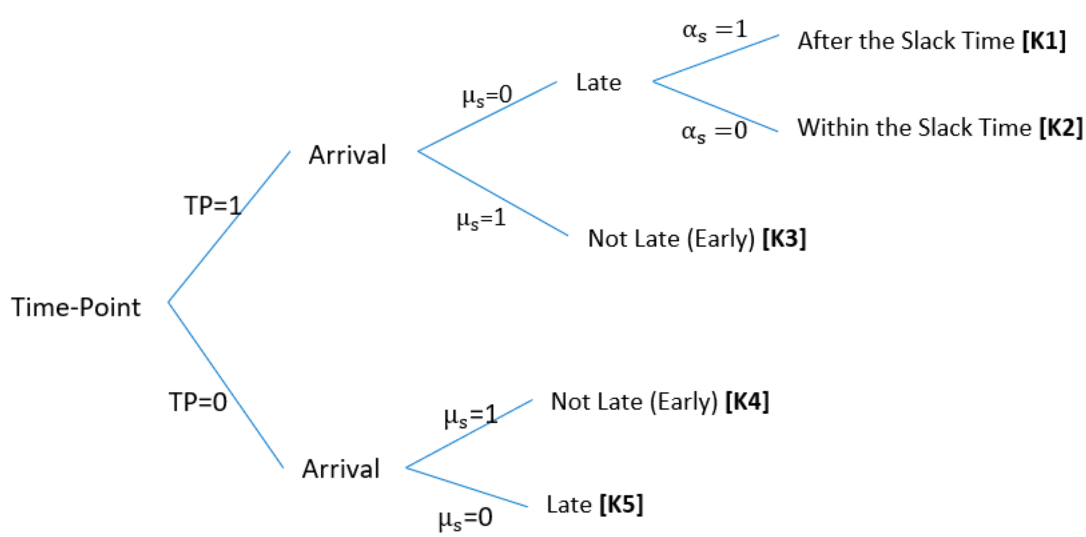

Figure 16 – The decision tree of all possible scenarios for a bus arriving at a stop.

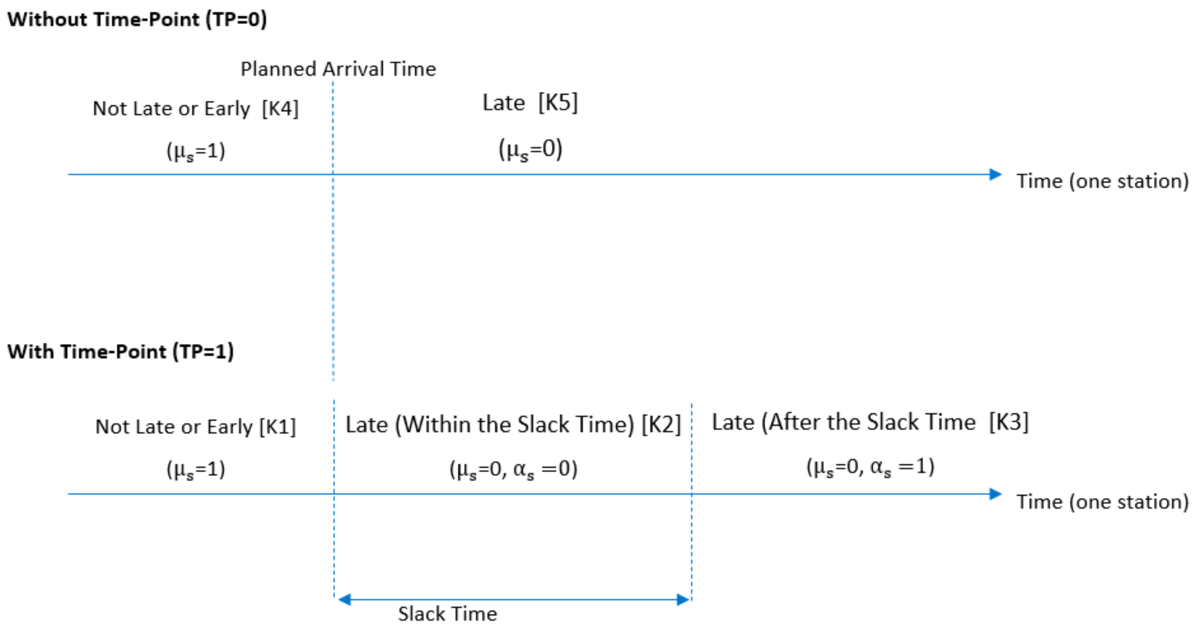

Figure 17 – Decision tree layout on the time axis

There is a cost associated with each scenario. After breaking down the sources of cost earlier in this section, the cost of occurrence of each scenario at stop s (C_{is}, i=1, …, 5) can be defined. C_{is} is defined as the occurred cost in case of a scenario K_s occurring on a stop I and denoted as K_{is}. For example, the associated cost at the 52nd stop (with no time point) for an early bus is addressed as $C_{4,52}$. Based on the cost break-down explained earlier, the cost for each scenario is defined as follows (see Equations (54) to (58)):

$$C_{1s} = \text{ Cost of a Late Bus} = C_{Ls} \tag{54}$$

$$C_{2s} = \text{Cost of the Through Passengers} + \text{Operation Cost} = C_{Ts} + C_{Os} \tag{55}$$

$$C_{3s} = \text{Cost of the Through Passengers} + \text{Operation Cost} = C_{Ts} + C_{Os} \tag{56}$$

$$C_{4s} = \text{Cost of an Early Bus} = C_{Es} \tag{57}$$

$$C_{5s} = \text{ Cost of a Late Bus} = C_{Ls} \tag{58}$$

For example, in the case that at stop “s” with time point (TP_s=1); if the bus is not late ($K_{3s} = 1$), the associated cost (C_{3s}) is the sum of the cost of through-passenger (C_{Ts}) and operation cost (C_{Os}). In other words, Equation (56) would be used to calculate the cost for this scenario.

6.2.4 *Defining piecewise constraints using μ_s, α_s , and K_{is} to define the decision tree and linearize the constraints.*

This model has a planned departure time for every bus stop. The planned departure time at stop “s” is shown as D_s. The planned departure time is when the bus is planned to leave a stop after boarding and alighting, and charging time is represented as the timetable to the users of the bus route. In this model, the earliest possible departure time T_s is defined for stop s. T_s indicates if a bus is early or late at stop “s” based on the pre-defined D_s. In terms of punctuality, a bus can be early or late. Notably, being on time is considered as early in this model. However, there is a zero chance that this happens from the probability theory point of view. To define the punctuality status of a bus at stop s μ_s is defined. μ_s is a binary variable ($\mu_s = 0, 1$) that takes value one for an early bus and zero for a late bus (see Equation (59)).

$$\mu_s = \begin{cases} 1 \quad \leftarrow \quad Early\ at\ statio``s"\ "s" & \leftarrow \quad 0 \leq D_s - T_s \\ 0 \quad \leftarrow \quad Late\ at\ stati``"\ "\text{s}" & \leftarrow \quad D_s - T_s < 0 \end{cases} \tag{59}$$

The main goal of this study is to develop a linear optimization model. Equation (59) is a piecewise function that should be linearized. Equation (59), which is one of the constraints of this model, is rewritten as follows to simplify the linearization procedure on how the value of ($D_s - T_s$) define the value of μ_s (see Equation (60)):

$$\mu_s = \begin{cases} 1 & \leftarrow \quad 0 \le (D_s - T_s) \\ 0 & \leftarrow \quad 0 > (D_s - T_s) \end{cases} \tag{60}$$

The piecewise Equation (60) can be replaced by Equations (61) and (62) as part of the linearization of this model.

$$(D_s - T_s)(\mu_s) \ge 0 \tag{61}$$

$$-(D_s - T_s)(1 - \mu_s) \ge 0 \tag{62}$$

To explain how Equations (61) and (62) work together; if $(D_s - T_s) > 0$ Equation (61) allows μ_s to take both values of zero and one and Equation (62) allows μ_s to take only the value of one. Therefore, it can be concluded that Equations (61) and (62) solve $\mu_s = 1$ in the case that $(D_s - T_s) > 0$. Also, if $(D_s - T_s) < 0$ Equation (62) allows μ_s to take both values of zero and one. But Equations (61) allows μ_s to take only the value of zero. Therefore, it can be concluded that Equations (61) and (62) solve $\mu_s = 0$ in the case that $(D_s - T_s) < 0$.

In the stops that are defined as time points, in the case that a bus is late $(D_s - T_s) < 0$, two options may occur. Therefore, α_s is defined as a binary variable ($\alpha_s = 0, 1$). If a bus is late within the slack time ($D_s - T_s < -ST$) the value of α_s is zero, and if a bus is late more than the slack time ($-ST < D_s - T_s$) the value of α_s is one (see Equation (63)).

$$\alpha_s = \begin{cases} 1 & \leftarrow \quad \text{Late more than the slack time at “t”p "s"} \quad \leftarrow \quad D_s - T_s < -ST \\ 0 & \leftarrow \quad \text{Late within the slack time at “t”p "s"} \quad \leftarrow \quad -ST < D_s - T_s \end{cases} \tag{63}$$

Equation (63), which is one of the constraints of this model, is rewritten as follows to simplify the linearization procedure on how the value of $(D_s - T_s + ST)$ define the value of α_s. In addition, α_s can be one when the time point exists in that stop ($TP_s = 1$). (see Equation (64)):

$$\alpha_s = \begin{cases} 1 & \leftarrow \quad D_s - T_s + ST < 0, TP_s = 1 \\ 0 & \leftarrow \quad 0 < D_s - T_s + ST \end{cases} \tag{64}$$

Having the additional condition for the value of TP_s for obtaining α_s, Equation (64) is rewritten in a more simplified version of a piecewise function as follows (see Equation (65)):

$$\alpha_s = \begin{cases} TP_s = 1, & \mathrm{D_s - T_s + ST} < 0 \ \rightarrow \ \alpha_s = 1 \\ TP_s = 1, & \mathrm{D_s - T_s + ST} \geq 0 \ \rightarrow \ \alpha_s = 0 \\ TP_s = 0, & \mathrm{D_s - T_s + ST} < 0 \ \rightarrow \ \alpha_s = 0 \\ TP_s = 0, & \mathrm{D_s - T_s + ST} \geq 0 \ \rightarrow \ \alpha_s = 0 \end{cases} \tag{65}$$

To linearize this piecewise function (Equation (65)), three equations are developed that replace the piecewise function (see Equations (66), (67), and (68)):

$$\mathrm{TP_s} \geq \ \alpha_s \tag{66}$$

$$-(\mathrm{D_s - T_s + ST})\ \alpha_s \geq 0 \tag{67}$$

$$(\mathrm{D_s - T_s + ST})(1 - \alpha_s) \geq 0 \tag{68}$$

Equations (66) make sure that $\alpha_s = 0$ when there is no time point at a stop ($TP_s = 0$), which is aligned with the third and fourth intervals of the piecewise function (Equation (65)). When the $TP_s = 1$, based on Equation (66), α_s can take both values ($\alpha_s = 0, 1$). Then, the combination of Equations (67) and (68) are to satisfy the first two intervals of Equation (65). If $(\mathrm{D_s - T_s + ST}) \geq 0$, Equations (68) allows α_s to take both values of zero and one. However, Equation (67) restricts α_s to be zero, which satisfy the second interval of the piecewise function. With the same logic, when $(\mathrm{D_s - T_s + ST}) < 0$ Equations (67) allows α_s to take both values of zero and one. However, Equation (68) restricts α_s to be one, which satisfy the first interval of the piecewise function. Therefore, the piecewise function in Equation (65) can be replaced by three Equations (66), (67), and (68) to have linear constraints rather than one non-linear constraint.

To illustrate how μ_s and $\boldsymbol{\alpha_s}$ break down all the scenarios that may occur when the bus arrives at a stop, Figure 16 is provided.

The value of K_{is}s can be obtained based on the values of TP_s, μ_s, and α_s (see Equations (69) to (73)).

$$K_{1s} = (TP_s)(1 - \mu_s)(\alpha_s) \tag{69}$$

$$K_{2s} = (TP_s)(1 - \mu_s)(1 - \alpha_s) \tag{70}$$

$$K_{3s} = (TP_s)(\mu_s) \tag{71}$$

$$K_{4s} = (1 - TP_s)(\mu_s) \tag{72}$$

$$K_{5s} = (1 - TP_s)(1 - \mu_s) \tag{73}$$

For example, if TP_s=1, $\mu_s = 0$, and $\alpha_s = 0$, the value of $K_{2s} = 1$ and the other K_{is}s are zero. Equations (69) to (73) can be rewritten as piecewise functions (see Equations (74) to (78)).

Defining the K_{is}s using piecewise functions is the first step to linearize them.

$$\begin{gathered} K_{1s} = \begin{cases} 1 & \leftarrow \quad TP_s = 1, \quad \mu_s = 0, \quad \alpha_s = 1 \\ 0 & \leftarrow \quad otherwise \end{cases} \\ \rightarrow K_{1s} = \begin{cases} 1 & \leftarrow \quad TP_s = 1, \quad 1 - \mu_s = 1, \quad \alpha_s = 1 \\ 0 & \leftarrow \quad otherwise \end{cases} \end{gathered} \tag{74}$$

$$\begin{gathered} K_{2s} = \begin{cases} 1 & \leftarrow \quad TP_s = 1, \quad \mu_s = 0, \quad \alpha_s = 0 \\ 0 & \leftarrow \quad otherwise \end{cases} \\ \rightarrow K_{2s} = \begin{cases} 1 & \leftarrow \quad TP_s = 1, \quad 1 - \mu_s = 1, \quad \alpha_s = 0 \\ 0 & \leftarrow \quad otherwise \end{cases} \end{gathered} \tag{75}$$

$$K_{3s} = \begin{cases} 1 & \leftarrow \quad TP_s = 1, \ \mu_s = 1 \\ 0 & \leftarrow \quad otherwise \end{cases} \tag{76}$$

$$K_{4s} = \begin{cases} 1 & \leftarrow \quad TP_s = 0, \quad \mu_s = 1 \\ 0 & \leftarrow \quad otherwise \end{cases}$$

$$\rightarrow K_{4s} = \begin{cases} 1 & \leftarrow \quad 1 - TP_s = 1, \quad \mu_s = 1 \\ 0 & \leftarrow \quad otherwise \end{cases} \tag{77}$$

$$K_{5s} = \begin{cases} 1 & \leftarrow \quad TP_s = 0, \quad \mu_s = 0 \\ 0 & \leftarrow \quad otherwise \end{cases}$$

$$\rightarrow K_{5s} = \begin{cases} 1 & \leftarrow \quad 1 - TP_s = 1, \quad 1 - \mu_s = 1 \\ 0 & \leftarrow \quad otherwise \end{cases} \tag{78}$$

The fact that K_{is}s can be defined uniquely by the values of TP_s, μ_s, and α_s allows us to define a linear objective function using their binary behavior. However, the piecewise functions (Equations (74) to (78)) are required to be linearized beforehand. Equations (74) can be replaced by two linear equations as follows:

$$TP_s + (1 - \mu_s) + \alpha_s \geq 3K_{1s} \tag{79}$$

$$(TP_s + (1 - \mu_s) + \alpha_s) - 2.5 \leq K_{1s} \tag{80}$$

when $TP_s = 1$, $\mu_s = 0$, and $\alpha_s = 1$, Equation (79) allows K_{1s} take both values of zero and one. However, Equation (80) restricts K_{1s} to be one, which satisfy the first interval of Equation (74). On the other hand, if at least one of the variables TP_s, $(1 - \mu_s)$, and α_s is not one, Equation (80) restricts the value of K_{1s} to be zero, which satisfies the second interval of the piecewise function.

With the same approach, K_{2s} can be linearized by being replaced with Equations (81) and (82).

$$TP_s + (1 - \mu_s) + (1 - \alpha_s) \geq 3K_{2s} \tag{81}$$

$$(TP_s + (1 - \mu_s) + (1 - \alpha_s)) - 2.5 \leq K_{2s} \tag{82}$$

Similarly, Equation (76) can be replaced by Equations (83) and (84) to linearize K_{3s}:

$$2.5 - (TP_s + \mu_s) \geq (1 - K_{3s}) \tag{83}$$

$$TP_s + \mu_s \geq 2K_{3s} \tag{84}$$

The same approach is used to linearize K_{4s} suing Equations (85) and (86):

$$2.5 - ((1 - TP_s) + \mu_s) \geq (1 - K_{4s}) \tag{85}$$

$$((1 - TP_s) + \mu_s) \geq 2K_{4s} \tag{86}$$

And, K_{5s} is linearized with the same technique (see Equations (87) and (88)):

$$2.5 - ((1 - TP_s) + (1 - \mu_s)) \geq (1 - K_{5s}) \tag{87}$$

$$(1 - TP_s) + (1 - \mu_s) \geq 2K_{5s} \tag{88}$$

Based on the departure time of stop s (T_s or D_s) and the travel time between stop s and stop "s+1" (TRV_s), the earliest possible departure time for stop "s+1" (T_{s+1}) can be obtained. For the stops without a time point ($TP_s = 0$), T_{s+1} can be obtained using the following Equation:

$$T_{s+1} = T_s + TRV_s \tag{89}$$

For the stops with time points ($TP_s = 1$) two conditions should be considered. If the bus arrives later than the slack time ($K_{1\mathrm{s}}$=1), Equation (89) can be used again to obtain T_{s+1}. But, when the bus arrives late within the slack time ($K_{2\mathrm{s}}$=1) or arrives early ($K_{3\mathrm{s}}$=1), T_{s+1} can be obtained using the following Equation:

$$T_{s+1} = D_s + TRV_s \tag{90}$$

Equations (89) and (90) can be combined and written within a piecewise function using the predefined K_{is}s as follows:

$$\begin{cases} T_{s+1} = D_s + TRV_s & \leftarrow \quad K_{2\mathrm{s}} + K_{3\mathrm{s}} = 1 \\ T_{s+1} = T_s + TRV_s & \leftarrow \quad K_{1\mathrm{s}} + K_{4\mathrm{s}} + K_{5\mathrm{s}} = 1 \end{cases} \tag{91}$$

Equation (91) is linearized and rewritten as follows (see Equation (92)):

$$T_{s+1} = (K_{2\mathrm{s}} + K_{3\mathrm{s}})(D_s + TRV_s) + (K_{1\mathrm{s}} + K_{4\mathrm{s}} + K_{5\mathrm{s}})(T_s + TRV_s) \tag{92}$$

6.2.5 Defining the objective function

$K_{\mathrm{i}s}$ are used later for developing the objective function of this optimization model. The cost function corresponding to each scenario of the decision tree is represented by $\mathrm{C}_{\mathrm{i}s}$, the cost of a bus arriving at a stop can be obtained using Equation (93).

$$OF_s = (TP_s)\Big((1-\mu_s)\big((\alpha_s)(C_{1s}) + (1-\alpha_s)(C_{2s})\big) + (\mu_s)(C_{3s})\Big) + (1-TP_s)\big((\mu_s)(C_{4s}) + (1-\mu_s)(C_{5s})\big) \tag{93}$$

The objective function for stop s can be rewritten as follows:

$$OF_s = (TP_s)(1-\mu_s)(\alpha_s)(C_{1s}) + (TP_s)(1-\mu_s)(1-\alpha_s)(C_{2s}) + (TP_s)(\mu_s)(C_{3s}) + (1-TP_s)(\mu_s)(C_{4s}) + (1-TP_s)(1-\mu_s)(C_{5s}) \tag{94}$$

Using the Equations (69) to (73), Equation (94) can be rewritten as follows:

$$OF_s = (K_{1s})(C_{1s}) + (K_{2s})(C_{2s}) + (K_{3s})(C_{3s}) + (K_{4s})(C_{4s}) + (K_{5s})(C_{5s}) \tag{95}$$

6.2.6 Linear optimization model

Having linearized constraints (section 6.2.4) and linearized cost function for stop s (see section 6.2.5), the linearized optimization model Z can be summarized as follows:

$$\text{Min } Z = (K_{1s})(C_{Ls}) + (K_{2s})(C_{Ts} + C_{Os}) + (K_{3s})(C_{Ts} + C_{Os}) + (K_{4s})(C_{\mathrm{E}s}) + (K_{5s})(C_{\mathrm{L}s})$$

$$K_{1s} + K_{2s} + K_{3s} + K_{4s} + K_{5s} = 1 \rightarrow \sum K_{is} = 1$$

$$\begin{cases} TP_s + (1 - \mu_s) + \alpha_s \geq 3K_{1s} \\ (TP_s + (1 - \mu_s) + \alpha_s) - 2.5 \leq K_{1s} \end{cases}$$

$$\begin{cases} TP_s + (1 - \mu_s) + (1 - \alpha_s) \geq 3K_{2s} \\ (TP_s + (1 - \mu_s) + (1 - \alpha_s)) - 2.5 \leq K_{2s} \end{cases}$$

$$\begin{cases} 2.5 - (TP_s + \mu_s) \geq (1 - K_{3s}) \\ TP_s + \mu_s \geq 2K_{3s} \end{cases}$$

$$\begin{cases} 2.5 - ((1 - TP_s) + \mu_s) \geq (1 - K_{4s}) \\ ((1 - TP_s) + \mu_s) \geq 2K_{4s} \end{cases}$$

$$\begin{cases} 2.5 - ((1 - TP_s) + (1 - \mu_s)) \geq (1 - K_{5s}) \\ (1 - TP_s) + (1 - \mu_s) \geq 2K_{5s} \end{cases}$$

$$T_{s+1} = (K_{1\mathrm{s}} + K_{3\mathrm{s}})(D_s + TRV_s) + (K_{2\mathrm{s}} + K_{4\mathrm{s}} + K_{5\mathrm{s}})(T_s + TRV_s)$$

It is important to note that the problem has the flexibility to direct the location of the time points search to match that of the charging stations' locations.

6.3 Numerical Results

The actual data of bus Route 3 of the City of Calgary as described in the previous chapters, is used to evaluate the model. The dataset includes information regarding the date, time, dispatch number, number of boarding and alighting, and the arrival and departure time of every bus at each stop for one year. Using the provided dataset, the average travel time between each consecutive stop and the average dwell time at each stop is obtained. Table 10 shows the model parameters and typical values adopted in the numerical example for model evaluation. Some of the provided values are like the ones provided in the previous chapters.

Table 10 – Model inputs

Symbol	Meaning	Typical Value
H	Route Headway (unit of time)	10 (min)
γ_{w0}	Operator cost ($/unit of time)	$30/hr
γ_{w1}	Value of passenger waiting time (waiting at the stop for a bus) ($/unit of time)	$15/hr
γ_{w3}	Value of passenger in-vehicle travel time ($/person/unit of time)	$5.1/passenger/hr (Hossain et al., 2015)

Based on the described model in the previous section, the model is linearized based on a constant initial slack time. The reason is that slack time has a discrete and integer nature. However, one of the main challenges of the optimum arrangement of the holding point problems is the value for the slack time. While working with the real data, it is a fair assumption that only round values would be considered for the slack time. Therefore, the optimum arrangement of time points for the integer slack time values of zero to five (positive slack times) and minus five to zero (negative slack times) are obtained separately. The result of the model applied to the data is in the following table (Table 11). It is noteworthy to emphasize the assumption that is made for this model that the stops with charging stations (obtained in Chapter 5) are time points. This assumption ensures that the schedule accommodates the time required for recharging and accounts for the additional time needed for charging during the route in the case of and early BEB. The first column represents the slack time for each run of the optimization model. The second column states the number of time points for the suggested optimal arrangement (including the pre-defined ones). The pre-defined time points are the charging stations, which are five in this case (stops #22, #40, #42, #71, #76) as obtained in the previous model presented in chapter 5. The third column represents the increase in the cost of run-time delay for each round trip due to using time points in addition to the ones having charging stations. The fourth and fifth columns state the cost of using only pre-defined time points and that of the optimal arrangement of the time points, respectively. The value in the fifth column is not constant, as the five pre-defined time points have different costs with different slack times. The last column states the impact of using the time points on the cost in percentage.

Table 11 – The output of the model for positive slack times

Slack (min)	Number of TPs	Extra delay (min)	Optimal Cost ($)	Total Cost: Only Charging Stations as TP ($)	Impact of using additional TPs on the cost
0	21	29.06	$ 116.14	$ 148.38	-22%
0.5	21	34.58	$ 105.45	$ 147.37	-28%
1	20	39.98	$ 96.19	$ 146.38	-34%
1.5	19	44.97	$ 92.10	$ 146.88	-37%
2	18	48.46	$ 88.33	$ 147.14	-40%
2.5*	16	47.26	$ 86.63	$ 147.93	-41%*
3	16	54.20	$ 91.75	$ 148.44	-38%
3.5	15	54.12	$ 96.77	$ 148.44	-35%
4	14	52.88	$ 100.96	$ 149.48	-32%
4.5	12	48.18	$ 104.21	$ 149.99	-31%
5	12	53.5	$ 107.99	$ 150.56	-28%

The result provided in Table 11 states that the optimal slack time for applying the holding control strategy is 2.5 minutes as it results in the optimum decrease in total cost. The suggested arrangement of the time points includes 16 time points (including five pre-defined ones). The suggested arrangement decreases the total cost of the transit service by 41%. Interestingly, the optimal cost for only applying the pre-defined time points is when the slack time is set to 1 minute.

In public transit scheduling, negative slack, also known as negative buffer time or negative layover time, refers to a situation where a public transit bus arrives at a time point earlier than its scheduled arrival time. Essentially, the vehicle arrives before the expected time, and similar to the positive slack time, this can lead to several implications and challenges in transit operations (such as passenger inconvenience, irregular service, bus, bunching, operational issues, and resource allocation). To mitigate the undesirable impacts of negative slack, transit agencies may employ several strategies, one of which is applying holding points. Therefore, the impact of deploying the negative slack time is obtained for the provided data using our developed model (see Table 12).

Table 12 – Output of the model for negative slack time

Slack (min)	Number of TPs	Extra delay (min)	Optimal Cost ($)	Total Cost: Only Charging Stations as TP ($)	Impact of using additional TPs on the cost
0	21	29.06	$ 116.14	$ 148.38	-22%
-0.5	13	23.45	$ 112.78	$ 149.38	-24.50%
-1	13	18.56	$ 110.88	$ 150.39	-26.28%
-1.5	13	14.05	$ 111.43	$ 151.40	-26.40%
-2	12	10.21	$ 113.92	$ 152.40	-25.25%
-2.5	10	7.00	$ 115.96	$ 154.85	-25.12%
-3	10	4.31	$ 119.80	$ 158.37	-24.35%
-3.5	10	1.81	$ 124.44	$ 161.89	-23.13%
-4	9	0.63	$ 137.29	$ 170.33	-19.40%
-4.5	9	0.13	$ 153.77	$ 181.16	-15.12%
-5	8	0.00	$ 171.58	$ 191.99	-10.63%

The result stated in Table 12 states that the optimal negative slack time is one minute with thirteen time points with a total cost of $110.88 per run. The obtained slack time states that the buses are arriving early at stations, up to 1 minute in some stations. Comparing the results in Table 11 and Table 12 state that the optimal slack time for Route 3 is 2.5 minutes with 16 time points. It is notable that the optimal cost of using the pre-defined time points increases within the negative slack times. It can be addressed as the bus does not arrive early at either of the predefined time points in the provided dataset.

To evaluate the model, some sensitivity analysis is performed. So, the slack time is set to 2.5 minutes (the optimal slack time), and the inputs of the model are changed separately as follows:

1. Testing two different values for the Operation Cost (γ_{wo}): Low (almost zero) and high value
2. Testing two different values for the value of waiting time for the passengers waiting for a bus at stops (γ_{w1})
3. Testing two different values for the value of waiting time for the passengers sitting in an ideal bus at a holding point (γ_{w3})

The result of re-running the model with the aforementioned changes is brought in Table 13.

Table 13 – The result of the sensitivity analysis

Scenario #	Input Parameters			Outputs				
	Operation Cost ($/min)	Cost of Waiting Passenger ($/min)	Cost of waiting in-vehicle Passenger ($/min)	Number of TPs	Extra delay (min)	Optimal Cost ($)	Total Cost: Only Charging Stations as TP ($)	Impact of using additional TPs on the cost
1 (benchmark)	0.5	0.25	0.085	16	47.26	$86.63	$147.93	41%
2	0.001	0.25	0.085	19	59.00	$58.93	$138.66	58%
3	100	0.25	0.085	5	18.56	$1857.70	$1,857.80	0%
4	0.5	0.00001	0.085	5	18.56	$26.09	$26.09	0%
5	0.5	100	0.085	77	91.26	$5157.30	$48762.00	89%
6	0.5	0.25	0.001	20	62.00	$46.79	$131.12	64%
7	0.5	0.25	100	5	18.56	$19892.0	$19905	0%

As explained earlier, the first three columns of Table 13 are the inputs that are changed (only one for each row). The rest of the columns in this table are similar to earlier tables, representing the optimal outcomes of the optimization model for time point arrangement (Table 11 and Table

12). The data in the first row is the optimal output represented in Table 11 and used as the benchmark. The optimization problem for the location of the time point in the stops is the trade-off between the operation cost plus the cost of in-vehicle waiting versus the cost of waiting at the stops. It is noticeable in the fourth row that by decreasing the value of waiting time at the stops (γ_{w3}= 0.0001), the model does not choose extra time points in addition to the pre-defined five time points at the charging stations. This scenario happens again when one of the operation cost (γ_{w0}) or in-vehicle waiting time value (γ_{w1}) increased to a large value (third row and last row of the table). On the other hand, when the value of waiting time at the stops is increased to a relatively large value (γ_{w3}= 100), the model chooses 77 time points. This value states that the buses of Route 3 miss at least one passenger at 72 stops (in addition to the pre-defined ones) if they do not use any time points. Also, it can be interpreted that the route service may increase by almost 91 minutes if the bus does not want to miss a single passenger.

When the operation cost is set to almost zero (γ_{w0}=0.001), the model finds the optimal answer in a trade-off between the waiting time cost and in-vehicle waiting time cost. When the in-vehicle waiting time cost is set to almost zero (γ_{w3}=0.001), the model finds the optimal answer in a trade-off between the waiting time cost and the operation cost. The comparison of the outcomes of the two scenarios (second and sixth row) reveals how much each of these factors is affecting the total cost to the model. In the first scenario, the model chooses 19 time points with a total cost of \$58.93/run, while in the second scenario, the model chooses 20 time points with a total cost of \$46.79/run. It can be inferred from this comparison that between the two mentioned factors (operation and in-vehicle waiting time), the in-vehicle waiting time of the passengers has a higher impact on the total cost of the model.

CHAPTER 7: SUMMARY AND CONCLUSIONS

The transition to battery-electric buses (BEBs) signifies a transformative shift in public transportation, bearing the promise of environmental sustainability. However, the transition is fraught with challenges, particularly in infrastructure planning and operational adjustments. This thesis has worked with a few critical aspects of this transition: the dispatch policy, the strategic placement of en-route charging stations, and optimal time points.

This chapter offers final reflections on potential avenues for subsequent research. Section 7.1 recapitulates the core findings and insights from Chapters 3, 4, 5, and 6. Section 7.2 provides the discussion and major findings, while section 7.3 lists the contribution of this work. Section 7.4 outlines prospective areas worth exploring in future studies.

7.1 Summary and Conclusion

In Chapter 3, we ventured into an exploration of the dispatch rate. We ascertained the optimal number of BEBs needed for a particular route, emphasizing the flexibility required in urban transport routes. Furthermore, the multifaceted examination of charging and refueling infrastructures unveiled the importance of diversifying charging alternatives, from single-ended plug-in facilities to the combination of charging facilities and charging stations at terminals. Notably, the numerical example underscored these alternatives' practical implications and tangible benefits. As urban transport undergoes rapid electrification, the insights gleaned from this chapter underscore the need for adaptive, efficient, and forward-thinking dispatch policies to enable improving and optimizing the harnessing of the potential of BEBs.

In Chapter 4, a linear deterministic model is developed to find the optimal location of the en-route charging stations for a one-way and two-way bus route. In Chapter 5, the suggested model is extended to a two-stage stochastic programming (SP) model to enable the model to address the weather-induced uncertainty in planning and scheduling the BEBs. It is the first time that the impact of weather conditions and the consequential uncertainty due to ambient temperature on the performance of the BEBs has been addressed in the literature. For the first time, this study proposes a stochastic model to find the en-route charging stations' optimum location and charging time for one bus route. The key advantage of the two-stage SP model in this study is its ability to tackle the weather-induced stochasticity of transit ridership and battery performance of the BEBs to optimize the location and charging time of the en-route charging station(s) considering the dwell time, extra operation cost, and electricity cost. Linear deterministic and SP models are formulated and evaluated on a one-way and a two-way bus route in Calgary using real data.

In Chapter 6, an optimization model is developed to find the optimal arrangement/location of the time points. The focus of this chapter was investigating the optimal placement of time points on a fixed urban bus route with scheduled service. A key aspect was the development of a methodology that integrates route-specific data using the linear optimization model, aiming for practical implications on operational routes. The major contribution of this chapter is developing a linear optimization model for the problem of the optimal time points. In addition, the adaptability of this model is a huge advantage. The cost function is structured to incorporate the temporal valuation assigned by passengers, which includes both the wait time arising from bus deviations—either early or late arrivals—and the durations expended at designated time points. Furthermore, operational expenditures, indicative of augmented travel durations due to intensified control interventions, are considered in the model. The optimization framework harnesses datasets from Automated Vehicle Location (AVL) and Automatic Passenger Count (APC) systems. The developed model is evaluated using the output of the two-stage SP model and the real data of the City of Calgary.

Put simply, the proposed models for the charging stations and the time points offer insights that can shape policy directions at both strategic and operational levels. Extending this model’s application across diverse urban contexts and scenarios can equip urban planners and stakeholders with tools to enhance their transportation networks. On the operational front, there is merit in oscillating time point allocations and schedules between day-parts or during weekend and weekday services. While this approach might necessitate a recalibration of Calgary Transit's performance metrics, the potential dividends for both the commuters and service providers are noteworthy.

7.2 Discussion and Major Findings

The findings from this study offer valuable insights into policy implications on several levels, such as strategic, planning, and operational. Urban planners and policymakers can use the outcome of this research, announced in Chapter 3, to develop strategic decisions related to the electrification of public transport. Determining the optimal number of BEBs for specific routes and the importance of route flexibility is subject to changes in population density, traffic patterns, and urban development. The diversification of charging alternatives and infrastructure, as highlighted in Chapter 3, can guide investment decisions to support the transition to BEBs, taking into account the distinctive operational requirements and passenger demand patterns unique to each route. The emphasis here is on versatility and adaptability, recognizing that a one-size-fits-all approach may not suit the varying needs and operational dynamics of different routes. By adopting a diversified approach, public transportation stakeholders can better position themselves to accommodate the burgeoning demand for electric buses while optimizing operational efficiency and sustainability.

The models take into account various costs and uncertainties, allowing policymakers to make informed decisions that balance cost-effectiveness and service quality in addition to the feasibility study before conducting the full transition to BEBs in the whole network. In addition, the integration of real data from Calgary provides practical applications of the models, especially in Calgary, in addition to serving as a reference for other urban contexts.

In a nutshell, the outcomes of this research offer practical tools for urban planners and stakeholders to enhance their transportation networks by testing different scenarios and implementing various sensitivity analyses to improve public transport service efficiency, sustainability, and adaptability. The findings provide a foundation for addressing the challenges and opportunities presented by the electrification of urban transport systems, such as the strategy or approach for the electrification of the transit service.

7.3 Contributions

For the first time, this work introduces a stochastic model designed to determine the most efficient location and optimal charging times for en-route charging stations along a BEB route. The utilization of a two-stage Stochastic Programming (SP) model in this study is particularly advantageous as it effectively addresses the stochastic variability induced by weather conditions on both transit ridership and the battery performance of Battery Electric Buses (BEBs). This model optimizes the placement and charging schedule of en-route charging stations, taking into consideration factors such as dwell time, additional operational costs, and electricity expenses. The research formulates and applies both linear deterministic and SP models based on real data to analyze bus routes, including one-way and two-way routes in Calgary. In addition, this work focuses on locating the optimum location of the time points of a bus route from the planning point of view. For the first time, this work introduces a linear model for modeling the problem of the location of the time points and gets input from a model for the location of the charging stations.

In addition to optimizing the placement and charging schedule of en-route charging stations, this work delves into the strategic planning of bus routes by focusing on determining the optimum location of time points. To address this aspect, our research introduces a linear model for the first time to model the problem of locating optimal time points within a transit route. This linear model also integrates seamlessly with our stochastic model designed for charging station optimization.

7.4 Key Limitations and Future Extensions

There are many prospective avenues for future research endeavors prompted by the constraints and limitations encountered during this study. The methodology developed in this research utilizes a discrete framework for modeling the decision tree for the stochastic model. This focuses on determining the optimal interval size to achieve the most precise model outcome. It could enhance its precision by crafting a model that pinpoints interval size and scenario count in the two-stage SP model. The scope of this study is circumscribed by its reliance on historical data. Utilizing such data does not fully capture the effects of schedule updates and optimizations on demand. Acquiring survey or experimental data regarding how schedule alterations due to fleet upgrades influence a route would bolster the development of a more grounded model using authentic data. An extension of this research could delve into determining the optimal arrangement of the charging stations and the time points when multiple bus routes overlap in their trajectories. Utilizing real-time data offers the potential to dynamically modulate holding controls throughout a given route. The literature is sparse regarding adjustments to scheduled services influenced by real-time data. Furthermore, the challenge of dynamically modifying time point placements remains underexplored and presents an avenue for further development. Another potential extension of this research could delve into anticipated progressions within the domain of interconnected transit vehicles and their corresponding infrastructure. Connected buses are becoming information hubs that generate, process, send, and receive vast amounts of real-time data. Incorporating this real-time information into the existing model could serve as a substitute for the historical data, potentially enhancing the model's output accuracy.

REFERENCES

Abdelaty, H., Foda, A., and Mohamed, M. (2023). The Robustness of Battery Electric Bus Transit Networks under Charging Infrastructure Disruptions. Sustainability, 15(4), 3642.

Aber, J. (2016). Electric Bus Analysis for New York City Transit. New York City.

Abkowitz, M., Eiger, A., and Engelstein, I. (1986). Optimal control of headway variation on transit routes. Journal of Advanced Transportation, 20(1), 73-88.

Adamski, A., Turnau, A., 1998. Simulation support tool for real-time dispatching control in public transport. Transport. Res. Part A: Policy Pract. 32 (2), 73–87.

Alamatsaz, K., Hussain, S., Lai, C., and Eicker, U. (2022). Electric Bus Scheduling and Timetabling, Fast Charging Infrastructure Planning, and Their Impact on the Grid: A Review. Energies, 15(21), 7919.

Alfa, A.S., Chen, M., 1995. Temporal distribution of public transport demand during the peak period. Eur. J. Oper. Res. 83, 137–153, 1984.

Ampountolas, K., Kring, M., 2015. Mitigating bunching with bus-following models and bus-to-bus cooperation. In: 2015 IEEE 18th International Conference on Intelligent Transportation Systems. IEEE, pp. 60–65.

An K. (2022). Battery electric bus infrastructure planning under demand uncertainty. Transportation Research Part-C, Vol. 111, pp. 572-587, 10.1016/j.trc.2020.01.009

and Environmental Engineering.

Ansari Esfeh, M., and Kattan, L. (2020). A new design and evaluation approach for managed lanes from a sustainability perspective. International Journal of Sustainable Transportation, 14(1), 35-55.

Ansari Esfeh, M., Wirasinghe, S. C., Saidi, S., and Kattan, L. (2021). Waiting time and headway modelling for urban transit systems–a critical review and proposed approach. Transport Reviews, 41(2), 141-163.

Asghari, M., Fathollahi-Fard, A. M., Mirzapour Al-e-hashem, S. M. J., and Dulebenets, M. A. (2022). Transformation and Linearization Techniques in Optimization: A State-of-the-Art Survey. Mathematics, 10(2), 283.

Bak, D. B. Bak, J. S. and Kim, S. Y. (2018). Strategies for Implementing Public Service Electric Bus Lines by Charging Type in Daegu Metropolitan City, South Korea. Sustainability, 10(10), 3386.

Bareilles, G., Laguel, Y., Grishchenko, D., Iutzeler, F., and Malick, J. (2020). Randomized Progressive Hedging methods for multi-stage stochastic programming. Annals of Operations Research, 295(2), 535-560.

Bartholdi III, J. J., and Eisenstein, D. D. (2012). A self-coördinating bus route to resist bus bunching. Transportation Research Part B: Methodological, 46(4), 481-491.

Basso, L.J., Guevara, C.A., Gschwender, A., Fuster, M., 2011a. Congestion pricing, transit subsidies and dedicated bus lanes: efficient and practical solutions to congestion. Transport Pol. 18 (5), 676–684.

Berrebi, S.J., Watkins, K.E., Laval, J.A., 2015. A real-time bus dispatching policy to minimize passenger wait on a high frequency route. Transport. Res. Part B:

Bi, Z., De Kleine, R., and Keoleian, G. A. (2017). Integrated Life Cycle Assessment and Life Cycle Cost Model for Comparing Plug-in versus Wireless Charging for an Electric Bus System. Journal of Industrial Ecology, 21(2), 344-355.

Bie, Y., Xiong, X., Yan, Y., and Qu, X. (2019). Dynamic headway control for high-frequency bus line based on speed guidance and intersection signal adjustment. Computer-Aided Civil and Infrastructure Engineering, 35, 4-25.

Birge, J. R., and Louveaux, F. (2011). Introduction to stochastic programming. Springer Science and Business Media.

Blynn, K., and Attanucci, J. (2019). Accelerating bus electrification: A mixed methods analysis of barriers and drivers to scaling transit fleet electrification. Transportation Research Record, 2673(8), 577-587.

Budde-Meiwes, H., Drillkens, J., Lunz, B., Muennix, J., Rothgang, S., Kowal, J., and Sauer, D. U. (2013). A review of current automotive battery technology and future prospects. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 227(5), 761-776.

Cats, O., (2014). Regularity-driven bus operation: Principles, implementation and business models. Transp. Policy 36, 223–230.

Chan H. L. and Sutanto, D. (2000). A new battery model for use with battery energy storage systems and electric vehicles power systems. in IEEE Power Engineering Society Winter Meeting, Volume 1, pp. 470-475.

Chan, C. C. (2007). The state of the art of electric, hybrid, and fuel cell vehicles. Proceedings of the IEEE, 95(4), 704-718.

Chen, X., Hellinga, B., Chang, C., Fu, L., 2015c. Optimization of headways with stop-skipping control: a case study of bus rapid transit system. J. Adv. Transport. 49

Chen, X., Yu, L., Zhang, Y., and Guo, J. (2009). Analyzing urban bus service reliability at the stop, route, and network levels. Transportation research part A: policy and practice, 43(8), 722-734.

City News (2023). In Chicago, adapting electric buses to winter's challenges. https://toronto.citynews.ca/2023/03/04/in-chicago-adapting-electric-buses-to-winters-challenges/. Accessed April 2023.

Cools, M., Moons, E., Creemers, L., and Wets, G. (2010). Changes in travel behavior in response to weather conditions: do type of weather and trip purpose matter?. Transportation Research Record, 2157(1), 22-28.

Cort´es, C.E., Jara-Díaz, S., Tirachini, A., 2011. Integrating short turning and deadheading in the optimization of transit services. Transport. Res. Part A: Policy Pract. 45 (5), 419–434.

Cort´es, C.E., Sa´ez, D., Milla, F., Nún˜ez, A., Riquelme, M., 2010. Hybrid predictive control for real-time optimization of public transport systems' operations based on

Cui, S., Zhao, H., Wen, H., and Zhang, C. (2018). Locating multiple size and multiple type of charging station for battery electricity vehicles. Sustainability, 10(9), 3267.

Daganzo, C. F. (2009). A headway-based approach to eliminate bus bunching: Systematic analysis and comparisons. Transportation Research Part B: Methodological, 43(10), 913-921.

Daganzo, C.F., Pilachowski, J., 2011. Reducing bunching with bus-to-bus cooperation. Transport. Res. Part B: Methodology. 45 (1), 267–277.

Das, S., and Pandit, D. (2015). Determination of level-of-service scale values for quantitative bus transit service attributes based on user perception. Transportmetrica A: Transport Science, 11(1), 1-21.

de Palma, A., Kilani, M., Proost, S., 2015. Discomfort in mass transit and its implication for scheduling and pricing. Transp. Res. Part B Methodol. 71, 1–18.

de Palma, A., Lindsey, R., 2001. Optimal timetables for public transportation. Transp. Res. Part B Methodol. 35 (8), 789–813.

de Palma, A., Lindsey, R., Monchambert, G., 2017. The economics of crowding in rail transit. J. Urban Econ. 101, 106–122.

De Palma, A., Rochat, D., 1999. Understanding individual travel decisions: results from a commuters survey in Geneva. Transportation 26 (3), 263–281.

Deng, T., and Nelson, J. D. (2011). Recent developments in bus rapid transit: a review of the literature. Transport Reviews, 31(1), 69-96.

Diab, E., Kasraian, D., Miller, E. J., and Shalaby, A. (2020). The rise and fall of transit ridership across Canada: Understanding the determinants. Transport Policy, 96, 101-112.

Dong, J., Liu, C., and Lin, Z. (2014). Charging infrastructure planning for promoting battery electric vehicles: An activity-based approach using multiday travel data. Transportation Research Part C: Emerging Technologies, 38, 44-55.

Eberlein, X. J., Wilson, N. H., and Bernstein, D. (2001). The holding problem with real–time information available. Transportation science, 35(1), 1-18.

Eberlein, X.J., 1995. Real-time control strategies in transit operations: Models and analysis. Ph.D. thesis, Massachusetts Institute of Technology, Department of Civil

Eboli, L., Mazzulla, G., 2008. A Stated Preference Experiment for Measuring Service Quality in Public Transport. Transportation Planning and Technology 31 (5), 509{523.

Esfeh, M. A., Kattan, L., Lam, W. H., Esfe, R. A., & Salari, M. (2020). Compound generalized extreme value distribution for modeling the effects of monthly and seasonal variation on the extreme travel delays for vulnerability analysis of road network. Transportation research part C: emerging technologies, 120, 102808.

Esfeh, M. A., Kattan, L., Lam, W. H., Salari, M., & Esfe, R. A. (2022). Road network vulnerability analysis considering the probability and consequence of disruptive events: A spatiotemporal incident impact approach. Transportation research part C: emerging technologies, 136, 103549.

Esfeh, M. A., Saidi, S., Wirasinghe, S. C., and Kattan, L. (2022). Waiting time and headway modeling considering unreliability in transit service. Transportation Research Part A: Policy and Practice, 155, 219-233.

Esmaeilnejad, S. Klumpenhouwer, W. Wirasinghe, S. C. and Kattan, L. (July 2022). Integrating En-Route Charging of Electric Bus with Holding Control on an Urban Route with Time-Points. Accepted for conference proceedings, and poster presentation, International Symposium on Transportation and Traffic Theory (ISTTT), Beijing, China.

Esmaeilnejad, S., Kattan, L., and Wirasinghe, C (2023). Optimal Charging Station Locations and Durations for a Transit Route with Battery-Electric Buses: A Two-Stage Stochastic Programming Approach with Consideration of Weather Conditions. Transportation Research Part C: Emerging Technologies. Volume 156, 104327: https://doi.org/10.1016/j.trc.2023.104327.

Fu, L., Yang, X., 2002. Design and implementation of bus-holding control strategies with real-time information. Transport. Res. Rec.: J. Transport. Res. Board 1791 (1), 6–12.

García, A., Monsalve-Serrano, J., Sari, R. L., and Tripathi, S. (2022). Life cycle CO_2 footprint reduction comparison of hybrid and electric buses for bus transit networks. Applied Energy, 308, 118354.

Gkiotsalitis, K., and Van Berkum, E. C. (2020). An analytic solution for real-time bus holding subject to vehicle capacity limits. Transportation Research Part C: Emerging Technologies, 121, 102815.

Gkiotsalitis, K., Wu, Z., Cats, O., 2019. A cost-minimization model for bus fleet allocation featuring the tactical generation of short-turning and interlining options. Transport. Res. Part C: Emerg. Technol. 98, 14–36.

Glaister, S., 1974. Generalised consumer surplus and public transport pricing. Econ. J. 84 (336), 849–867.

Glaister, S., 1986. Bus deregulation, competition and vehicle size. J. Transport Econ. Pol. 217–244.

Glaister, S., Collings, J.J., 1978. Maximization of passenger miles in theory and practice. Journal of Transportation Economics. Pol. 304–321.

Gwilliam, K., Nash, C., Mackie, P., 1985. Deregulating the bus industry in Britain: a rejoinder. Transport Rev. 5 (3), 215–222.

H ̈all, C. H., Ceder, A.A., Ekstr ̈om, J., Quttineh, N.H., 2019. Adjustments of public transit operations planning process for the use of electric buses. Journal of Intelligent Transportation Systems: Technology, Planning, and Operations 23, 216–230.

Hans, E., Chiabaut, N., and Leclercq, L., 2015. Investigating the irregularity of bus routes: highlighting how underlying assumptions of bus models impact the regularity results. J. Adv. Transport. 49 (3), 358–370.

Helber, S., Broihan, J., Jang, Y. J., Hecker, P., and Feuerle, T. (2018). Location planning for dynamic wireless charging systems for electric airport passenger buses. Energies, 11(2), 258.

Hensher, D. A., Wei, E., and Balbontin, C. (2022). Comparative assessment of zero emission electric and hydrogen buses in Australia. Transportation Research Part D: Transport and Environment, 102, 103130.

Herna ́ndez, D., Mun ̃oz, J.C., Giesen, R., Delgado, F., 2015. Analysis of real-time control strategies in a corridor with multiple bus services. Transport. Res. Part B:

Hickman, M. D. (2001). An analytic stochastic model for the transit vehicle holding problem. Transportation Science, 35(3), 215-237.

Hofmann, M., O'Mahony, M., 2005. The impact of adverse weather conditions on urban bus performance measures. In: Proceedings of the 8th International IEEE Conference on Intelligent Transportation Systems, Vienna, Austria, pp. 431–436.

Horrox, J., and Casale, M. (2019). Electric buses in america: Lessons from cities pioneering clean transportation. US PIRG Education Fund. Accessed June, 6, 2021.

Hossain, M. S., Hunt, J. D., and Wirasinghe, S. C. (2015). Nature of influence of out-of-vehicle time-related attributes on transit attractiveness: a random parameters logit model analysis. Journal of Advanced Transportation, 49(5), 648-662.

Hu, H., Du, B., Liu, W., and Perez, P. (2022). A joint optimisation model for charger locating and electric bus charging scheduling considering opportunity fast charging and uncertainties. Transportation Research Part C: Emerging Technologies, 141, 103732.

Huang, D., Gu, Y., Wang, S., Liu, Z. and Zhang, W. (2020) A Two-Phase Optimization Model for the Demand-Responsive Customized Bus Network Design. Transportation Research Part C, 111, 1-21.

Huang, M., and Li, J. Q. (2016). The shortest path problems in battery-electric vehicle dispatching with battery renewal. Sustainability, 8(7), 607.

Jansson, J.O., 1980. A simple bus line model for optimisation of service frequency and bus size. J. Transport Econ. Pol. 53–80.

Jara-Díaz, S., Fielbaum, A., Gschwender, A., 2020. Strategies for transit fleet design considering peak and off-peak periods using the single-line model. Transp. Res. Part B Methodol. 142, 1–18.

Jara-Díaz, S., Gschwender, A., 2003. Towards a general microeconomic model for the operation of public transport. Transport Rev. 23 (4), 453–469.

Jara-Díaz, S., Gschwender, A., 2005. Making pricing work in public transport provision. In: Button, K.J., Hensher, D.A. (Eds.), Handbook of Transport Strategy, Policy and Institutions. Elsevier Ltd, Amsterdam, pp. 447–459.

Jiang, Y., Wang, L., Fang, J., Li, R., Han, R., and Wang, Y. (2020). A high-efficiency ZVS wireless power transfer system for electric vehicle charging WithVariable angle phase shift control. IEEE Journal of Emerging and Selected Topics in Power Electronics, 9(2), 2356-2372.

Klumpenhouwer, W., and Wirasinghe, S. C. (2018). Optimal time point configuration of a bus route-A Markovian approach. Transportation Research Part B: Methodological, 117, 209-227.

Koehler, L.A., Kraus Jr, W., 2010. Simultaneous control of traffic lights and bus departure for priority operation. Transport. Res. Part C: Emerg. Technol. 18 (3), 288–298.

Kraus, M., 1991. Discomfort externalities and marginal cost transit fares. J. Urban Econ. 29 (2), 249–259.

Kuby, M., Lim, S., 2005. The flow-refueling location problem for alternative-fuel vehicles. Socio-Economic Planning Sciences 39, 125–145.

Kuby, M., Lim, S., 2007. Location of alternative-fuel stations using the Flow-Refueling Location Model and dispersion of candidate sites on Arcs. Networks and Spatial Economics 7, 129–152.

Kühne, R. (2010). Electric buses–An energy-efficient urban transportation means. Energy, 35(12), 4510-4513.

Kunith, A. Mendelevitch, R. and Goehlich, D. (2017). Electrification of a city bus network - An optimization model for cost-effective placing of charging infrastructure and battery sizing of fast-charging electric bus systems. International Journal of Sustainable Transportation, 11(10), 707–720.

Lamb, W. F., Wiedmann, T., Pongratz, J., Andrew, R., Crippa, M., Olivier, J. G., Minx, J. (2021). A review of trends and drivers of greenhouse gas emissions by sector from 1990 to 2018. Environmental research letters, 16(7), 073005.

Laskaris, G., Seredynski, M., and Viti, F. (2020). Enhancing bus holding control using cooperative ITS. IEEE Transactions on Intelligent Transportation Systems, 21(4), 1767-1778.

Lee, J. H., Chakraborty, D., Hardman, S. J., and Tal, G. (2020). Exploring electric vehicle charging patterns: Mixed usage of charging infrastructure. Transportation Research Part D: Transport and Environment, 79, 102249.

Lesley, L. J. S. (1975, April). The role of the timetable in maintaining bus service reliability. In Proceedings of operating public transport symposium (pp. 36-53).

Li, J. Q. (2016). Battery-electric transit bus developments and operations: A review. International Journal of Sustainable Transportation, 10(3), 157-169.

Li, W., Li, Y., Deng, H., and Bao, L. (2018). Planning of electric public transport system under battery swap mode. Sustainability, 10(7), 2528.

Liang, Y. Zhang, X. Xie, J. and Liu, W. (2017). An optimal operation model and ordered charging/discharging strategy for battery swapping stations. Sustainability, 9(5), 700. doi:10.3390/su9050700

Lim, S., Kuby, M., 2010. Heuristic algorithms for siting alternative-fuel stations using the Flow-Refueling Location Model. European Journal of Operational Research 204, 51–61.

Lin, Y., Zhang, K., Shen, Z. J. M., Ye, B., and Miao, L. (2019). Multistage large-scale charging station planning for electric buses considering transportation network and power grid. Transportation Research Part C: Emerging Technologies, 107, 423-443.

Lindgren, Juuso, and Peter D. Lund. "Effect of extreme temperatures on battery charging and performance of electric vehicles." Journal of Power Sources 328 (2016): 37-45.

Liu Zh., Ziqi Song, and Yi He (2018). Planning of Fast-Charging Stations for a Battery Electric Bus System under Energy Consumption Uncertainty. Transportation Research Record Volume 2672, Issue 8, pp 96-107. https://doi.org/10.1177/0361198118772953.

Liu, H., and Wang, D. Z. (2017). Locating multiple types of charging facilities for battery electric vehicles. Transportation Research Part B: Methodological, 103, 30-55.

Liu, H., Skabardonis, A., Zhang, W.-b., 2003. A dynamic model for adaptive bus signal priority. In: 82nd Transportation Research Board Annual Meeting, Washington, DC.

Liu, T., and Ceder, A. A. (2020). Battery-electric transit vehicle scheduling with optimal number of stationary chargers. Transportation Research Part C: Emerging Technologies, 114, 118-139.

Liu, Z., Song, Z., and He, Y. (2018). Planning of fast-charging stations for a battery electric bus system under energy consumption uncertainty. Transportation Research Record, 2672(8), 96-107.

Liu, Z., Yan, Y., Qu, X., Zhang, Y., 2013. Bus stop-skipping scheme with random travel time. Transport. Res. Part C: Emerg. Technol. 35, 46–56.

Luethi, M., Weidmann, U., and Nash, A. (2007). Passenger arrival rates at public transport stations. In TRB 86th Annual Meeting Compendium of Papers (pp. 07-0635). Transportation Research Board.

Luo, L., Gu, W., Zhou, S., Huang, H., Gao, S., Han, J., ... and Dou, X. (2018). Optimal planning of electric vehicle charging stations comprising multi-types of charging facilities. Applied energy, 226, 1087-1099.

Ma, X., Miao, R., Wu, X., and Liu, X. (2021). Examining influential factors on the energy consumption of electric and diesel buses: A data-driven analysis of large-scale public transit network in Beijing. Energy, 216, 119196.

Madansky, A. (1960). Inequalities for stochastic linear programming problems. Management science, 6(2), 197-204.

Marguier, P., 1985. Bus route performance evaluation under stochastic conditions. Ph.D. thesis. Massachusetts Institute of Technology, Cambridge, MA.

Martin, P. (2018). Habit Analysis: Using Decision Analysis to Take Control of Habits. Stanford University.

Medina, R. M., Cervone, G., and Waters, N. M. (2017). Characterizing and predicting traffic accidents in extreme weather environments. The Professional Geographer, 69(1), 126-137.Miles, J. and Potter, S. (2014). Developing a viable electric bus service: The Milton Keynes demonstration project. Research in Transportation Economics, 48, 357-363.

Mohamed, M., Farag, H., El-Taweel, N., and Ferguson, M. (2017). Simulation of electric buses on a full transit network: Operational feasibility and grid impact analysis. Electric Power Systems Research, 142, 163-175.

Mohring, H., 1972. Optimization and scale economies in urban bus transportation. Am. Econ. Rev. 62 (4), 591–604.

Mohring, H., 1983. Minibuses in urban transportation. J. Urban Econ. 14 (3), 293–317.

Muñoz, J.C., Cortés, C.E., Giesen, R., Sáez, D., Delgado, F., Valencia, F., Cipriano, A., 2013. Comparison of dynamic control strategies for transit operations. Transport. Res. Part C: Emerg. Technol. 28, 101–113.

Nash, C.A., 1978. Management objectives, fares and service levels in bus transport. J. Transport Econ. Pol. 70–85.

Nash, C.A., 1982. Economics of Public Transport. Longman Group UK Limited.

Newell, G. F. (1974). Control of pairing of vehicles on a public transportation route, two vehicles, one control point. Transportation Science, 8(3), 248–264.

Newell, G.F., 1971. Dispatching policies for a transportation route. Transport. Sci. 5 (1), 91–105.

Ngo, N. S. (2019). Urban bus ridership, income, and extreme weather events. Transportation research part D: transport and environment, 77, 464-475.

Nicholas, M., 2019. Estimating electric vehicle charging infrastructure costs across major U.S. metropolitan areas. International Council on Clean Transportation, 11.

Oldfield, R., Bly, P., 1988. An analytic investigation of optimal bus size. Transp. Res. Part B Methodol. 22 (5), 319–337.

Osuna, E. E., and Newell, G. F. (1972). Control strategies for an idealized public transportation system. Transportation Science, 6(1), 52-72.

Panah, P. G., Bornapour, M., Hemmati, R., and Guerrero, J. M. (2021). Charging station Stochastic Programming for Hydrogen/Battery Electric Buses using Multi-Criteria Crow Search Algorithm. Renewable and Sustainable Energy Reviews, 144, 111046.

Panda, B., Rad, F. M., and Rajabi, M. S. (2023). Wireless Charging of Electric Vehicles Through Pavements: System, Design, and Technology. Handbook of Smart Energy Systems, 1-26.

Pels, E., Verhoef, E., 2007. Infrastructure pricing and competition between modes in urban transport. Environ. Plann. 39, 2119–2138.

Petit, A., Ouyang, Y., and Lei, C. (2018). Dynamic bus substitution strategy for bunching intervention. Transportation Research Part B, 115, 1–16.

Rahman, M. (2011). Real time bus information provision: users' views and some aspects of bus travel time in Calgary (Master's thesis, University of Calgary).

Reyes, J. R. M. D., Parsons, R. V., and Hoemsen, R. (2016). Winter happens: The effect of ambient temperature on the travel range of electric vehicles. IEEE Transactions on Vehicular Technology, 65(6), 4016-4022.

Rietveld, P., Dekkers, J., van Woudenberg, S., 2002. Choice of frequency and vehicle size in rail transport. Implications for marginal external costs. Eur. J. Transport Infrastruct. Res. 2 (2), 95–111.

Rogge, M., Wollny, S., Sauer, D.U., 2015. Fast charging battery buses for the electrification of urban public transport-A feasibility study focusing on charging infrastructure and energy storage requirements. Energies 8,4587–4606.

Rupp, M., Handschuh, N., Rieke, C., and Kuperjans, I. (2019). Contribution of country-specific electricity mix and charging time to environmental impact of battery electric vehicles: A case study of electric buses in Germany. Applied energy, 237, 618-634.

S´aez, D., Cort´es, C.E., Milla, F., Núnez, A., Tirachini, A., Riquelme, M., 2012. Hybrid predictive control strategy for a public transport system with uncertain demand.

Schoenberg, S., Buse, D. S., and Dressler, F. (2021). Coordinated electric vehicle re-charging to reduce impact on daily driving schedule. In 2021 IEEE Intelligent Vehicles Symposium (IV) (pp. 1180-1187). IEEE.

Sebastiani M.T., Luders R. and Fonseca K.V.O., 2016. Evaluating electric bus operation for a real-world BRT public transportation using simulation optimization. IEEE Trans. Intell. Transp. Syst., 17 (10) (2016), pp. 2777-2786, 10.1109/TITS.2016.2525800.

Senevirante, P. N. (1990). Analysis of on-time performance of bus services using simulation. Journal of Transportation Engineering, 116(4), 517-531.

Shapiro, A., Dentcheva, D., and Ruszczynski, A. (2021). Lectures on stochastic programming: modeling and theory. Society for Industrial and Applied Mathematics.

Shih, C., and Nicholls, S. (2011). Modeling the influence of weather variability on leisure traffic. Tourism Analysis, 16(3), 315-328.

Skabardonis, A., 2000. Control strategies for transit priority. Transp. Res. Rec. 1727 (1), 20–26.

Small, K., Rosen, H., 1981. Applied welfare economics with discrete choice models. Econometrica 49 (1), 105–130.

Strathman, J., Dueker, K., Kimpel, T., Gerhart, R., Turner, K., Taylor, P., Callas, S., Griffin, D., Hopper, J., 1999. Automated bus dispatching, operations control, and

Su, L., Zhang, J., Wang, C., Zhang, Y., Li, Z., Song, Y., Jin, T., Ma, Z., 2016. Identifying main factors of capacity

Sun, A., Hickman, M., 2005. The real–time stop–skipping problem. J. Intell. Transport. Syst. 9 (2), 91–109.

Sun, Y. K. (2020). Promising all-solid-state batteries for future electric vehicles.

Sweda, T. M., Dolinskaya, I. S., Klabjan, D., 2017. Adaptive routing and recharging policies for electric vehicles. Transportation Science 51, 1326–1348.

Tang, X., Lin, X., and He, F. (2019). Robust scheduling strategies of electric buses under stochastic traffic conditions. Transportation Research Part C: Emerging Technologies, 105, 163-182.

The Global e-Sustainability Initiative, "#SMARTer2030 - ICT Solutions for 21st Century Challenges. Brussels, Belgium, Rep. B-1040, June 2015. [Online]. Available: http://smarter2030.gesi.org/downloads/Full report2.pdf.

Tirachini, A., Hensher, D.A., Rose, J.M., 2014. Multimodal pricing and optimal design of urban public transport: the interplay between traffic congestion and bus crowding. Transp. Res. Part B Methodol. 61, 33–54.

Translink, 2023. TransLink begins battery-electric bus fleet expansion. https://www.translink.ca/news/2023/february/translink%20begins%20battery-electric%20bus%20fleet%20expansion. Accessed April 2023.

Turnquist, M. A. (1981). Strategies for improving reliability of bus transit service. Transportation Research Record, 818, 7–13.

van Oort, N., Boterman, J., van Nes, R., 2012. The impact of scheduling on service reliability: trip-time determination and holding points in long-headway services. Public Transport 4 (1), 39–56.

Velandia Vargas, J. E., Falco, D. G., da Silva Walter, A. C., Cavaliero, C. K. N., and Seabra, J. E. A. (2019). Life cycle assessment of electric vehicles and buses in Brazil: effects of local manufacturing, mass reduction, and energy consumption evolution. The International Journal of Life Cycle Assessment, 24, 1878-1897.

Verbrugge, B., Hasan, M. M., Rasool, H., Geury, T., El Baghdadi, M., and Hegazy, O. (2021). Smart integration of electric buses in cities: A technological review. Sustainability, 13(21), 12189.

Walters, A.A., 1982. Externalities in urban buses. J. Urban Econ. 11 (1), 60–72.

Wang X., Song Z., Xu H., Wang H. (2023). En-route fast charging infrastructure planning and scheduling for battery electric bus systems. Transportation Research Part D: Transport and Environment Volume 117. https://doi.org/10.1016/j.trd.2023.103659.

Wang, S., Lu, C., Liu, C., Zhou, Y., Bi, J., and Zhao, X. (2020). Understanding the energy consumption of battery electric buses in urban public transport systems. Sustainability, 12(23), 10007.

Wang, X., Ding, Y. L., Deng, Y. P., and Chen, Z. (2020). Ni-rich/Co-poor layered cathode for automotive Li-ion batteries: promises and challenges. Advanced Energy Materials, 10(12), 1903864.

Wang, Y. W., and Lin, C. C. (2013). Locating multiple types of recharging stations for battery-powered electric vehicle transport. Transportation Research Part E: Logistics and Transportation Review, 58, 76-87.

Wang, Y., Huang, Y., Xu, J., Barclay, N., 2017. Optimal recharging scheduling for urban electric buses: A case study in Davis. Transportation Research Part E: Logistics and Transportation Review 100, 115–132.

Wang, Y., Ma, W., Yin, W., Yang, X., 2014. Implementation and testing of cooperative bus priority system in connected vehicle environment: case study in taicang city, china. Transp. Res. Rec. 2424 (1), 48–57.

Wei, R. Liu, X. Ou, Y. and Fayyaz, S. K. (2018). Optimizing the spatiotemporal deployment of battery electric bus system. Journal of Transport Geography, 68, 160–168.

Weiss, A., Ansari Esfeh, M., Esmaeilnejad, S., Chan, K., & Kattan, L. (2023). Accounting for preference heterogeneity for high occupancy toll lanes in a Canadian city: a latent class approach. Transportation Letters, 15(8), 980-991.

Wirasinghe, S. C. (1990). Re-Examination of Newell's dispatching policy and extension to a bus route with many to many time varying demand. In M. Koshi (Ed.), Transportation and traffic theory (pp. 363–378) Elsevier.

Wirasinghe, S. C., and Liu, G. (1995). Determination of the number and locations of time points in transit schedule design—Case of a single run. Annals of Operations Research, 60(1), 161-191.

Wu, W., Liu, R., Jin, W., 2017. Modelling bus bunching and holding control with vehicle overtaking and distributed passenger boarding behaviour. Transport. Research Part B: Methodology. 104, 175–197.

Xie, Y., Guo, H., Li, W., Zhang, Y., Chen, B., and Zhang, K. (2021). Improving battery thermal behavior and consistency by optimizing structure and working parameter. Applied Thermal Engineering, 196, 117281.

Xuan, Y., Argote, J., and Daganzo, C. F. (2011). Dynamic bus holding strategies for schedule reliability: Optimal linear control and performance analysis. Transportation Research Part B: Methodological, 45(10), 1831-1845.

Xylia, M., Leduc, S., Patrizio, P., Kraxner, F., Silveira, S., 2017. Locating charging infrastructure for electric buses in Stockholm. Transportation Research Part C: Emerging Technologies 78, 183–200.

Yao, E., Liu, T., Lu, T., and Yang, Y. (2020). Optimization of electric vehicle scheduling with multiple vehicle types in public transport. Sustainable Cities and Society, 52, 101862.

You, P., Yang, Z., Zhang, Y., 2015. Efficient battery charging schedule of battery-swapping station for electric buses. 2015 European Control Conference, ECC 2015, 316–321.

Yu, Y., Ye, Z., Wang, C., 2015. Study of bus stop skipping scheme based on modified cellular genetic algorithm. In: CICTP 2015, pp. 2397–2409.

Zheng, H., Song, M., and Shen, Z. (2021). The evolution of renewable energy and its impact on carbon reduction in China. Energy, 237, 121639.

Zheng, Y. Dong, Z. Y. Xu, Y. Meng, K. Zhao, J. H. and Qiu, J. (2014). Electric vehicle battery charging/swap stations in distribution systems: Comparison study and optimal planning. IEEE Transactions on Power Systems, 29(1), 221–229. doi:10.1109/TPWRS.2013.2278852.

Zhou Y., Ghim Ping Ong, Qiang Meng, Haipeng Cui (2023). Electric bus charging facility planning with uncertainties: Model formulation and algorithm design. Transportation Research Part C: Emerging Technologies, Volume 150, 2023, 104108, https://doi.org/10.1016/j.trc.2023.104108.

Zhu, Z.H., Gao, Z.Y., Zheng, J.F., Du, H.M., (2016). Charging station location problem of plug-in electric vehicles. Journal of Transport Geography 52, 11–22.

Zivanovic, Z. Nikolic, Z. (2012). The application of electric drive technologies in citybuses. In: Stevic Z editor. New generation of electric vehicles.

APPENDIX 1: BIG M LINEARIZATION

Integer-programming formulations can represent non-linear functions. One of the most useful techniques is the "Big M" technique. This technique is explained in an example. There is a fixed cost in almost all the objective functions for the minimization problems. For example, the cost of producing "n" units of a product consists of a fixed cost "K" (for setting up the manufacturing), and a variable cost "c" for each unit produced. We assume that the manufacturer has a maximum capacity of "M" units. We define a new binary variable "y" as an indicator for the condition that the fixed cost is incurred. Therefore, y=1if x>0 and y=0 when x=0. In this case, the cost function and the constraints look as follows:

$$\mathrm{Z} = Ky + cx \tag{96}$$

$$\begin{aligned} \text{s.t.} \quad & x \leq My \\ & 0 \leq x \qquad\qquad (97) \\ & y = 0,1 \end{aligned}$$

In this optimization model, the maximum capacity of the manufacturer is used to apply the big M technique (first constraint). As required, with the current constraints x=0 when the fixed cost is not acquired (where y should be zero). Without implementing the "big M" technique, the constraints do not imply that y = 0 if x=0. In addition, when x=0, the minimization will clearly select y=0, so the fixed cost is not acquired. And, in the case that y=1, then the added constraint becomes x<M (which reflects the maximum capacity of the manufacturer).

APPENDIX 2: BRIEF INTRODUCTION ON PROGRESSIVE HEDGING (PH) ALGORITHM

Progressive Hedging (PH) is an iterative algorithm used to solve large-scale stochastic optimization problems. It is instrumental in solving problems in which there is significant uncertainty, such as those encountered in finance, energy, and environmental management. PH is based on the idea of decomposing a large stochastic optimization problem into smaller subproblems that are easier to solve. It does this by breaking down the original problem into a series of smaller subproblems, each of which is solved iteratively. At each iteration, the subproblems are solved by using the solutions from the previous iteration as a starting point.

The PH algorithm proceeds in the following steps:

1. Decomposition: The original problem is decomposed into a series of smaller subproblems, each of which is a scenario-specific optimization problem.

2. Initialization: An initial solution is computed for each subproblem.

3. Iteration: The algorithm proceeds through a series of iterations. At each iteration, the following steps are performed:

a. Fixing: One scenario is fixed, and the other scenarios are ignored. This is done by adding penalty terms to the objective function that penalize solutions that deviate from the scenario being fixed.

b. Optimization: The subproblem with the fixed scenario is solved. The solution is used as a starting point for the next iteration.

c. Aggregation: The solution from the subproblem with the fixed scenario is aggregated with the solutions from the previous iterations to form a new set of solutions.

d. Unfixing: The scenario that was fixed in step (a) is released, and the next scenario is fixed. Steps (b) through (d) are repeated for each scenario.

4. Termination: The algorithm terminates when the solutions have converged, or when a specified termination criterion is met.

The PH algorithm is particularly useful in solving problems with a large number of scenarios, as it allows the problem to be decomposed into smaller subproblems that can be solved more easily. However, the algorithm can be computationally expensive, as it requires solving a large number of optimization problems. Nonetheless, it is a powerful tool for solving large-scale stochastic optimization problems with significant uncertainty.

A2.1 A sample stochastic optimization problem to explain how PH is working:

Consider the following stochastic optimization problem:

$$\min c^T x$$

$$Ax \geq b$$

$$x \geq 0$$

where c is a vector of costs, x is a vector of decision variables, A is a matrix of constraints, and b is a vector of constraint values. Assume that the costs and constraints are uncertain and depend on a set of scenarios S. Specifically, the cost vector c and the constraint matrix A are assumed to be different for each scenario in S.

We can solve this problem using PH as follows:

Here's an example of how to solve this problem using PH for a small problem with two scenarios, where c_1=[1, 2], c_2=[2, 1], A_1=[-1, -1; -1, 1], A_2=[-1, 1; -1, -1], b_1=[-1; -1], b_2=[-1; 1].

Decomposition: We decompose the problem into two subproblems:

$$\min \begin{bmatrix} 1 \\ 2 \end{bmatrix} x$$

$$\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} x \geq \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$x \geq 0$$

And

$$\min \begin{bmatrix} 2 \\ 1 \end{bmatrix} x$$

$$\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} x \geq \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$x \geq 0$$

Initialization: We initialize each subproblem with an initial feasible solution, such as x_1 = [1; 1] and x_2 = [1; 1].

Iteration: We fix scenario 1 and solve the subproblem associated with that scenario. We add a penalty term to the objective function that penalizes solutions that deviate from scenario 1. The penalty term takes the form $alpha \times ||\mathrm{x} - x_1||$, which is a parameter that controls the strength of the penalty term. We use an optimization solver, such as a linear programming solver (CPLEX in our study), to solve the following problem:

$$\min \begin{bmatrix} 1 \\ 2 \end{bmatrix} x + alpha \times \left\| x - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\|$$

$$\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} x \geq \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$x \geq 0$$