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Assessing Alternative Optimum Bus Operations Strategies Considering Route Demand, Pattern, and Crowding

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Assessing Alternative Optimum Bus Operations Strategies Considering Route Demand, Pattern, and Crowding

by

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ABSTRACT

This thesis developed four mathematical models using continuum approximation and applied them to an urban bus route. Alternative operating strategies are compared to conventional all-stop operations. The comparison of skip-stop and express-local schemes with stop skipping designs and on-demand strategies with flexible stopping patterns are conducted to determine the most efficient bus operating service under various conditions. Each alternative has a total cost that includes walking time, waiting time, in-vehicle travel time, and transferring between lines, in addition to operating costs. This thesis also considers the impact of the COVID-19 pandemic on public transit. As a result, we incorporated the crowding disutility based on the loading factor and the denied boarding costs into the optimization models. First, we solved the theoretical case and determined the most efficient bus operating strategy for various ranges of passenger demand and average trip length. Next, we solved the continuous optimization models by optimizing bus headway and stop spacing. Additionally, this thesis conducted sensitivity analyses of various conditions, including determining the most efficient strategy under fleet size constraints as well as the sensitivity of passengers to crowding and travel times using numerical examples. The solution proposed in this thesis is responsive to changes in demand, trip patterns, and passenger sensitivity to cost components. The model is applied to a bus route in Calgary, Canada, and provides an optimal bus dispatching scheme for two scenarios, with and without considering crowding discomfort. Results show that on-demand services have the lowest generalized costs in scenarios with low demand. In the case of higher demand and longer trips, conventional all-stop systems are preferred. Under high demand and longer passenger trips, skip-stop and express-local services can lower overall system costs. Considering crowding measures, the lowest cost alternative option shifts from

conventional services to strategies with stop-skipping designs, such as skip-stop and express-local policies. Express-local strategy dominates other services when the fleet size is limited, and crowding is considered.

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Chapter 1: Introduction

1.1 Background

Public transit provides a sustainable mode of transportation that encourages a positive shift from private cars to more environment-friendly modes and helps to improve air quality, especially in densely urbanized areas. Disruptions may, however, alter transit system demand and trip patterns. There are many factors changing public transit features, including extreme weather conditions, fluctuating demand, and changes in travel patterns over time and space.

The COVID-19 pandemic, decrease or increase in gas prices, and more working from home are disrupting passenger travel patterns and changing demand levels. Public transit may also be affected by passengers' sensitivity to a variety of factors; for example, during harsh weather, waiting at open bus stops can be uncomfortable, while during the COVID-19 epidemic, passengers may be sensitive to crowding and proximity to others inside buses.

Increasing costs of running conventional transit services and reduced revenue when demand decreased have forced some agencies to cut or decrease their services. In order to lower their costs, transit agencies reduced their services and, in some cases, stopped operating several bus routes. In addition, transit agencies face new challenges due to the unpredictability of disruptions such as pandemics, gas prices, commuting demand, and transit use.

Different disruptions will have different impacts over time and space. Due to uncertainty relating to demand levels and passenger travel patterns, transit operations planning needs to be more adaptive. Traditional transit schemes are the most convenient and preferable form of service because the vehicles stop at all the stops along the transit corridor, but do not perform well at very low or high levels of demand. However, the traditional transit system is not very adaptive, especially with the conventional all-stop system. It can either be very expensive for the service provider or provide a very low level of service for its passengers. Along with unpredicted disruptions, transit demand and trip patterns are changing from peak to off-peak hours, and from rural to urban areas. Public transit is further affected by this, as is the need for a safe and reliable service, specifically for those who rely on public transit as their only mode of transportation.

As such, the purpose of this study is to examine the overall system's efficiency including both passenger and operator costs by developing adaptive strategies. The choice of most suitable transit service, among a set of adaptive strategies, can vary with the stage of the pandemic and new variants of the virus, overall demand and demand patterns, trip length patterns, and the level of users’ concerns and perceived risk.

1.2 Research motivation

Passenger demand and trip patterns are changing over time and space. For different times of day (e.g., peak, and off-peak periods), days of the week (e.g., weekdays and weekends), and different times of year (e.g., holiday season), demand for public transit, and patterns of transit trips are changing. Gas price fluctuations, COVID-19 pandemics, and more working from home also alter transit demand and trip patterns. While COVID-19 disruption decreased transit demand due to more sensitivity of passengers to crowding and duration of exposure, disruptions such as an increase in gas price would attract more users to the transit system.

As an example of COVID-19 disruption, some agencies reduced their service or suspended some lines due to the high costs of operating vehicles for a limited number of passengers. This, however, would not be a long-term solution for agencies and would decrease transit attraction for users.

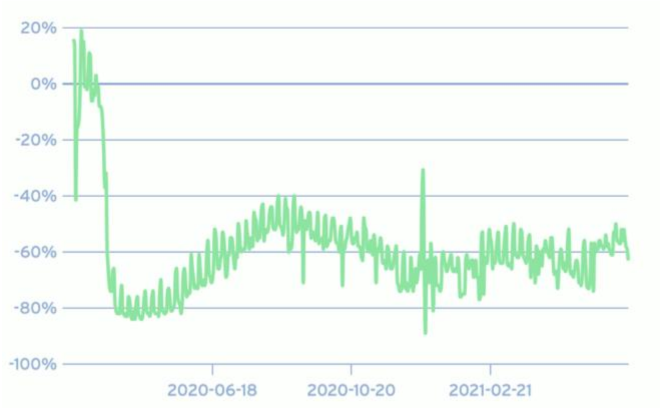


Figure 1.1. Change in public transit demand in Calgary[1]

Figure 1.1 illustrates the variation in transit demand in Calgary and the impact of COVID-19 on the transit demand since Spring 2020. As a result, a comprehensive solution is required to address pandemic conditions without reducing bus capacities and adjust to demand variations.

Additionally, transit demand, passengers' sensitivity to crowding and duration of exposure, and travel patterns are changing during different phases of the pandemic. In addition to considering pandemic disruptions, an adaptive tool would also be beneficial for dealing with low-demand areas or demand based on time of day, weekday, and year. Any changes in transit features and different disruptions, increase the need for an adaptive tool to determine the most efficient type of operating strategy. As a result, transit planners will be able to adjust to changes in ridership, and users will be able to travel with a safe and reliable mode of transportation.

1.3 Research objective

The objective of this thesis is to find the most efficient bus operating strategy in response to the different disruptions. This thesis proposes a framework consisting of four different bus operating services: all-stop, skip-stop, express-local and on-demand strategy. The proposed

[1] https://transitapp.com/coronavirus

framework is designed to adapt to sudden changes in demand as well as passengers' sensitivity to waiting, walking, riding time, and loading factor levels. In addition, this study considers the impact of trip patterns and sensitivity of passengers to different components of transit trips. Particularly, the study considered impact of COVID-19 by considering onboard crowding and denied boarding costs. The cost function for each alternative service includes the cost of agency and passengers and optimizes bus headway and stop spacing.

In this thesis, we are creating a framework consisting of several transit services that will be adaptive to unforeseen changes in demand. The scope of this thesis is not to attract transit users, but based on the findings of this study, agencies can determine the bus operation strategy that will minimize costs and can be adapted to changes in demand levels and trip patterns (e.g. trip length).

1.4 Contribution

The present thesis has successfully developed and implemented four alternative operating services under varying demand, passenger trip length, and users' cost components. The study compares alternative transit services with fixed stops and on-demand services with flexible stops is conducted in this study. Additionally, the effects of disruptions on total users' costs, and increased sensitivity to riding time and crowding are examined. The denied boarding cost is also included in the comparison models.

This thesis follows the adopted fashion in the literature on continuous models (Wirasinghe & Ghoneim, 1981; Kikuchi & Vuchic, 1982; Daganzo, 1984; Gu et al., 2016). Past literature including Gu et al. (2016) developed all-stop, skip-stop, and express-local models to minimize the total generalized cost of the system. They neglected to consider the differences in users' cost factors (such as waiting, walking, and travel time) and minimized users' average time spent. In addition, the operating cost in the optimization model only considers the cost of operating one vehicle for an hour and one kilometer. The following important extensions to the literature are proposed in this thesis: (i) distinguishing between users' costs, such as walking, waiting, riding, and transferring, to describe impacts on users' perceptions under different disruptions such as COVID-19, (ii) developing an on-demand cost function and adding to the comparison framework, (iii) considering the total time of operating vehicles for the agency costs, (iv) considering the onboard crowding penalty, and (v) the cost of rejecting passengers due to capacity constraints. Extensions can provide valuable insight into the impact of COVID-19 on operating public transit and users' susceptibility.

1.5 Thesis organization

This thesis is organized into five chapters as below.

Chapter One provides general background information, research motivation, research objectives, thesis contributions, as well as a thesis organization.

Chapter Two presents a comprehensive literature review of public transit systems, alternative operating strategies, crowding in transit vehicles, impacts of COVID-19 on public transit, effective measures to cope with passengers' sensitivity to COVID-19, and a discussion of the literature review.

Chapter Three describes the main assumptions of this study and each cost component of alternative operating strategies including walking, waiting, riding, transfer, crowding, denied boarding and operating costs. Also, describes the development of four alternative strategies (mathematical model) to optimize the bus headway and stop spacing of each service.

Chapter Four describes the application of a comparison framework, by comparing alternative services for different demand and trip lengths. The most efficient service, as well as optimal bus headway and stop spacing for various services, is determined. The models' sensitivity to different levels of in-vehicle riding time with and without costs of crowding and denied boarding is also analyzed. We conduct a case study on a bus line in Calgary, and compare the ridership levels for 2019, 2020, and 2021.

Chapter Five presents the summary of results, as well as limitations and recommendations for future work.

Chapter 2: Literature review

This chapter reviews the literature relevant to various transit operating services optimizing total generalized costs. The aim of this thesis is to investigate alternative operating strategies and apply them along the transit line. Using a comparison framework, we also aim to determine the most effective operating strategy under various conditions. Additionally, this thesis applies the COVID-19 pandemic conditions to select the most desirable option during the outbreak. Thus, we also review studies that propose solutions to COVID-19 disruptions to address users' concerns about public transit use during pandemics.

This chapter is organized as follows: section 2.1 investigates fixed -route alternative transit services, section 2.2. describes flex-route transit models, section 2.3. reviews studies with comparison models, section 2.4. summarizes the methods to address COVID-19 effects on public transit services, and section 2.5. provides a discussion of the literature.

2.1 Fixed-route operating strategies

In busy transit corridors, fixed-route services, are the most cost-effective type of operations because of predetermined routes, schedules, and high capacity of vehicles (Quadrifoglio et al., 2007). In the fixed-route transit systems, passengers need to walk a certain distance to the nearest fixed stop to take the fixed-route services (Wirasinghe & Ghoneim, 1981). Esfeh et al. (2022) report half of the headway waiting time is commonly assumed in the literature, but this might not be realistic. Waiting time cost is affected by the type of schedule (scheduled or frequency-based), the frequency of the service (high-frequency or low-frequency), and passengers (planning or non-planning) (Esfeh et al., 2022).

Several alternative transit services are defined in this category of operations with different patterns of dwelling at stops. Vehicles operating in conventional all-stop service are dwelling at all the stops in the corridor. However, other services, namely skip-stop and express-local, are scheduled to reside at a set of stops that is less than or equal to the number of stops. In the following subsections, we describe the mentioned services.

2.1.1 Skip-stop service

In the skip-stop strategy, buses only visit a fixed subset of stops to increase operating speed, decrease passenger travel time, and avoid overcrowding. This strategy can be applied when a bus is behind schedule or maintaining a passenger loading factor below a predetermined value. There are several patterns for skipping the stops in this policy. In one pattern, multiple routes visit transfer stops, and other stops are shared between the routes in such a way that each non-transfer stop is served by only one of the routes. (Suh et al., 2002; Freyss et al., 2013; Gu et al. 2016; Abdelhafiez et al., 2017). An example of this type of skip-stop policy, Yang et al. (2019) presented an AB-type skip-stop operating strategy minimizing total passenger travel time to reduce excessive waiting times at transfer stations. The authors considered that passengers can always board the first-arriving train, and overcrowding is not an issue in their work. In response to the COVID-19 pandemic, Salama and McGarvey (2022) proposed an AB-type of skip-stop strategy and minimized users' travel times. The researchers evaluated the trade-off between passenger time in the vehicle as a safety measure and the number of direct trips as a measure of passengers' satisfaction. Their results show that skip-stop strategies can reduce ride time in vehicles by 34% when compared with conventional all-stop strategies.

In Freyss et al., (2013), continuous models were used to examine scenarios of skip-stop service in which metro trains on each of two routes served all non-transfer stops and all transfer stops along a single-track corridor. Based on the assumptions in the work, stop spacing is given and limited passenger capacity is not considered. Gu et al. (2016) was inspired by Freyss et al., (2013) to propose their skip-stop scheme with more than two routes coexisting along a corridor. In such a strategy, stops are divided into four categories. Their study also allows for schedule coordination of routes at transfer stations which reduces passenger transfer costs. Gu et al.'s model is constrained by limited passenger-carrying capacity and a minimum headway to avoid bus queueing at transfer stations. Results illustrate that in moderately high travel demand in bus routes to very busy rail systems, skip-stop service can reduce the system's total cost by about 10% compared to the conventional all-stop service.

Stop-skipping approaches generally address the problem by considering the passenger-related costs, such as walking time, waiting time, in-vehicle travel time, etc., and the overall costs of operating buses (Leiva et al., 2010; Freyss et al., 2013; Gu et al., 2016; Fan et al, 2018). Leiva et al. (2010) proposed an optimization problem using a skip-stop strategy minimizing user and agency costs by optimizing the line frequency. The authors considered three different scenarios: (i) no capacity constraints and considering transfers, (ii) considering both capacity constraints and transfers, and (iii) considering different vehicle types. Results revealed a 10% reduction in cost function value for all scenarios compared with situations that skip-stop is not implemented in the line. Also, they found that when passengers have longer trip lengths and higher demand variability, skip-stop service operates better in the system.

Fu et al. (2003) proposed a real-time stop-skipping operating strategy minimizing total users' waiting time. First, they find the line patterns based on the previous two trips and actual passenger demand. Then they determine the stops that need to be skipped with a rolling time horizon approach. The authors argued that the skip-stop strategy is operating more effectively in a situation with higher passenger demand, shorter headway, and moderate variability in passenger trip time.

Some previous works studied skip-stop service as a control strategy. Lack of control strategies in the system would result in bus bunching, increased headway variance, and consequently, users' waiting times (Ibarra-Rojas et al., 2015). To solve these issues in transit systems, Liu et al. (2013) tried to address skip-stop and deadheading problems on a bus line by minimizing passengers' waiting and riding time and the total operating cost of bus company using random travel time between every two stops. Their model forced the vehicles to serve the stations which have been skipped by the last bus. Skip-stop service also guarantees a minimum level of service and is easy to implement (Fu et al., 2003). Passengers can be informed of this service schedule by monitoring stop-skipping information so that passengers would board the right bus to reach their destinations. Stop-skipping can correct service inconsistencies due to the inherent travel time and passenger demand variations but might increase passenger waiting time at the skipped stations (Chen et al., 2015). Rail congestion has also been reduced by using skip-stop strategies. Using skip-stop reduces runtimes and passenger journey times, and increases train productivity and service reliability, according to Zhou et al. (2020). Using a microscopic agent-based simulation model, they assessed the effectiveness of different strategies for improving the performance of heavy rail systems.

2.1.2 Express-local service

With the express-local strategy, two independent routes serve one bus route, so that local bus routes visit all stops along the line, while express bus routes visit a subset of stops (Gu et al., 2016; Li et al., 2019; Feng et al., 2020; Teng et al., 2021; Tang et al., 2021). By increasing the number of trips between suburb areas to the city center, the need to provide an efficient service to passengers increases. Emerging this new type of urbanization and longer passengers' trips enhance the attractiveness of express-local service.

Teng et al. (2021) focused on the varying departure times and dwell times of express/local trains. Their study assumes that the express trains only visit the high-demand stations and will overtake other stops, resulting in longer dwell time for the local trains at the overtaking stops with lower demand. Then, they proposed an improved Multinomial logit model to solve the problem of express/local trains to build the passenger flow assignment. Their express/local mode noticeably decreases passenger in-vehicle travel time but increases their waiting time compared to conventional all-stop mode. Teng et al. (2021) also analyzed the sensitivity of passengers to levels of crowding and its impact on their route choice.

Li et al. (2019) developed a mixed-integer non-linear programming model to jointly set train schedules and stopping patterns from minimizing passenger ride time in an express/local setting. Using this strategy, passengers' travel time was reported be decreased by 11.31% in the real-world case study, while increasing users' waiting times at skipped stations.

Thilakaratne and Wirasinghe (2016) proposed converting a long regular bus route into a bus route with full length and a BRT route along a segment of the corridor with limited stops. They optimize the headways of regular and BRT routes and choose the termini of BRT segments and BRT stops. As a result of this transition, the average speed and maximum flow of passengers on the route can be increased.

Another type of express service is zonal stopping. In this system, a corridor is divided into several zones. There is an inbound zone express route that picks up passengers within its area and heads to the CBD (Central Business District), skipping all other stops in between, and an outbound route that does the reverse. By collecting trips with similar origins, destinations, and departure times, zonal express can minimize passengers' travel and transfer time (Furth, 1986; Larrain et al., 2015; Moon et al., 2021).

Since the COVID-19 pandemic has emerged, passengers are now more sensitive to onboard crowding, and maintaining social distance has also become a critical issue on public transit. Based on the pandemic-imposed capacity, Gkiotsalitis (2021) developed an algorithm to predict dynamically the number of stops a public transportation vehicle will skip. Through reviewing previous research on skip-stop and express-local strategies, they can suggest alternative bus operating strategies to reduce the adverse health effects of COVID-19. This study tries to propose route options to transit users. Thus, they can take the shortest route to decrease their travel time and the duration of exposure in the age of the COVID-19 pandemic.

2.2 Flexible-route operating strategies

When demand is sparse in an area or fluctuates over the time of a day, the flexible-route strategy works well (Daganzo, 1984; Koffman, 2004; Quadrifoglio et al., 2008; Shen et al., 2017; Wang et al., 2018). Demand fluctuates by time of day (peak and off-peak hours), by day of the week (weekdays or weekends) and by time of year (e.g., holidays). Therefore, demand could vary not only across different areas, but also at different times.

Using fixed-route strategies would lead to longer travel times and walking distances, demonstrating the need for a flexible service type. Changing the transit network design is not a feasible solution to every challenge planners face every year (Ceder & Wilson, 1986). Flexible-route strategies provide door-to-door service to passengers who want to be picked up or dropped off at their desired location, as well as to those with physical disabilities (Quadrifoglio et al., 2008).

According to Kim and Schonfeld (2012), agencies may switch between conventional and flexible modes of transportation as demand fluctuates over time. They proposed incorporating fixed-route and flex-route operations in a hybrid transit system. Their system provides both fixed and flexible services, which is beneficial for both passengers and transit agencies.

In a hybrid context, Sipetas and Gonzales (2021) proposed a flexible hybrid service on a fixed route corridor. It allows vehicles to deviate from the fixed route to serve passengers in a flexible manner. Their study assumes many-to-one trip patterns for passengers, and they identify two types of passengers: (i) fixed-stop users who board or alight at fixed stops along the fixed route and (ii) curb-to-curb passengers who are either picked up or dropped off curb-to-curb by a vehicle that deviates to the passengers' desired locations upon curb-to-curb requests. It was also assumed that there would be no service request rejection, and that vehicles would return to the fixed line after serving each curb-to-curb request. This assumption would result in higher operating costs for the agency and make their on-demand service design less appealing.

A demand-responsive connector which is also known as a feeder service is one of the flexible-route transit services. Quadrifoglio and Li (2009) proposed an analytical modeling framework to find the most appropriate service between fixed-route transit (FRT) and demand-responsive connector (DRC) under different conditions and when the agency needs to switch between two policies based on the derived critical demand. The critical demand densities depend on a wide range of factors, like region layout, vehicle speed, and travel times. They analyzed one-vehicle and two-vehicle cases, and they assumed that dwell time at fixed stops is longer at flexible stops.

2.3 Selection of different schemes

Several previous studies compared conventional fixed-route transit services with flexible-route schemes based on demand density (Luo & Schonfeld 2007; Chien et al. 2010; Alshalalfah & Shalaby 2011; Nourbakhsh & Ouyang 2012; Edwards & Watkins 2013; Qiu et al. 2015 b). In these models, a critical demand density was determined to switch between fixed and flexible schemes and select the most effective operating strategy.

Most of previous studies compared alternative transit schemes in an empirical method by focusing on case studies with fixed location of stops, limiting in parametric analysis (Tétreault et al., 2010; Leiva et al., 2010). However, few studies investigated alternative transit strategies operating with parametric analysis. According to Kikuchi and Vuchic (1982), three alternatives transit services were compared using a numerical example based on the demand density. In their study, a continuum approximation was used to develop all-stop, stop on-call, and stop on-demand. The comparison model provides a demand-driven strategy for selecting the most cost-efficient service considering the agency and user costs.

Daganzo (1984) compared three alternative operating strategies with a continuous approach: (i) Fixed-route transit (FRT), (ii) door-to-door dial-a-ride transit (DDDART), and (iii) checkpoints dial-a-ride transit (CPDART). Door-to-door dial-a-ride services provide an on-demand service to pick up and drop off passengers at their desired locations. In a checkpoint dial-a-ride system, characteristics of both fixed-route (FRT) and on-demand systems (DDDART) are combined. There are a limited number of checkpoints along the routes where passengers can be picked up and dropped off on demand, like a door-to-door system. When requests for boarding or alighting at checkpoints are not made, checkpoint stops can be skipped, and routes shortened. Passengers must walk to and from the checkpoints, so this system is similar to fixed routes. Their research indicates that with higher demand, the FRT system is most cost-effective, and when demand is lower, the on-demand system (DDDART) is most efficient. Fixed-route services provide the same level of service as checkpoint dial-a-ride, but without requiring transfers; therefore, Daganzo (1984) concluded that checkpoint services aren't as attractive to both passengers and transit agencies when only 5% of the total cost is saved compared to FRT. In other words, CPDART will only compete with FRT and DDDART at moderate demand levels.

More recently, Gu et al. (2016) compared skip-stop and express-local strategies with the conventional all-stop service. By comparing these strategies for a variety of circumstances, passengers could choose between schemes for minimizing travel time. By using continuous models, they minimized the total generalized costs of each scheme to determine the lowest generalized cost options, under varying combinations of demand and the average passengers' travel distance for wealthy cities as well as those with lower wages. According to their study, rail transit systems are difficult to implement skip-stop and express-local service. Most studies in this area examined how express-local strategies could be applied on rail transit systems, but Gu et al. (2016) provided an analysis of the costs of implementing these strategies on bus, BRT, and railways. Based on their results, all-stop service remains the lowest-cost option in lower demand. Also, with skip-stop schemes, BRT is more cost-effective for more cases, and express-local services are more efficient in rail systems to accommodate high demand levels.

2.4 COVID-19 measures in public transit

Emerging COVID-19 disease significantly affected the public transit system in urban areas. The COVID-19 pandemic has rapidly transformed ridership behavior all over the world. Low-income groups of people are probably the most susceptible group to economic shifts who rely on transit service more than other groups (Wilbur et al. 2020). When COVID-19 remains a concern, agencies are encouraged to evaluate the current situation and determine an adaptable policy in order to ensure that passengers receive adequate and reliable service. At early stages of the COVID-19 pandemic, some surveys indicated that public transit is the riskiest mode of transportation, since close contact between people in transit equipment and stations cannot always be avoided (Tirachini & Cats, 2020). Several factors are contributing to enhancing the risk of contagion at public transit stations and in vehicles environment; (i) shared surfaces such as seats, doors, handrails, and other surfaces that easily transfer viruses and germs, (ii) passengers are traveling in small and confined places, the more onboard crowding, the more risk of contamination, and (iii) lack of rapid access control to identify sick passengers and workers (UITP, 2020).

In the event of an outbreak, new variants of the virus, business closures, or staff working at home, passengers refused to use transit, which resulted in agencies losing revenue. The dramatic drop in transit ridership forced transit agencies to implement new regulations, alter their service spans, cancel some routes, and close selected stops. For example, the night tube service was canceled, and 40 metro stations were closed in Transport for London (TfL) as a result of the reduced ridership. In addition, 19 metro stations were closed by Washington Metropolitan Area Transit Authority (WMATA) and service frequencies were reduced from 10 trains per hour to 3–4 trains per hour during peak hours. They also cut the services after 9 pm to limit the operation hours (WMATA, 2020). Operators of trains and buses have reduced their frequencies to less than one-third of pre-pandemic levels (Tan. 2020). Some service providers imposed capacity limits to ensure that distance regulations are respected. The Regional Transportation District of Denver restricted passengers to 15 per city bus and 30 per train (Hughes, 2020). Furthermore, due to staff getting infected with the new variant of COVID-19 (omicron variant), Calgary Transit had to make unexpected service changes and reduce services. Bus routes are mostly affected, with some being cancelled or coming less frequently (Calgary News, 2022).

A survey was conducted in the city of Gdansk, in Poland, to investigate the impacts of COVID-19 on behavioral perception and attitudes towards passengers' willingness to return to transit travels. Their results revealed that 75% of them want to return to transit when the pandemic situation becomes stable. This further indicates that the return of travelers to transit use depends on the levels of safety and perceived convenience during the pandemic (Przybylowski et al., 2021).

Pollock et al. (2021) recently conducted a Stated Preference (SP) survey in Calgary, Canada, to capture passengers' behavior toward transit use. They used a Bayesian D-efficient design to generate the SP scenarios and used the survey results to estimate a multinomial logit model. The results of their estimation model showed that transit agencies can attract users by decreasing in-vehicle crowding and implementing mandatory mask-wearing policies (Pollock et al., 2021).

2.4.1 Modelling the spread of disease

Various studies have investigated the role of public transportation in the spread of airborne diseases. Goscé and Johansson (2018) explored the link between the use of public transportation and the spread of airborne infections in urban environments. They studied a large number of journeys on the London underground, which is crowded at certain times, and used Oyster card (electronic tickets) data to find passengers' routes. From TfL (Transport for London), they mathematically derive the time it takes to move in a system of two connected stations, then extend it to the whole line. Therefore, they evaluate the number of contacts and new infections in some selected stations, then used real data on influenza-like illnesses (ILI) to show the correlations between the use of the underground and new ILI infections. Also, their model applies to the very early stages of contagion in environments smaller than the usual scale. They showed a correlation between the use of the London underground and the spread of ILI, mainly showing this correlation in environments with high numbers of infections. Areas with higher ILI cases are also areas whose inhabitants spend more time in the underground network.

Qian et al. (2021) presented a multi-city investigation of communicable diseases percolating among metro travelers. They used smart card data from three megacities in China and developed a simulation model based on the observed metro network layout, demand density, and mobility patterns. This model was used to build the contact networks, and based on this network, the spread of disease can be modeled. Their approach first constructs the contact networks during travel and then embeds the disease percolation process among travelers into the contact network. Qian et al. (2021) aimed to identify how human mobility shapes contact networks during users' travel and observe its subsequent impacts on the threshold of disease contagion among passengers. They concluded that the travel time and the contact duration are positively correlated. That is, travelers with a longer contact duration would have a longer travel time.

Kucharski and Cats (2020) developed an epidemiological model to examine 3000 ride-sharing travelers in Amsterdam. They considered a contact network instead of the shareability graph. The existing contact network is obtained through bipartite matching, where travelers are assigned to a ride. However, links in the shareability graph represent potential rather than the actual connection between travelers. They determined a minimum threshold of 60 seconds for their model to get infected by contacts. Infected travelers go to quarantine and stop traveling, which means all rides that include them as a member will become invalid. Therefore, the number of feasible rides will decrease. They considered the following assumptions: 1) Drivers do not contribute to the virus spreading, and 2) travelers make a maximum of two trips per day. Travellers might become less likely to share, yet ridesharing may offer a less crowded and still affordable alternative to mass transit. Therefore, it is important for service providers to offer less-crowded services to attract more passengers.

As a short-term solution to the airborne disease problem, the impacts of confined spaces and indoor places are also investigated. Following the emergence of COVID-19, some regulations have been recommended, such as maintaining physical distance, wearing masks, washing hands, and sanitizing surfaces. In many cities, public transit agencies followed these regulations by regularly cleaning the transit vehicles and shared surfaces, blocking some seats to maintain social distancing, placing some screens between the driver and passengers, and improving ventilation. To increase ridership, Pollock et al. (2021) advised transit agencies to adopt masking policies and decrease in-vehicle crowding. Additionally, the authors argued that safety policies, such as backdoor boarding and daily deep cleaning, would not be likely to attract transit riders. These steps, however, can only provide short-term solutions and cannot be used as an adaptive measure to address any pandemic situation.

2.4.2 Crowding penalty

Avoiding crowded conditions in the public transit system can prevent the spread of the virus. Public transit policies aim to increase ridership; however, COVID-19 can make it unsafe due to crowded areas, close contact, and closed areas (Kim et al., 2021). The crowding in the public transit system has three impacts on users: 1) onboard discomfort, 2) denied boarding, and 3) service irregularities (Cats et al., 2016). Although human contact remains inevitable in the public transit system, distancing measures can be applied to maintain adequate distances between passengers and reduce crowding (Musselwhite et al, 2020; Tirachini & Cats, 2020). After the COVID-19 pandemic, fear of infection, more teleworking, proving online services, and adapting to the changes in travel behavior, would discourage passengers from transit use and result in a shift from public transit to private modes and active modes of transportation (Abdullah et al., 2020; Moslem et al, 2020)

It is necessary to manage the crowding in the public transit system at the operational level to reduce the negative effects of infectious diseases (Tirachini & Cats, 2020). Wardman and Whelan (2011) investigated the influence of crowding and loading factor on the value of time spent sitting or standing. According to their findings, crowding affects the value of time spent seated when the load factor is around 50%. Devasurendra et al. (2022) solve the crowding problem by finding the optimum headway, considering a trade-off between the users' and agency costs, in the context of COVID-19 health-related issues. They modeled the perceived health risks using the crowding penalty factor, and the associated crowding discomfort is assumed to be affected by the severity of a pandemic phase. The congestion in public transit causes crowding, denied boardings, and decreased service reliability. Hence, transit agencies are interested in capturing denied boarding to minimize the number of passengers who are left behind. (Cats et al., 2016; Ma et al., 2019; Yap & Cats, 2021).

Klumpenhouwer and Wirasinghe (2016) developed two optimization models that minimize total generalized cost and a cost of crowding model that is derived from supporting psychological research. Their models find the optimal length of trains and platforms for many-to-one commuting trips. They considered a linear and a quadratic relationship between crowding cost and users' in-vehicle riding time cost. Their research shows that crowding has an effect on operational decisions. For example, when demand grows, longer train vehicles are needed, but longer trains will increase the system's operating costs.

Tirachini et al. (2013) reviewed the impact of crowding on the quality and comfort of users' trips such as in-vehicle travel cost-saving, waiting cost, and passengers' wellbeing as well as operating and planning decisions namely vehicle size, route choice, fare, optimal frequency, and operating speed. Authors found that an increase in levels of crowding would enhance passengers' anxiety, stress, and feeling of risk to their personal safety and security. They also tried to illustrate the impact of crowding on the estimated value of travel time savings and demand prediction. The data from Sydney were used to predict the crowding cost function based on the number of available seats and density of standees per square meter.

Hörcher et al. (2017) presented a method to estimate the crowding cost of passengers in a transit system according to a revealed preference route choice framework. They used APC (Automated Passengers Counting) and AVL (Automated Vehicle Location) data to control the fluctuation in the level of crowding over the entire metro journey. The model also controls the variation of the probability of finding a seat and the density of standing passengers per square meter. Their results revealed that the stated choice model might overestimate the crowding cost, compared to the revealed preference values.

Cats et al. (2016) presented a dynamic representation of demand and supply in a transit system; thus, it was possible to track variation in passenger loading under varying vehicle capacities. They proposed a cost-benefit analysis consisting of three travel times affected by congestion: (i) more waiting time because of denied boarding, (ii) longer waiting time because of irregularity in service schedules, and (iii) additional discomfort due to onboard crowding. Their results showed that the dynamic congestion model added 60% to the cost benefits, while conventional static models only capture the travel time savings. Moreover, their findings showed that failing to express the dynamic congestion model might underestimate the system's benefits, which are essentially designed to increase the vehicles' capacity rather than decrease travel times, such as constructing a high-capacity transit system, redesigning vehicle capacity, or increasing service frequency.

2.4.3 New service patterns

The transit system abruptly changed after the COVID-19 pandemic began spreading, and specialists tried to find solutions. Using a mathematical approach, Gkiotsalitis (2021) has proposed a service pattern model that adapts to recent changes caused by the COVID-19 pandemic. For each dispatching vehicle operating for a particular trip, this service establishes a set of skipped stops. Based on the dynamic model, different service patterns are proposed for each trip of a transit line, which uses actual passenger demand information to determine which stops can be skipped to guarantee passenger loading remains less than the pandemic-imposed capacity. In addition, they secured a specific level of service with an all-stop strategy. This means that stops that aren't visited by a vehicle will be served by the next vehicle. The authors evaluated 1000 scenarios with varying demand levels and demonstrated that the associated service pattern could decrease passenger loading per vehicle under the pandemic capacity. Their results, however, indicate a considerable number of stops (4 out of 13 stops) need to be skipped, and a significant number of passengers would be left behind. Therefore, their model is suitable for low-demand areas because missing some stops will not result in many unserved passengers. In addition, a lower headway will make the model more attractive since the skipped passengers must wait for the next vehicle to board.

During the early stages of the pandemic, measures such as maintaining physical distance were recommended to reduce the risk of spreading the disease. Gkiotsalitis and Cats (2021) investigated the impacts of different social distancing policies on total agency and users' costs, including waiting and revenue loss costs. This study applied to the metro network of Washington D.C. and determines the optimal fleet allocation while considering the variation in capacity limits of different distancing policies that may result in not all demands being satisfied. According to the results, all 122 vehicles can serve all passengers within the 1-meter distance requirements. Their study found that stricter distance policies resulted in a higher rate of denied boarding.

2.5 Discussion

To the best our knowledge, no studies have been conducted for alternative operating strategies that enable a detailed comparison between alternative transit services with fixed stops and an on-demand design with flexible stops. The perks of applying both fixed and flexible services can be revealed where transit demand fluctuates, and demand patterns are unpredictable.

In this thesis we developed continuous models that can be used by transit agencies to develop designs and policies that minimize generalized costs. In addition, they can be expressed as simple analytical forms that indicate operational, and user costs. This thesis compared alternative transit services under varying demands, passenger trip lengths, and users' costs factors while responding to the crowding and denied boarding issues as result of different disruptions such as COVID-19 epidemic.

Chapter 3: Methodology

This study compares alternative bus operations strategies running along a fixed route, with difference stopping patterns. The transit system assumed in this thesis is a single fixed bus route with fixed or flexible stops. Trip patterns for all the proposed strategies are many-to-many, which means passengers can both board and alight at each stop along the bus route. Both directions of bus routes were assumed to have equal demand (balanced inbound and outbound routes). For each alternative service, an optimization model is developed that determines the headway and stop spacing with a continuous approach. The five alternative services were designed to minimize total generalized costs, including users and agency costs. Users' costs include walking to and from the nearest stop, waiting at the origin stop to board the first arriving vehicle, in-vehicle travel time, and transferring between routes. Fuel costs, drivers' wages, vehicle operating costs, and vehicle maintenance costs are reflected in agency costs.

Our approach includes modeling the cost functions of each alternative strategy, then optimizing bus headway and stop spacing to calculate the total cost of the service under varying demand and average user trip lengths. Then, the total cost of each service is compared to find the strategy with the lowest generalized cost. Crowding and denied boarding costs are also taken into consideration (Figure 3.1).

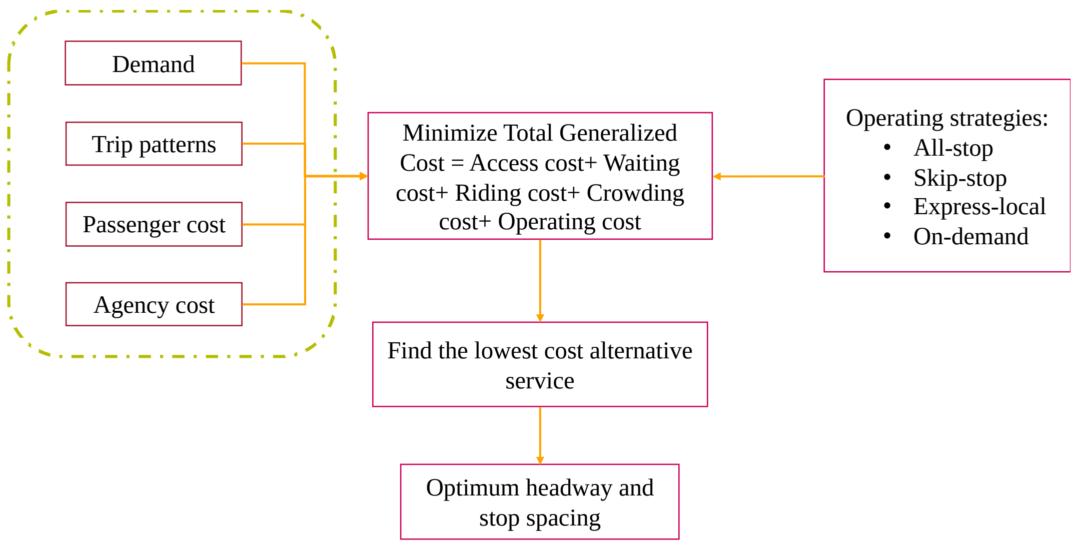


Figure 3.1. Study framework

3.1 Model assumptions and contributions

The main assumptions in this study are: (i) vehicles are running on bi-directional routes, (ii) passenger boarding demand is assumed to be uniformly distributed along the route in each direction with p passengers per hour, (iii) passenger trip length is uniformly distributed over $[0, L]$ in the corridor, (iv) passengers walk to and from the nearest bus stop to their origin and destinations, respectively, in parallel to the bus route (Wirasinghe & Ghoneim, 1981), independent of the type of alternative bus operating service, (v) in each service, vehicles are dispatched with a uniform headway, (vi) dwell time at each stop is assumed to be constant, τ (hour), including vehicle acceleration, deceleration, and loading and unloading passengers, as in Sivakumaran et al. (2014), (vii) alternative services can be applied along the bus route without the necessity of new infrastructure, (viii) passenger's walking to the destination is not considered as a travel option, and (ix) passengers take the buses in first-come, first-served order.

We develop the total cost functions of four competing operating policies, inspired by the work of Gu et al. (2016). However, Gu et al. (2016) did not differentiate between users' cost components and converted them into time units. Therefore, they assumed that passengers were equally sensitive to waiting, walking, travel time, and transfer costs. However, the time that passengers spend onboard has a different utility from the time they wait for a bus or walk to a stop. Specifically, with the emergence of the COVID-19 epidemic, riding time is assumed to be less desirable than pre-COVID. Hence, this study distinguishes between users' cost elements by considering unit costs of walking, waiting, riding, and transferring, and emphasizes passengers' perception of a pandemic. Here, we summarized the definition of each cost component:

1. Passengers' walking time cost: the disutility in terms of monetary cost for time of walking to and from a bus stop per passenger-hour,
2. Passengers' waiting time cost: the disutility in terms of monetary cost for average time of waiting for a bus to board per passenger-hour,
3. Passengers' riding time cost: the disutility in terms of monetary cost for passenger time spending in a transit vehicle to reach their destination per passenger-hour,
4. Passengers' transfer cost: the disutility in terms of monetary cost for each time transferring between transit line per passenger-transfer,
5. Operating cost of agency: Cost for one hour of operating a bus in one direction per vehicle-hour.

The COVID-19 pandemic increased passengers' sensitivity to close contact on board. Thus, we try to determine the crowding penalty based on the loading factor and minimize the crowding cost that better reflects the consequences of the COVID-19 pandemic. Furthermore, this study captures the cost of denied boarding that occurs due to capacity constraints. Drivers would leave some passengers behind to prevent overcrowding in a confined space due to the risk of infection. The optimization model takes into account the cost of denied boarding for agencies to determine the most efficient bus operating strategies.

3.2 Proposed alternative bus operations strategies

3.2.1 All-stop service

In all-stop service bus stops at all the bus stops along the corridor wherever demand for boarding and/or alighting exists. As we assumed a uniform distribution for transit demand there's demand for boarding and alighting at all the stops. Thus, based on such assumption, bus stops at all the stops in this service. Total generalized cost in all-stop operating strategy consists of walking, waiting, riding, and operating costs. Passengers access the nearest bus stop and encounter an average walking distance of $\frac{s}{4}$, where s is the stop spacing in km. As such, the passengers' average access time to origin stops or from destination stops is $\frac{s}{4v_w}\gamma_a$, where v_w is the passenger's walking speed in km/h and is assumed to be considerably lower than the speed of the vehicles. To convert this term into access cost, we defined γ_a as a unit cost of walking per passenger hour ($/passenger-hour) and multiplied it by the average access time.

Passengers walk to the nearest origin stop and wait kH at the origin stop; where H is the bus headway and k is a value between 0.5 and 1, which accounts for the average waiting time (Saidi et al., 2016). Users' waiting cost is then $kH\gamma_w$, where γ_w is the unit cost of waiting per passenger-hour.

Users are traveling an average trip length of $\bar{l}$ in the system and their riding time along $\bar{l}$ at cruising speed of v, which is $\frac{\bar{l}}{v}$. Also, dwelling times at the stops entailing their trip is the average number of stops for each passenger multiplied by the mean dwell time at each stop, τ in hour; so, the dwell time is $\frac{\bar{l}\tau}{s}$. Thus, the cost of riding time is $\left(\frac{\bar{l}}{v} + \bar{l}\frac{\tau}{s}\right)\gamma_r$. Here, γ_r represents the cost of one hour of riding for one passenger in this transit system.

On the agency side, vehicles are running the entire route with length of L with a cruising speed of v. The operating cost of all-stop system per vehicle and per passenger is:

$$C_{A\_AS} = \frac{\lambda}{pH}\left(\frac{L}{v} + L\frac{\tau}{s}\right) \tag{3.1}$$

We defined λ as the unit cost of operating time per vehicle in one direction. The operating time is then divided by the hourly demand of the bus route is pH, to find the operating cost of each vehicle per passenger.

Passengers are more sensitive to onboard crowding due to the COVID-19 pandemic. This study also considers passenger costs of in-vehicle crowding. Any occupancy level exceeding a predefined threshold decreases the convenience of a trip in a vehicle for passengers.

According to Moccia and Laporte (2016), the load factor can be assumed as a function of hourly demand density, the average passengers' trip length, and bus capacity, and the crowding disutility is a piecewise linear function of the load factor.

$$\delta = \begin{cases} 1 + \rho(\theta - \theta_{min}), & \theta > \theta_{min} \\ 1 & , otherwise \end{cases} \tag{3.2}$$

In equation (3.2), δ is the crowding penalty and ρ is a slope value, and the crowding disutility linearly increases with ρ. Based on this definition, θ is

$$\theta = \frac{\bar{l}pH}{LK} \tag{3.3}$$

Where $\bar{l}$ is the average trip length of travelers, p is hourly route demand, H is the headway, L is bus route length, and K is the capacity of the vehicles.

The crowding cost is then multiplied by the cost of in-vehicle riding time to account for the inconvenience of onboard crowding. The cost of passengers riding time in all-stop system (C_{R_AS}) is:

$$C_{R\_AS} = \left(\frac{\bar{l}}{v} + \bar{l}\frac{\tau}{s}\right)\gamma_r \times \delta \tag{3.4}$$

According to Pollock et al. (2021), the impacts of onboard crowding were included as the sensitivity to the levels of riding time. The sensitivity to travel times increases as the loading factor increases. Through a stated preference survey in Calgary, Canada, Pollock et al. (2021) defined three levels of crowding and three sensitivity levels to ride time. In the results section of our case study, we will include values for three levels of loading factor and associated riding time costs.

Some passengers may not be allowed to board when demand exceeds the vehicle's crush capacity. The number of rejected demands is approximated in equations (3.5) and (3.6):

$$p_d = \begin{cases} pH - K_C, & pH > K_C \\ 0 & , otherwise \end{cases} \tag{3.5}$$

$$C_d = \frac{p_d \gamma_d}{p} \tag{3.6}$$

Here p_d represents the number of denied boarding passengers due to the capacity constraint, and p is the passengers flow on the all-stop route. We assume that if passengers missed their first bus due to capacity constraint, they would wait for the next arriving vehicle to reach their destination. Equation (3.6) represents the cost component regarding the denied boarding passengers in the all-stop system. This equation finds the cost of denied boarding per passenger (p), as other cost components are defined, where γ_d is the cost of rejecting one passenger.

When route demand is exceeding the bus crush capacity, only a subset of passengers can board the arriving vehicle and other passengers need to wait for a full headway for the next bus. Therefore, the bus can carry no more than its crush capacity (K_C). This would update the riding cost term to account for the passengers who could board the bus. The extra waiting time included in the trips of denied boarding passengers is $p_d H \gamma_w$, where p_d is the number of passengers who are left behind.

We divide each cost component by p to find the total cost of each service per passenger.

The total generalized cost of all-stop strategy is:

$$\min C_{AS} = \frac{s}{2v_w}\gamma_a + kH\gamma_w + \left(\frac{p_d}{p}\right)H\gamma_w + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + \frac{\bar{l}\tau}{s}\right)\gamma_r\delta + \frac{\lambda}{pH}\left(\frac{L}{v} + L\frac{\tau}{s}\right) + C_d \tag{3.7}$$

subject to:

$$H \geq H_{min} \tag{3.8}$$

The first term in equation (3.7) is the cost of passengers walking to and from the nearest bus stop. The second and third terms described the cost of waiting time for all and the denied boarding passengers, respectively. The riding cost of each passenger in the all-stop strategy is given in the fourth term of equation (3.7); it shows that only K_C passengers can ride the bus. This term is multiplied by δ to add the extra inconvenience due to bus crowding. The last two terms are the operating cost of the system and the denied boarding penalty, respectively.

In equations (3.7) and (3.8), the crowding disutility and denied boarding penalties capture the pandemic state of the transit system. The constraint described in equation (3.8) ensures that bus headway must be greater than a minimum value, H_{min}.

3.2.2 Skip-stop service

In this service, multiple routes are provided along the bus line. Each route visits the transfer stop and a set of stops in such a way that each stop is served by only one of the routes. m is the number of routes, and each route is serving n stop between two consecutive transfer stops (Figure 3.2). Each dot in Figure 3.2 with the letter A, B, and C, represents which route is visiting that stop. Vehicles on each route serve $\frac{n+1}{mn+1}$ stops in one direction of the bus route and skip $(m-1)n$ intermediate stops.

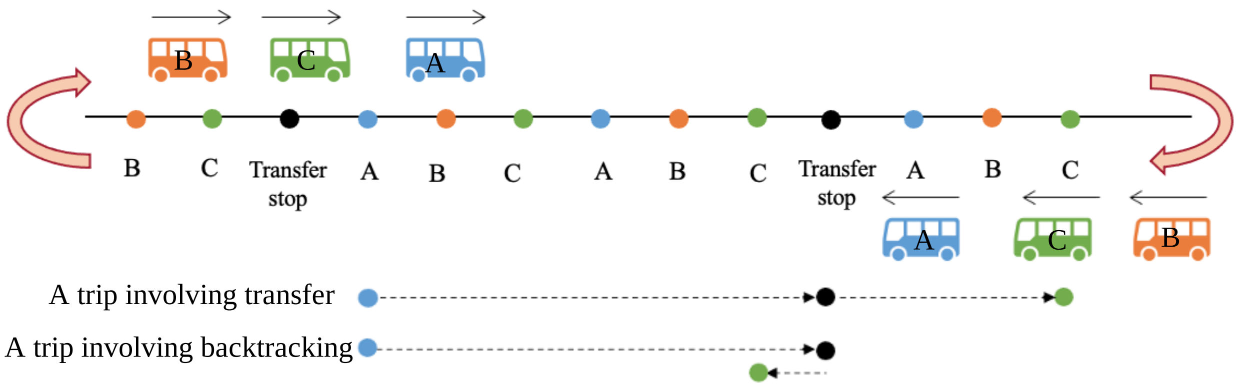


Figure 3.2. A skip-stop system with 3 routes and 2 stops between each transfer stop

According to different patterns of ODs along with passengers' trips, four different types of trips are categorized in skip-stop service:

Type 1: both origin and destination are transfer stops. The probability of a station being a transfer stop is $\frac{1}{mn+1}$; thus, the probability of a trip belonging to this type is $\frac{1}{(mn+1)^2}$. In this group, we consider that passengers board the first arriving vehicle, regardless of which stops the vehicles on that route are serving, and then make a transfer if needed.

Type 2: both the origin and destination of a trip are along one route, but one or both stops are not at the transfer stop. Passengers in this group are assumed to take the first arriving vehicle of their desired route; thus, no transfer is needed during their trip. In this type, the probability of either origin or destination is a transfer stop is $2 \times m \times \frac{1}{mn+1} \times \frac{n}{mn+1}$. The probability of a trip starting and ending at non-transfer stops is $m \times \frac{n}{mn+1} \times \frac{n}{mn+1}$ where $\frac{n}{mn+1}$ is the probability of a station being a non-transfer stop. Therefore, trips of type 2 occur with the probability of $\frac{2mn+mn^2}{(mn+1)^2}$.

Type 3: both the origin and destination are within two consecutive transfer stops but visited by distinct routes. Passengers in this group must backtrack. Passengers' trip lengths are uniformly distributed over $[0,2\bar{l}]$, and therefore the number of stations visited by each trip is uniformly distributed over $\left[0,\frac{2\bar{l}}{s}\right]$. The probability of selecting a random non-transfer origin stop in both directions is $\frac{4\bar{l}}{s}$, and among them, $(m-1)n$ are belonging to this trip type. Thus, the probability is $\frac{mn}{mn+1} \times \frac{(m-1)n}{\frac{4\bar{l}}{s}}$.

Type 4: origin and destination of a trip are located at two stations with different routes, and one or two transfer stops are in the middle. In this type, passengers need to transfer between routes to get to their destinations. The probability of occurrence of this type of trip is the difference between 1 and the summation of the above probabilities, which is $\frac{(m-1)mn^2}{(mn+1)^2} - \frac{mn}{mn+1} \times \frac{(m-1)n}{\frac{4\bar{l}}{s}}$.

The cost of riding time in this strategy consists of two parts: an average in-vehicle ride time and intervening stops along a passenger's route and the expected backtracking distance. For the first part, like in the previous section, passengers traveling an average distance of $\bar{l}$ with a cruising speed of v and their average riding time is $\left(\frac{\bar{l}}{v}+\bar{l}.\frac{\tau}{s}.\frac{n+1}{mn+1}\right)$, in which $\bar{l}.\frac{\tau}{s}.\frac{n+1}{mn+1}$ is the number of stations entails in users' trips, and the term $\frac{n+1}{mn+1}$ represents the number of stations visiting by each route.

Any passenger trip which starts and ends at non-transfer stops on different routes (mk) and resides between two consecutive transfer stops (trip type 3) includes a backtracking distance. Gu et al. (2016) defined X_1 and X_2 as the distances from an origin or destination stop to the nearest transfer stop (as shown in Figure 3). We can find the backtrack distance Z by assuming X_1 and X_2 as continuous variables with a uniform distribution over $\left(0,\frac{(mn+1)s}{2}\right)$ as follows:

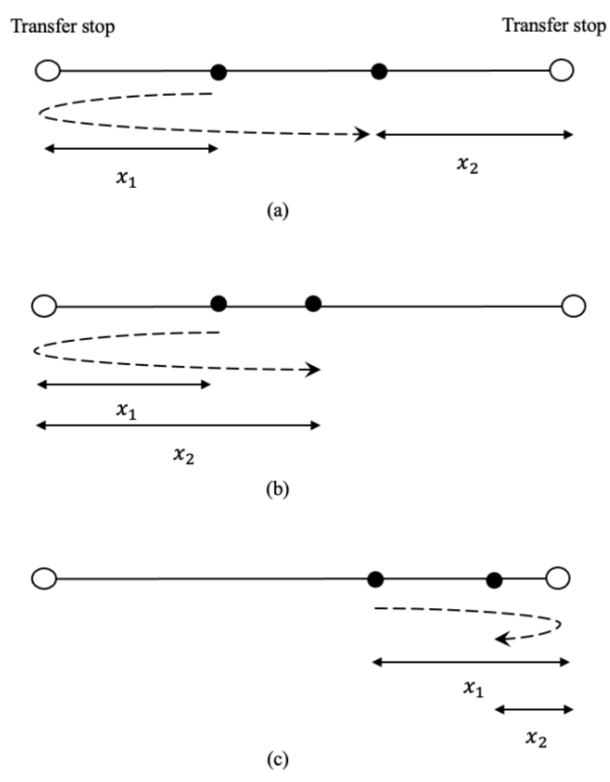


Figure 3.3. Three cases of backtracking

$$E[d] \approx 2\int_{x=0}^{\frac{(mn+1)s}{2}} \Pr(\min(x_1, x_2) \geq x)\, dx = 2\int_{x=0}^{\frac{(mk+1)s}{2}} \Pr(x_1 \geq x) \Pr(x_2 \geq x)\, dx =$$

$$2\int_{x=0}^{\frac{(mn+1)s}{2}} \left(1 - \frac{x}{\frac{(mn+1)s}{2}}\right)^2 dx = \frac{(mn+1)s}{3} \tag{3.9}$$

To find the expected backtracking distance, the probability of trip type 3 is multiplied to $E[d]$:

$$D_{bt\_ss} = \frac{mn}{mn+1} \times \frac{(m-1)\mathrm{n}}{\frac{4\bar{l}}{s}} \times \frac{(mn+1)s}{3} = \frac{m(m-1)n^2}{\frac{12\bar{l}}{s}} s \tag{3.10}$$

The passengers' total cost in the skip-stop system extends the work in Gu et al. (2016) on the cost of crowding and denied boarding and differentiating between different users' cost components. The users' cost in the skip-stop scheme is given in equation (3.11):

$$C_{U\_SS} = \frac{s}{2v_w}\gamma_a + mkH\gamma_w + \left(\frac{p_d}{p}\right)H\gamma_w + \left(\frac{K_C}{p}\right)\left(\bar{l} + \frac{m(m-1)n^2}{\frac{12\bar{l}}{s}} s\right)\left(\frac{1}{v} + \frac{\tau}{s} \cdot \frac{n+1}{mn+1}\right)\gamma_r \delta + \frac{(m-1)mn^2}{(mn+1)^2}\gamma_t + C_d \tag{3.11}$$

The first term in equation (3.11) represents the cost of users' walking to and from the nearest bus stop. The second term describes the cost of waiting in the skip-stop system, where we assume the schedule coordination for multiple routes at the transfer stops and the headways on all the routes are equal. We assume that passengers wait kH for the suitable route to board, where k is a value between 0.5 and 1. The third term is the cost of additional waiting (a full headway, H) for denied boarding passengers (p_d). Taking crowding into account, the fourth term in equation (3.11) indicates the cost of riding time. The cost of crowding is obtained based on the crowding penalty we defined in equation (3.2). We multiplied the crowding cost component by users' ride time cost to represent the situation where passengers are susceptible to close contact in a confined place.

The cost of transferring between routes and rejecting passengers due to capacity constraints are illustrated in the last two terms of the equation (3.11), respectively.

As described before, we distinguish between various users' cost elements by determining unit cost factors of walking, waiting, riding, and transfer; thus, each term in equation (3.11) delivers a specified weight, e.g. γ_a represents the inconvenience of one hour walking for passengers.

The number of denied boarding in this strategy is calculated in equation (3.12), and the associated cost is given in equation (13):

$$p_d = \begin{cases} Q - K_C, & Q > K_C \\ 0 \quad , & otherwise \end{cases} \tag{3.12}$$

$$C_d = \frac{p_d \gamma_d}{p} \tag{3.13}$$

Here Q is the passenger flow on skip-stop routes. If the backtracking were not included in the model, the passenger flow would be pH. Accounting for backtracking trips results in higher passenger flow near a transfer stop; because passengers of type 3 will first arrive at a transfer stop and then turn back to reach their non-transfer stop. The extra flow (p_{bt}) is equal to the portion of passengers type 3 divided by the number of transfer stops, which is given by equation (3-14):

$$p_{bt} = \frac{p.\frac{mn}{mn+1}.\frac{(m-1)n}{\frac{4\bar{l}}{s}}}{1/(mn+1)} \tag{3.14}$$

Then the passenger flow is equal to equation (3.15):

$$Q = p\left(H + \frac{m(m-1)n^2}{\frac{4\bar{l}}{s}}\tau\right) \tag{3.15}$$

The total operating cost per bus and per passenger trip in the skip-stop strategy is given in equation (3.16):

$$C_{A\_SS} = \frac{\lambda}{pH}\left(\frac{L}{v} + L.\frac{\tau}{s}.\frac{n+1}{mn+1}\right) \tag{3.16}$$

The term $\frac{L}{v}$ indicates the total operating hour in one direction, and $\frac{L}{s}.\frac{n+1}{mn+1}$ in equation (3.16), is the number of stops visiting by each route in each direction; thus, the total additional time due to stopping at stops in the skip-stop scheme is $L.\frac{\tau}{s}.\frac{n+1}{mn+1}$. We divide the total cost of operating by H to find the cost of operating each bus and by p, to find the agency cost per passenger.

Thus, the total generalized cost of the skip-stop strategy is the sum of user costs and agency costs:

$$\min C_{SS} = \frac{s}{2v_w}\gamma_a + mkH\gamma_w + \left(\frac{p_d}{p}\right)H\gamma_w + \left(\frac{K_C}{p}\right)\left(\bar{l} + \frac{m(m-1)n^2}{\frac{12\bar{l}}{s}}s\right)\left(\frac{1}{v} + \frac{\tau}{s}.\frac{n+1}{mn+1}\right)\gamma_r\delta + \frac{(m-1)mn^2}{(mn+1)^2}\gamma_t + \frac{\lambda}{pH}\left(\frac{L}{v} + L.\frac{\tau}{s}.\frac{n+1}{mn+1}\right) + C_d \tag{3.17}$$

subject to:

$$mH \geq H_{min} \tag{3.18}$$

The cost function defined in equation (3.17) is the total cost of the skip-stop scheme per passenger, including the cost of crowding and denied boarding. Equation (3.18) ensures that the headway at a transfer stop respects the minimum headway constraint.

3.2.3 Express-local

In the express-local policy, two distinct routes serve one bus route. According to Figure 3.4, a local route serves all the stops along the line, including local stops labeled as L and express stops labeled as E, and an express route stops at every N stop, as shown in Figure 3.4. The same bus capacity is assumed for both express and local vehicles, and backtracking is not assumed in this service.

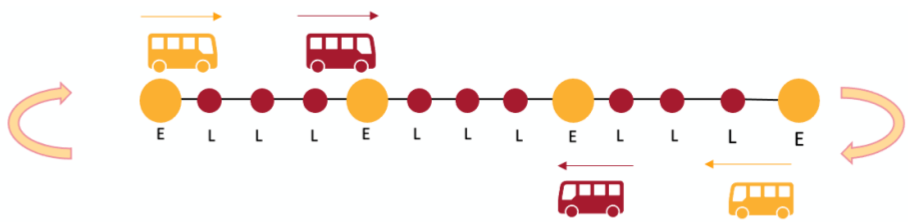


Figure 3.4. Express-local system with N=4

Based on different combinations of origin and destination, three types of passenger trips are defined (Gu et al. 2016):

Type 1: Traveling between two express stops. This type of trip occurs with the probability of $\frac{1}{N^2}$, where both origin and destination are at express stops, and passengers can board the first arriving vehicle.

Type 2: Traveling between an express and a local stop. This trip type is happening when either origin or destination is either express or local, with the probability of $\frac{2(N-1)}{N^2}$, in which $\frac{N-1}{N}$ is the probability that a stop is visited only by a local route. This group of passengers can either take a local-only route and reach their destination stop or board an express route and then transfer to a local route or vice versa. We will further explore the details.

Type 3: Traveling between two local stops. This type of trip occurs with the probability of $(\frac{N-1}{N})^2$. Passengers of this type have two options: the first option is to take a local-only route, and the second option is to take local-express-local and make two transfers in the middle.

We expand the models in Gu et al. (2016) by defining unit costs of walking (γ_a), waiting (γ_w), riding (γ_r), and transfer (γ_t) to distinguish between user-related costs. The present express-local system extends Gu et al. 's (2016) analysis of users' and agencies' costs. User costs are explored separately for each group of trips:

Type 1: Passengers travel via express-only route, and the users' cost is similar to all-stop service.

The total user cost for this type is:

$$C_{p\_EE} = \frac{s}{2v_w}\gamma_a + kH_E\gamma_w + \left(\frac{\bar{l}}{v} + \bar{l}\frac{\tau}{Ns}\right)\gamma_r \quad (3.19)$$

The first term in equation (3.19) is the cost of walking for this type of passengers. The second term is the average cost of waiting for express route, in which H_E is the headway on the express route and γ_w is the cost of 1 hour of waiting per passenger. This type of passenger trip has the riding cost shown in third term of equation (3.19). The average trip time is $\frac{\bar{l}}{v}$, and $\tau\frac{\bar{l}}{Ns}$ is the total additional time due to stopping at stops in a trip length of $\bar{l}$.

Type 2: The passengers in this trip type can choose between two options, for example, in traveling from an express to a local stop: taking the local-only route or taking the express route and then transfer to the local route. The total user cost of this type is similar to the trips including local route at first and then transferring to the express route, thanks to the symmetry; thus, in this section, we only formulate the cost of the first type.

By taking the local-only route, the passenger-related costs are access/egress cost + waiting cost + riding cost. The total cost of this type is:

$$GC_{Type2\_LL} = \frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \left(\frac{\bar{l}}{v} + m\tau\right)\gamma_r \quad (3.20)$$

By choosing the second option, passengers face: walking cost, waiting cost for both routes, riding cost, and transfer from one route to another: The total generalized cost of this type of trip is:

$$GC_{Type2\_EL} = \frac{s}{2v_w}\gamma_a + k(H_L + H_E)\gamma_w + \left(\frac{\bar{l}}{v} + (M + m')\tau\right)\gamma_r + \gamma_t \tag{3.21}$$

The relation of M, m and m' can be described below:

$$m = MN + m' \ ;1 \leq m' \leq N - 1 \tag{3.22}$$

Passengers would choose the local-only route when the cost of the first option is lower than the second:

$$GC_{Type2\_LL} \leq GC_{Type2\_EL} \tag{3.23}$$

Which results in:

$$M(N-1)\tau\gamma_r \leq kH_E\gamma_w + \gamma_t \tag{3.24}$$

Thus:

$$M \leq M_0 \equiv \left\lfloor \frac{kH_E\gamma_w + \gamma_t}{(N-1)\tau\gamma_r} \right\rfloor \tag{3.25}$$

This term, $M \leq M_0$, can also be expressed as $m \leq N(M_0 + 1) - 1$. M_0 is the maximum number of express stops in the second option, assuming that the local-only option is the most desirable option. Also, $\lfloor x \rfloor$ means an integer value that is not greater than x.

The local-only trips occur with the probability of p_0. Since the number of stops visiting in a trip is uniformly distributed over $[0, \frac{2\bar{l}}{s}]$, we have:

$$p_0 \approx \min\left\{1, \frac{N(M_0+1)-1}{\frac{2\bar{l}}{s}}\right\} \tag{3.26}$$

The probability of choosing the second option, express-local or local-express, is therefore $1 - p_0$. Therefore equation (3-37) can be rewritten as equation (3-47):

$$GC_{Type2\_LL} = \frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \frac{E[l|local-only]}{v} \times \gamma_r + \tau \times E[m|m \leq N(M_0+1) - 1]\gamma_r \tag{3.27}$$

In equation (3-46), the last term is the number of stops visiting in the local-only option and is equal to $\frac{1}{2}N(M_0+1)\tau \times \gamma_r$.

The user cost of the second type of trip is:

$$GC_{Type2\_EL} = \frac{(N+1)s}{4v_w}\gamma_a + k(H_L+H_E)\gamma_w + \frac{E[l|express-first]}{v} \times \gamma_r + \gamma_t + \tau \times E[M + m'|M \geq M_0]\gamma_r \tag{3.28}$$

Here, both M and m' follow a uniform distribution over $[M_0+1, \frac{2\bar{l}}{Ns}]$ and $[1, N-1]$, respectively.

Thus, in equation (3-42) the last term is equal to $\frac{M_0+1+\frac{2\bar{l}}{Ns}+N}{2}\tau\gamma_r$.

In total, the generalized cost of trips type 2 in the express-local service is given by equation (3-48):

$$C_{p\_EL\_EL} = p_0\left(\frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \frac{E[l|local-only]}{v}\gamma_r + \frac{1}{2}N(M_0+1)\tau\gamma_r\right) + (1-p_0)\left(\frac{s}{2v_w}\gamma_a + kH_L + H_E\gamma_w + \frac{E[l|express-first]}{v}\gamma_r + \frac{M_0+1+\frac{2\bar{l}}{Ns}+N}{2}\tau\gamma_r + \gamma_t\right) = \frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \left(\frac{\bar{l}}{v} + p_0.\frac{1}{2}N(M_0+1)\tau\right)\gamma_r + (1-p_0)\left(kH_E\gamma_w + \gamma_t + \frac{M_0+1+\frac{2\bar{l}}{Ns}+N}{2}\tau\gamma_r\right) \tag{3.29}$$

Type 3: When traveling between two local stops, passengers can either take a local-only route or a local-express-local route. Following are the costs and probabilities associated with each type of route.

Among the costs of the local-only route for passengers are access and egress to and from a local station, waiting time, and in-vehicle ride time.

Thus, the passengers' total cost on the local-only route option is:

$$GC_{Type3\_LL} = \frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \left(\frac{\bar{l}}{v} + m\tau\right)\gamma_r \quad (3.30)$$

Here, m is the number of local stops visited for this group of passengers.

Total generalized cost on a local-express-local route includes walking cost, waiting for both local and express routes, riding cost, and transfer cost.

Therefore, the total cost of a local-express-local option is:

$$GC_{Type3\_LEL} = \frac{s}{2v_w}\gamma_a + (H_L + kH_E)\gamma_w + \left(\frac{\bar{l}}{v} + (M + m')\tau\right)\gamma_r + 2\gamma_t \quad (3.31)$$

In this case, M is the number of express stops, and m' is the number of local stops visited by each trip.

Thus, passengers would choose the local-only option if:

$$GC_{Type3\_LL} \leq GC_{Type3\_LEL} \quad (3.32)$$

By substituting equations (3.30) and (3.31) in equation (3.32), we have:

$$M(N-1)\tau\gamma_r \leq k(H_E + H_L)\gamma_w + 2\gamma_t \quad (3.33)$$

Which results in:

$$M \leq M_1 \equiv \left\lfloor \frac{k(H_E+H_L)\gamma_w + 2\gamma_t}{(N-1)\tau\gamma_r} \right\rfloor \quad (3.34)$$

The probability and the passenger cost in the local-only route, can be calculated according to their OD patterns (Gu et al., 2016).

Thus, the first group of the local-only trips occurs with the probability of p_1:

$$p_1 = \min\left\{1, \frac{\frac{(N-1)(N-2)}{2}}{\frac{2\bar{l}(N-1)^2}{Ns}}\right\} = \min\left\{1, \frac{N(N-2)s}{4\bar{l}(N-1)}\right\} \quad (3.35)$$

Therefore, the average cost of passengers in this trip type is the sum of walking cost, waiting cost, and riding cost.

Equation (3.30) can be approximated by equation (3.36) for the first group and (3.39) for the second group:

$$GC_{Type3\_LL\_group1} = \frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \frac{E[l|first\ group\ of\ local-only]}{v}\gamma_r + \tau E[m|first\ group\ of\ local-only]\gamma_r \quad (3.36)$$

Where the number of local stops visiting by this type of trip is equal to equation (3.37) (Gu et al., 2016):

$$E[m|first\ group\ of\ local-only] = \frac{N}{3} \quad (3.37)$$

Similarly, the second group of the local-local trips has the probability of p_2:

$$p_2 = \min\left\{1 - p_1, \frac{(N-1)^2(M_1+1)}{\frac{2\bar{l}(N-1)^2}{Ns}}\right\} = \min\left\{1 - p_1, \frac{N(M_1+1)s}{2\bar{l}}\right\} \quad (3.38)$$

The cost of passengers in the second group of local-only trips is including walking cost, waiting cost, and in-vehicle riding cost, given in equation (3-60):

$$GC_{Type3\_LL\_group2} = \frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \frac{E[l|second\ group\ of\ local-only]}{v}\gamma_r + \tau \times E[m|second\ group\ of\ local-only]\gamma_r \quad (3.39)$$

Here m is calculated in equation (3.40):

$$E[m|second\ group\ of\ local-only] = E[MN + m'|second\ group\ of\ local-only] = N \times E[M|second\ group\ of\ local-only] + E[m'|second\ group\ of\ local-only] = N\frac{0+M_1}{2} + E[m'|second\ group\ of\ local-only] = \frac{NM_1}{2} + N \quad (3.40)$$

In equation (3.40), m' is the total number of local stops at both ends of the trip visiting in the second group of trips in the local-only route. Since the expected number of local stops at each end is $\frac{N}{2}$, m' is equal to N.

The last group of passengers in local-to-local trips takes a local-express-local route. This type of trip occurs with the probability of $1 - p_1 - p_2$. The average cost of passengers, given by equation 3.31, can be approximated as:

$$GC_{Type3\_LEL} = \frac{s}{2v_w}\gamma_a + (H_L + kH_E)\gamma_w + \frac{E[l|local-express-local]}{v}\gamma_r + 2\gamma_t + \tau \times E[M + m'|M > M_1]\gamma_r \quad (3.41)$$

Where the last term in equation (3.41) is equal to:

$$E[M + m'|M > M_1] = E[M|M > M_1] + E[m'|M > M_1] = \frac{(M_1+1)+\left(\frac{2\bar{l}}{Ns}-1\right)}{2} + N \quad (3.42)$$

Therefore, the average cost of passengers on the local-to-local trip (type 3 in express-local service) is the sum of equations (3.36), (3.39), and (3.41):

$$C_{p\_LL} = \frac{s}{2v_w}\gamma_a + kH_L\gamma_w + \left(\frac{\bar{l}}{v} + p_1.\frac{1}{3}N\tau + p_2.\frac{(M_1+2)N}{2}\tau\right)\gamma_r + (1-p_1-p_2)\left(H_L + kH_E\gamma_w + 2\gamma_t + \left(\frac{\frac{2\bar{l}}{Ns}+M_1}{2} + N\right)\tau\gamma_r\right) \quad (3.43)$$

The users' costs also improved by adding the cost of crowding and denied boarding formulated in equations (3.44) to (3.47). The load factor on each vehicle of the express-local system would be calculated separately since we defined two distinct routes with different headways:

$$\theta_E = \frac{\bar{l}Q_E}{LK} \quad (3.44)$$

$$\delta_E = \begin{cases} 1 + \rho(\theta_E - \theta_{min}), \theta_E > \theta_{min} \\ 1 \qquad\qquad\qquad\quad , \ otherwise \end{cases} \quad (3.45)$$

Equation (3.45) finds the crowding disutility based on the loading factor on an express bus, which is shown by equation (3.44). The crowding penalty of express and local routes will then be multiplied by the cost of riding time on each route.

$$\theta_L = \frac{\bar{l}Q_L}{LK} \quad (3.46)$$

$$\delta_L = \begin{cases} 1 + \rho(\theta_L - \theta_{min}), \theta_L > \theta_{min} \\ 1 \qquad\qquad\qquad\quad , \ otherwise \end{cases} \quad (3.47)$$

Equation (3.47) calculates the crowding penalty for a bus on a local route using the loading factor defined in equation (3.46).

Q_E and Q_L indicate maximum passenger flow on express and local routes, respectively. We formulated Q_E and Q_L like in Gu et al. (2016). First, passenger flow on the local route is calculated here:

The passenger flow on each route in the express-local system is extended for an hourly route demand. In trips of type two, a portion of passengers, $\tilde{Q}_{L_{EL}}$, are being served by the local route, with the probability of $\frac{2(N-1)}{N^2}$. A portion of $\tilde{Q}_{L_{LL}}$ in trips of type 3 taking the local route, with the probability of $\left(\frac{N-1}{N}\right)^2$.

The $\tilde{Q}_{L_{EL}}$ consists of passengers on the local-only route which is $\frac{2(N-1)}{N^2} p_0 p \frac{1}{2} N(M_0+1)\tau$, and for those passengers taking either the express-local or the local-express trip is $\frac{2(N-1)}{N^2}(1-p_0)p\frac{N\tau}{2}$.

The $\tilde{Q}_{L_{LL}}$ consists of passengers traveling between two local stops in such a way:

(i) Traveling via a local-only route between two neighboring express stops, $\left(\frac{N-1}{N}\right)^2 p_1 \frac{N}{3}\tau$,

(ii) The local-only trips include at least one express stop, $\left(\frac{N-1}{N}\right)^2 p_2 p \frac{(M_1+2)N}{2}\tau$, and

(iii) The local-express-local type of trips, $\left(\frac{N-1}{N}\right)^2 (1-p_1-p_2)pN\tau$.

Thus, the passenger flow on the express route is:

$$Q_E = p\left(H_E - \frac{2(N-1)}{N^2}\left(\frac{N\tau}{2} + p_0.\frac{NM_0\tau}{2}\right) - \left(\frac{N-1}{N}\right)^2.\left(N\tau - p_1.\frac{2Ns}{3} + p_2.\frac{(M_1-1)N\tau}{3}\right)\right) \quad (3.48)$$

The passenger flow on the local route can be obtained based on the maximum passenger flow near an express stop because many passengers make a transfer at an express stop. Since the express stops are the same, we divide the total number of express stops visited by passengers on the local route by the number of express stops to calculate the maximum flow for different types of trips.

For an express-local or local-express route, passengers choose the local-only option if $E \leq M_0$, when E is the number of express stops passed by a trip. According to the uniform distribution of E over $[0, M_0]$, the average number of express stops spanned over by a trip is $\frac{M_0+1}{2}$, the value $\frac{1}{2}$ added to this term accounts for the origin or destination express stop. Therefore, the number of express stops visited by this portion of local trips is $pp_0\frac{2(N-1)}{N^2}\frac{M_0+1}{2}$ per hour.

If $M > M_0$, passengers take the local route and then make a transfer at the nearest express stop; thus, the number of express stops visited by a local route is $\frac{1}{2}$. This portion of local trips visits $p(1-p_0)\frac{2(N-1)}{N^2}\frac{1}{2}$ express stops per hour.

For the local-express-local option, if $M \leq M_1$, users opt the local-only route. Thus, the number of express stops visiting by this type of trip is $pp_2\left(\frac{N-1}{N}\right)^2\frac{M_1+1}{2}$ per hour. Those passengers who make two transfers in their trip will visit $p(1-p_1-p_2)\left(\frac{N-1}{N}\right)^2$ express stops.

Therefore, the maximum passenger flow on the local route is:

$$Q_L = \frac{p.p_0.\frac{2(N-1)}{N^2}.\frac{M_0+1}{2}+p.(1-p_0).\frac{2(N-1)}{N^2}.\frac{1}{2}+p.p_2.\left(\frac{N-1}{N}\right)^2\frac{M_1+1}{2}+p.(1-p_1-p_2).\left(\frac{N-1}{N}\right)^2}{\frac{1}{N\tau}} = \frac{p\tau(N-1)}{N}\left(1+p_0M_0+\frac{p_2(N-1)(M_1-1)}{2}+(1-p_0)(N-1)\right) \tag{3.49}$$

We calculate the number of denied boarding passengers for each route, separately in equations (3.50) and (3.52).

$$p_{d\_E} = \begin{cases} Q_E - K_C, & Q_E > K_C \\ 0 \quad\quad\quad , & otherwise \end{cases} \tag{3.50}$$

$$C_{d\_E} = \frac{p_{d\_E}\gamma_d}{p} \tag{3.51}$$

$$\lambda_{d\_L} = \begin{cases} Q_L - K_C, & Q_L > K_C \\ 0 \quad\quad\quad , & otherwise \end{cases} \tag{3.52}$$

$$C_{d\_L} = \frac{p_{d\_L}\gamma_d}{p} \tag{3.53}$$

Equations (3.51) and (3.32) are the cost of denied boarding on express and local routes, respectively.

User costs, including waiting cost, walking cost, riding cost, and transfer cost between routes, are explored separately for each group of trips.

The cost of agency of the express-local strategy is given in equation (3.54):

$$C_{A\_EL} = \frac{\lambda}{p}\left(\frac{1}{H_E}\left(\frac{L}{v} + L\frac{\tau}{Ns}\right) + \frac{1}{H_L}\left(\frac{L}{v} + L\frac{\tau}{s}\right)\right) \tag{3.54}$$

The agency cost of an express-local system is similar to the agency cost of an all-stop system, but express and local routes have different headways (H_E and H_L, respectively). Therefore, the total generalized cost in the express-local service is:

$$C_{p\_EL} = \frac{1}{N^2}C_{p\_EE} + \frac{2(N-1)}{N^2}C_{p\_EL} + \left(\frac{N-1}{N}\right)^2 C_{p\_LL} + \frac{\lambda}{p}\left(\frac{1}{H_E}\left(\frac{L}{v} + L\frac{\tau}{Ns}\right) + \frac{1}{H_L}\left(\frac{L}{v} + L\frac{\tau}{s}\right)\right) \tag{3.55}$$

Therefore, the total generalized cost in the express-local service is:

$$\min C_{EL} = \frac{1}{N^2}\left[\frac{s}{2v_w}\gamma_a + kH_E\gamma_w + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + \bar{l}\frac{\tau}{Ns}\right)\gamma_r\delta_E\right] + \frac{2(N-1)}{N^2}\left\{\frac{s}{2v_w}\gamma_a + p_0\left[kH_L\gamma_w + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + \frac{1}{2}N(M_0+1)\tau\right)\gamma_r\delta_L\right] + (1-p_0)\left[k(H_E+H_L)\gamma_w + \gamma_t + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + \frac{N}{2}\tau\right)\gamma_r\delta_L + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + \frac{M_0+1+\frac{2\bar{l}}{Ns}}{2}.\tau\right)\gamma_r\delta_E\right]\right\} + \left(\frac{N-1}{N}\right)^2\left\{\frac{s}{2v_w}\gamma_a + (p_1 + p_2)(kH_L\gamma_w) + \left(\frac{K_C}{p}\right)\left[(p_1+p_2)\frac{\bar{l}}{v} + p_1\left(\frac{1}{3}N\tau\right) + p_2\left(\frac{(M_1+2)N}{2}\tau\right)\right]\gamma_r\delta_L + (1-p_1-p_2)\left[H_L\gamma_w + kH_E\gamma_w + 2\gamma_t + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + \frac{\frac{2\bar{l}}{Ns}+M_1}{2}\tau\right)\gamma_r\delta_E + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + N\tau\right)\gamma_R\delta_L\right]\right\} + \frac{\lambda}{p}\left[\frac{1}{H_E}\left(\frac{L}{v} + L\frac{\tau}{Ns}\right) + \frac{1}{H_L}\left(\frac{L}{v} + L\frac{\tau}{s}\right)\right] + \left(\frac{p_{d_E}}{p}\right)H_E\gamma_w + \left(\frac{p_{d_L}}{p}\right)H_L\gamma_w + C_{d_E} + C_{d_L} \tag{3.56}$$

subject to:

$$H_E \geq H_{min} \tag{3.57}$$

$$H_L \geq H_{min} \tag{3.58}$$

The optimization model of the express-local scheme in equation (3.56) is inspired by Gu et al. (2016). However, we defined unit costs of passengers (e.g., $\gamma_a, \gamma_w, \gamma_r$, and γ_t). Besides, we account for the crowding for each route (δ_E and δ_L), while also considering denied boarding cost (C_{d_E} and C_{d_L}) and the associated additional waiting cost. We also considered the operating cost of vehicles for total operating hours in one direction of bus route. Based on the constraints of this mode (equations (3.57) and (3.58)), headways on each route should be greater than a minimum headway.

3.2.4 On-demand service

In an on-demand strategy, passengers can board or alight anywhere upon their request along the fixed bus route. For this flexible service, we considered a fixed bus line, with no fixed stop along its route. This would allow vehicles to pick up and drop off passengers wherever they want. We also assume that each time a vehicle stops, only one passenger would be picked up or dropped off. The location of stops is flexible, and passengers do not need to walk to a fixed stop.

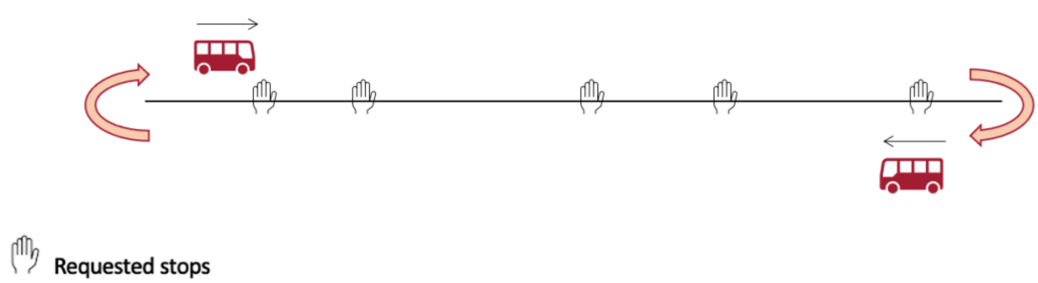


Figure 3.5. on-demand system

The total generalized cost in this service is the sum of users' waiting and riding costs, and the cost of operating vehicles. The waiting time cost is equal to $kH\gamma_w$, where headway will be optimized.

The riding cost of passengers, traveling with on-demand service, is:

$$C_{R\_O} = \left(\frac{\bar{l}}{v} + \tau.\frac{\bar{l}}{\acute{s}}\right)\gamma_r \tag{3.59}$$

The first term in equation (3.59) shows the in-vehicle riding time and the second term is the additional time due to the bus stopping per request. In this equation, $\acute{s}$ is the approximated space between each two flexible bus stops and is equal to equation (3.60):

$$s' = \frac{L}{2pH} \tag{3.60}$$

We assumed that only one passenger could board or alight the bus at each stopping. Therefore, for a total number of pH passengers in one direction of a bus route, and for many-to-many trip patterns, the total number of stoppings is $2pH$.

The cost of denied boarding is assumed to be calculated as in previous services:

$$p_d = \begin{cases} pH - K_C, & pH > K_C \\ 0 & , \; otherwise \end{cases} \tag{3.61}$$

$$C_d = \frac{p_d \gamma_d}{p} \tag{3.62}$$

We formulate the agency cost of the on-demand service for the total operating time per bus and per passenger, like other strategies:

$$C_{A\_O} = \frac{\lambda}{pH}\left(\frac{L}{v} + \tau.\frac{L}{s'}\right) \tag{3.63}$$

The total cost of on-demand service is:

$$\min C_O = kH\gamma_w + \left(\frac{p_d}{p}\right)H\gamma_w + \left(\frac{K_C}{p}\right)\left(\frac{\bar{l}}{v} + \tau.\frac{\bar{l}}{s'}\right)\gamma_r\delta + \frac{\lambda}{pH}\left(\frac{L}{v} + \tau.\frac{L}{s'}\right) + C_d \tag{3.64}$$

$subject\ to$:

$$H \geq H_{min} \tag{3.65}$$

The on-demand system includes waiting, riding, crowding, and denied boarding costs for passengers, as well as operating costs for vehicles. Due to denied boarding, only a limited number of passengers can board the bus; rejected passengers must wait for the next bus.

3.3. Summary

In this chapter, four mathematical models were developed to optimize bus stop spacing and headway along a hypothetical bus route. These models were designed for many-to-many travel patterns. The models were established based on minimization of the average cost of passengers, and agency. Our optimization models are expressed in equations (3.7)- (3.8) for all-stop system, (3.17)- (3.18) skip-stop, (3.56)- (3.58) express-local, and (3.64)- (3.65) for on-demand scheme. The cost of each system includes the average walking time, waiting time, in-vehicle travel time, transfer, crowding, and denied boarding costs, as well as the operating cost. Then we solved the comparison problem under different conditions.

3.4. Solution algorithm

Because the optimization models of alternative strategies are non-convex, they cannot be solved using a closed-form solution. We want to compare alternative operating strategies for different demand levels and passengers' trip lengths in this thesis, not just optimize each alternative operating strategy separately. This requires a solution algorithm that optimizes each strategy by headway and stop spacing, as well as comparing the total cost of different systems under different conditions to find the most efficient bus operating strategy. Due to the difficulty of solving non-convex optimization forms, we used an exhaustive search method in Python to find the optimal headway and spacing. Within the first two dimensions of the solution algorithm, we search for the optimum headway and stop spacing numerically. In the third dimension, we search for the lowest cost alternative service for a wide range of demand and passengers' average trip lengths.

The next chapter provides the results of parametric and numerical analysis and identifies the most efficient strategy. Sensitivity analyses are also performed to investigate the effects of different factors on operating costs, such as different crowding levels and unit values of riding time. A case study of Calgary is also presented.

Chapter 4: Results and case study

In this section, we parametrically and numerically quantify the performance of the proposed alternative operating services. We investigate the impacts of various parameters on the optimal designs under varying route demand and passengers' average trip length. A parametric analysis is first conducted. Then, we present some numerical examples. A sensitivity analysis is then conducted on the impact of crowding, additional riding costs caused by COVID-19 pandemic, and denied boarding on the overall system conversions. The application of the proposed models and the parametric analysis are then examined with real data from the City of Calgary, Canada.

4.1 Optimum adaptive strategy analysis

In this section, we conduct a parametric analysis of the cost functions of alternative operating services by deriving the function of the borderline between each two alternative strategies. We consider each optimization model in a 3-D space, taking values of demand per hour per route (on the x-axis) and average trip length (on the y-axis) which results in a total system cost value on the third dimension (z). At the boundary of their contours, the cost of each service is similar. In other words, by putting the cost functions of two operating strategies equal, we can find the function of the borderline, where the cost values of services are equal.

We undertake a comparison between bus routes operating all-stop, skip-stop, express-local and on-demand policies to identify the most efficient bus operating service. If we set the equation of their cost functions equal, we can determine the boundary between two alternative services. As an example, by setting the cost functions of all-stop and on-demand services (equations (7) and (32)) equal to each other (without considering crowding and denied boarding cost in cost functions), we can develop the following equation of the boundary contour line between on-demand and all-stop services:

$$\bar{l} = \frac{s.L}{\gamma_r \tau (L - 2pHs)} \left[\frac{\lambda}{pH} \left(2\tau pH - \tau . \frac{L}{s} \right) - \frac{s}{2v_w} \gamma_a \right] \quad (4.1)$$

The red contour line drawn in Figure 4.1, illustrates the boundary line between on-demand and all-stop strategies (equation (4.1)).

The rest of the boundary contour lines are derived in the same way as equation (4.1). We then plot all the borderlines under different combinations of p and $\bar{l}$, to find which service performs better. Figure 4.1 shows the range of route demand on the x-axis, and $\bar{l}$ is increasing from 1 to 15 km, on the y-axis.

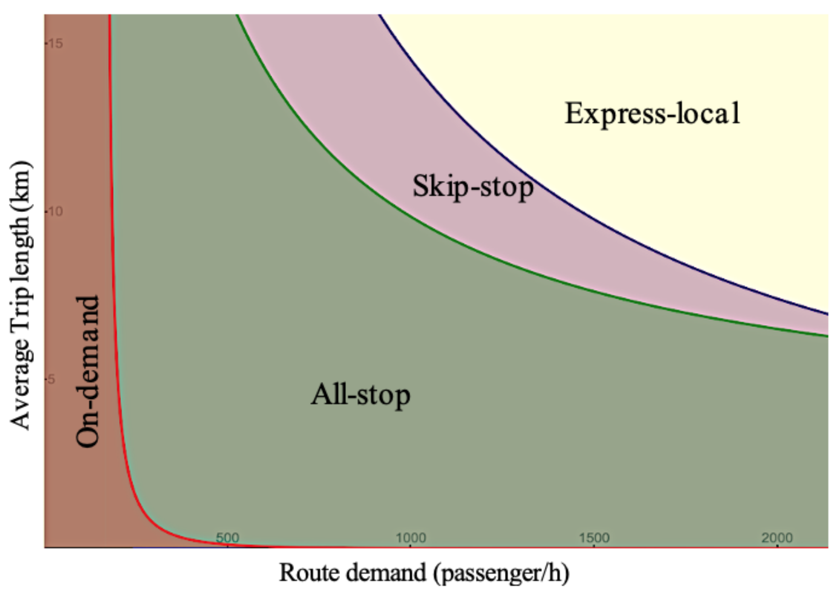


Figure 4.1. System conversions based on parametric analysis, without costs of crowding and denied boarding

Figure 4.1 delineates a demand range of (0 to 2000] passengers per hour on the x-axis, and an average trip length of (0, 15] km on the y-axis. The areas dedicated to each service show the combinations of p and $\bar{l}$ that generate the lowest cost operations strategy. According to Figure 4.1, each contour line is a borderline that determines in which area alternative services perform better. On-demand service remains the lowest cost option for the smallest demands. This is because under low demand conditions the vehicle can only stop upon passengers' request and save the additional time due to stopping at fixed stops. Thus, on-demand service produces the lowest cost scenario in small demand rates.

As demand grows, the all-stop service becomes more desirable in lower values of $\bar{l}$. This system provides a conventional operating service with a fixed stopping pattern. By stopping at all stops along the bus route in the all-stop system, the overall cost of the scheme increases. Therefore, when passengers' average trip length increases, the entire cost of system increases. Skip-stop and express-local strategies provide more efficient services in higher values of p and $\bar{l}$. In busy transit corridors with longer passenger trips, these policies are the most cost-effective, as they skip a set of stops, allow transfers between bus lines during passengers' trips, and minimize their travel time. In the next section, we present numerical examples of the system conversions to validate the findings from analytical modeling and parametric results.

4.2 System conversion

The values of input parameters in the theoretical network are obtained from different sources and are listed below:

Table 4.1. Values of input parameters

Parameter	Definition	Value	Unit
γ_a	Hourly passenger access time cost	15	\$/passenger-hour
γ_r	Hourly passenger ride time cost (for an empty bus)	9.5	\$/passenger-hour
γ_w	Hourly passenger waiting time cost	20	\$/passenger-hour
γ_t	Value of passenger transfer cost	0.1	\$/passenger-transfer
λ	Value of operating a vehicle per hour	117.95	\$/vehicle-hour
γ_d	Value of rejecting passengers	5	\$/passenger
v_w	Passenger's average walking speed	2	km/hour
v	Vehicle's average cruising speed	25	km/hour
L	Bus route length	15	km
τ	Passengers loading/unloading time	0.008	hour
m	Number of routes in skip-stop service	3	-
n	Number of non-transfer stops per route between two neighboring transfer stops in skip-stop service	2	-
N	Number of spacing between two express stops in express-local strategy	4	-
H_{min}	Minimum headway	2	minute

Based on the values of table 4.1, system conversion is performed numerically for an average passengers' trip length of $\bar{l} \in [1, L]$ km. For each alternative service, optimal headway and stop spacing is obtained from their cost function, and the lowest cost scenario is found under varying p and $\bar{l}$.

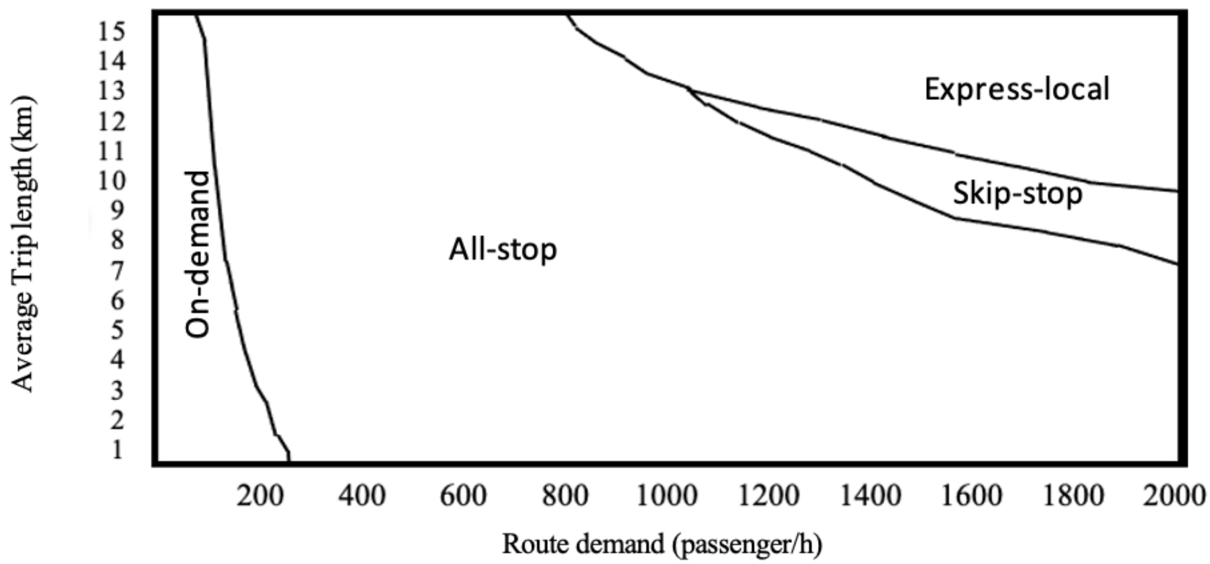


Figure 4.2. System conversions, without the costs of crowding and denied boarding

Figure 4.2 illustrates the conversions for different combinations of p and $\bar{l}$ that produce the lowest generalized costs without crowding and denied boarding. The x-axis in Figure 6 shows the hourly demand along the bus route, and it changes from 20 passengers/hour to 2000 passengers/hour. The average passengers trip length is shown on the y-axis in Figure 6, increasing from 1 km to 15 km.

The result in Figure 4.2 shows that on-demand service produces the lowest cost for the smallest levels of p, as delineated on the left side of Figure 4.2. This is because, in on-demand service, the bus only stops upon passengers' requests, and by increasing the number of passengers, the number of flexible stops along a route would be increased; thus, for many passengers, the cost of the on-demand system increases. According to Figure 4.2, as demand grows, all-stop service becomes the preferred service, compared to on-demand service. By increasing $\bar{l}$, the demand range which produced the lowest cost in all-stop decreases. This trend occurs because, in strategies with skipping patterns, more stops can be skipped in higher $\bar{l}$ outperforming all-stop service.

When demand is low, services like skip-stop and express-local are less competitive than regular all-stop services because of the relatively high agency costs. As demand increases, operating costs in all-stop service increase and produce a greater total cost. At higher values of demand and trip length, skip-stop and express-local strategies are the most cost-efficient options. Transit agency costs decrease with increase in demand, therefore coordinated skip-stop and express-local services are preferred in higher demand due to the patterns of skipping and coexistence of more than one route in their system.

The express-local service also outperforms all other services in higher demand, but not for smaller trip length. This is because, in the express-local strategy, the ratio of the total number of stops over the number of express stops is large, which means more stops can be skipped under this service compared to the skip-stop strategy.

Table 4.2 shows the values of optimum headway, stop spacing, and fleet size for alternative operating strategies, without considering crowding and denied boarding costs, for bur route length of 15 km (L=15 km), demand level of 400 passengers per hour (p=400 passenger/h), and average trip length of 10 km ($\bar{l}$=10 km):

Table 4.2. The number of fleets without considering cost of crowding and denied boarding

Strategy	**All-stop**	**Skip-stop**	**Express-local**		**On-demand**
			Express	Local	
Optimum headway	**9 min**	5 min	10 min	9 min	4 min
Optimum spacing	**0.5 km**	0.4 km	0.4 km		-
Total cost (per passenger)	**$10.3**	$11.25	$11.5		$11.8
Fleet size	**12**	53	8	12	31

The lowest cost option for p=400 passenger/h and $\bar{l}$=10 km is all-stop service, shown in bold in Table 4.2. Skip-stop strategy utilizes the most fleets since it runs three parallel routes in its system, according to Table 4.2.

4.3 Optimal number of routes in skip-stop and stop spacing between express routes

During the analysis, we have assigned a constant value to the number of skip-stop routes (m=3) and the spacing between express stops (N=4). A study of different values of m and N is performed in order to calculate the total costs of skip-stop and express-local strategies to examine the effects of these parameters on the total cost of the system. Below are the results of the lowest cost option:

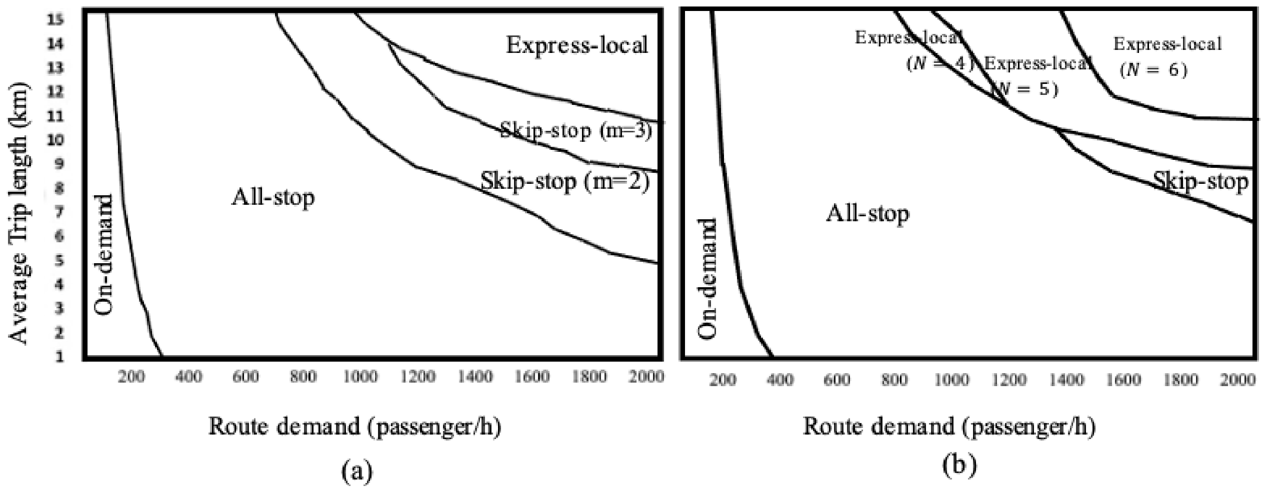


Figure 4.3. conversions without costs of crowding and denied boarding, to find the optimal values of: (a) number of routes in skip-stop service (m) and (b) spacing between express stops (N)

Conversions of Figure 4.3 (a) show the most efficient service under different combinations of p and $\bar{l}$, in addition to illustrating the impact of changing the number of routes in the skip-stop strategy (m) on the shape of the contours. When demand and average trip length increase, the optimal number of skip-stop routes provided in the strategy increases from 2 to 3. Skip-stop service providing m=2 parallel bus lines is preferable for a wide range of passenger demand, as is evident in Figure 4.3 (a).

Skip-stop with $m = 3$ is preferred only for very high values of p and a smaller range of $\bar{l}$. Values of m greater than 3, result in a higher generalized cost compared to the express-local strategy. Thus, they are not efficient choices for passengers.

Figure 4.3 (b) displays the system conversion for the most efficient bus operating strategy under varying p and $\bar{l}$, when the average number of stop-spacings between two consecutive express stops, N, increases from 4 to 6. From Fig 4.3 (b), we can observe that by increasing the value of N and for a high range of p and $\bar{l}$, the total generalized cost of the express-local strategy decreases.

In other words, when express stops are farther from each other, the cost of operating express-local policy decreases.

4.4 Sensitivity to riding time

Transit users are more susceptible to crowding during the COVID-19 pandemic. Thus, we conduct sensitivity analyses based on unit costs of riding time (γ_r) to evaluate the system conversion under different levels of passengers' sensitivity to crowding during the pandemic.

For one hour of passenger riding time, Pollock et al. (2021) estimated three different values depending on the level of crowding on a vehicle: (i) \$9.5 per passenger for an empty bus; (ii) \$18.5 per passenger for a semi-crowded bus, and (iii) \$30 per passenger for a crowded bus. Based upon these values of riding time cost for each level of bus loading factor, we determined which operation is most efficient under each scenario.

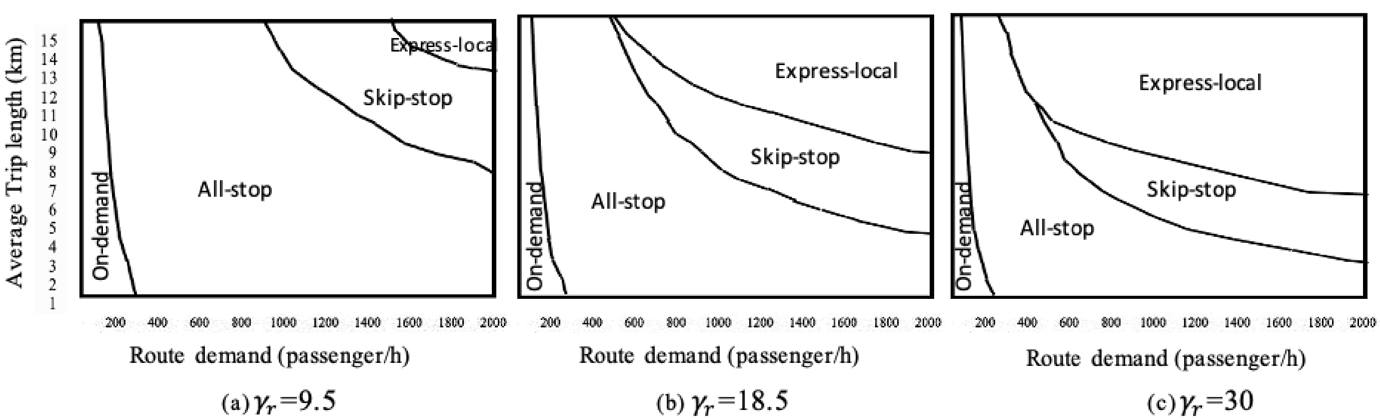


Figure 4.4. conversions with different values of riding time, without crowding and denied boarding costs: (a) γ_r=9.5, (b) γ_r=18.5, and (c) γ_r=30

Cost comparisons of alternative schemes for three levels of riding costs are summarized in Figure 4.4. The crowding cost is not captured in this output. Nonetheless, passengers' sensitivity to one hour of riding (duration of exposure) increases based on severity levels of pandemic or the impact of crowding on virus transmission as determined by Pollock et al. (2021). In other words, even when buses are not fully crowded, the severity of the pandemic, as well as the spread of diseases, result in passengers' perception of greater susceptibility to infection during their travel time (Pollock et. al, 2021; Basnak et al., 2022).

As the riding cost increases, passengers prefer to spend less time onboard. From Figure 4.4, we can observe that the express-local strategy becomes more efficient, as the sensitivity to riding time increases. Since this service skips a number of stops, and provides several transfers between routes, thus, passengers can move between different routes to decrease their in-vehicle ride time.

As expressed in equation (3.3), the crowding level can be approximated with the loading factor, which is a function of hourly route demand, average passengers' trip length, length of the bus route, and capacity of vehicles. In this study, we assumed that crowding cost increases linearly with the increase in load factor. The cost functions of alternative strategies are minimized, accounting for crowding disutility for transit users. In this case, we consider $\theta_{min} = 0.289$ (Pollock et al., 2021) and $\rho = 1$, and the values of other parameters remain the same. The value of θ_{min} is calculated by Pollock et al. (2021) and the slope factor is estimated to be 1 (ρ =1), where crowding increases linearly with the loading factor. Pollock et al. (2021) determined three levels of bus load factor indicating onboard crowding level. The slop factor can then be approximated by finding the slope of a best-fitted line of three riding times as a function of the loading factor.

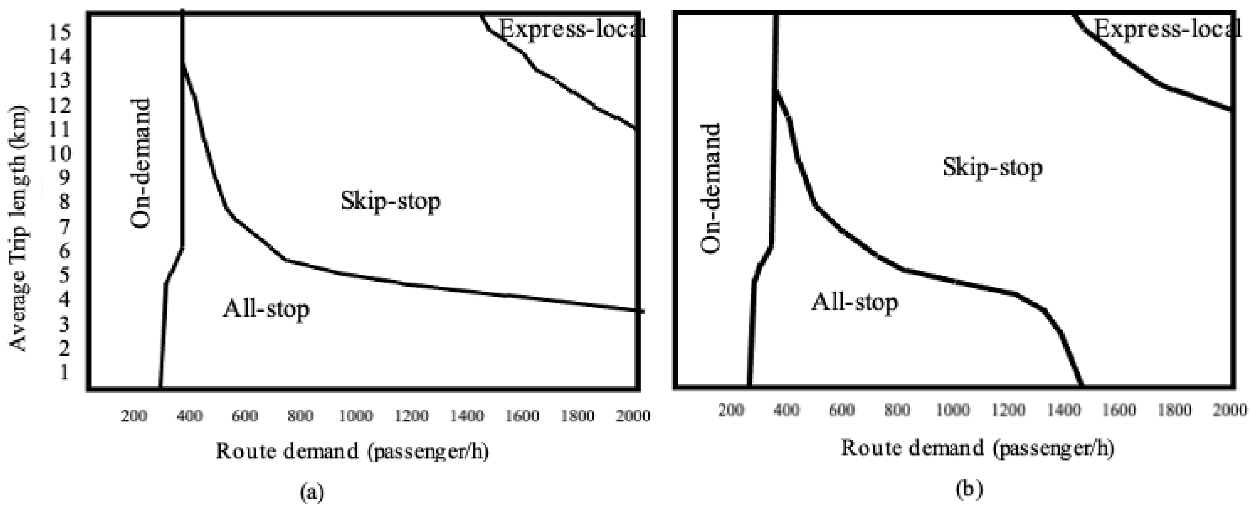


Figure 4.5. Conversions with considering crowding cost; (a) without, and (b) with denied boarding costs

Figure 4.5 (a) shows the lowest cost service with contour lines when the capacity constraint is applied, and we define a crowding cost when the loading factor is greater than θ_{min}. At longer average trip lengths, the shape of the on-demand contour line has changed, and all-stop services become less efficient when crowding costs are considered. The headway does not differ much between a system with on-demand and one with all-stop for longer trips (higher $\bar{l}$). However, because of the longer roundtrip time of the on-demand system, it employs a larger fleet to offset its higher cost. With on-demand service, the fleet is almost twice as large as with all-stop service. Therefore, Figure 4.5 illustrates that the contour line for the on-demand strategy changed from Figure 4.2, and on-demand strategy becomes more attractive for longer average trips than an all-stop system.

According to the conversions in Figure 4.5 (a), considering on-board crowding, skip-stop service is the most efficient option in a wide range of p and $\bar{l}$. This is because, in the skip-stop strategy, m distinct routes serve all the stops along the route, and each vehicle would skip a set of stops; thus, it will reduce users' riding time. Also, the hourly demand divided between m routes and vehicles on each route would only carry a portion of $1/m$ of the passengers.

To track the number of denied boardings and minimize the associated cost in the optimization models, we will compare the total cost of alternative operating strategies. With crowding and denied boarding costs considered, Figure 4.5 (b) shows the most efficient bus operating strategy. As a result of including the denied boarding charges, the contour lines of skip-stop strategy are changing. Comparing Figure 4.5 (a) and (b), when the denied boarding costs are considered, we can see that skip-stop service becomes more attractive than all-stop, in very high demand situations. With a higher number of routes in skip-stop service in comparison to other services (m=3), the number of denied boardings in this service would be lower than in other strategies; thus, when agencies are concerned with the number of served passengers, they should consider skip-stop services more often to avoid rejecting passengers from boarding.

According to Figure 4.5 (b), express-local is the second-best service that performs well in high demand rates. The express-local strategy runs two parallel routes on a bus route and can serve more passengers than the all-stop service and result in fewer denied boardings. By comparing Figure 4.5 (a) and (b), we can conclude that when transit users are more sensitive to the level of occupancy, they prefer to ride those services with a stop skipping pattern to avoid crowding.

Table 4.3 shows the optimum headway, stop spacing, and fleet size of different operating strategies, considering crowding and denied boarding costs, for L=15 km, p=400 passengers/h and $\bar{l}$=10 km.

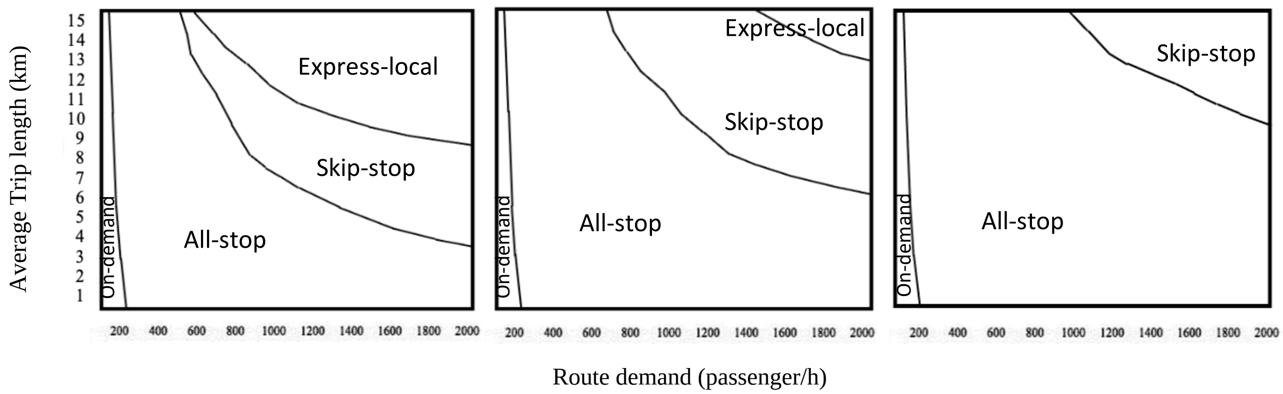
Table 4.3. The number of fleets with considering cost of crowding and denied boarding

Strategy	**All-stop**	**Skip-stop**	**Express-local**		**On-demand**
			Express	Local	
Optimum headway	4 min	**4 min**	5 min	8 min	3 min
Optimum spacing	0.6 km	**0.4 km**	0.5 km		-
Total cost (per passenger)	$12.3	**$12.1**	$13.6		$13.05
Fleet size	24	**66**	14	13	37

As we compare Table 4.2 and Table 4.3, we find that by considering the passengers' sensitivity to travel times and crowding, the lowest cost option changes from all-stop to skip-stop. As a result, when passengers are sensitive to crowding, services with skipping patterns reduce the system's cost. Furthermore, more fleet is required for all strategies to avoid overcrowding and address passengers’ concerns during a pandemic.

4.5 Sensitivity to transfer cost

Our optimization models of skip-stop and express-local strategies considered the disutility of passengers transferring between lines, as well as the additional waiting time associated with uncoordinated routes in the express-local system. To evaluate the system conversion based on different levels of passengers' sensitivity to transfers between lines, we performed a sensitivity analysis based on unit transfer costs (γ_t).



(a) γ_t=0.1 (b) γ_t=0.5 (c) γ_t=1

Figure 4.6. Conversions with different values of transfer cost, without crowding and denied boarding costs: (a) γ_t=0.1, (b) γ_t=0.5, and (c) γ_t=1

Figures 4.6 (a-c) show the optimum scheme when we increase the unit cost of transfer from 0.1 to 1. In this output, we considered the value of one hour of waiting to be \$15 (γ_w=\$15). As such, the additional waiting time is already considered under the waiting time for the skip-stop and express location services in equations XXX and XX presented in Chapter 3. The sensitivity to transfer, only considers the discomfort in addition to the extra waiting for transferring. As γ_t changes, the contour lines of skip-stop and express-local strategies change. As described in the methodology chapter, passengers in an express-local system would be allowed to make two transfers, but in a skip-stop scheme, they would only be allowed to make one. The express-local strategy is, therefore, more affected by transfer costs than the skip-stop strategy. Thus, express-local service becomes less appealing as transfer between lines becomes less desirable.

4.6 Fleet size constraint

In this section, we explore the fleet size constraint in alternative operating transit systems. In some cases, finding the bus headway by minimizing the cost function of each service results in a higher number of fleets than is realistic. As such, we investigated the available fleet size constraint and determine the bus headway under an optimal fleet.

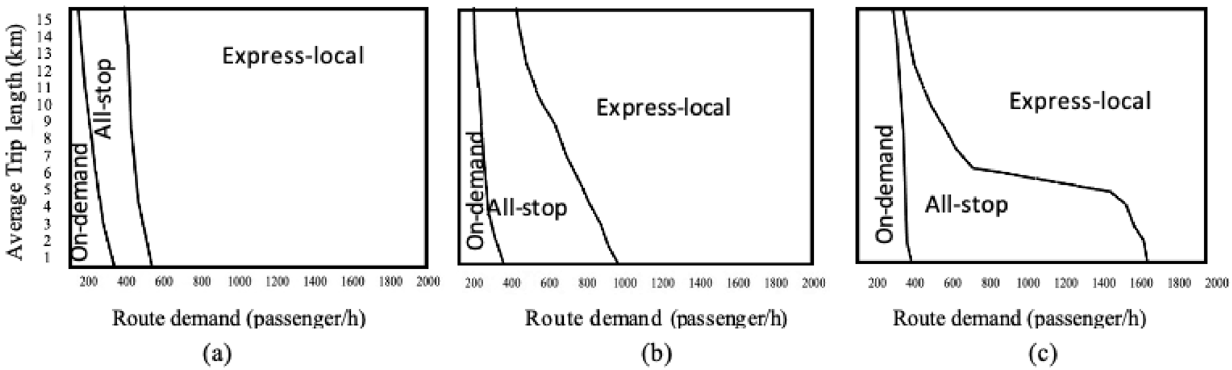


Figure 4.7. Fleet size constraints with crowding and denied boarding costs; (a) available fleet= 3, (b) available fleet= 9, and (c) available fleet= 15

Figures 4.7 (a-c) show the optimum scheme when we apply different fleet size constraints between 3 and 15. When considering onboard crowding and denied boarding costs, the express-local strategy is the most efficient service, even in low fleet availability conditions. Based on the definition of the skip-stop strategy, m (here 3) parallel routes are serving one bus line; therefore, compared to other services, we need a larger fleet to operate this service. The optimal headway for L=15 km, p=800 passengers per hour and $\bar{l}$=10 km, and for each fleet size constraint is given in table 4.4:

Table 4.4. Optimal values of headway and spacing on each bus route for different fleet size

Fleet size	All-stop		Skip-stop		On-demand	Express-local		
	headway	spacing	headway	spacing	headway	Headway on Express route	Headway on Local route	spacing
M≤3	33 min	0.9 km	72 min	1.2 km	28 min	**23 min**	**32 min**	**0.6 km**
Total cost (per passenger)	$26.14		$101.2		$74.2	**$23.7**		
M≤9	11 min	0.7 km	26 min	1.1 km	10 min	**10 min**	**11 min**	**0.6 km**
Total cost (per passenger)	$23.2		$57.5		$65.8	**$19.3**		
M≤15	8 min	0.6 km	17 min	0.7 km	6 min	**7 min**	**8 min**	**0.4 km**
Total cost (per passenger)	$22.8		$48.3		$65.8	**$16.5**		
No constraint	2 min	0.6 km	**2 min**	**0.4 km**	2 min	3 min	6 min	0.4 km
Total cost (per passenger)	$11.97		**$11.1**		$12.4	$13.4		

Under each row, the most efficient scheme is shown in bold. Under fleet size constraints, express-local appears to be the most desirable strategy. On the other hand, skip-stop is more efficient when fleet size isn't a constraint, and when crowding and denied boarding costs are taken into account (see Figure 4.5 (b)). According to values in Table 4.4, for this sample combination of p and $\bar{l}$, an increase in available fleet decreases the optimal headway on each alternative operating bus route.

4.7 Discussion

The results of this study differ from previous studies in the literature several aspects: (i) we defined on-demand transit policy, and it is an efficient strategy in lowest demand levels, compared to Gu et al.'s outputs where all-stop service remains the lowest cost strategy in low demand levels, in addition, (ii) by considering passengers' sensitivity to travel time and minimizing onboard crowding, skip-stop scheme with 3 routes can dominate all schemes with the majority of combinations of p and $\bar{l}$ (see Figure 4.5 (a)), (iv) we also defined denied boarding cost to minimize the number of rejected passengers leading to skip-stop becoming more desirable, particularly in high demand levels (see Figure 4.5 (b)).

4.8 Case study

We will test the optimization models on a real bus route in the City of Calgary, Canada. In this paper, we use the system parameters and operating data from Calgary Transit to test system performance under each bus operating policy and show how the selection of different operational strategies changes in a real transit system, before and after the COVID-19 pandemic. Calgary Transit (CT) provides several transit services including bus, bus rapid transit (BRT), and light rail transit (LRT). For the purpose of our case study, we focus on bus route 43, from McKnight-Westwinds LRT Station to Chinook LRT Station Eb, in Calgary. The route layout is presented in Figure 4.8:

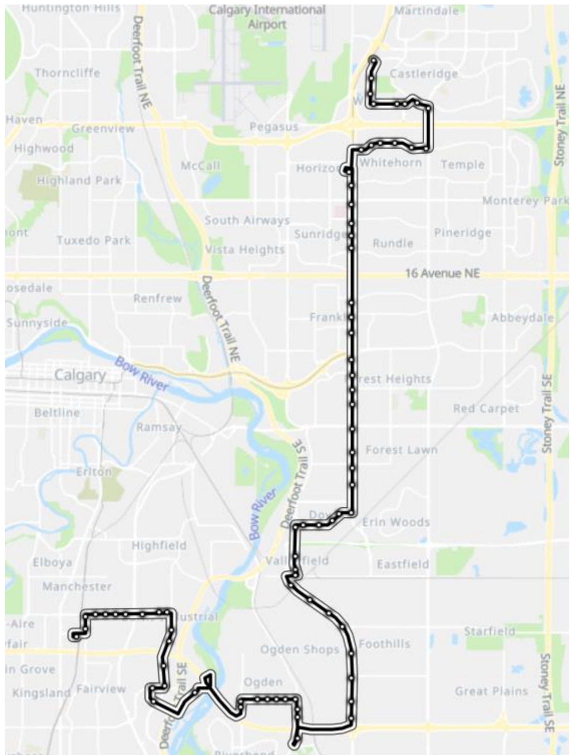


Figure 4.8. Topology of route 43 in the City of Calgary[2]

The hourly demand of a week for this bus route, from April 22nd to 28th in 2019, 2020 and 2021 is provided in Figure 4.9:

[2] https://moovitapp.com/calgary_ab-1162/lines/43/31606073/4815868/en?customerId=4908&ref=2&poiType=line

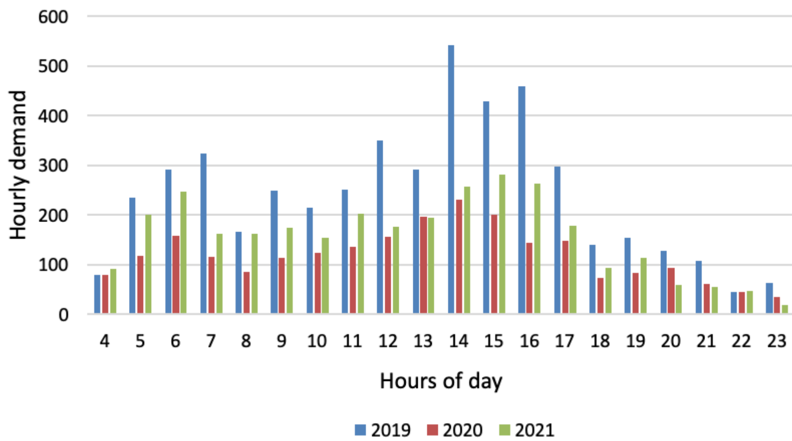


Figure 4.9. The average hourly demand for bus route 43 (a week in April 2019, 2020, and 2021)

As shown in Figure 4.9, the average hourly demand for bus route 43 is given for a week in April 2019, 2020, and 2021. Before the pandemic (2019) the average passenger demand for morning peak hours (6-9 am) is 258 passengers per hour, and for the afternoon peak (3-6 pm) is about 332 passengers per hour. As we can see from Figure 4.9 after emerging the COVID-19 pandemic (2020), demand has dramatically dropped to an average of 119 passengers per hour in the morning peak, and 142 passengers in the evening peak. As we expected a new normal after adapting to the COVID-19 conditions, we can see a growing trend of use of public transit, as such returning to schools and reopening some businesses in the fall of 2021. The average demand of 187 in the morning peak, and 205 in the afternoon peak, shows an increasing trend of transit demand, catching up with the pre-COVID trend.

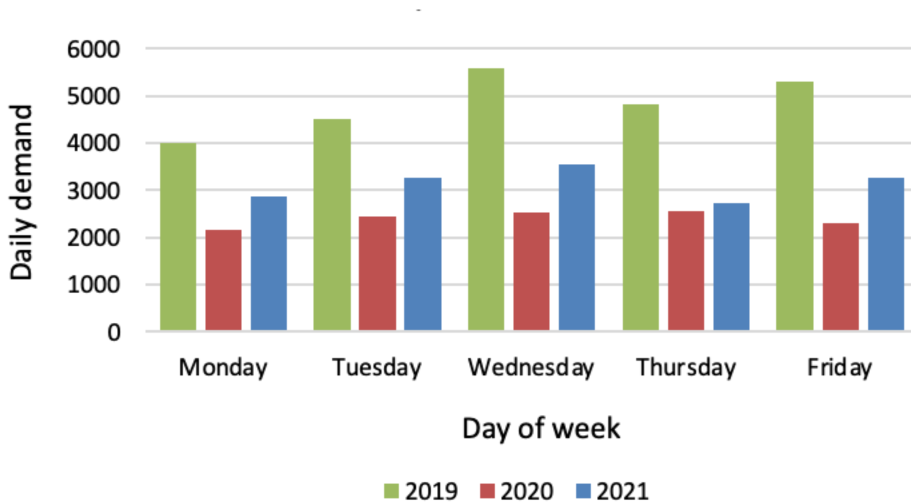


Figure 4.10. daily demand of route 43 for a week in April (2019, 2020, and 2021)

The columns in Figure 4.10 show the daily demand for route 43 for a week in April in 2019, 2020 and 2021. Figure 4.10 shows a similar pattern to Figure 4.9; after the emergence of the COVID-19 pandemic, demand for weekdays starts to decline in 2020, and then grows in 2021, in the post-pandemic. According to Figure 4.10, there is not much variation in demand from day to day. Upon returning to normal and increasing remote work, the demand for different days of the week might change. Consequently, alternate operating strategies can be helpful for transit agencies in adapting their daily plans depending on day-to-day variations in demand.

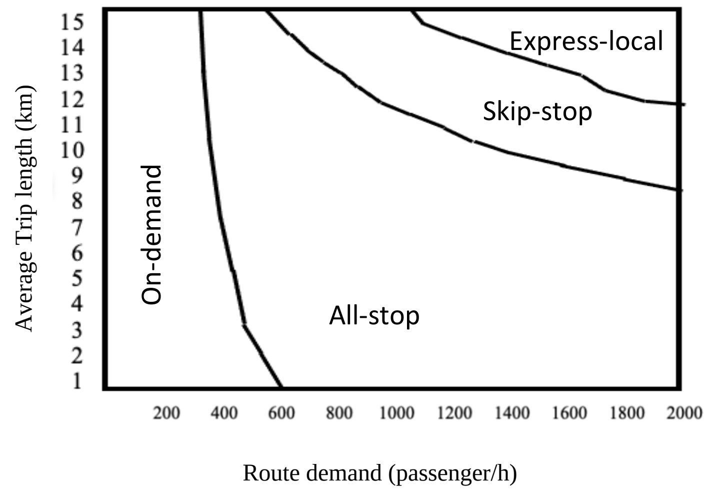


Figure 4.11. conversions for bus route 43 in Calgary, without crowding and denied boarding costs

We used the GTFS and AVL data for route 43 to obtain some of the model parameters such as route length (31.8 km) and average stop spacing (0.4 km). Figure 4.11 shows system conversion before the pandemic. Due to the fixed location of bus stops on route 43 in the CT network, the average stop spacing on all-stop, skip-stop, and express-local routes cannot be optimized for these services as a decision variable anymore. Thus, on-demand design is an effective strategy for a wider range of trip lengths and demand, compared to the contour lines of theoretical results in Figure 4.2.

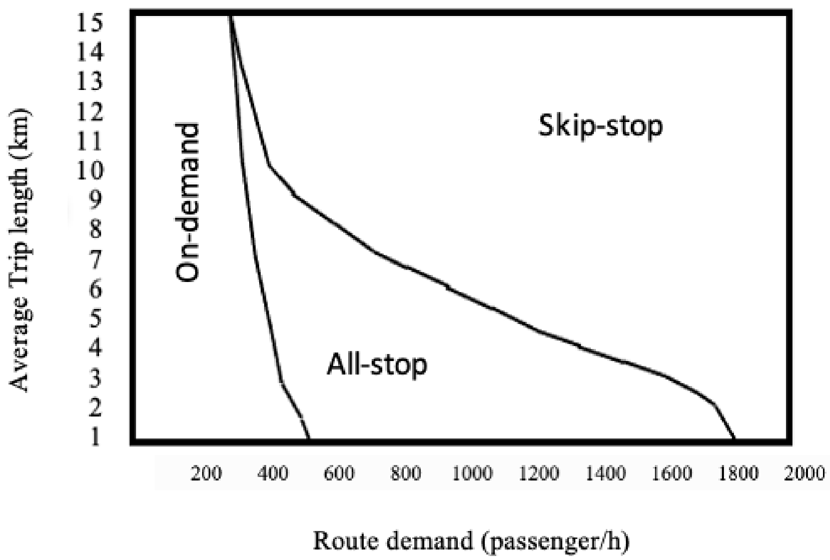


Figure 4.12. conversion for bus route 43 with the costs of crowding and denied boarding

Figure 4.12 presents conversion for an optimal operating strategy that captures the crowding disutility and denied boarding. When comparing Figure 4.12 with Figure 4.11, it is evident that the skip-stop strategy provides a more efficient service to users who are sensitive to onboard crowding due to route coordination and skipped patterns, and because congestion can be distributed across multiple routes in a skip-stop system.

According to Figure 4.12, conventional operational strategies cannot address passengers' concerns in crowded transit vehicles due to the emerging COVID-19 pandemic. As the sensitivity to crowding increases, the lowest cost option shifts from all-stop service to skip-stop service with schedule coordination. According to our theoretical results, we found that the use of a skip-stop system can be a better option, through more frequent service, stopping patterns, and dispersing the crowd over several routes. As a result, skip-stop dominates express-local service on route 43 when crowding penalty is considered. In addition, on bus route 43 (with a route length of 31.8 km), the operating costs of express-local are higher than other strategies, which results in outperforming skip-stop over express-local.

According to the average demand for route 43 in Calgary, it appears to be fluctuating by the time of day, by day of the week, as well as by the year, due to the COVID-19 pandemic. Based on the adaptive operational strategies' models outlined in this paper, Calgary Transit experienced less than 300 passengers per hour in 2021, which can be accommodated by on-demand service. The return to normal would result in more passengers using transit, and demand levels of 500 passengers/h or more would be experienced (as pre-COVID). Therefore, an all-stop strategy would be recommended. As demand grows, the hourly demand for this bus route is likely to increase to over 700 passengers/h post-COVID, at which point skip-stop and express-local strategies is recommended to be implemented.

Chapter 5: Conclusion

5.1 Research summary and conclusions

In this thesis, four alternative operating strategies were formulated and compared in a framework. The cost functions of alternative services include the cost of walking time, waiting time, travelling time, and transfers between lines, as well as the operating costs of the agency. Using the proposed framework in this thesis, we can suggest the most cost-effective service considering variations in demand and trip patterns, as well as changes in users' sensitivity to cost factors. The models developed in this thesis can be applied to disruptions such as severe weather conditions, long term impacts of COVID-19 with mix of in-person/remote travel patterns, changes in travel patterns and trip distances, and rises in gas prices. In this study, costs of crowding were also included to take into account passengers' sensitivity to crowding during a pandemic on transit vehicles. The reduction of vehicles' capacity due to crowding discomfort results in vehicles leaving passengers behind to alleviate overcrowding. In order to avoid leaving many passengers behind, the cost of denied boarding was included in the cost functions of different services, to create a trade-off between the level of onboard crowding and the level of service.

Conventional all-stop service is often compared to flexible on-demand service. We extend the literature by including more service designs in the present study. This paper develops models of alternative operating strategies that minimize the generalized costs of all-stop, skip-stop, express-local, and on-demand services. We proposed a framework that compares conventional all-stop service with skip-stop, on-demand, and express-local strategies and suggests the most efficient scheme under a wide range of demand and passengers' average trip length. The demand-adaptive policies in this study combine fixed- and flexible-route transit strategies for any variations in demand and trip patterns during peak and off-peak hours and areas experiencing demand fluctuations.

According to the parametric analysis, on-demand service has the lowest generalized costs for low demand scenarios. In intermediate levels of demand and longer trip lengths, all-stop service dominates on-demand. With high demand and longer passengers, skip-stops and express-local services can reduce overall system costs compared to conventional all-stop services.

Despite passengers' longer waiting times in skip-stop systems, passengers are distributed to different vehicles. Thus, a skip-stop strategy becomes more desirable, given the high level of passengers' sensitivity to crowding, for a wide range of trip lengths and levels of demand.

Traditional all-stop bus operations are most efficient when passengers are not sensitive to the level of crowding, the fleet is limited, and demand is low. In situations where passengers are more sensitive to crowding (i.e., during a pandemic), express-local becomes the most cost-efficient option for various combinations of demand and average trip length, and under limited fleet size constraints.

Optimal area of on-demand service changed as the sensitivity to crowding increased. Utilizing larger fleet helped on-demand service to be more system efficient than all-stop strategy in longer trip lengths and higher demand levels, skip-stop was identified as the lowest cost system when passengers are sensitive to crowding. It becomes even more efficient when agencies define a penalty for denied boardings.

Transit planners and operators can use this framework to select the most efficient bus operating strategy based on rapid changes in travel demand patterns due to any disruption to transit sector, remote and in-person activities and through different time of day, and day of the week. Public transit systems in densely urbanized areas and sprawling suburban areas need to determine their most effective operational strategies to improve their performance and service levels. In addition to urban residential areas, our optimization models can be applied to rural regions with low demand rates.

A case study is conducted on bus route 43 in Calgary to demonstrate the applicability of the proposed model. Calgary Transit can apply all the services along one bus route without adding any extra infrastructure. Based on transit demand, average passengers' trip length, and sensitivity to the pandemic level, Calgary Transit suggests the most efficient alternative service to its users.

5.2 Limitations and future work

A limitation of the study is the use of uniform demand distributions. Transit demand does not always disperse uniformly over transit routes. In this thesis, the unbalanced demand for inbound and outbound trips is not considered. In addition, we assumed a fixed route for all strategies. However, on-demand services can deviate from the fixed route to serve passengers door-to-door. However, we intentionally did not consider such service to be able to compare all services together. The dwell time at each stop is also assumed to be a constant value, which is a simplified assumption. Additionally, this thesis did not consider attracting users to transit, and reliability is not taken into account. Variability in dwell time and generally uncertainty in the operations can affect the most desired more of operations under different demand and travel pattern scenarios.

Studying the non-uniform distribution of demand and dwell times that vary with demand could be a future focus of research. A future study can consider zonal stopping policies and penalize bus stops for overcrowding. Moreover, the results of this study can be applied to transit networks with multiples routes operating under different operating strategies.

5.3 Thesis contribution

The contributions of this thesis are as follows:

- We developed cost functions of all-stop, skip-stop, express-local, and on-demand operating services.
- We defined the unit cost of users' costs to distinguish between different components of users' costs such as walking, waiting, riding, and transferring.
- We developed a comparison framework that compares fixed and flexible policies under various demand levels and trip lengths.
- We emphasized on passengers' sensitivity to crowding by defining crowding disutility on buses.
- We defined penalties for denied boarding to maintain adequate levels of service.
- We conducted sensitivity analysis for different levels of bus loading.
- We conducted analysis of some fleet size constraint scenarios under crowding conditions
- In response to fluctuations in demand and trip patterns, we created a generic and adaptive tool to improve public transit operations. This tool can easily be applied to other transit systems by considering proper unit cost and demand factors.

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