THE UNIVERSITY OF CALGARY

THE PERMANENT TIDE AND ITS EFFECTS ON THE DEFORMABLE EARTH

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "The Permanent Tide and Its Effects on the Deformable Earth," submitted by Dale A. G. Arden in partial fulfillment of the requirements for the degree of Master of Engineering.

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Abstract

The three-body problem is used as the starting point for development of the permanent tide-generating potential. The Earth responds like an inviscid fluid to this stationary disturbance. To more accurately estimate the response of the deformable Earth to this potential, second order fluid Love numbers are calculated: $k_2^f = 0.932\,448$ and $h_2^f = 1 + k_2^f = 1.932\,448$.

Instead of trying to find a compromise between the sometimes conflicting requirements of geodesy and geodynamics, a two-geoid system is recommended. The so-called mean geoid, which includes the direct and indirect effects of the permanent tide, is suggested for geodesy because it is the closest to mean sea level and requires the fewest corrections. The direct permanent tide-generating potential is not harmonic everywhere outside the Earth because it is caused by luni-solar masses. However, in geodesy, harmonicity is required only of the anomalous potential. By adding the permanent tide-generating potential to the normal potential, the anomalous potential will maintain its harmonicity even though the gravitational potential is not harmonic. This is the same strategy used in dealing with the non-harmonic rotational potential. For geodynamics, a geoid is preferred which is free of all effects due to external masses. This geoid is called the zero geoid. Its potential is the same as the mean geoid. A third geoid, named here the deformed geoid, was recommended by the IAG in 1983 [14]. It includes the indirect effects of the permanent tide but does not include the direct effects. The deformed geoid is a possible alternative to the mean geoid but has some disadvantages: the potential of the deformed geoid is different from the potential of the mean geoid or zero

geoid; it does not coincide with mean sea level; and some geodetic observations have to be corrected to remove the effects of the direct permanent tide-generating potential.

Corrections are derived to transform certain geodetic observations (ie. surface gravity, heights and satellite orbits) and three of the four independent parameters required to define the normal ellipsoid (ie. J_2 and the Earth's semi-major axis and rotation rate). The fourth constant, GM_E, is independent of the reference figure.

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Chapter 1

INTRODUCTION

The study of Earth tides has a short but distinguished history in the geosciences. According to Melchior [25], Ximenes in 1757 was the first person to suggest that the direction of the vertical changes due to the gravitational attraction of the sun and moon. In 1844, C. A. Peters published the first correct calculations. Early in the nineteenth century, the concept that the Earth may deform under the influence of tidal potentials was first put forward; and in 1876, Lord Kelvin pointed out that observations taken on the surface of the Earth are affected by the tidal deformations. G. Darwin compared the amplitudes of observed long-period (ie. semi-monthly and monthly) tides with theoretical values in 1883. He found that the observed amplitudes were about two thirds of the theoretical values and concluded that the solid Earth deformed by an amount equal to about one third that of the liquid oceans. This is within 10% of present-day estimates. This discovery led Lord Kelvin to derive an estimate of the rigidity of the Earth which was near that of steel. Since that time, great improvements in measurement technology have expanded both the quantity and quality of tidal observations.

The knowledge gained through observing tidal phenomena has played an important role in the advancement of geodesy, geophysics and astronomy. By comparing these observed phenomena with the corresponding phenomena computed for a certain Earth model, valuable information on the rheology of the Earth can be collected. Earth tides are unique in this role because the magnitude and direction of the tidal forces can be accurately computed. The difficulty in the analysis arises from the complex structure of the Earth and the intricate interactions between various components; eg. crustal loading due to ocean tides and the effects due to the Earth's liquid core.

In 1959, Honkasalo demonstrated that the tidal signal has a nonzero mean. This 1959 report was published in 1964 [12]. In it, Honkasalo computed the long-term mean of the usual tidal correction for surface gravity observations. He derived an expression for the correction to surface gravity which is due to the effects of the permanent tide-generating potential. The permanent tide differs significantly from its periodic counterparts in that it cannot be directly observed. Periodic tides have a time-dependent influence on all observations taken on or near the Earth's surface. Long-term observational sessions can be analysed to separate the different components. However, the permanent tide shows up only as a latitude-dependent bias in the tidal record and cannot be separated from other zonal effects (such as rotation) using the observational record. As Honkasalo demonstrated, a theoretical estimate of the permanent tide is possible. Since the magnitude of the permanent tide cannot be directly measured, assumptions regarding the response of the Earth to very long-term disturbances are required. Honkasalo and other authors (eg. [10], [36], [13], [24]) assumed that the Earth responds elastically to the permanent tide-generating potential. However, a number of observed phenomena suggest that the Earth's response to excitation of very low frequencies is similar to that of an inviscid fluid. The lack of correlation between topography and gravity is evidence of isostatic compensation which, along with the agreement between predicted and observed rotational deformation, serves to support this theory (eg. [16]). The use of fluid Love numbers to describe the disturbances due to the permanent tide-generating potential is now generally accepted as being the most appropriate (eg. [21], [3] and [45]).

1.1 A BRIEF HISTORY OF THE PERMANENT TIDE IN GEODESY

Tidal corrections are applied to surface gravity observations to remove the effects of the tide-generating potential. Honkasalo [12] was the first to point out that the reference geoid of these corrected gravity data does not coincide with mean sea level. He suggested that the effects due to the permanent tide-generating potential should be added back onto the corrected gravity data to bring the reference geoid into coincidence with mean sea level. This secondary correction to surface gravity data has been called the *Honkasalo correction.* Honkasalo computed an estimate of the correction assuming elastic deformations. His advice was not implemented until the International Gravity Standardization Net 1971 (IGSN71) was compiled in the early 1970's [26]. The Honkasalo correction was included in the final adjustment of the network. The controversy began almost immediately and at the XVII General Assembly of the International Union of Geodesy and Geophysics in Canberra, 1979 (IUGG XVII), the International Association of Geodesy (IAG) recommended that the Honkasalo correction not be applied to gravity data. Furthermore, they recommended that IGSN71 data be re-corrected by removing the Honkasalo correction. This resolution, along with an article by M. Heikinnen in Bulletin Géodesique [10] in the same year, initiated a spirited discussion through a series of Bulletin Géodesique articles by Heikinnen, M. Ekman and E. Groten ([3], [8], [4], [9]). The basic arguments presented in this series are summarized in the following paragraphs.

Heikinnen [10] agreed with the IUGG XVII resolution: he concluded that the Honkasalo correction " *is harmful and should be avoided*". He drew his conclusion primarily from the argument that by including the Honkasalo correction, masses outside of the geoid contributed to the Earth's gravity potential, causing the potential to lose its harmonicity, thus eliminating the possibility of using Stokes' formula for computing the geoid (see eg. [11]).

In reply to [10], Groten [8] pointed out that even though the forces acting on the Earth due to the permanent tide-generating potential are well determined, the Earth's response to these forces is not well understood. He argued that elastic Love numbers were inappropriate for the permanent tide, but since secular Love numbers cannot be observed, no attempt to describe the permanent tidal potential should be made. In addition, Groten suggested that the direct permanent tide-generating forces should be removed from gravity observations but the indirect effects due to mass rearrangement and displacement of the observer should remain in the gravity signal. This argument was partially based on the knowledge that the second-degree zonal harmonic coefficient, C_{20}, found from satellite orbit analysis contains only the indirect effects of the permanent tidal potential.

Ekman [3] supported Groten's position. He added that if the Heikinnen-IAG proposition was to be accepted, then the rotation rate of the Earth would have to be corrected to be consistent with the gravity observations. Ekman also pointed out that Groten's geoid is consistent with the observed rotation rate. Further, he suggested that the Heikinnen-IAG definition of gravity could not be used on celestial bodies with large permanent tidal deformations (eg. the moon).

In response to [3], Groten [9] added that the model Earth should deviate as little as possible from the real Earth to avoid difficulties in, for example, deriving fluid Shida numbers to correct astronomical observations or in determining the sea surface topography.

The IAG, in consideration of this discussion, passed a new resolution at the next IUGG General Assembly in Hamburg in August, 1983 (IUGG XVIII). This new resolution followed the recommendations of Groten and Ekman, stating that the indirect permanent tidal potential should remain in the geoid, but the direct permanent tide-generating potential should be removed..

One of the most active groups in the study of the permanent tidal potential is the Research Institute of Geodesy, Topography, and Cartography at Zdiby, Czechoslovakia. Šimon and Zeman [39] and [40] support Honkasalo's original concept that the direct and indirect effects of the permanent tide should be included in the geoid. They have argued that the physical realization of the geoid contains the total permanent tidal deformation, direct and indirect, and, in addition, they voiced concern about the accuracies of the estimates available for the magnitude of the permanent tidal deformation. Using these two arguments, they support the notion that the optimum geoid for geodetic purposes is one with all periodic components removed and all permanent effects in place.

In practice, the geoid is defined on the continents by using gravity anomalies in Stokes' formula. Heikinnen's argument against the Honkasalo correction was based on the premise that the addition of masses outside the geoid caused the gravity potential to lose its harmonicity and prevented the use of Stokes' formula. Šimon and Zeman circumvent this problem by adding the direct permanent tide-generating potential to the normal potential, ie. the potential of the normal ellipsoid. The anomalous potential, being the difference of the actual and normal potentials, is then free of the external masses and is thus harmonic. Since it is the anomalous potential which must be harmonic, Stokes' formula can be safely applied even with the luni-solar masses in place. A recent publication by this group [47] illustrates a procedure allowing users to compute corrections to existing gravity anomalies which will remove the effects of the permanent tide without changing established normal heights.

1.2 EXTENSION OF THE PERMANENT TIDAL THEORY

The following chapters were written with two major goals in mind: first of all, to explain the appropriateness of using a hydrostatic model of the Earth in quantifying the deformation resulting from the permanent tide-generating potential; then to demonstrate how this deformation should be handled in dealing with a number of geodetic parameters. The need for a consistent datum for all geodetic quantities is stressed.

There are several possibilities available for defining the geoid. Past studies have implied that one geoid is required for all the Earth sciences. However, conflicting priorities in, for example, geodesy and geodynamics make such a unified system difficult to realize. Inevitably, compromises are required. Alternatively, two geoids can be used. In geodesy, the geoid should be as close as possible to the actual mean sea level in order to reduce the number of corrections required for geodetic observations. And, more importantly, all geodetic parameters should have one, consistent datum. When studying the rheology of the Earth in a geodynamic setting, a geoid which is isolated in space is most appropriate. Otherwise, the reference Earth models would have to be altered to account for the external masses. In regard to the permanent tidal deformation, the requirements of geodesy and geodynamics clearly contradict one another.

Chapter 2 presents a complete development of the permanent tide-generating potential starting from the three-body problem, on to the total tide-generating potential, then to the permanent tide-generating potential and a qualitative description of the resulting deformation.

Chapter 3 outlines the justification for the use of the fluid Love numbers in describing the permanent tidal deformation, then shows the development of the first-order and second-order theories in the estimation of the fluid Love numbers. The chapter ends by quantifying the effects of the permanent tide using the second-degree fluid Love numbers.

Chapter 4 is a detailed analysis of the effects of the permanent tide on a number of geodetic parameters. Under the assumption that two geoids are required, one for geodesy and one for geodynamics, corrections to the Earth's rotation rate, gravity observations, heights, harmonic coefficients, and satellite orbits are developed so that all these parameters are internally consistent regardless of the geoid being used. Three different geoids are discussed: the geoid recommended by Honkasalo (the mean geoid), the geoid recommended by the IAG at IUGG XVIII (the deformed geoid), and the geoid which is completely free of permanent tidal effects (the zero geoid). The first two are suitable for geodetic purposes; the third is appropriate for geodynamics.

Chapter 2

DEVELOPMENT OF THE PERMANENT TIDAL POTENTIAL

Ocean tides have been a closely observed phenomenon ever since man began using the oceans and seas as a means of transportation. Over the last couple of centuries, the amplitudes and periods of the ocean tides have been carefully studied thereby enabling the prediction of future tides. Very early in the study of the tidal phenomenon, observers noticed the correlations between the periods of the tides and the positions of the sun and moon. Today, these correlations are very well understood because of the long tidal histories available, improved processing and analysis methods, and new and more precise observations of tidal phenomena.

The forces responsible for the deformations of the fluid part of the Earth's surface originate with the gravity fields of the sun and moon. These same forces also act on the solid part of the Earth. Advances in gravimetry and artificial satellite technology have shown that the surface of the solid Earth is deformed in a manner similar to that of the oceans [42]. Estimates of these deformations are often used to estimate physical properties of the solid earth [25].

Careful mathematical development of an expression for the potential at the Earth's surface due to the gravitational fields of the sun and moon shows that there is a component which is permanent (or stationary). In the subsequent development, it will be shown that this permanent tide-generating potential has

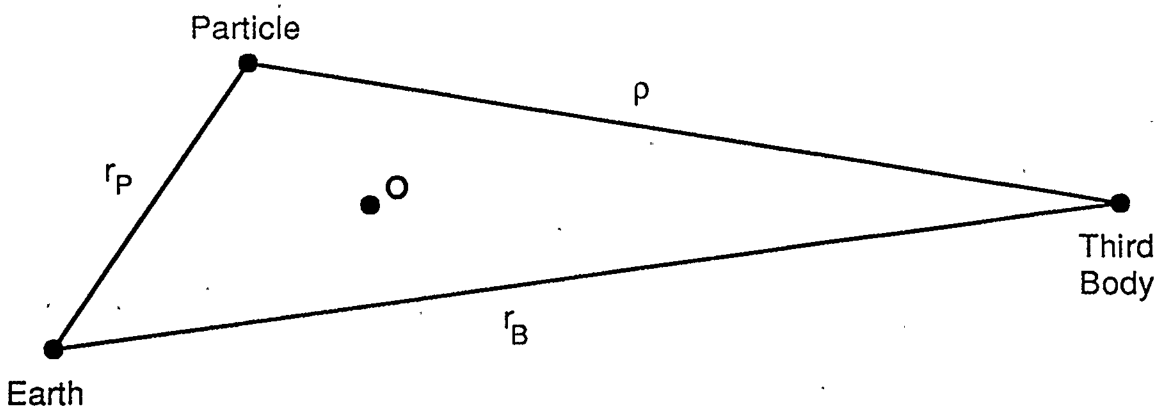

Figure 2.1: The Three-Body Problem

exactly the same kind of effect as the potential due to the rotation of the Earth. The result is an increased flattening of the reference figure of the Earth.

2.1 THE THREE-BODY PROBLEM

The potential field due to the gravity of a celestial body can be derived with the so-called *three-body problem*. In this case, the first body is the Earth, the second is a particle of negligible mass on or near the Earth's surface, and the third body is either the sun or the moon. This three-body system is assumed to be isolated in space. Figure 2.1 illustrates the system.

The centre of mass of the three-body system is at point O in Figure 2.1. A non-rotating reference system centred at O will be inertial. The problem will be simplified by assuming that all bodies are spherically symmetrical and can be considered point masses. This is not true of the acceleration of the particle due to the gravitational attraction of the Earth, but it is an adequate approximation in view of the subsequent development. Using Newton's laws, the equations of motion can be written in this inertial system [18]. Since this is not a very convenient system, the equations of motion are transformed to a Cartesian non-inertial, non-rotating system centred at the centre of mass of the Earth. The acceleration of particle P in this new system is given by

$$\vec{\mathbf{a}}_P = \sum_{i=1}^{3} \left[-\frac{GM_E}{r_P^3} x_i^P + \frac{Gm_B}{\rho^3}(x_i^B - x_i^P) - \frac{Gm_B}{r_B^3} x_i^B \right] \vec{\mathbf{e}}_i, \tag{2.1}$$

where

- subscript P refers to the particle, E refers to the Earth and B refers to the third body,
- $\vec{\mathbf{a}}_P$ is the vector of accelerations,
- G is the Newtonian gravitational constant,
- M_E is the mass of the Earth,
- m_B is the mass of the third body,
- r_P is the distance from E to P,
- ρ is the distance from P to B,
- r_B is the distance from E to B,
- x_i^B is the i^{th} coordinate of P in the Earth-centred system,
- x_i^P is the i^{th} coordinate of B in the Earth-centred system,
- $\vec{\mathbf{e}}_i$ is the unit vector in the i^{th} coordinate direction.

The first term represents the acceleration of the particle due to the gravitational attraction of the Earth; the second term represents the acceleration of the particle due to the gravitational attraction of the third body; and the third term represents the acceleration of the Earth (ie. the coordinate system) due to the gravitational attraction of the third body.

The gravity field is conservative, implying that the accelerations in equation 2.1 can be written as the gradient of a scalar function, ie. potential. Defining the scalar potential at point P due to the three-body configuration as V_P,

$$grad\, V_P = \vec{\mathbf{a}}_P, \tag{2.2}$$

where

$$grad\, V_P = \sum_{i=1}^{3} \frac{\partial V_P}{\partial x_i^P} \vec{\mathbf{e}}_i = \frac{\partial V_P}{\partial x_i^P} \vec{\mathbf{e}}_i.$$

Note that the convention of summation over repeated subscripts will be used from this point on.

By using the knowledge that an infinitesimal change in potential is equal to the work done in moving a unit mass along the infinitesimal vector $d\vec{\mathbf{r}}$, an expression for V_P can be derived (see eg. [18]);

$$V_P = \frac{GM_E}{r_P} + \frac{Gm_B}{\rho} - \frac{Gm_B}{r_B^3} \vec{\mathbf{r}}_P \cdot \vec{\mathbf{r}}_B. \tag{2.3}$$

Implicit in this formula is the assumption that the potential is zero at infinity.

2.2 THE TIDE-GENERATING POTENTIAL DUE TO A THIRD BODY

The primary concern of the current discussion is the potential, U_B, due to the third body. Since we are not presently interested in the central gravitational potential of the Earth, equation 2.3 can be simplified by deleting the first term (which is due to the central field) and expanding the dot product in the last term;

$$U_B = Gm_B \left[\frac{1}{\rho} - \frac{r_P \cos\psi}{r_B^2} \right], \tag{2.4}$$

where ψ is the geocentric angle between P and B. At this point, the problem has effectively been reduced to a two-body problem (the particle and the sun or moon). The reference system is at the centre of mass of the Earth which is accelerating under the influence of the gravitational attraction of the sun or moon.

The $1/\rho$ term can be written in terms of r_P, r_B, and ρ by using the cosine law;

$$\frac{1}{\rho} = \frac{1}{r_B} \left[\left(\frac{r_P}{r_B} \right)^2 - 2\frac{r_P}{r_B} \cos\psi + 1 \right]^{-1/2}. \tag{2.5}$$

Then, by using the binomial expansion, the right-hand side of equation 2.5 can be written as an infinite sum of Legendre polynomials;

$$\frac{1}{\rho} = \frac{1}{r_B} \sum_{n=0}^{\infty} \left(\frac{r_P}{r_B} \right)^n P_n(\cos\psi), \tag{2.6}$$

where P_n is the n^{th} degree Legendre polynomial (see eg. [11]). Substituting equation 2.6 into equation 2.4 and dropping terms which are independent of the position of P (since they do not contribute to the forces at P), the required form of the third-body potential is realized;

$$U_B = \frac{Gm_B}{r_B} \sum_{n=2}^{\infty} \left(\frac{r_P}{r_B} \right)^n P_n(\cos\psi). \tag{2.7}$$

2.3 SIMPLIFICATION OF THE TIDE-GENERATING POTENTIAL

Equation 2.7 developed in the last section is difficult to use for two reasons : it contains an infinite series and the angle ψ is not a convenient coordinate. We begin this section by simplifying U_B in these two areas.

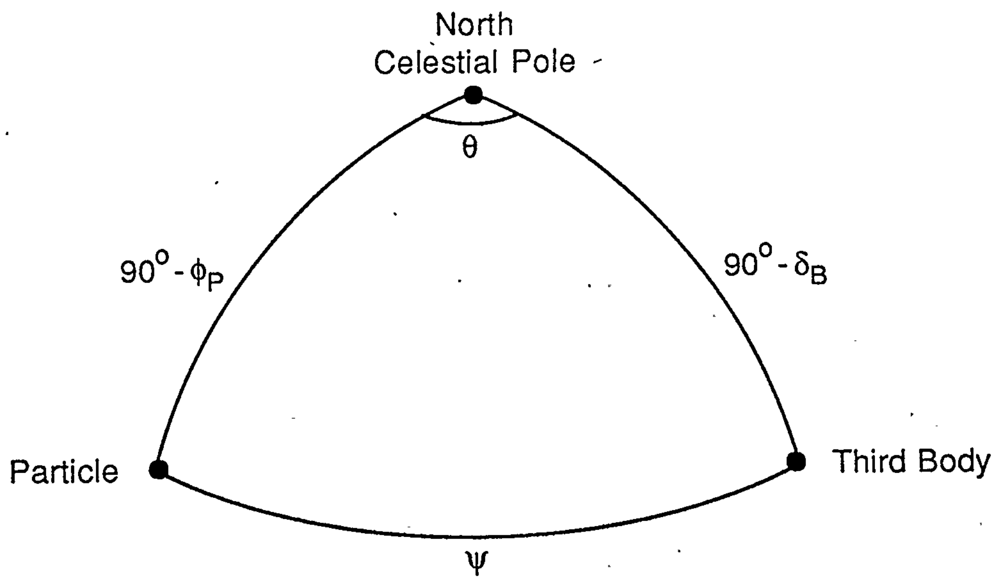

Figure 2.2: **The Spherical Triangle for Coordinate Conversion**

First of all, the geocentric angle ψ will be replaced by more common coordinates. Earth-fixed points, such as P, are often given in terms of latitude (ϕ) and longitude (λ). Celestial points may be conveniently described by their right ascension (α) and declination (δ). By extending all these coordinates up to the celestial sphere and using spherical trigonometry, a function relating ψ with δ and ϕ can be developed. Figure 2.2 shows the spherical triangle used to make the conversion. Using the cosine law,

$$\cos\psi = \sin\phi_P \sin\delta_B + \cos\phi_P \cos\delta_B \cos\theta,$$

where θ is the difference in longitude between P and B.

The $\cos\psi$ term shows up as the argument in the Legendre polynomials. Therefore, the Legendre polynomials must also be converted. The conversion is well-known and can be found in many references (eg. [7], p. 348). For any n,

$$P_n(\cos\psi) = P_n(\sin\phi_P)P_n(\sin\delta_B) + 2\sum_{m=1}^{n} Q_{nm}(\sin\phi_P, \sin\delta_B, \cos m\theta), \tag{2.8}$$

where

$$Q_{nm} = \frac{(n-m)!}{(n+m)!}P_{nm}(\sin\phi_P)P_{nm}(\sin\delta_B)\cos m\theta,$$

and P_{nm} is the Legendre function of degree n, order m (see eg.[11]).

Equation 2.7 contains the expression $(r_P/r_B)^n$ inside the summation. Since $r_P << r_B$, the infinite sum can safely be replaced by second degree terms only [25]. For example, if P is on the surface of the Earth and B is the moon, $r_P/r_m \doteq 0.02$. Note that for the highest Earth-orbiting satellites $r_P/r_m \doteq 0.10$. In this case, higher degree terms may be necessary, but it should be noted that the long-term average of all odd degree terms (eg. n=3) is zero. By limiting the summation to second degree terms and expanding the Legendre polynomials, equation 2.7 can be re-written in the simplified form

$$U_B = \frac{Gm_B}{r_B}\left(\frac{r_P}{r_B}\right)^2 (S+T+Z), \tag{2.9}$$

where $S = \frac{3}{4}\cos^2\phi_P\cos^2\delta_B\cos 2\theta$,

$T = \frac{3}{4}\sin 2\phi_P \sin 2\delta_B \cos\theta$,

$Z = \frac{9}{4}(\sin^2\phi_P - \frac{1}{3})(\sin^2\delta_B - \frac{1}{3})$.

S is the sectorial term (ie. $m = n$); T is the tesseral term (ie. $m \neq n$ and $m \neq 0$); Z is the zonal term (ie. $m = 0$). A brief overview of the effects which these components have on the Earth are summarized in the following paragraphs (see eg. [25]).

The sectorial term separates the Earth into four equal sectors with nodal lines at the meridians 45° to either side of the third body's meridian.The amplitude is therefore zero at the poles. It has its maximum at the equator. Since the period of $\cos 2\theta$ is about half the Earth's rotation rate, these tides are semi-diurnal. Sectorial redistribution of mass does not affect either the position of the rotation axis or the rotation rate (length of day). However, secondary effects of these tides due to internal friction and energy dissipation do contribute to a gradual slowing of the Earth's rotation rate and corresponding secular reduction in the size of the moon's orbit (see eg. [21]).

The tesseral term has nodal lines along meridians at 90° to either side of the third body and also along the equator. Again the function is zero at the poles. However, it reaches its maximum magnitude at 45° latitude. These are diurnal tides. Tesseral mass redistribution causes changes in the instantaneous rotation axis but none in rotation rate.

The third component in the tidal expression of equation 2.9 is the zonal term: it depends only on latitude. The nodal lines are at $\phi_P = \pm 35°16'$. The effect is maximum at the poles. The zonal function is symmetric about the Earth's rotation axis. Its frequency is dependent only on δ_B and r_B whose periods are of the order of weeks to years, indicating that zonal tides have longer periods than sectorial or tesseral tides. Zonal tides have no effect on the position of the rotation axis but do cause length of day fluctuations.

In the next section, the permanent part of the tidal signal will be extracted from the expression for the zonal tides.

2.4 THE PERMANENT TIDE-GENERATING POTENTIAL

In equation 2.9, an expression is given for the second-order terms in the expansion of the tide-generating potential. All higher order terms are assumed to be negligible. This is an adequate assumption at or near the surface of the Earth. Part of this potential is permanent at the Earth's surface. In other words, if the tide-generating potential is averaged over a very long period of time, the result will be nonzero. In this section, the permanent potential will be computed by averaging the the potential of equation 2.9 over a sufficiently long time. This can be done by using the integral equation

$$\overline{U_B} = \frac{1}{\tau} \int_0^\tau U_B \, dt, \tag{2.10}$$

where τ is the longest period in U_B. An overbar will be used to indicate the permanent part of any parameter.

$\overline{U_B}$ will thus be free of all periodic components. Since S and T in equation 2.9 are both functions of θ and assuming $dt/d\theta$ is constant, $\int_0^\tau T\,dt = \int_0^\tau S\,dt = 0$. This is an adequate assumption because θ is dominated by the Earth's rotation rate and because the orbits of the moon and Earth are almost circular and have small inclinations. The permanent tide-generating potential will therefore include only the zonal term.

For a point fixed on the surface of the Earth, the zonal potential is a function of r_B and δ_B. The parameters Gm_B, r_P and ϕ_P are assumed constant. Furthermore, r_P and r_B can be replaced by mean values and δ_B can be simplified.

The geocentric distance, r_P,can be replaced by the spherical approximation

$r_P \doteq R$, where R is the radius of the sphere which has the same volume as the globally best-fitting Earth ellipsoid with semi-major axis, a_e, and flattening, f;

$$R = a_e \sqrt[3]{1-f}.$$

The maximum error introduced by this assumption is $O(10^{-9}\,ms^{-2})$ in gravity which is about 0.3% of the size of the corrections for the permanent tide (see δg_z or δg_d in Table 3.3).

The moon's orbit around the Earth and the Earth's orbit around the sun are both elliptical. The Earth-moon and Earth-sun distances are therefore functions of their respective orbital positions. However, since both orbits have small eccentricities($e_m = 0.0549$ and $e_E = 0.017$ according to [18]), the distances between centres of mass are almost constant, and the mean distance, $\overline{r_B}$, can be substituted for $r_B(t)$ in equation 2.10. Again, the maximum error introduced in gravity is $O(10^{-9} ms^{-2})$. Values for the mean distances are listed in Table 2.1 along with their sources. Note that the Earth-moon distance, r_m, is computed using the formula

$$r_m = \frac{a_e}{\sin \Pi},$$

where $\sin \Pi$ is the sine of the equatorial parallax. The value for $\sin \Pi$ is taken from [25]; the value for a_e is the GRS80 value taken from [13].

Equation 2.10 can now be written in the simplified form

$$\overline{U_B} = \frac{9}{4}\frac{Gm_B}{\overline{r_B}}\left(\frac{R}{\overline{r_B}}\right)^2 (\sin^2 \phi_P - 1/3)\frac{1}{\tau}\int_0^\tau (\sin^2 \delta_B - 1/3)\, dt. \qquad (2.11)$$

To evaluate the term in the integral, it is necessary to consider the spherical triangle in Figure 2.3. Using spherical trigonometry (see eg. [28]),

Table 2.1: Tidal Constants

Parameter	Symbol	Value	Source
Earth semi-major axis	a_e	6 378 137 m	[13]
Inverse Earth flattening	f^{-1}	298.257	[13]
Earth mean radius	R	6 371 000 m	[13]
Earth-sun distance	$\overline{r_s}$	$1.495\ 978\ 70 \cdot 10^8$ km	[1]
Earth-moon distance	$\overline{r_m}$	384 399 km	[25]
G· mass of Earth	GM_E	$3.986\ 005 \cdot 10^{14}\ m^3 s^{-2}$	[1]
G · mass of sun	Gm_s	$1.327\ 124\ 38 \cdot 10^{20}\ m^3 s^{-2}$	[1]
G · mass of moon	Gm_m	$4.902\ 794 \cdot 10^{12}\ m^3 s^{-2}$	[1]
Earth rotation rate	Ω	$7.292\ 115 \cdot 10^{-5}\ rad\ s^{-1}$	[13]
Dynamical form factor	J_2	$1082.63 \cdot 10^{-6}$	[13]
Mean Earth gravity	$\overline{g}$	$9.8203\ ms^{-2}$	[13]
Mean obliquity of ecliptic	$\overline{\epsilon}$	23°21.′5	[44]

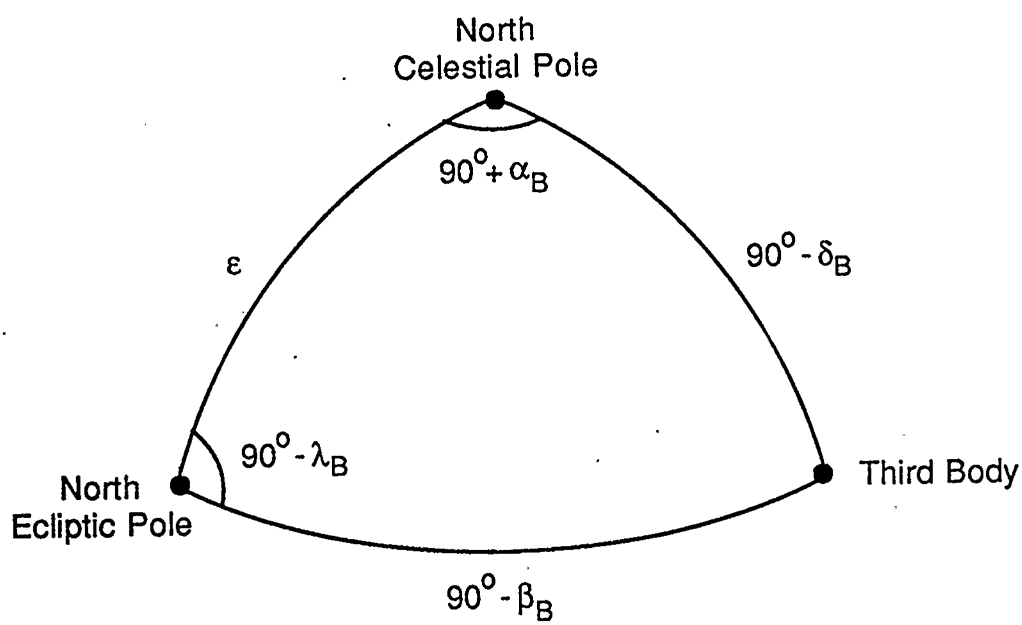

Figure 2.3: Spherical Triangle to Extract Constant Part of δ_B

$$\sin\delta_B = \cos\beta_B \sin\lambda_B \sin\epsilon + \sin\beta_B \cos\epsilon, \tag{2.12}$$

where

- β_B is the ecliptic latitude of B,
- λ_B is the ecliptic longitude of B,
- ϵ is the obliquity of the ecliptic.

For the sun, $\beta_s \doteq 0$. For the moon, β_m has a period of about 28 days, ranges between $\pm 5^\circ.15$ [18] and has a zero mean over the time interval τ. The ecliptic longitudes, λ_s and λ_m, are also periodic with zero means. The obliquity, ϵ, varies from $22^\circ.10$ to $24^\circ.51$ over a period of 41 000 years [44]. Since it changes slowly over a small range, it can be replaced by a constant representative value. To be consistent with previous assumptions, the long-term mean will be used. The mean obliquity is given in [44] and is listed in Table 2.1. In this case, the maximum relative error is about 3%.

Equation 2.11 can now be evaluated by using equation 2.12 and assuming that $\beta_B \doteq \overline{\beta_B} = 0$ and $\epsilon \doteq \overline{\epsilon}$ and $d^2\lambda/dt^2 \doteq 0$;

$$\overline{U_B} = \frac{9}{4}\frac{Gm_B}{\overline{r_B}}\left(\frac{R}{\overline{r_B}}\right)^2 (\sin^2\phi - 1/3)(1/2\sin^2\overline{\epsilon} - 1/3).$$

So that the permanent part of the tide-generating potential due to the masses of the sun and moon is

$$\overline{U} = (K_s + K_m)(3\sin^2\phi - 1) = 2(K_s + K_m)P_{20}(\sin\phi), \tag{2.13}$$

where

$$K_B = \frac{3}{4}\frac{Gm_B}{\overline{r_B}}\left(\frac{R}{\overline{r_B}}\right)^2 (1/2\sin^2\overline{\epsilon} - 1/3).$$

The contributions from the sun and moon can be added because potential is a scalar.

Table 2.2 lists the values of K_s, K_m and K. All parameters in equation 2.13 are listed in Table 2.1. Note that $K_s/K_m \doteq 0.4592$ indicating that the permanent tide-generating potential due to the sun is about one-half that of the moon.

Table 2.2: **Constants for Evaluating Permanent Tidal Potential**

Parameter	Value
K_s	-0.307 40 $m^2\,s^{-2}$
K_m	-0.669 36 $m^2\,s^{-2}$
$K = K_m + K_s$	-0.976 76 $m^2\,s^{-2}$

2.5 THE PERMANENT TIDAL POTENTIAL

In equation 2.13, we now have an expression for the permanent part of the tide-generating potential. If the Earth was a rigid body, this perturbing potential would deform the equipotential surfaces of the Earth but not the physical surface. However, as explained in Chapter 3, the Earth does deform under the influence of applied forces. The deformation response of the solid Earth depends on both the frequency and the wavelengths of the applied forces. At one end of the spectrum, the Earth responds elastically to high frequency disturbances; at the other end of the spectrum, it behaves like an inviscid fluid when exposed to very long-term stresses. The radial deformation due to a disturbing potential of degree n and the additional potential due to this mass rearrangement can be described by the Love numbers, h_n and k_n. The present discussion will begin under the assumption that the disturbances due to the permanent tide-generating potential can be described by second-order fluid Love numbers. A more complete discussion of the Love number operators and the justification for using the present assumption is given in the next chapter.

Let us first consider the effect of the permanent tide-generating potential on gravity measurements taken on the Earth's surface. It is important to understand that since the disturbance is permanent, it cannot be seen in a time series of measurements. In other words, if a series of gravity measurements taken over some period of time are corrected for all other tidal effects, the remaining signal will be constant and no information on the permanent tide can be gained through, for example, spectral analysis. Therefore, at the present time, there is no way of verifying the theoretical results through observation.

It is also important to understand and quantify the permanent tidal potential when the question of choosing the proper reference surface for a certain purpose arises. The most common reference surface in geodesy is the equipotential surface defined by mean sea level. Following past tradition [15], the equipotential surface which most closely approximates the mean sea level will be called the *mean* geoid. Since the permanent tide is independent of time, uncorrected geodetic observations will include effects due to this disturbing potential. However, if certain properties of the Earth are being compared with a model to determine rheological parameters (eg. [32]), then all external effects must be removed (including the direct and indirect effects of the permanent tide). Hereafter, this figure will be referred to as the *zero* geoid [15]. Some geodesists have recommended a geoid which includes the indirect permanent tidal potential but excludes the direct permanent tide-generating potential. This figure will be given the new name of *deformed* geoid.

The deformed geoid has the same physical shape as the mean geoid but has a different potential (as shown in chapter 3). Thus, in one case we want a reference figure which includes all or part of the permanent tidal disturbances, and in the other case we need a reference figure which excludes all external disturbances.

Surface gravity observations are a very common tool in geodesy and geophysics. Through Stokes' equation and the choice of an appropriate normal ellipsoid, they can be used to determine the geoid (see [11]). It is therefore important to be able to correct such observations so they are related to the correct reference surface. The disturbance of the gravity vector due to the direct permanent tide-generating potential given by equation 2.13 can be written as the gradient of the potential (similar to equation 2.2);

$$grad\,\overline{U} = \delta\vec{\mathbf{g}}_d,$$

where $\delta\vec{\mathbf{g}}_d$ is the gravity disturbance due to the direct potential $\overline{U}$. Since $d\overline{U} = grad\,\overline{U} \cdot d\vec{\mathbf{x}}$ and taking (for example) $dx_1 = dr$ (where dr is in the radial direction away from the Earth's centre of mass), the magnitude of the gravity disturbance in the radial direction is

$$\delta g_d = -\frac{d\overline{U}}{dr}. \tag{2.14}$$

Recall that the potential difference between the mean geoid and the deformed geoid is the direct potential ie. $W_d = W_m - \overline{U}_d$ where W_d and W_m are the total potentials referred to the deformed and mean geoid, respectively and $\overline{U}_d \equiv \overline{U}$. This correction, δg_d, should therefore be applied to transform gravity observations which refer to the mean geoid, g_m, to gravity observations which refer to the deformed geoid, g_d : $g_d = g_m - \delta g_d$. Substituting equation 2.13 into equation 2.14 and replacing r by

R to show we are on the surface of the Earth gives

$$\delta g_d = -\frac{2\overline{U}}{R}. \tag{2.15}$$

Similarly, the radial deformation of an equipotential surface of the deformed geoid relative to the corresponding equipotential surface of the mean geoid is

$$u_d^e = \frac{\overline{U}}{g}. \tag{2.16}$$

This result is analogous to Bruns' formula in physical geodesy (see eg. [11]). Since the physical surfaces of the mean geoid and deformed geoid coincide but the equipotential surfaces do not (by equation 2.16), and since, by definition, the mean geoid coincides with mean sea level, it follows that mean sea level is not an equipotential surface in the deformed geoid system. This observation is a common source of confusion.

The physical surface of the real Earth deforms along with the equipotential surfaces under the influence of the permanent tide-generating potential. This physical rearrangement of mass induces a further movement of the equipotential surface as well as a change in the position of an observer who is sitting on the surface. The difference between the deformed geoid and the zero geoid is due solely to the indirect effects of these deformations. The difference between the mean geoid and the zero geoid is due to these indirect effects plus the direct effects described above. The difference in potential between the zero geoid and the mean geoid can be written as

$$\overline{U}_z = \overline{U} + \overline{\delta U} - g u_r, \tag{2.17}$$

where $\overline{\delta U}$ is the potential change due to the mass rearrangement and gu_r is the change in the central gravitational field due to the vertical movement of the observer, u_r. If W is the total gravity potential as before, then $W_z = W_m - \overline{U}_z$.

Love [23] showed that the n^{th} degree terms of $\overline{\delta U}$ and u_r are proportional to the n^{th} degree term of $\overline{U}$. Recall that $\overline{U}$ is given in terms of second degree terms only. Thus,

$$u_r = h_2^f \frac{\overline{U}}{g}, \tag{2.18}$$

and

$$\overline{\delta U} = k_2^f \overline{U}, \tag{2.19}$$

where h_2^f is the second degree displacement fluid Love number and k_2^f is the second degree potential fluid Love number. For the moment, it is sufficient to consider these terms simply as constants. They will be discussed at length in the next chapter.

Substituting equations 2.18 and 2.19 into equation 2.17 gives an expression for $\overline{U}_z$ in terms of $\overline{U}$ only;

$$\overline{U}_z = (1 + k_2^f - h_2^f)\overline{U}. \tag{2.20}$$

The gravity corrections which will transform observed gravity observations referring to the mean geoid to gravity observations referring to the zero geoid can be written in a similar form. Begin by substituting equation 2.17 into equation 2.14,

$$\delta g_z = -\left(\frac{d\overline{U}}{dr} + \frac{d\overline{\delta U}}{dr} - \frac{dg}{dr}u_r\right), \tag{2.21}$$

where $g_z = g_m - \delta g_z$. The first term is the result of masses outside the Earth's surface and is given by equation 2.15. The second term is the result of masses within the Earth's surface. Therefore, according to eg. [18], p. 64,

$$\frac{d(\overline{\delta U})}{dr} = -\frac{3\overline{\delta U}}{R}.$$

And, since

$$g \doteq \frac{GM_E}{R^2},$$
$$\frac{dg}{dr} \doteq -\frac{2g}{R}.$$

Substituting these terms into equation 2.21 yields

$$\delta g_z = -(1 - 3/2k_2^f + h_2^f)\frac{2\overline{U}}{R}. \tag{2.22}$$

Or,using equation 2.15,

$$\delta g_z = (1 - 3/2k_2^f + h_2^f)\delta g_d. \tag{2.23}$$

It is important to recall that this expression describes the gravity disturbance for an observer on the Earth's deforming surface due to the gravitational attraction of the sun and moon. It includes the direct effect (δg_d), the change due to the mass rearrangement ($-3/2\, k_2^f\, \delta g_d$), and the change due to the vertical movement of the observer ($h_2^f\, \delta g_d$).

It is also interesting to note that an equipotential surface of the mean geoid is lifted relative to the zero geoid by an amount equal to

$$u_z^e = (1 + k_2^f)\overline{U}/g. \tag{2.24}$$

Chapter 3

THE EARTH'S RESPONSE TO THE PERMANENT TIDE

Frequencies observed in stresses applied to the Earth range from those of seismic waves [21], to the lag in the Earth's response to periodic tidal perturbations [25], on to the Earth's delayed response to deglaciation [33] and the isostatic compensation of surface topography [16], and finally to the permanent effects due to tidal forces and diurnal rotation. Table 2.3 in [21] shows that dispersive effects are negligible up to periods of 430 days (the Chandler wobble period). However, an estimate of k_2 for long-period tidal perturbations (ie. 18.6 years) in [22] is 20% larger than the current best estimate of the elastic Love number. This evidence indicates that the response of the Earth to applied stress is frequency dependent.

In the past, elastic Love numbers were used to describe disturbances at all tidal frequencies. However, certain geophysical observations indicate that under the influence of very long period disturbances, the Earth behaves like an inviscid fluid. The agreement between predicted and observed rotational deformation supports this hypothesis. Another observation confirming this statement is the lack of correlation between topography and surface gravity measurements. This phenomenon has been studied since the middle of the 19^{th} century when two different models were developed to explain this fact (see eg. [11]). Both models require *isostatic compensation* of the topography: in other words, the topography is considered a mass floating on the fluid surface of the Earth. The models differ in their assumptions regarding the densities of the topography and the denser underlayer. Heiskanen and Moritz [11] state that the Earth is about 90% isostatically compensated, indicating a fluid-like response to long-term loading.

According to [22], the Earth's response approaches that of a fluid at periods of between 20 000 years and 200 000 years, assuming a mantle viscosity of between $2.5 \cdot 10^{21}$ and $2.5 \cdot 10^{22}$ poise. This assumption has been verified by several recent studies: [38] using deglaciation-induced polar wander; [46] using rotational data, polar wander and nontidal deceleration of the Earth's rate of rotation; and [37] using observations to LAGEOS.

In [32], an estimate of J_2 for a hydrostatic model of the Earth was computed and compared to the observed value of J_2 which had been corrected for isostatically compensated topography, permanent tidal deformation and mass anomalies associated with on-going deglaciation. The results indicate that the corrected, real Earth very closely approaches the predicted hydrostatic shape. It is pointed out in [6] that the differences in J_2 are similar in magnitude to some other nonhydrostatic coefficients and need no special explanation. Lambeck, in [20], shows that randomly distributed mass anomalies can contribute to second degree terms of the expansion of the Earth's gravity field.

The permanent tide-generating potential developed in the last chapter is assumed to have an infinite period and is therefore expected to induce a fluid-like response in the Earth. This assumption neglects the 41 000-year period in the obliquity of the ecliptic. However, since changes in the amplitude of the obliquity are small and have little effect on the permanent tide, and since its period is within the limits given by [22], a fluid response can still be safely assumed. The secular decrease in the Earth-moon distance (eg. [21]) has also been ignored in the development of the permanent tide-generating potential. This will produce a very slow secular drift in the constant, K_m (in equation 2.13), but will not affect the periodicity or the assumption of a fluid-like behavior.

Since the Earth reacts like a fluid to the permanent tide-generating potential, fluid Love numbers are required to describe the resulting disturbances. The next section will cover the first-order development of the fluid Love numbers; then, to ensure adequate accuracy in the final results, the development will be expanded to second order.

3.1 THE HYDROSTATIC THEORY OF THE EARTH

The hydrostatic theory of the Earth was first developed by Clairaut in the middle of the 18^{th} century. At that time, it was necessary to assume such an Earth model to allow a determination of the size and shape of the Earth using astronomical observations. However, today we have accurate information describing the Earth's shape, gravity field and, to a lesser degree, density distribution. With this new information, the hydrostatic theory can be tested and some mechanical properties of the Earth can be determined.

For an arbitrary body in the process of deformation, the equations of motion ([16] or [30]) are

$$\rho a_i = \frac{\partial p_{ij}}{\partial x_j} - \rho \frac{\partial W}{\partial x_i}, \tag{3.1}$$

where

- $i, j = 1, 2, 3$,
- ρ is the density,
- a_i is the i^{th} component of the acceleration vector,
- p_{ij} is the stress tensor,
- W is the gravity potential.

If the body is in the hydrostatic state and in equilibrium, then $p_{ij} = p\delta_{ij}$, where δ_{ij} is the Kronecker delta, and $a_i = 0$. It can be shown that, in this situation, the surfaces of equal density coincide with equipotential surfaces. Also, W and p are functions of ρ alone ([30] or [16]).

Now, assume we have an almost spherical, self-gravitating, slowly rotating body in hydrostatic equilibrium. The density distribution within this body is a function of the distance from the centre of mass so that any internal equipotential (or equal density) surface can be written as

$$r = Rx\left[1 + \sum_{n=0}^{\infty} f_{2n} P_{2n}(\cos\theta)\right], \tag{3.2}$$

(see [30]) where

- r and θ are spherical coordinates,
- R is the mean radius of the external equipotential surface,
- x is the mean radius of an internal equipotential surface in units of R,
- $\rho = \rho(x)$,
- $f_{2n} = f_{2n}(x)$.

Note that since the only disturbing potential is due to the body's rotation, the equipotential surfaces can be described by zonal terms alone.

3.1.1 The First-Order Hydrostatic Theory

The first order approximation to equation 3.2 is given by

$$r = Rx\left[1 + f_2(x)P_2(\cos\theta) + O(f_2^2)\right]. \tag{3.3}$$

Kopal [19] shows that, since mass is conserved in the deformation process, $f_0 = -1/5f_2^2$, and, therefore, f_0 does not appear in equation 3.3. The governing differential equations and boundary conditions are given in [30] . Knowledge of the body's mass, rotation rate and density distribution is also required for a first order solution. The solution to the first order problem for the Earth at the external equipotential surface (x=1) is given in [30] and is listed in Table 3.1.

Table 3.1: **Zonal Coefficients of the Hydrostatic Earth**

Order	$-f_2 \cdot 10^3$	$f_4 \cdot 10^6$
First [30]	2.243	—
Second [32]	2.229 257	4.472

Since we now know the deformations due to the disturbing potential, we can derive an expression for the fluid Love numbers. The potential due to the Earth's rotation, Ω, is

$$\Phi = \frac{1}{2}\Omega^2 r^2(1 - \sin^2\phi). \tag{3.4}$$

At the Earth's surface, $x = 1$ and therefore,

$$r^2 = R^2[1 + 2f_2P_2(\sin\phi) + f_2^2P_2^2(\sin\phi)].$$

Neglecting terms of $O(f_2^2)$ and substituting r^2 into equation 3.4 gives

$$\Phi = \frac{1}{2}\Omega^2 R^2[1 + 2f_2P_2(\sin\phi)](1 - \sin^2\phi).$$

Or, since

$$1 - \sin^2\phi = \frac{2}{3}[1 - P_2(\sin\phi)],$$

we can write

$$\Phi = \frac{1}{3}\Omega^2 R^2 \sum_{n=0}^{2} \Phi_{2n} P_{2n}(\sin\phi), \tag{3.5}$$

where

$\Phi_0 = 1 - 2/5 f_2,$

$\Phi_2 = -1 + 10/7 f_2,$

$\Phi_4 = -36/35 f_2.$

The radial deformation due to the rotation can be written as

$$u_r(\Phi) = r - R = R f_2 P_2(\sin\phi). \tag{3.6}$$

We can now substitute the second-degree part of equation 3.5 along with equation 3.6 into equation 2.18. By substituting $\overline{g}$ for g and solving for the second-order displacement Love number, h_2^f, we get

$$h_2^f = \frac{3\overline{g} f_2}{\Omega^2 R(-1 + 10/7 f_2)}.$$

The result obtained by taking the first-order solution for f_2 from Table 3.1 and all other parameters from Table 2.1 is given in Table 3.2.

There is a simple relationship between k_2^f and h_2^f which is important in the subsequent analysis. Equation 2.18 gives the radial deformation of the physical surface referred to the zero geoid relative to the physical surface referred to the mean geoid. In addition, equation 2.24 gives the corresponding deformation of the

Table 3.2: **The Second Degree Fluid Love Numbers**

Order	h_2^f	$k_2^f = h_2^f - 1$
First	1.957	0.957
Second	1.932 448	0.932 448

equipotential surface. Since, by definition, the deformation of the physical surface is equivalent to the deformation of the equipotential surface on a hydrostatic body, $h_2^f = 1 + k_2^f$.

Substituting h_2^f and k_2^f into equation 2.20 shows that the potential at any point on the surface of the zero geoid is equivalent to the potential of the same point on the mean geoid;

$$\overline{U_z} = (1 + k_2^f - h_2^f)\overline{U} = 0. \tag{3.7}$$

In Chapter 2, it was shown that the potential of the deformed geoid is not the same as the potential of the mean geoid. It is, therefore, also different from the potential of the zero geoid.

3.1.2 The Second-Order Hydrostatic Theory

The second-order hydrostatic theory is developed in a manner similar to that of the first-order theory except that terms on the order of f_2^2 are included in the development. This means that the f_0 and f_4 terms must be added;

$$r = Rx[1 + f_0 + f_2 P_2(\sin\phi) + f_4 P_4(\sin\phi) + O(f_2^3)].$$

This second-order problem is solved in [32] assuming that the trace of the Earth's inertia tensor, GM_E, and R are invariant under deformation. Table 3.1 lists the zonal coefficients determined by the second-order external theory of [32].

Again following a procedure similar to that used in the first-order development, the second-order Love numbers can be computed. They are listed in Table 3.2. Using the Love numbers in this table, we can see that using first-order estimates will introduce errors of 0.8% in the gravity disturbance (from equation 2.22), 1.3% in the deformations (from equation 2.18), and 2.6% in the tidal potential (from equation 2.19) .

3.2 DEFORMATION OF THE EARTH DUE TO THE PERMANENT TIDE-GENERATING POTENTIAL

We are now at a point where the disturbances of the Earth's gravity field and shape due to the permanent tide-generating potential can be estimated. In Chapter 2, an expression for the permanent tide-generating potential as well as expressions for the resulting disturbances at the Earth's surface were developed. In the previous section of this chapter, fluid Love numbers were shown to be appropriate in describing the effects of the permanent tide-generating potential on the real Earth. Hydrostatic theory was then utilized to quantify the second-order fluid Love numbers.

Using the development in section 2.5 along with the second order fluid Love numbers derived above, the corrections necessary to transform the reference figure from the mean geoid to the deformed or zero geoid can be quantified. The discussion, at this point, is limited to gravity corrections, surface deformations, and disturbing potentials. Since all observations are assumed to be taken in the mean geoid system (ie. with the periodic tidal effects removed), the mean geoid will be

Table 3.3: **Corrections to Observed Geodetic Parameters**

Correction	Estimate
δg_d	$30.66(3\sin^2\phi - 1)\ \mu gal$
u_d	0
u_d^e	$-0.099(3\sin^2\phi - 1)\ m$
$\overline{U}_d$	$-0.9768(3\sin^2\phi - 1)\ m^2\,s^{-2}$
δg_z	$47.03(3\sin^2\phi - 1)\ \mu gal$
u_z	$-0.192(3\sin^2\phi - 1)\ m$
u_z^e	$-0.192(3\sin^2\phi - 1)\ m$
$\overline{U}_z$	0

used as the basis for all corrections. In every case, the corrected value is equal to the 'observed' (ie. mean geoid) value minus the correction. All terms are defined in section 2.5. The second order Love numbers are listed in Table 3.2. The estimated corrections are given in Table 3.3. In all cases, the corrections have their maximum magnitude at the poles; the magnitude is zero at $\phi = \pm 35°16'$; and at the equator, the correction is one-half the polar value (with opposite sign).

It is interesting to note that the shape of the permanent tidal deformation is the same as the dominant, second-order deformation due to the Earth's rotation. However, $u(\Phi_2)$ is four orders of magnitude larger than $u(\overline{U})$. Just as the Earth's rotation increases its flattening, the mean geoid and deformed geoid have a larger flattening than the zero geoid.

In the past, the elastic Love numbers were used to describe the disturbances due to the permanent tide. However, it has been shown in the preceding section that for long-period tides like the 18.6-year tide or the permanent tide, elastic Love numbers are inappropriate. As an example, the elastic Love numbers, $h_2^e = 0.60$

and $k_2^e = 0.30$, can be used to compute u_z and δg_z giving

$$u_z(elastic) = -0.059(3\sin^2\phi - 1)\ m,$$

and

$$\delta g_z(elastic) = 34.68(3\sin^2\phi - 1)\ \mu gal.$$

The errors introduced by using elastic Love numbers to describe the disturbances due to the permanent tide amount to as much as 26 cm in radial displacement and 23 μgal in gravity. With today's goal of a 10 centimetre geoid, these errors are unacceptable.

Chapter 4

THE DISTURBING EFFECTS OF THE PERMANENT TIDE

As explained in earlier chapters, the permanent tide-generating potential causes fluid-like deformations resulting in changes of the shape of the geoid. The geoid which includes none of the effects of the permanent tide has been named the zero geoid; the geoid which includes the direct and indirect effects of the permanent tide has been named the mean geoid. A third geoid, named the deformed geoid, includes the indirect permanent tidal potential but not the direct permanent tide-generating potential. The deformed geoid has the same physical shape as the mean geoid but has a different potential. The choice of the proper geoid depends on the purpose at hand.

Equally important as the choice of a geoid is the requirement that all parameters and observations be consistent. In the past, there has been a lack of consistency (eg. [13]). The object of this chapter is to attempt to alleviate some of this past confusion and to show how certain geodetic parameters and observations can be corrected so they all refer to the same geoid.

For the purpose of clarifying the subsequent discussion, the total gravity potential of the Earth can be written as

$$W(t) = V + \Phi + U_0(t) + T_0(t) + \overline{U} + \overline{T}, \tag{4.1}$$

where

- V is the gravitational potential due to the mass distribution of the Earth (including the rotational flattening and mass anomalies),
- Φ is the potential due to the Earth's mean rotation,
- $U_0(t)$ is the direct time-dependent tide-generating potential,
- $T_0(t)$ is the tidal potential due to $U_0(t)$ plus the potential due to associated rotational disturbances,
- $\overline{U}$ is the direct permanent tide-generating potential,
- $\overline{T}$ is the permanent tidal potential due to $\overline{U}$ plus the potential due to associated rotational disturbances,
- $U = U_0(t) + \overline{U}$ and $T = T_0(t) + \overline{T}$.

A reference figure of the Earth which is independent of time is the first objective. Therefore, the time-dependent components of W(t) must be removed. The potential, independent of time, is derived by simply subtracting $U_0(t)$ and $T_0(t)$ from equation 4.1 yielding

$$W_m = V + \Phi + \overline{U} + \overline{T}. \tag{4.2}$$

W_m is the potential of the mean geoid. This step is complicated by the present gravity reduction procedure, as discussed in section 4.3.

The permanent tide-generating potential, $\overline{U}$, is due to masses outside of the boundary defined by the physical surface of the Earth and according to Resolution No. 16 of IAG at IUGG XVIII in Hamburg in 1983 [14], it should be removed. The tidal potential, $\overline{T}$, is due to masses within the required boundary and need not be removed at this stage even though the mass displacement giving rise to $\overline{T}$ is caused by external masses. The resulting potential defines a surface which is independent of time and encloses the suggested boundary surface;

$$W_d = V + \Phi + \overline{T}. \tag{4.3}$$

W_d is the potential of the deformed geoid.

In some instances (for example, in geodynamics), a reference Earth which is isolated in space may be required (eg. [32]). If this is the case, $\overline{T}$ must also be removed since it arises from third-body gravitation, and

$$W_z = V + \Phi. \tag{4.4}$$

W_z is the potential of the zero geoid.

It is also important to note that observations of the Earth's gravity field may be affected differently depending on the location of the observer: gravity observations on the surface of the solid Earth are affected by the mass rearrangement due to tidal deformations plus a displacement of the observer; gravity observations above the surface (ie. in an aircraft or space vehicle) are affected only by the mass rearrangement.

4.1 ORBITAL PERTURBATIONS

Earth-orbiting satellites are useful tools in geodesy and geophysics. Since any orbiting satellite is held in orbit by the Earth's gravitational field, satellites can be used to determine long-wavelength features of the gravity field or to determine geophysical properties of the Earth. So once again the need for a consistent reference datum for the gravity field is apparent to allow satellite-derived features or properties to be used with surface data. In this section, a brief summary of satellite orbit dynamics will be given, followed by an analysis of the effects expected on satellite orbital parameters due to the permanent tidal potential. A complete development of satellite orbit dynamics can be found in [17].

4.1.1 Satellite Orbit Dynamics

For the purposes at hand, the total potential of the Earth's gravitational field can be written in the form

$$V = \frac{GM_E}{r} + R,$$

where the disturbing function, R (not to be confused with the mean radius), contains all terms of V except the central part. Any body orbiting a spherically symmetric body (ie. where R=0) will have an elliptical orbit. Therefore, since $R << V$ on the Earth, an elliptical coordinate system will be the most convenient to describe a satellite's orbit around the Earth. The six well-known Keplerian elements (a, e, i, ω, Ω and M) are most often used. Their description can be found in any textbook on orbital dynamics (eg. [17]).

In the presence of a non-central field (ie. one where $R \neq 0$), the Keplerian elements are not constant but change in time. However, where deviations from the unperturbed orbit are small, Keplerian elements are still the most convenient coordinates. The rates of change of the elements in time are now required to allow calculation of the osculating (ie. instantaneous) orbit.

The energy function acting on a satellite is given by

$$F = \frac{GM_E}{r} + R - \kappa = \frac{GM_E}{2a} + R, \tag{4.5}$$

where κ is the kinetic energy of the satellite. Because the satellite motion is described by Keplerian elements, the gravitational potential and equations of motion must be converted to Keplerian elements. The equations of motion in terms of Keplerian elements are

$$[s_\ell, s_k]\frac{ds_k}{dt} = \frac{\partial F}{\partial s_\ell}, \tag{4.6}$$

where the s_i's are the six Keplerian elements of the osculating orbit, and

$$[s_\ell, s_k] = \frac{\partial x_i}{\partial s_\ell}\frac{\partial \dot{x}_i}{\partial s_k} - \frac{\partial \dot{x}_i}{\partial s_\ell}\frac{\partial x_i}{\partial s_k}$$

are the *Lagrange brackets.* The complete set of twelve Lagrange brackets is given in [17]. Substituting the Lagrange brackets into equation 4.6 gives six simultaneous differential equations which are known as the *Lagrange Planetary Equations.*

The next step is to transform $R(\phi, \lambda, r)$ to $R(s_k)$. Again, the complete development is given in [17]. At this point, the discussion will be restricted to the second-degree zonal harmonic term. In Keplerian elements,

$$R_{20} = \frac{GM_E}{r}\left(\frac{a_e}{r}\right)^2 C_{20}\sum_{p=0}^{2} F_{20p}(i)\sum_{q=-2}^{2} G_{2pq}(e)\cos[(2-2p)\omega + (2-2p+q)M],$$

where F_{nmp} is the inclination function and G_{npq} is the eccentricity function.

Since M is periodic over one orbit, secular terms will be independent of M. This implies that when integrating the Lagrange Planetary Equations over long periods of time, the largest perturbations will be due to terms where (2-2p+q)=0. The inclination and eccentricity functions are listed in Tables 1 and 2 in [17]. Since $G_{20-2} = G_{222} = 0$, the only term contributing to the secular perturbations is p=1, q=0. Taking F_{201} and G_{210} from these tables, the part of the second-degree disturbing potential causing secular perturbations in the satellite's orbit is given by

$$R_{20} = \frac{GM_\Gamma}{r}\left(\frac{a_e}{r}\right)^2 C_{20}(3/2\sin^2 i - 1/2)(1-e^2)^{-3/2}.$$

The Lagrange Planetary Equations are now reduced to

$$\begin{aligned}
\dot a &= 0,\\
\dot e &= 0,\\
\dot i &= 0,\\
\dot\omega &= \frac{3na_e^2C_{20}}{4(1-e^2)^2a^2}(1-5\cos^2 i),\\
\dot\Omega &= \frac{3na_e^2C_{20}}{2(1-e^2)^2a^2}\cos i,\\
\dot M &= n - \frac{3na_e^2C_{20}}{4(1-e^2)^{3/2}a^2}(3\cos^2 i - 1),
\end{aligned} \tag{4.7}$$

where $n = \sqrt{GM_E/a^3}$. Observations of $\dot\Omega$ are routinely used to provide estimates of C_{20}.

4.1.2 Orbital Disturbances Due to the Permanent Tidal Deformation

The published value of C_{20} ($= -J_2$) includes the effects of the indirect permanent tidal potential but does not include effects due to the direct permanent tide-generating potential [5]: it refers to the deformed geoid. To correct C_{20} so that it refers to the other two geoids, the following well-known expansion can be used;

$$W(R) = \frac{GM_E}{R}\left[1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\sin\phi)\right].$$

Since $W_m = W_d + \overline{U}$ and $\overline{U} = 2KP_{20}(\sin\phi)$, the correction to get C_{20} referred to the mean geoid is

$$\delta C_{20}^m = -\frac{R}{GM_E}2K.$$

Using numerical values from Tables 2.1 and 2.2,

$$\delta C_{20}^{m} = 3.12 \cdot 10^{-8},$$

where $C_{20}^{m} = C_{20}^{d} + \delta C_{20}^{m} = 1082.661 \cdot 10^{-6}$. Similarly,

$$\delta C_{20}^{z} = \frac{R}{GM_E} 2k_2^f K = -2.91 \cdot 10^{-8}, \tag{4.8}$$

where k_2^f is taken from Table 3.2 and $C_{20}^{z} = C_{20}^{d} + \delta C_{20}^{z} = 1082.601 \cdot 10^{-6}$.

Estimates of δC_{20}^{z} are also given in [36] and in [21] as an errata to [36]. The estimate given in [36], $-2.79 \cdot 10^{-8}$, is very close to that derived here. However, the errata in [21] states that the value quoted in [36] was derived using the elastic Love number and gives a 'corrected' estimate of $-8.69 \cdot 10^{-8}$ which seems to be incorrect. Unfortunately, there is not enough information in either reference to allow any suggestions as to the source of the problem.

By differentiating equations 4.7, the orbital disturbances due to the various components of the permanent tide can be estimated;

$$\begin{aligned}
\delta\dot{\omega} &= \frac{3na_e^2\delta C_{20}}{4(1-e^2)^2a^2}(1-5\cos^2 i), \\
\delta\dot{\Omega} &= \frac{3na_e^2\delta C_{20}}{2(1-e^2)^2a^2}\cos i, \\
\delta\dot{M} &= \frac{3na_e^2\delta C_{20}}{4(1-e^2)^{3/2}a^2}(3\cos^2 i - 1).
\end{aligned} \tag{4.9}$$

To illustrate the magnitude of the errors, the LAGEOS satellite will be used. The necessary orbital parameters for LAGEOS are given in eg. [37]. They are: $a = 1.227 \cdot 10^7$ metres, $e = 0.004$, and $i = 109°.9$. To correct C_{20} to refer to the zero geoid, for example, δC_{20}^{z} should be used. The corresponding corrections to orbit parameters are given by substituting this value into equations 4.9;

$$\delta\dot{\omega}_z = 1.152 \cdot 10^{-12}\, rad\, s^{-1} = 0.''021\, /\, day,$$

$$\delta\dot{\Omega}_z = -1.865 \cdot 10^{-12}\, rad\, s^{-1} = -0.''033\, /\, day,$$

$$\delta\dot{M}_z = 1.787 \cdot 10^{-12}\, rad\, s^{-1} = -0.''032\, /\, day.$$

For these three elements, the fractional change in $\dot{s}_k$ is the same as the fractional change in C_{20}, ie.

$$\frac{\delta \dot{s}_k}{\dot{s}_k} = \frac{\delta C_{20}}{C_{20}} \doteq 3 \cdot 10^{-5}.$$

Similar computations can be performed using δC_{20}^m.

Since the permanent tidal deformation is independent of time, the disturbance in C_{20} does not change with time, ie. $\delta\dot{C}_{20} = 0$. As a consequence, it is easy to see that there are no accelerations in the Keplerian elements due to the permanent tide, ie. $\ddot{s}_k = 0$.

As with other geodetic observations, it is important that observations of satellite orbit perturbations be referred to the proper geoid. The corrections must include the potential changes as well as the displacements of the physical surface if the observer is sitting on the physical surface. Ranging measurements (including satellite laser ranging and satellite altimetry) are particularly sensitive to displacements of the observer (eg. [24]).

4.2 ROTATIONAL DISTURBANCES

The permanent tide-generating potential causes two indirect disturbances in the Earth's gravity field: the disturbance due to the mass rearrangement and the resulting change in rotation rate. The permanent tide-generating potential itself has no direct effect on the Earth's rotation. In this section, a general discussion of rotational dynamics will be followed by an analysis of the effects due specifically to the permanent tidal deformation.

4.2.1 Rotational Dynamics

Only a brief overview of the problem of rotational dynamics will be given here. See [21] or [29] for a more complete discussion. In an inertial reference system, the dynamics of a rotating body are described by the relationship

$$\vec{\mathbf{L}} = \dot{\vec{\mathbf{H}}},$$

where torque is defined by

$$\vec{\mathbf{L}} = \int_v \rho \vec{\mathbf{x}} \wedge \ddot{\vec{\mathbf{x}}} dv$$

and angular momentum by

$$\vec{\mathbf{H}} = \int_v \rho \vec{\mathbf{x}} \wedge \dot{\vec{\mathbf{x}}} dv;$$

and where

- ρ is density,
- $\vec{\mathbf{x}}$ is the position vector in the inertial system, and
- v is the volume of the body.

If we now choose a coordinate system which is fixed to the body and therefore rotating,

$$\vec{\mathbf{L}} = \frac{\partial \vec{\mathbf{H}}}{\partial t} + \vec{\Omega} \wedge \vec{\mathbf{H}}, \tag{4.10}$$

where $\vec{\mathbf{L}}$ and $\vec{\mathbf{H}}$ are now in the rotating system and $\vec{\Omega}$ is the vector of angular velocities with which the axes turn about themselves (not to be confused with the Keplerian element of the last section).

If the deformations within the body are small, it can be assumed that $\vec{\Omega}$ is the same for all particles within the body. Then, in tensor notation,

$$H_i = I_{ij}\Omega_j + h_i, \tag{4.11}$$

where the inertia tensor,

$$I_{ij} = \int_v \rho(x_k x_k \delta_{ij} - x_i x_j)dv,$$

and

$$h_i = \int_v \rho\varepsilon_{ijk} x_j \dot{u}_k dv.$$

- $\dot{u}_k$ is the velocity in the direction of the k^{th} coordinate,
- δ_{ij} is the Kronecker delta,
- ε_{ijk} is the alternating tensor,

where

$$\begin{aligned}\varepsilon_{ijk} = {} & 0 \text{ if any two subscripts are equal,} \\ & = +1 \text{ if the subscripts are in even order (ie. 1,2,3,1,2 \ldots),} \\ & = -1 \text{ if the subscripts are in odd order (ie. 3,2,1,3,2 \ldots).}\end{aligned}$$

Substituting equation 4.11 into equation 4.10 gives the so-called *Liouville equations*;

$$L_i = \frac{\partial}{\partial t}[I_{ij}(t)\Omega_j(t) + h_i(t)] + \varepsilon_{ijk}\Omega_j[I_{kl}(t)\Omega_l(t) + h_k(t)]. \tag{4.12}$$

If the Liouville equations are applied to the rotating Earth, we can assume a mean rotation about the x_3-axis with small perturbations. The rotation vector, $\vec{\Omega}$, can then be written in terms of these perturbations;

$$\Omega_1 = \Omega m_1, \quad \Omega_2 = \Omega m_2, \quad and \quad \Omega_3 = \Omega(1 + m_3).$$

These small perturbations in the rotation imply small perturbations in I_{ij} and h_i as well;

$$I_{11} = A + c_{11}(t), \; I_{22} = A + c_{22}(t), \; I_{33} = C + c_{33}(t),$$

and $I_{ij} = c_{ij}(t), \; i \neq j$.

Substituting these quantities into equation 4.12 and neglecting all terms which include products or squares of the small quantitites c_{ij}/C, m_i, and $h_i/(\Omega C)$, the equations of motion reduce to

$$\begin{aligned} \frac{\dot{m}_1}{\sigma_r} + m_2 &= \psi_2, \\ \frac{\dot{m}_2}{\sigma_r} - m_2 &= -\psi_1, \\ \dot{m}_3 &= \dot{\psi}_3, \end{aligned} \tag{4.13}$$

where

$$\begin{aligned} \psi_1 &= \frac{\Omega^2 c_{13} + \Omega \dot{c}_{23} + \Omega h_1 + \dot{h}_2 - L_2}{\Omega^2(C - A)}, \\ \psi_2 &= \frac{\Omega^2 c_{23} - \Omega \dot{c}_{13} + \Omega h_2 - \dot{h}_1 - L_1}{\Omega^2(C - A)}, \\ \psi_3 &= \frac{-\Omega^2 c_{33} - \Omega h_3 + \Omega \int_0^t L_3 \, dt}{\Omega^2 C}, \\ \sigma_r &= \frac{C - A}{A}\Omega. \end{aligned}$$

The ψ_i terms are known as *excitation functions* and can be used to describe all possible sources of rotational changes. The excitation functions are determined by geophysical observations and theory. The rotational disturbances, m_i, are determined through astronomical observations. The Liouville equations can be used to explain rotational disturbances in terms of geophysical data. Alternatively, they may be used to deduce certain geophysical properties from observations of the Earth's rotational variability (see [21] or [29]). The following section uses the first approach to determine the disturbance in the Earth's rotation resulting from the permanent tide.

4.2.2 The Rotational Disturbance Due to the Permanent Tidal Deformation

Since there is a need for at least two geoids, one with the permanent tidal deformation and one without, all geodetic and geophysical parameters must be defined in terms of the geoid which is appropriate for the purposes at hand. The Earth's rotation rate is one important parameter. Given the disturbing potential (equation 2.13) and the rotational dynamics (equation 4.13), the effect of the permanent tidal deformation on the Earth's mean rotation rate can be computed.

Let us begin by choosing the x_3-axis so that it coincides with the axis of maximum moment of inertia and assume that the permanent tidal potential is the only source of rotational disturbance. Now, since the disturbing deformation, u_r, is rotationally symmetric about x_3, $c_{11} = c_{22}, c_{ij} = 0$ when $i \neq j$, and $m_1 = m_2 = 0$. Since the deformation is permanent, $\dot{u}_k = 0$ which means that $h_i = 0$. In addition, the permanent tidal potential applies no torque to the Earth so that $L_i = 0$. The

Liouville equations (4.13) now reduce to one equation;

$$\dot{m}_3 = \frac{\dot{c}_{33}}{C},$$

or

$$m_3 = \frac{c_{33}}{C}.$$

Recall that

$$\Omega_3 = \Omega(1 + m_3),$$

or

$$m_3 = \frac{\Omega_3 - \Omega}{\Omega} = \frac{\delta\Omega}{\Omega}.$$

It then follows that

$$\delta\Omega = c_{33}\frac{\Omega}{C}. \tag{4.14}$$

Rochester and Smylie [35] showed that the trace of the inertia tensor, I_{ij}, is invariant under a wide range of deformations. In other words, $\sum_i c_{ii} = 0$. From rotational symmetry, $c_{11} = c_{22}$. Therefore, $c_{33} = -2c_{11}$.

The second-degree zonal harmonic coefficient, C_{20}, can be written in terms of I_{ii} (see [21]);

$$C_{20} = -\frac{1}{M_E R^2}[I_{33} - 1/2(I_{11} + I_{22})].$$

Differentiating both sides,

$$dC_{20} = -\frac{1}{M_E R^2}[dI_{33} - 1/2(dI_{11} + dI_{22})].$$

Using the rotational symmetry again, $dI_{11} = dI_{22}$, so that

$$dC_{20} = -\frac{1}{M_E R^2}\left[dI_{33} - \frac{dI_{11}}{dI_{33}}dI_{33}\right].$$

And since $dI_{11}/dI_{33} = c_{11}/c_{33} = -1/2$,

$$dC_{20} = -\frac{3}{2M_E R^2} dI_{33},$$

or

$$\delta I_{33} = c_{33} = -\frac{2}{3} M_E R^2 \, \delta C_{20}. \tag{4.15}$$

We can now get an expression for the change in mean rotation rate due to the permanent tidal deformation by substituting equation 4.15 into equation 4.14;

$$\delta\Omega = -\frac{2}{3}\Omega \, \delta C_{20} \frac{M_E R^2}{C}. \tag{4.16}$$

From [21], $C/(M_E R^2) = 0.3306$. The approach taken here requires that the geoid coincide with mean sea level.

The observed rotation rate is appropriate for the mean geoid or the deformed geoid. As discussed in earlier sections, the deformed geoid has the same mass distribution as the mean geoid, but it is not coincident with mean sea level. Therefore, to compute the rotation rate appropriate for use with the zero geoid, δC_{20} must be computed from the mean geoid to the zero geoid. This can be done in a manner similar to that used in Section 4.1;

$$\delta C_{20}^{mz} = \frac{R}{GM_E} 2(1 + k_2^f) K = -6.03 \cdot 10^{-8},$$

and substituting this vàlue into equation 4.16 ,

$$\delta\Omega_z = 8.873 \cdot 10^{-12} \; rad \; s^{-1},$$

corresponding to a 10.484 millisecond reduction in length of day (l.o.d.) when compared to the observed value. Now,

$$\Omega_z = \Omega + \delta\Omega_z = 7.292\,115\,89 \; rad \; s^{-1},$$

where $\Omega = \Omega_m = \Omega_d$.

Past studies (eg. [4]) computed Ω_z using equation 4.8, giving a reduction in l.o.d. of only 5 milliseconds. This error most likely occurred because the published value of C_{20} contains the indirect effects of the permanent tide but not the direct effects. Thus, δC_{20}^{z} is appropriate for correcting the published C_{20} to take it to the zero geoid, but δC_{20}^{mz} must be used to compute the correction needed to derive the rotation rate for the zero geoid.

4.3 GRAVITATIONAL DISTURBANCES

The corrections required to reduce surface gravity observations (which refer to the mean geoid) to gravity values referring to the deformed or zero geoids have been derived in Chapter 2. Here, a systematic method for applying these gravity corrections will be illustrated and some special cases will be examined.

In equation 2.21, the three components of the gravity corrections used to reduce values on the mean geoid to values on the zero geoid are discussed. The three components are: the part due to the direct potential ($\overline{U}$), the part due to the mass rearrangement ($\overline{\delta U}$), and the part due to the radial displacement of the observer ($-gu_r$). The prefix in equation 2.22, $(1-3/2 \cdot k_2^f + h_2^f)$, is often called the *gravimetric factor* and represents these three parts in order. If a subset is required for some specific purpose (eg. to reduce mean geoid values to the deformed geoid), the three terms can be mixed and matched as necessary. As another example, an observer above the Earth's surface will not experience any displacement and therefore the third term in equation 2.22 should be dropped;

$$\delta \hat{g}_z(r) = -(1 - 3/2\, k_2^f)\frac{2\overline{U}}{r} = -\frac{0.7659}{r}(3\sin^2\phi - 1)\; m\, s^{-2}.$$

Near the surface, $r \doteq R$, $\delta \hat{g}_z(R) = 12.02\,(3\sin^2\phi - 1)\ \mu gal$. For a satellite with an orbit similar to LAGEOS with r= 12 000 kilometres, $\delta \hat{g}_z(r) = 6.38\,(3\sin^2\phi - 1)\ \mu gal$. And for a satellite with an orbit similar to NAVSTAR with r= 26 000 kilometres, $\delta \hat{g}_z(r) = 2.95\,(3\sin^2\phi - 1)\ \mu gal$. The current measurement errors of airborne gravimeters are well above these values (about 1000 microgals according to [43]).

Starting with the total potential given in equation 4.1, the usual tidal corrections to surface gravity observations (δg_1) are applied in an effort to remove the complete tidal spectrum using elastic Love numbers (see [12] or [40]);

$$\delta g_1 = -(1 - 3/2\, k_2^e + h_2^e)\frac{\partial U}{\partial r},$$

where U is given by equation 2.9. Since the elastic gravimetric factor is smaller than the fluid factor (1.15 vs 1.534), only part of the permanent tidal effect is removed in this procedure. The corrected observations, therefore, refer to some intermediate geoid which has no physical meaning, being somewhere between the zero geoid and the mean geoid. To replace that part of the permanent tidal effect removed in the first step, the so-called *Honkasalo correction*, $-\delta g_z(elastic)$, (see [12]) must be applied. The same elastic Love numbers that were used in the first step must be used to compute this correction. For example, the correction given by Honkasalo [12] is $\delta g_z^H(elastic) = 37\,(3\sin^2\phi - 1)\ \mu gal$. A value of $\delta g_z^A(elastic) = 34.68\,(3\sin^2\phi - 1)\ \mu gal$ was derived in section 2.5. This has brought us to an identifiable reference surface, namely, the mean geoid.

To transform the gravity observations to the deformed geoid system (as recommended by IAG [14]), it is necessary to apply the correction, δg_d, given in Table 3.3. We now have the reference potential given by equation 4.3: the direct effects of the permanent tide-generating potential have been eliminated, but the effects due to the mass rearrangement and displacement of the observer are still present in the corrected gravity observations.

The zero geoid, which is the reference figure of an Earth isolated in space, is often required in geophysical applications. The step from the deformed geoid to the zero geoid requires the application of the correction $(\delta g_z - \delta g_d) = 16.10\,(3\sin^2\phi - 1)\ \mu gal$. It is simpler, in practice, to apply the correction δg_z directly to the gravity observations referred to the mean geoid.

The process can be summarized as follows:

1. Collect the raw gravity observations, g_{obs};
2. Apply the usual tidal corrections: $g_1 = g_{obs} - \delta g_1$;
3. Apply the Honkasalo correction to get the mean geoid: $g_m = g_1 + \delta g_z(elastic)$;
4. Remove the direct effect of the external masses to get to the deformed geoid: $g_d = g_m - \delta g_d$; and
5. Remove the effects of the mass rearrangement and displacement of the observer to get to the zero geoid: $g_z = g_d - (\delta g_z - \delta g_d)$; or alternatively: $g_z = g_m - \delta g_z$.

Šimon and Zeman [40] recommend an alternative procedure for tidal gravity corrections which has significant advantages over the present methods. Instead of correcting all tidal frequencies with the same (elastic) gravimetric factor (as in [12]), they recommend a correction of the form

$$\delta g = -\sum_i D_i A_i \cos(\theta_i + \alpha_i),$$

where

- i is a specific tidal frequency,
- D_i is the gravimetric factor for frequency i,
- A_i, θ_i are the amplitude and phase,
- α_i is the phase shift.

One of the advantages of this method is the ability to use the proper Love numbers for each frequency. Three groups would be sufficient to cover all tidal periods: periods from 0 to 430 days, the 18.6-year period, and the permanent tide. The phase lags of the periodic tides (see eg. [25]) can be included in the α_i term. In addition, particular frequencies can be added or dropped from the summation as required. For example, to remove all the effects of the permanent tide from the observations, the permanent tidal correction (eg. δg_z) would be included; to leave the permanent tidal effects in the observations, the permanent tidal correction would be removed from the summation. Similarly, part of the permanent tidal effects could be removed and part could be left in the gravity values (using eg. δg_d).

The tilt of the equipotential surfaces relative to the physical surface as a result of tidal disturbances is developed in eg. [25]. Along a meridian, it has the form

$$\delta g_h = (1 + k - h)\frac{1}{gR}\frac{\partial U}{\partial \phi}. \tag{4.17}$$

In the present case, $1 + k_2^f - h_2^f \doteq 0$, so that $\delta g_h = 0$. Tilt of the equipotential surfaces relative to the distant stars involves the Shida number ℓ and is beyond the scope of this discussion. Again reference is made to [25] for details.

Resolution No. 15 passed by the International Association of Geodesy (IAG) at the XVII General Assembly of the International Union of Geodesy and Geophysics in Canberra, December 2–14, 1979 states [13]:

> **considering** the importance of a simple and unambiguous way of treating the permanent tidal deformation due to the attraction of the Sun and Moon,
>
> **resolves** that the tidal effect be removed completely from all geodetic observations, without restoring the permanent deformation, and that, consequently, the so-called *Honkasalo correction* should not be applied to observed gravity.

This resolution recommended the adoption of a geoid which includes the non-elastic part of the permanent tidal potential. It is internally inconsistent in requiring that the tidal effect be completely removed and that the Honkasalo correction not be applied. As shown above, the Honkasalo correction must be applied whether or not the permanent tidal potential is to be included in the geoid. The inconsistency arises from the incorrect assumption that the Earth responds elastically to disturbances at all frequencies.

The International Gravity Standardization Net 1971 (IGSN71) was originally compiled with the Honkasalo correction applied. Upon the passage of Resolution No. 15, the IAG recommended that IGSN71 gravity data be 'corrected' by removing the Honkasalo correction (see [13]). As illustrated in the preceding discussion, the resulting geoid has no physical meaning.

The next IUGG General Assembly in Hamburg in 1983 partially corrected this situation by passing IAG Resolution No. 16 which suggested that the direct permanent tidal effects should be removed and the indirect effects should be left in all gravity values (ie. the deformed geoid was recommended).

4.4 HEIGHT ERRORS DUE TO THE PERMANENT TIDAL POTENTIAL

In geodesy, two basic height systems are used : ellipsoidal heights which use a mathematical figure (the ellipsoid) as datum and spirit levelled heights which use a physical surface (the geoid) as datum. Because of the different datum types, the permanent tide affects the two systems differently.

4.4.1 Definition of the Normal Ellipsoid and Ellipsoid Heights

An ellipsoid height is defined as the distance above some pre-determined ellipsoid of revolution measured along the ellipsoid normal. The permanent tide has an effect on certain parameters used to define a reference ellipsoid, so it may also affect ellipsoidal heights. However, re-defining a reference ellipsoid to better deal with permanent tidal effects can easily be accommodated by a knowledgeable user.

A reference ellipsoid is defined by its origin, axes orientation and four other independent parameters. For example, GRS80 is centred at the Earth's centre of mass, its z-axis passes through the Conventional International Origin, and the primary meridian passes through the zero meridian of the Bureau International de l'Heure adopted longitudes. The four independent parameters are GM_E, J_2, Ω, and a_e. These four fundamental constants were recommended by the Special Study Group No. 5.39 of IAG [27]. GM_E is the parameter which provides the scale for the GRS80 system. It is independent of the permanent tide. J_2, Ω and a_e are perturbed by the effects of the permanent tide-generating potential. The perturbations in all three have been computed in earlier sections. Note that $\delta J_2 = -\delta C_{20}$. Also, from Table 3.3, $\delta a_e^z = u_z(\phi = 0) = 0.192$ metres. As the need for a 10 centimetre geoid becomes greater, care will be required to ensure that the permanent tidal deformation is properly dealt with in defining a_e. With satellite observations being used almost exclusively for the estimation of a_e, the situation can be confusing. The positions of ground stations tracking any satellite will be disturbed by the total permanent tidal deformation as given by u_z in Table 3.3. Usually, the direct effect of the permanent tide-generating potential is removed from satellite orbits [5]. There is, therefore, the possibility of inconsistencies in the estimates of a_e unless the data reduction process is fully understood and carefully handled. This problem occurs in many satellite observation schemes and will become increasingly important as accuracies increase.

One of the main objectives of geodesy is the determination of the Earth's gravity field (which consists of the gravitational potential and the rotational potential). The rotational potential is not harmonic so the gravity potential of the real Earth is not harmonic either. Geodetic boundary value problems are linearized by defining a normal reference figure (eg. an ellipsoid) which closely approximates the geoid. The difference between the actual potential and the normal potential is called the anomalous potential. To solve the geodetic boundary value problems, the anomalous potential must be harmonic. By defining the normal ellipsoid so that it includes the disturbances due to the Earth's rotation, the anomalous potential can be made harmonic even if uncorrected geodetic observations (eg. gravity anomalies) are used. The effects of the permanent tide are similar to the rotational disturbances but with much smaller magnitudes. Therefore, the same procedure can be used for the permanent tide as is currently used for the rotation (ie. use uncorrected observations and add the effects of the permanent tide to the normal ellipsoid).

Current and past recommendations of the IAG do not take advantage of this possibility. IAG Resolution No. 15 passed at Canberra in 1979 [13] recommended that gravity observations be corrected to remove the total tidal potential (including the permanent part). At Hamburg, in 1983, the IAG passed another resolution which recommended that the direct permanent tide-generating potential should be removed from gravity observations and the indirect permanent tidal potential should remain. This second recommendation is essentially a compromise solution which removes the external masses which cause the Earth's gravity potential to lose its harmonicity but leaves the mass rearrangement due to those masses in order to keep the geoid closer to the actual mean sea level.

The Czech group (eg. [47]) has recommended that gravity observations should not be corrected for any permanent tide effects, direct or indirect. The anomalous potential will remain harmonic if, as a minimum, the direct permanent tide-generating potential is added to the normal ellipsoid.

Table 4.1 lists all possible combinations of observations and normal ellipsoids

Table 4.1: **Possible Geodetic Observations and Normal Ellipsoids**

Option	Observations	Ellipsoid	Anomalous Potential	Reference
1.	D+I	D+I	—	—
2.	D+I	D	I	[39],[40],[47]
3.	I	I	—	[14]
4.	I	—	I	[14]
5.	—	—	—	[13]
6.	—	I	-I	[13]

which will maintain an harmonic anomalous potential. The D's in the table indicate where the direct permanent tide effects are included, and the I's indicate where the indirect effects are included. The first two options in the table refer to the mean geoid, options 3 and 4 refer to the deformed geoid, and the last two options refer to the zero geoid.

The first option in Table 4.1 has not been recommended in the past. However, it has some advantages over the other options when used as the geodetic datum. Its main advantages are:

1. The normal ellipsoid parameters can be determined using uncorrected observations.

2. Geodetic observations do not have to be corrected for the permanent tide effects.

3. The geoid coincides with mean sea level.

4. The Earth's surface potential is not changed in the presence or absence of the total direct and indirect permanent tide effects because $\overline{U_z} = 0$.

The most important advantages apply equally to the first two options in Table 4.1. Only advantage 1 is unique to option 1, and the advantage is small relative to option 2: as shown earlier, the four fundamental constants of the normal ellipsoid can be transformed fairly easily to whatever system is required. However, if the normal ellipsoid is being re-defined, the direct and indirect effects of the permanent tide should be included if only for the sake of consistency.

The last two advantages listed above are significant in the following discussion of spirit levelled heights. The last advantage also has implications in studies of other geodynamic phenomena. For example, Zeman and Šimon [48] compute the effect of the permanent ocean tide on the solid Earth tide. Since the physical surface deforms just as much as the equipotential surfaces (in the mean geoid system), the permanent Earth tide has the same magnitude as the permanent ocean tide and no additional loading due to the permanent ocean tide would be expected. However, in [48], elastic Love numbers were used, resulting in non-zero estimates of the indirect disturbances on the Earth tide due to the ocean tide. If fluid Love numbers are substituted into the development of [48], then all indirect permanent ocean tide effects are zero.

To be consistent with other geodetic parameters, ellipsoid heights should be corrected as indicated under the 'Observations' column of Table 4.1. Disturbances of ellipsoid heights due to the permanent tide are caused entirely by the indirect permanent tidal deformation. Therefore, if observations are to include the indirect effects, no corrections to ellipsoid heights are required. For example, if option 1, 2, 3 or 4 is being used in the geodetic system, then observed, uncorrected ellipsoid heights should be used. If option 5 or 6 is being used, then all effects due to the permanent tide should be removed from observed ellipsoid heights using u_z in Table 3.3. In doing so, the permanent tidal deformation is being removed from the physical surface of the Earth. Ellipsoid heights are derived exclusively from satellite observations. Therefore, as mentioned above, precautions are necessary to ensure that the permanent tide is handled consistently in computing satellite orbits and ground coordinates.

4.4.2 Spirit Levelled Heights

The primary observables in spirit levelling are the small levelling increments, δn, measured along a levelling line. However, because the equipotential surfaces within the Earth are not parallel, $\delta n \neq \delta H$ (where δH is the corresponding change in height). In fact, there is no simple, direct geometrical relationship between δn and δH. In addition, $\oint dn \neq 0$ indicating that $\int dn$ is path dependent. This path dependency is a serious deficiency in a height system and leads us to try to find an alternative approach. See [11] for a more complete discussion of the problem of spirit levelled heights.

While there is no simple relationship between changes in height and the levelling increments, there is one between changes in potential and the levelling increments: $dW = -g\,dn$. The potential change along a levelling line between points A and B can be determined by integration;

$$W_B - W_A = -\int_A^B g\,dn \,. \tag{4.18}$$

It is thus necessary to observe both the levelling increments and the gravity along the line from A to B to compute the potential change. By adding the gravity observations, we overcome a major obstacle: now, $\oint g\,dn = 0$ so that $\int g\,dn$ is independent of the path.

Taking B in equation 4.18 to be a point on the geoid, then

$$C_A = W_0 - W_A = \int_0^A g\,dn\;, \tag{4.19}$$

where C_A is the *geopotential number* of A. The geopotential number is the basis of all spirit levelled heights. Orthometric height is defined as the distance from the surface point, A, along the plumbline to the corresponding point on the geoid, A_0. The orthometric height of A, H_A, is related to the C_A through the expression

$$H_A = \frac{C_A}{\tilde{g}}, \tag{4.20}$$

where

$$\tilde{g} = \frac{1}{H}\int_0^H g\,dH$$

is the mean gravity between A_0 and A.

Since the geoid defined by uncorrected geodetic observations (ie. the mean geoid) is not necessarily the geoid required as the reference figure, corrections to observed spirit levelled heights may be required. The following discussion will quantify these corrections.

If the zero geoid is the required reference figure, the geopotential number is unchanged relative to the mean geoid (because $W_z = W_m$). Any change in height will therefore be due to changes in gravity (if any). Dynamic heights will change only if the normal gravity changes. Normal heights will change with normal gravity and with surface deformations. Since the geopotential number does not change, the change in height is inversely proportional to the change in gravity (from equation 4.20). Using orthometric heights as an example, the change in height when changing from the mean geoid to the zero geoid is

$$\delta H_z = \frac{\delta g_z}{\tilde{g}} H.$$

Since $\delta g_z \leq 9.4 \cdot 10^{-7}\ ms^{-2}$ and $H \leq 10^4\ m$, $\delta H_z \leq 10^{-3}\ m$. However, since the limits on δg_z and H cannot occur simultaneously on the real Earth, a more realistic limit is $\delta H_z \leq 10^{-4}\ m$. This error is smaller than other systematic errors in spirit levelling so it is safe to conclude that there are no significant changes in spirit levelled heights between the mean geoid and the zero geoid.

The second case to be considered is the height difference between the mean geoid and the deformed geoid. In this case, since the potential changes (ie. $\overline{U}_d \neq 0$), the geopotential numbers will change. In addition, while the Earth's physical surface does not deform (relative to the mean geoid), the gravity values will again change. Differentiating equation 4.20,

$$\delta H_d = \frac{\tilde{g}\delta C_A - C_A \delta\tilde{g}}{\tilde{g}^2},$$

where

$$\delta C_A = \overline{U}_0 - \overline{U}_A = 3K(\sin^2\phi_0 - \sin^2\phi_A),$$

and (from Table 3.3)

$$\delta\tilde{g} = \delta g_d = 30.66(3\sin^2\phi - 1)\ \mu gal.$$

The maximum magnitude of δH_d can be estimated by applying some simplifications. Let $\tilde{g} \doteq 9.82\ m\,s^{-2}$ and $C_A \doteq gH \doteq 9.82H\ m^2\,s^{-2}$. Then,

$$\delta H_d \doteq \frac{\delta C_A}{9.82} - \frac{H\delta\tilde{g}}{9.82}.$$

Taking $H \leq 10^4$, the maximum magnitude of the second term is about $6 \cdot 10^{-4}$ metres and is insignificant. The first term has a maximum magnitude of about 0.30 metres. Again, this is unrealistically large since it implies a levelling line from the equator to the poles or vica versa. More realistically, using $\phi_0 = 45°$ and $\phi_A = 55°$, $\delta H_d \doteq -0.05$ metres. This may still be a significant effect for very precise spirit levelling lines with large changes in latitude. δH_d is dependent on the difference in latitude between the origin of the levelling line, ϕ_0, and the latitude of the point, ϕ_A. If $\phi_0 = \phi_A$, then $\delta H_d \doteq 0$.

Implicit in the above discussion is the assumption that all observations are referred to the same datum. If, as an example, the zero geoid is the required datum and the gravity observations refer to the mean geoid, then a systematic effect on the order of 10^{-4} metres will result. Again, errors of this magnitude are insignificant.

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

The permanent tide is a small disturbance which affects the shape and gravity potential of the Earth. It is stationary in time and dependent on latitude. It cannot be directly observed. However, since the tidal forces can be accurately modelled, the direct permanent tide-generating potential can be accurately computed. The Earth is a deformable body which, under the influence of very long-term disturbances (such as the permanent tide), behaves like an inviscid fluid. The indirect effects of the permanent tide due to these fluid-like deformations can be described by fluid Love numbers. A second order theory developed in [32] was used in Chapter 3 to compute accurate estimates of the fluid Love numbers. The second degree displacement fluid Love number, h_2^f, was calculated to be 1.932 448, and the second degree potential fluid Love number, $k_2^f = h_2^f - 1 = 0.932\ 448$. These estimates require some assumptions regarding physical properties of the Earth. Since the fluid Love numbers cannot be verified by observation, there has been a reluctance on the part of some people to use them at all (eg. [14] and [8]).

Partially because of this reluctance, there are at least three different geoids arising from different treatments of the permanent tide. The IAG recommends that the geoid should include the indirect effects of the permanent tide (to keep the physical surface coincident with mean sea level) but should not include the direct effects (since the direct potential is not harmonic). This has been named the deformed geoid. A Czech group has reinforced Honkasalo's original concept

[12] by suggesting that the geoid should contain the direct and indirect effects of the permanent tide (eg. [47]). This is usually called the mean geoid. This group has argued that use of the mean geoid eliminates the need for permanent tidal corrections of geodetic observations. And, since harmonicity is required only of the anomalous potential, if the direct effects are included in the normal ellipsoid, the anomalous potential will maintain it harmonicity. These two geoids are most suitable for geodesy. Neither requires any knowledge of the Earth's response to the permanent tide. However, the mean geoid has several advantages over the deformed geoid when used for geodetic purposes. The most important of these advantages is that geodetic observations do not have to be corrected for the effects of the permanent tide when the mean geoid system is used. In addition, the mean geoid coincides with mean sea level, and the potential on the mean geoid is the same as the potential on the zero geoid. The deformed geoid is defined so that its physical surface is the same as the physical surface of the mean geoid, but the potential of the deformed geoid is different from the potential of the mean geoid. Thus, the deformed geoid does not coincide with mean sea level.

A third geoid, called the zero geoid, is more appropriate for geodynamics and geophysics. The zero geoid is free from all effects of the permanent tide, direct and indirect. It is the geoid representing the Earth isolated in space. The effects of external masses will corrupt analyses of the Earth's rheology, rotation, shape, etc.

Previous authors have advanced the notion that a single geoid is required for all the geosciences. Since the needs and objectives of, for example, geodesy and geodynamics are often contradictory, the recommendation that different geoids be used for different purposes is presented here. In particular, the mean geoid (with a corresponding normal ellipsoid which includes the effects of, at least, the direct permanent tide-generating potential) is recommended for geodesy, and the zero geoid is recommended for geodynamics and global geophysics.

Surface observations are considered to be collected in the mean geoid system. Corrections to transform gravity observations and heights from the mean geoid to the zero geoid are derived in Chapter 4. Assuming fluid Love numbers, the gravity correction is $\delta g_z = 47.03(3\sin^2\phi - 1)\ \mu gal$, and the correction to orthometric heights, $\delta H_z \doteq 0$, where $g_z = g_m - \delta g_z$ and $H_z \doteq H_m$. Similar corrections are given to transform these observations to the deformed geoid system.

The estimated radial surface deformation due to the permanent tide is given by $u_z = -0.192(3\sin^2\phi - 1)$ metres where the radial geocentric distance, $r_z = r_m - u_z$. This shows that there is an additional flattening of the mean geoid relative to the zero geoid. The shape of this deformation is the same as the rotational deformation but with much smaller magnitude.

Satellite-based observations are generally taken in the deformed geoid system (they include the indirect effects but not the direct effects). Great care must be taken to ensure that all aspects of satellite observations (eg. satellite coordinates and ground station coordinates) are in the same system. Corrections to orbit parameters and to C_{20} are also given in Chapter 4: $\delta C_{20}^z = -2.91 \cdot 10^{-8}$ and $\delta C_{20}^m = 3.12 \cdot 10^{-8}$, where $C_{20}^i = C_{20}^d + \delta C_{20}^i$.

The mean rotation of the Earth is also different for different geoids because of the changes in the inertia tensor due to the mass displacements. Since the mean geoid and deformed geoid have the same physical shape, they also have the same rotation rate. This is the observed value. The correction for the zero geoid is

$\delta\Omega_z = -8.873 \cdot 10^{-12}\, rad\, s^{-1}$ where $\Omega_z = \Omega_m - \delta\Omega_z = \Omega_d - \delta\Omega_z$. This corresponds to a 10.484 millisecond reduction in length of day. Errors in past computations of $\delta\Omega_z$ (eg. [4]) were due to the use of the factor k_2^f instead of $(1 + k_2^f) = h_2^f$. It is the shape of the physical surface of the Earth which determines the rotation rate and not the shape of the equipotential surface.

It is very important that all observations and constants in a geodetic system be referred to the same datum. With the information contained in the previous chapters, this is readily achieved. The normal ellipsoid used to linearize geodetic boundary value problems is defined by its origin, orientation and four independent parameters: GM_E, a_e, Ω and J_2. The first is not dependent on the reference figure and supplies the scale to the system. The second, a_e, depends on the physical deformation at the equator. Corrections to the last two parameters, Ω and J_2 have also been calculated. The normal ellipsoid can, therefore, be defined as required by the constraints of the geoid chosen and the need to maintain the harmonicity of the anomalous potential. As explained above, geodetic surface and satellite observations can be corrected so they refer to the proper geoid.

In summary, one primary purpose of this thesis is to clarify some of the past confusion surrounding the permanent tide. As observational accurracies increase, a consistent treatment of the permanent tide becomes more important. Two geoids have been recommended: one for geodesy which includes the direct and indirect effects of the permanent tide and another for geodynamics and geophysics which is completely free of permanent tidal effects. These conclusions do not agree with the current recommendations of the IAG, but they are well-founded on the arguments presented here.

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