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# An Analysis of the Bayesian Classification of Monetary Assets in the United States

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "An Analysis of the Bayesian Classification of Monetary Assets in the United States" submitted by Carmen Anderson in partial fulfilment of the requirements of the degree of Master of Arts.

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## Abstract

Using monthly time series data from 1959 to 2002, this paper attempts to cluster the Federal Reserve's 26 monetary assets according to mathematical properties, rather than by similarities regarding their liquidity. The new classes are formed using a Bayesian procedure entitled AutoClass, which groups the assets according to their various attributes such as asset quantities, user costs, velocity, and growth rates. This classification results in three distinct groups being formed. Thereafter, monetary aggregates are constructed using simple sum, divisia, and currency equivalent techniques. Using these new aggregates, a formal analysis of the interaction between money, prices, output and the interest rate follows, paying particular attention to the causal and cyclical relationships present.

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## Dedication

*To Craig, for your understanding*

*and encouragement.*

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A theory has only the alternative of being right or wrong.
A model has the third possibility: it may be right, but irrelevant.

Manfred Eigen, The Physicist's Conception of Nature

# CHAPTER ONE: INTRODUCTION

Monetary theory has played a role in macroeconomic literature for decades, most often used by central banks to determine the result from using different operational objectives. Currently, many central banks implement an inflation target after many unsuccessful years of money growth targeting. However, the relationships between the economic variables in the system each play a role in determining the attainment of a chosen target. The interactions between the interest rate, price level, money supply, nominal output and real output will all be discussed in this paper. Whether it stems from causality or common cycles, the interaction among the key macroeconomic variables will continue to be a frequent topic in the literature.

The central purpose of this paper is to reconstruct the monetary asset groupings in the United States according to statistical similarities, construct the new monetary aggregates, and examine the relationship between the monetary aggregates and the interest rate, the price level, nominal output, and real output. The AutoClass program, based on Bayesian cluster analysis, determines the optimal number of groups by finding the "natural" classes within the data. This type of classification requires variables to be grouped into an unknown number of optimal groups, rather than by arranging them into a set of preexisting classes. Unsupervised clustering becomes more complicated when the variables of interest contain many attributes used for grouping. The attributes we use to classify the 26 assets monetary assets include the asset quantity, user cost, velocity and growth rate. Bayesian cluster analysis will be implemented in this paper since typical classification programs cannot handle the multi-dimensional structure of our monetary asset variables.

These new classes of monetary assets will share statistical similarities, rather than the shared liquidity properties that exist in the current grouping outlined by the Federal Reserve Board. The monetary asset groupings published by the Federal Reserve have changed significantly over the years, being redefined roughly 12 times. In the 1960's, M1 became the focus of government policy-makers due to its value as a medium of exchange, as outlined by Friedman and Schwartz (1963). In the 1970's, the Federal Reserve developed M2, M3, M4, and M5 with the prospect of finding a better predictor of the interaction of money with other key policy variables. They believed that stimulating the economy through money targeting could be achieved if the proper monetary asset grouping was found.

Currently, the Federal Reserve constructs six nested levels of monetary assets: M1A, M1, MZM, M2, M3, and L. They range from the very liquid M1A, which contains currency, demand deposits, and travelers checks, to L which contains more long-term assets such as U.S. savings bonds. We will compare our results with the classes currently published by the Federal Reserve to see if any comparison can be made. Although the objective of policy-makers has changed from monetary growth targeting to inflation targeting, the need for appropriate monetary asset groupings has remained in tact. Now, monetary assets are used as intermediate targets, in order to reach the ultimate goal of a stable inflation rate.

After implementing the AutoClass program, our asset groupings will share the same statistical distribution, which we expect will detail the relationship between money and other key macroeconomic variables more accurately. This interaction becomes important for policy-makers when predicting the future impact of central banking decisions.

Once the new classes have been formed, we will construct three versions of monetary aggregates for each class. The simple sum, divisia, and currency equivalent indexes, which have been examined previously in the literature, will be used for further econometric testing purposes. The simple sum aggregates, although exhibiting problems with the assumption that all assets in a group are perfect substitutes, have been widely used by central banks for decades. The development of the divisia aggregates, as introduced by Barnett (1980), allowed for a more theoretical approach using a weighting technique. In furthering the study for a more comprehensive aggregation method, Rotemberg et al. (1995) developed the currency equivalent index, a utility-based aggregate. We will examine each method of aggregation in Chapter three, which is followed by a study of the time series properties pertaining to each aggregate.

In Chapter four, each of our three aggregate versions will first be examined for stationarity, as detailed by Serletis (2001). The results from the unit root tests are imperative when developing models regarding the interaction between money and the policy-makers target variable. The relationship between the monetary aggregates and the key macroeconomic variables such as prices, output and the interest rate is developed using cointegration and causality tests. The linear combination of two non-stationary variables must be stationary for cointegration to be present.

Causality was introduced by Granger (1969), demonstrating that a variable causes another only if the additional information provided helps in the prediction of the first. We implement the single equation causality test, as introduced by Granger (1969) in addition to the multi-equation method as developed by Sims (1992). We are specifically interested in whether our monetary aggregates cause the macroeconomic variables of interest. The implications of causality assist policy-makers in discovering how the relationship exists between these key variables in the economy.

We also examine the existence of common or codependent features within the data, to distinguish whether long run cycles are present, and hence useful for policy objectives. In similar form to cointegration, common (synchronized) cycles exist when a common feature is present in each series, but is not within the linear combination of the two. Codependence, or an unsynchronized cycle, is represented when the linear combination of the two variables has an order with a lower moving average than the others.

In the next chapter, we will explain the AutoClass procedure, the data used, and the resulting classification of the monetary assets formed. In chapter three, the three versions of monetary aggregates will be constructed, which will allow us to run subsequent econometric tests. Chapter four examines the time series properties of the newly formed aggregates, developed in Serletis (2001). Common and codependent cycle analysis, our final test of how money is related to output, prices, and the interest rate, will be discussed in Chapter five. Finally, in the last chapter, we will conclude our results, and the implication for further research concerning monetary groupings.

# CHAPTER TWO: THE CLASSIFICATION OF MONETARY AGGREGATES

## *Bayesian Cluster Analysis*

The cluster analysis of different subjects has been studied frequently in previous economic literature. Kontkanen et al. (1996) examined the Bayesian model selection for real world applications. They used this type of classification to cluster a data vector regarding different medical subjects into distinct regions. Ardic (2002) further employs this method in her analysis of grouping 105 countries according to their socioeconomic similarities. We are specifically interested in the application of unsupervised classification of the monetary assets in the United States.

The Bayesian system, based on the finite (or classical) mixture model, searches for the most probable number of classes, given the attributes of the data and the prior expectations. Each region (class) in the model is comprised of a different statistical distribution that the data can be sorted into. Determining the model class and the associated class parameters are the two steps in the Bayesian clustering technique that are outlined by Kontkanen et al. (1996). Cheeseman and Stutz (1996) continue to make use of these steps in their development of the AutoClass procedure, which is utilized in Ardic (2002) and subsequently in this paper.

The first step, determining the appropriate probability model class, can be approximated using three different methods. Kontkanen et al. (1996) adapt two methods using either the Bayesian Information Criterion (BIC) or the Akaike Information Criterion (AIC) to determine the model class. However, their methods are only practical when we are interested in using the incomplete data vector. The third method, introduced by

Cheeseman and Stutz (1996) in the AutoClass program, implements the conditional binary probability of observing an attribute, and a Gaussian probability if the attribute is actually observed, to assist in choosing the appropriate model class.

The data set, associated priors, and the EM algorithm are used to estimate the MAP (maximum a posteriori) values. The iteration of the EM algorithm is continued until there is convergence of the MAP parameters to a stable maximum point. Then, the posterior probabilities for each model class are compared to find the optimal number of classes.

For this paper, we implement the AutoClass procedure, which is discussed in detail in the following section.

### *The AutoClass Program*

The automatic classification program (AutoClass)[1] for cluster analysis, developed by Cheeseman and Stutz (1996), uses unsupervised classification, since it aims to find natural classes within the cases, rather than assuming a specific number of groups preexists. Using attributes of the cases as inputs, this Bayesian system of classification constructs the maximum log probability to automatically determine the optimal number of classes.

As indicated earlier, AutoClass uses the classic mixture model, which is useful when there is unobserved heterogeneity present in the data, and was developed by Everitt

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[1] Source: http://ic.arc.nasa.gov/ic/projects/bayes-group/autoclass/autoclass-c-program.html#Obtaining

and Hand (1981) and Titterington et al. (1985). This mixture model can be translated to a Bayesian form by addressing the priors of the parameters. Let $i$ index the number of cases, which need to be clustered into the optimal number of $j$ classes $(C)$, which is unknown. Let $X_i$ denote the cases of data, where each case contains $k$ attributes, with the entire data set $X$ comprising of $(i*k)$ attributes. $F_j$ are the mathematical forms and $\theta_j$ are the parameter values for each of the $j$ distributions. Let $\lambda = (\theta, p)$ be the set of the parameters for the entire model where $S$ is the space of possible probability distribution functions. Each case belongs to only one class, with the probability:

$$P(X_i \in C_j \mid \lambda_j, F_j, S) = p_j \qquad \text{where } 0 \le p_j \le 1 \quad \text{and} \quad \sum_{j=1}^{n} p_j = 1 \tag{2.1}$$

To examine the prior and the likelihood portion of the equation separately, this interclass model can be rewritten as a joint probability of the data and parameter values:

$$\begin{aligned} P(X, \lambda \mid F, S) &= P(\lambda \mid F, S) P(X \mid \lambda, F, S) \\ &= P(\lambda \mid F, S) \prod_i \sum_j p_j P(X_i \mid X_i \in C_j, \theta_j, F_j) \end{aligned} \tag{2.2}$$

The AutoClass program uses the Gaussian distribution for real-valued attributes, whereby the log-Gaussian form is constructed by applying $log(X_{ik} - min_k)$ from Aitchison and Brown (1957) to account for possible negative values.

Once we have input the attribute values, a search is conducted for the joint probability of the maximum a posteriori (MAP) parameter values $\lambda$ and the form of $F$, which is accomplished with the application of the EM algorithm. These two searches can be observed as:

$$P(\lambda \mid X,F,S) = \frac{P(X,\lambda \mid F,S)}{P(X \mid F,S)} = \frac{P(X,\lambda \mid F,S)}{\int P(X,\lambda \mid F,S)d\lambda} \tag{2.3}$$

$$\begin{aligned} P(F \mid X,S) &= \frac{P(F \mid S)P(X \mid F,S)}{P(X \mid S)} = \frac{P(F \mid S)\int P(X,\lambda \mid F,S)d\lambda}{P(X \mid S)} \\ &\propto \int P(X,\lambda \mid F,S)d\lambda \\ &= P(X \mid F,S) \end{aligned} \tag{2.4}$$

To accommodate the restriction that each case belongs to only one class, once the distributions for the clusters *(F)* and the probabilities ($p_j$) are estimated, equation 2.2 can be rewritten as:

$$P(X,\lambda \mid F,S) = P(\lambda \mid F,S)\prod_i \sum_{X_i \in C_j} p_j\, P(X_i \mid \lambda_j, F_j) \tag{2.5}$$

If the number of classes ($j$) was known, we would maximize equation 2.5, which is not possible with AutoClass since it assumes unsupervised classification. This requires us to construct weighted class assignments given the estimated set of $F$ and $\lambda$. These normalized class assignments can be written in the form:

$$w_{ij} = P(X_i \in C_j \mid \lambda, F, S) \propto p_j P(X_i \mid X_i \in C_j, \lambda_j, F_j) \tag{2.6}$$

Since our monetary asset data contains real-valued attributes, these assignments give us our weighted class number, mean, and variance due to the log-Gaussian model:

$$w_j = \sum_i w_{ij} \qquad \beta_{jk} = w_j^{-1}\sum_i w_{ij}X_{ik} \qquad \sigma_{jk}^2 = w_j^{-1}\sum_i w_{ij}^{-1}(X_{ik} - \beta_{jk})^2 \tag{2.7}$$

These statistics are used to reestimate equation 2.5 and the associated probabilities. This EM algorithm is repeated until the MAP parameters converge to a maximum stationary point.

The next section will explain the data set used for the AutoClass procedure, describing the specific attributes used to classify the monetary assets.

### *Data*

Anderson and Kavajecz (1994) outline the importance of precise monetary asset data in their review of the history of monetary aggregates in the United States. Considering some version of money is used frequently throughout the economic literature, the proper form is imperative. Anderson and Kavajecz (1994) believe that the perfect monetary aggregates would be comprised of only very liquid assets. Furthermore, the frequency of the data collection was also discussed. They theorized that more frequent collection, such as daily observations, would assist in a more accurate analysis of the monetary assets.

In 1980, the Federal Reserve constructed data back to 1959, using new definitions for the monetary assets. The data available prior to 1959 was collected from different sources, using different definitions. Therefore, this paper only uses data from the period 1959:01 to 2002:12 so that our empirical results are not compromised.

Thus, the monthly time series data for each of the 26 monetary assets was gathered from the research division of the Federal Reserve Bank of St. Louis (FRED)[2]. The research division, in their Monetary Services Index (MSI), produces the monetary asset quantities, as well as their respective user costs. The MSI is examined in further detail in the Anderson et al. (1997a, 1997b, 1997c) papers.

The data contains many missing values since some of the assets did not exist during certain points over the 44-year period; see Table 2.1. AutoClass is able to handle the missing values with use of the EM algorithm, as indicated in the previous section. This allows the missing data to be treated as valid, rather than an error in collection. This allows AutoClass to cluster the entire data set, not just the data that is observed.

The four attributes used by AutoClass to classify each of our 26 monetary asset cases, include the monetary asset value ($x_i$), the growth rate $(\mu_i)$, the user cost ($\pi_i$), and the velocity ($V_i$). Anderson et al. (1997c) establish that it is imperative to discern between nominal and real monetary asset quantities and user costs. Since the MSI database produces nominal asset quantities and real user costs, the formulas in chapter three will be accurate.

The monetary asset quantity ($x_i$) was extracted from the Federal Reserve's MSI database, as indicated above. The data used is not modified for seasonality, since the seasonally adjusted data could result in spurious results in our chapter five analysis of the comovement of the variables, as shown by Hecq et al. (1998) and Cubadda (1999).

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[2] Source: http://research.stlouisfed.org/msi/

The annual growth rate of a monetary asset, due to seasonality concerns, is first constructed monthly as:

$$\mu_i = 1200 * (\log x_{it} - \log x_{i,t-1}) \qquad where\ i = 1,2,....26 \qquad (2.8)$$

The user cost ($\pi_i$), also available from the MSI database, is the present value of the interest forgone of holding that asset. Established in Barnett (1978), the user cost formula is constructed from that asset's own rate of interest. It follows:

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t} \qquad (2.9)$$

where $r_{it}$ is the asset's own rate of interest and $R_t$ is the benchmark rate of return. The own rates of return are discussed in detail in Anderson et al. (1997c). For some assets, such as deposits, the rates of return are simply the interest rate paid by banks and thrifts. However, the interest rate data is not available for other assets, which compels Anderson et al. (1997c) to adopt methods such as calculating implicit rates, finding proxies by use of regression analysis, as well as filling in missing data.

The benchmark rate ($R_t$), which is available in the MSI database, is the proxy for an asset that holds no value as a medium of exchange. In furthering Barnett et al (1984), the method to calculate the benchmark rate of return used by Anderson et al. (1997c) incorporates Moody's BAA bonds. According to Anderson et al. (1997c), the benchmark rate is formed by:

$$R_t = \max\{r_{it}(i=1,2,...,n), r_{BAA\,t}\} + c \tag{2.10}$$

where c is some small constant. This constant was not included in the Barnett et al. (1984) version, but this allows the benchmark rate to be higher the other rates of return with certainty.

Note that these own rates of return, and therefore user costs, are affected by shocks to the rate of inflation. As expected inflation rises, Barnett (1982) expects the user cost to increase, causing a movement from monetary assets to consumer products. The link between inflation and money will be further discussed in chapter four.

In order to calculate the velocity of a given asset, it is necessary to collect the nominal GDP data. We have calculated this by using the price level and a proxy for real GDP. These series will be used in the chapters wherein we discuss the relationship among the various macroeconomic variables of interest. The consumer price index (CPI)[3], extracted as a monthly series from the Bureau of Labor Statistics, is used as an indicator for inflation. The variable used as a proxy for real GDP is the Industrial Production Index ($GDP_{real}$)[4], extracted from the Federal Reserve Bank of St. Louis, which measures the change in manufacturing, mining, gas, and electric utilities in the United States. This monthly index allows us to calculate the nominal GDP variable, as follows:

$$GDP_{nom} = CPI * GDP_{real} \tag{2.11}$$

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[3] Source: http://research.stlouisfed.org/fred2/series/CPIAUCNS/9
[4] Source: http://research.stlouisfed.org/fred2/series/INDPRO

Therefore, velocity, the rate at which money is circulated, can be calculated from the Quantity theory of money formula *MV=PY*, as:

$$V_i = \frac{CPI * GDP_{real}}{x_i} = \frac{GDP_{nom}}{x_i} \qquad where\ i = 1,2,\ldots 26 \qquad (2.12)$$

The next section will examine the results from AutoClass for our monetary data, which will aid in constructing the newly classified monetary aggregates implementing the divisia, simple sum, and currency equivalent techniques.

**TABLE 2.1: The 26 Monetary Assets from the Federal Reserve**

| Monetary Asset | Period |
|---|---|
| Currency | 1959:01 - 2002:12 |
| Travelers checks | 1959:01 - 2002:12 |
| Demand deposits | 1959:01 - 2002:12 |
| Other checkable deposits at commercial banks | 1974:01 - 1985:12 |
| Other checkable deposits at thrift institutions | 1959:01 - 1985:12 |
| Super now accounts at commercial banks | 1983:01 - 1985:12 |
| Super now accounts at thrift institutions | 1983:01 - 1985:12 |
| Other checkable deposits and super now accounts at banks | 1986:01 - 2002:12 |
| Other checkable deposits and super now accounts at thrifts | 1986:01 - 2002:12 |
| Money market deposit accounts at commercial banks | 1982:01 - 1991:08 |
| Money market deposit accounts at thrift institutions | 1982:01 - 1991:08 |
| Savings deposits at commercial banks | 1959:01 - 1991:08 |
| Savings deposits at thrift institutions | 1959:01 - 1991:08 |
| Savings deposits and money market deposit accounts at banks | 1991:09 - 2002:12 |
| Savings deposits and money market deposit accounts at thrifts | 1991:09 - 2002:12 |
| Retail money funds | 1973:02 - 2002:12 |
| Small denomination time deposits at commercial banks | 1959:01 - 2002:12 |
| Small denomination time deposits at thrift institutions | 1959:01 - 2002:12 |
| Repurchase agreements | 1959:10 - 2002:12 |
| Eurodollars | 1959:01 - 2002:12 |
| Large denomination time deposits | 1959:01 - 2002:12 |
| Institutional money funds | 1974:01 - 2002:12 |
| Saving bonds | 1959:01 - 2002:12 |
| Short term treasury securities | 1959:01 - 2002:12 |
| Bankers acceptances | 1959:01 - 2002:12 |
| Commercial paper | 1959:01 - 2002:12 |

*Classifying the Monetary Assets*

The entire program used 176 attributes per case, the 44 annual averages of each of the four variables: quantity, user cost, velocity, and growth rate. Each case belongs to its respective class with a probability of one, displaying the robustness of the results. From AutoClass, the log posterior of possible classifications is given in Table 2.2, which shows that the largest probability belongs to three classes.

**TABLE 2.2: Log Probability Results**

| Number of Classes | Log Probability |
|---|---|
| 2 | -33877.264 |
| 3 | -33660.219* |
| 4 | -33771.350 |
| 5 | -33827.509 |

Table 2.3 reports how the 26 monetary assets are grouped according to the Bayesian classification, while Table 2.4 compares our results with the Federal Reserve's monetary groupings. The main difference between the two systems of classification is that our classification is not nested, but rather three distinct groups of assets. The Federal Reserve concentrates on six nested levels of aggregation: M1A, M1, MZM, M2, M3, and L. Note that our class three is identical to the Federal Reserve's M1 excluding currency, traveler's checks and demand deposits, or their M1A. The small and large denomination time deposits are now grouped together in our class two, whereas the Federal Reserve's classification separates these two assets. It appears that these two variables share statistical properties although they may have different values in terms of liquidity. Similarly, the

assets denoted as 'savings deposits' at commercial banks and thrifts are now isolated from the 'savings deposits and money market deposit accounts' at banks and thrifts. This is contradictory to the usual cluster used in the United States, MZM, which groups all of the aforementioned assets together. Although not usually assembled in the same group, the institutional money funds and the retail money funds are now found collectively in our class one. This seems intuitive that some money funds may indeed move together over time, sharing common properties.

Throughout the decades, the Federal Reserve changed its view on the appropriate monetary aggregate to use in money demand analysis. In the 1960's, M1 became the aggregate of interest due to its medium of exchange property. However, M2 became more widely accepted in the last two decades due to its seemingly close interaction with the price level, as described by Belongia and Batten (1992). This paper does not attempt to chose the optimal new aggregate for policy-makers, only to introduce a new method to create the levels of aggregation according to mathematical properties.

This classification according to statistical similarities allows us to group the 26 monetary assets according to "natural" classes within the data. In the next section, we will construct the three versions of the monetary aggregates for each of our three classes, resulting in nine different aggregates being formed. These aggregates will be our focus in the remainder of the paper, when discussing the relationship between money and other key macroeconomic variables. These monetary aggregates will enable us to proceed with the tests for unit roots, cointegration, serial correlation, Granger causality, as well as common and codependent cycles.

| **Table 2.3 : AutoClass Results** | | |
|---|---|---|
| **Class 1** | **Class 2** | **Class 3** |
| Travelers checks | Currency | Other checkable deposits at commercial banks |
| Money market deposit accounts at commercial banks | Demand deposits | Other checkable deposits at thrift institutions |
| Money market deposit accounts at thrift institutions | Savings deposits at commercial banks | Super now accounts at commercial banks |
| Savings deposits and money market deposits at banks | Savings deposits at thrift institutions | Super now accounts at thrift institutions |
| Savings deposits and money market deposits at thrifts | Small denomination time deposits at banks | Checkable deposits and super now accounts at banks |
| Retail money funds | Small denomination time deposits at thrifts | Checkable deposits and super now accounts at thrifts |
| Repurchase agreements | Large denomination time deposits | |
| Eurodollars | Saving bonds | |
| Institutional money funds | Short term treasury securities | |
| Bankers acceptances | Commercial paper | |

**TABLE 2.4: A Comparison Of The Federal Reserve And AutoClass Groupings**

**M1A**
Currency
Demand deposits
Travelers checks

**M1 = M1A + the following**
Other checkable deposits at commercial banks
Other checkable deposits at thrift institutions
Super now accounts at commercial banks
Super now accounts at thrift institutions
Checkable deposits and super now accounts at banks
Checkable deposits and super now accounts at thrifts

**MZM = M1 + the following**
Money market deposit accounts at commercial banks
Money market deposit accounts at thrift institutions
Savings deposits at commercial banks
Savings deposits at thrift institutions
Savings deposits and money market deposits at banks
Savings deposits and money market deposits at thrifts
Retail money funds

**M2 = MZM + the following**
Small denomination time deposits at commercial banks
Small denomination time deposits at thrift institutions

**M3 = M2 + the following**
Repurchase agreements
Eurodollars
Large denomination time deposits
Institutional money funds

**L = M3 + the following**
Saving bonds
Short term treasury securities
Bankers acceptances
Commercial paper

**Class 1**
Travelers checks
Money market deposit accounts at commercial banks
Money market deposit accounts at thrift institutions
Savings deposits and money market deposits at banks
Savings deposits and money market deposits at thrifts
Retail money funds
Repurchase agreements
Eurodollars
Institutional money funds
Bankers acceptances

**Class 2**
Currency
Demand deposits
Savings deposits at commercial banks
Savings deposits at thrift institutions
Small denomination time deposits at commercial banks
Small denomination time deposits at thrift institutions
Large denomination time deposits
Saving bonds
Short term treasury securities
Commercial paper

**Class 3**
Other checkable deposits at commercial banks
Other checkable deposits at thrift institutions
Super now accounts at commercial banks
Super now accounts at thrift institutions
Checkable deposits and super now accounts at banks
Checkable deposits and super now accounts at thrifts

# CHAPTER THREE: MONETARY AGGREGATION PROCEDURES

### *Historical Practices*

Anderson et al. (1997a) reviewed early aggregation theory, which established the use of utility functions to construct consistent aggregates. They illustrate how the quantity of monetary assets demanded by consumers was previously found from the solution to the consumer's utility maximization problem, shown by:

$$\max U(x_1, x_2, \ldots, x_n, q_i, q_2, \ldots, q_m) \quad \textit{subject to} \sum_{i=1}^{n} \pi_i x_i + \sum_{j=1}^{m} p_j q_j = Y \tag{3.1}$$

where $x_i$ and $\pi_i$ and are the monetary asset's quantity and user cost, while $q_j$ and $p_j$ are the consumer durable's quantity and price.

However, aggregation theory is difficult to implement in practice due to the existence of unknown parameters in the utility function. Therefore, most central banks employ the simple sum method to construct monetary aggregates, including the Federal Reserve in the United States. However, the fundamental problem associated with the simple sum technique is that it assumes that all component assets are perfect substitutes. In the literature on monetary aggregation techniques, Friedman and Schwartz (1970) explain the danger associated with using the simple sum method, and report the need to incorporate the weights of each asset. They believed that this weighted sum idea attempts to give each asset a 'moneyness' measure, based on that asset's value as a medium of exchange. The theories behind the Friedman and Schwartz (1970) paper gave rise to numerous studies for superior methods.

The discussion on the disadvantages of using simple sum aggregates was continued with Barnett (1980), who pioneered new aggregation literature with his research on using statistical number theory to quantify the monetary aggregates. The fundamentals behind index number theory were actually established by Fisher (1922). Statistical index number theory introduces the use of user costs, or the price of the monetary assets, instead of relying on the knowledge of unknown parameters. This allows the assets to be weighted differently in the aggregate, in proportion to their user costs. This is in contrast to the simple sum method, which assumes identical weightings for each asset regardless of their value as a medium of exchange.

In studying the statistical index number literature, Anderson et al. (1997a) outlined the two steps to construct the money demand functions. This two-step procedure originated from Barnett's (1980) assumption that a subutility function involving only monetary assets can be separated from the remainder of the maximization problem. First, each consumer chooses the monetary asset allocation, but not quantities, and the desired amount of consumer durables. In the second step, the optimal quantity of each asset is chosen, in order to complete the transactions selected in the first step. Barnett et al. (1992) define the 'aggregator function' as the aforementioned subutility function, which is the utility received from holding only monetary assets. They also point out that a statistical index number can be used to estimate the aggregator function, at the optimal level of monetary asset quantities.

The divisia index, a product of the statistical index number theory, was first introduced by Tornqvist (1936) and Theil (1967), and is now widely examined throughout the literature. The divisia aggregation method is discussed in detail in Barnett (1980), emerging as a direct result of Diewert's (1976) theory on superlative index numbers. The definition of 'superlative', according to Diewert (1976), is when an index number is consistent with a flexible aggregator function. In other words, the quantity index can be estimated using the second-order approximation of the translog function. According to this theory, the divisia index can be denoted as 'Diewert-superlative'.

The currency equivalent index, an additional aggregation technique, was introduced and examined in detail by Rotemberg et al. (1995). They define this aggregate as the amount of currency required to complete the transactions done by the entire aggregate. The development of this monetary aggregation method specifically revolved around the definition of user cost. It is known that currency yields a higher user cost than other monetary assets due to the additional utility received from its increased liquidity. In similar form to the divisia index, the practice of attaching weights to each asset in the index is implemented, to accommodate the fact that each asset has a different user cost. Therefore, any assets that do not collect interest have a unitary weighting, while the weights of other assets lie within the range of zero and one, depending on their interest rate relation to the benchmark rate of return. The benchmark asset is the proxy for an asset which holds no liquidity value. However, the central problem associated with the currency equivalent aggregate is that it is very sensitive to the rates of return.

The divisia and currency equivalent indexes share one important optimal portfolio property. In either method, if the quantity of an asset held is altered, only the rate of return is affected. The fundamental distinction between the divisia and the currency equivalent index is the time period being estimated. The divisia index measures the expenditure on monetary services in the current period (flow of services), while the currency equivalent index discounts the future levels, as well as incorporating the current expenditure level (stock of services). Although similar in construction, these two methods also differ in that an additional assumption must be made for the currency equivalent aggregate. Rotemberg et al. (1995) point out that the currency equivalent index also assumes that the transactions completed with currency are distinct from those completed by other assets.

The proper form of a monetary aggregate becomes significant since decisions are very often made based on that aggregate being an intermediate target, to reach the ultimate objective of a stable inflation rate. If the relationship between money and prices, output, or the interest rate is misrepresented, the intermediate targeting procedure will not have the planned effect.

In this paper, we continue the Friedman and Schwartz (1970) original notion concerning the advantages of using weighted average monetary aggregates. Although the divisia index seems more frequent in the literature than the currency equivalent index, it is imperative to examine the impact of implementing both of these concepts. Applying Diewert's (1976) and Barnett's (1980) theories on statistical index numbers, we construct the divisia and currency equivalent indexes in the following subsections. However, since the simple sum method is still widely used in practice, we first present our simple sum aggregates to be used as a comparison to the more complex methods. This paper is structured in similar form to Serletis (2001), which compares the money demand analysis using all three indexes as well.

### *The Simple Sum Index*

The simple sum index (SS) has been used for decades by the central banks. This index sums each nominal monetary asset in each period as:

$$SS_t = \sum_{i=1}^{n} x_{it}^{nom} \tag{3.2}$$

For the purposes of our calculations, we will produce three simple sum aggregates, denoted as SS1, SS2, and SS3; one for each class of monetary assets resulting from the AutoClass procedure described in the previous chapter.

However, the main problem with the simple sum index, as indicated earlier, is that it assigns a unitary weight to each asset, regardless of the relative price. Due to differences in liquidity, and hence user costs, each monetary asset is not a perfect substitute for another.

Hence, we will also be discussing the two alternatives to the simple sum method which incorporate a weighting procedure: the divisia and currency equivalent indexes.

### *The Divisia Index*

The divisia index (DIV) uses the weighted average of the component asset's growth rate to accommodate the differences in user costs. The index, as shown in Serletis (2001), is formed by:

$$DIV_t = \log M_t^D - \log M_{t-1}^D = \sum_{i=1}^{n} s_{it}^* (\log x_{it}^{nom} - \log x_{i,t-1}^{nom}) \tag{3.3}$$

where the growth rate is found using the monetary asset value ($x_i$) as well as the average expenditure share ($s_{it}^*$) from the two periods.

The expenditure share, expressed using the asset quantity and user cost, is found from the following relation:

$$s_{it} = \frac{\pi_{it} x_{it}}{\sum \pi_{jt} x_{jt}} \tag{3.4}$$

where $x_i$ is the nominal asset quantity and $\pi_i$ is the associated real user cost. Thus, the average expenditure is calculated using:

$$s_{it}^* = \frac{1}{2}(s_{it} + s_{i,t-1}) \tag{3.5}$$

Using equation 3.3, we construct divisia indexes for each our our new classes, denoted as DIV1, DIV2, and DIV3.

### *The Currency Equivalent Index*

The currency equivalent (CE) aggregation technique, as denoted in Serletis (2001), is formulated as:

$$CE_t = \sum_{i=1}^{n} \frac{R_t - r_{it}}{R_t} x_{it}^{nom} \tag{3.6}$$

The above index is merely the simple sum index with the addition of a weighting mechanism. Rotemberg et al. (1995) define this weight as the increase in utility from services completed by asset *i*, proportional to those completed by currency. The currency equivalent indexes, CE1, CE2, and CE3, are similar to the divisia indexes, with the exception of incorporating the present discounted value of the expenditure on monetary services.

***Empirical Results***

After constructing the three versions of our aggregates for each of our three classes, the mean and standard deviation of each of our nine monetary aggregates are reported in the table below. As can be seen in Figures 3.1 and 3.2, the logged aggregate values do not vary significantly, though perhaps the currency equivalent versions demonstrate more volatility than the simple sum and divisia techniques. In Figure 3.3, there seems to exist very little discrepancy between the aggregation methods in terms of logged values, perhaps due to the time frame of the data collected for those assets.

**TABLE 3.1: Statistics**

| Series | Mean | Std Dev |
|---|---|---|
| SS1 | 4.142 | 1.188 |
| DIV1 | 3.651 | 1.026 |
| CE1 | 3.871 | 1.281 |
| | | |
| SS2 | 2.606 | 0.315 |
| DIV2 | 2.530 | 0.287 |
| CE2 | 2.635 | 0.370 |
| | | |
| SS3 | 4.355 | 1.587 |
| DIV3 | 4.310 | 1.546 |
| CE3 | 4.379 | 1.614 |

**Figure 3.1: Class 1 Monetary Aggregates**

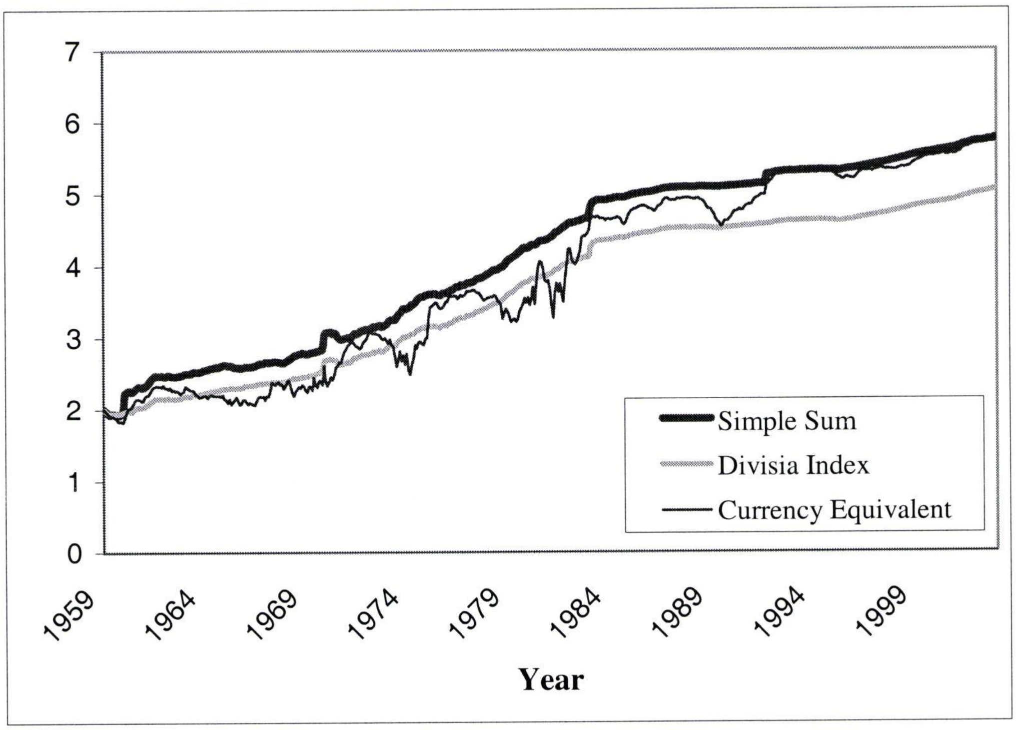


**Figure 3.2: Class 2 Monetary Aggregates**

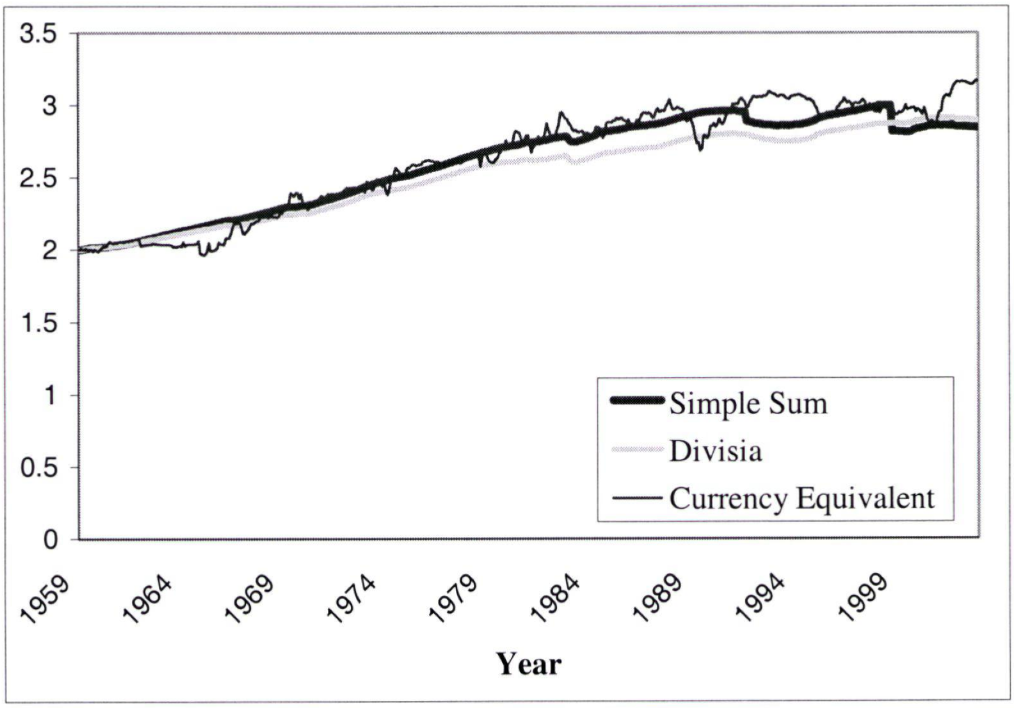
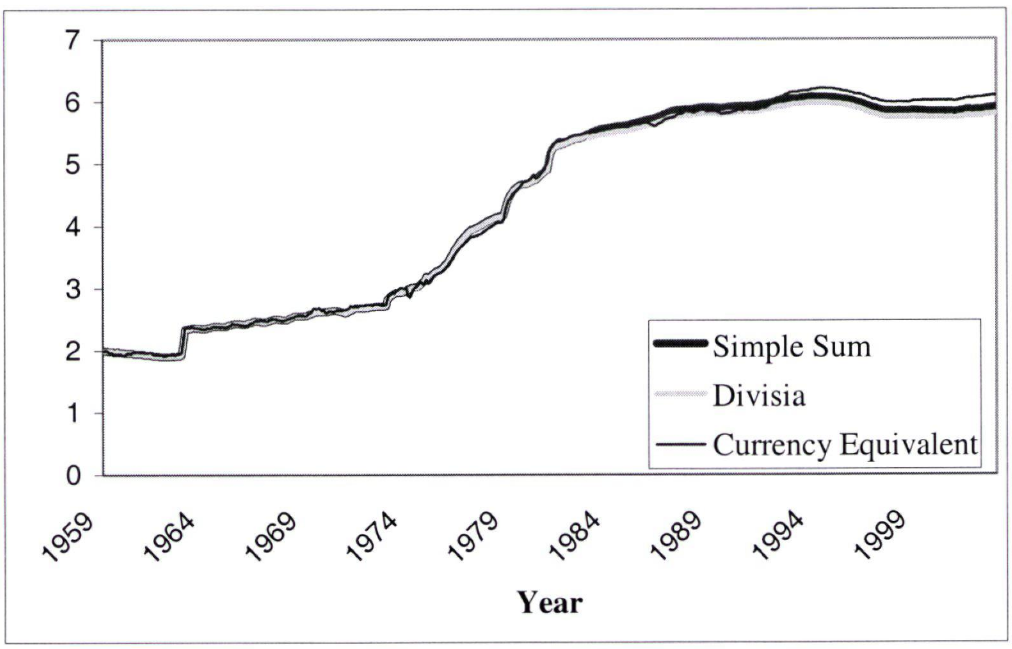



**Figure 3.3: Class 3 Monetary Aggregates**



# CHAPTER FOUR: TESTING THE MONETARY AGGREGATES

## *Review of Developments*

In this chapter, we will discuss the relationship of price, output and the interest rate with money supply. This topic has been covered frequently in the macroeconomic literature. The use of econometrics to find how that relationship exists was developed by Granger (1969), who used past and present values for the variables of interest to test for causality. Sims (1972) was more specifically interested in which direction the causation occurred. He applied the vector autoregression (VAR) methodology to the bivariate model of money and output. He noted that there did exist a positive relationship between money and output, but believed that this should not allow you to conclude that money solely cause output. He supposed that there could be some form of reverse causation as well. Sims' (1992) paper adopted a four-variable VAR method, with the addition of the interest rate and price, to determine the strength of the results from the Granger causality tests, He found that the effect of money on output was still causal, but the impact was greatly reduced from the two-variable model. Serletis (2001) rejected this money to output causal theory when using the four-variable model, with 11 of his 15 monetary aggregates. However, the narrower monetary aggregates, such as M1 and M1+ did seem to support the causality theory.

This relationship between money and prices is extremely useful for policy-makers in determining their operating targets. Over the past decade, inflation targeting has become the objective of choice by many central banks, including the Federal Reserve in the United

States. This price variable target can be achieved by utilizing different intermediate targets, such as money. This implies the knowledge about the money to price relationship is vital in maintaining this target. In the past, the central bank has utilized a money growth rate target, which assumes that the prices or interest rate to money association is perhaps more important.

We adopt Sims' (1992) VAR framework, as further developed by Serletis (2001), to identify the interaction among the four variables of interest. Baharumshah and Tan (1999) also incorporate the VAR method into their analysis of the monetary assets in Malaysia. This approach by Sims (1992) also enables the examination of the potential forecasting value. This is appropriate in this paper seeing as we want to evaluate the response of each variable from shocks in the monetary aggregate. This can be accomplished with analyzing the impulse response functions from the VAR technique.

In the next sections, we will examine the time series properties of the data, such as stationarity and cointegration. Once we are aware of the stochastic trends within the data, we can proceed with the tests for causality using Granger's (1969) single equation method and Sims' (1992) VAR method.

### *Data*

As indicated above, the monthly variables of interest include the monetary aggregates, the price level, the nominal interest rate, as well as the nominal and real forms of output. We have already discussed the relevant data sources for the price level and output in chapter two. Therefore, the only key variable we are adding to the equations is the nominal interest rate. We have collected the three-month Treasury bill rate[5] from the Federal Bank of St. Louis' database, for the relevant years 1959:01 until 2002:12.

### *Unit Root Tests*

Prior to further analysis regarding the interactions of the macroeconomic variables, we must first determine whether there exists a unit root in any of the series. In looking at the stationarity of the nine monetary aggregates, price level, and forms of GDP, we used three different tests: the Weighted-Symmetric, augmented Dickey-Fuller (1979), and the Phillips-Perron (1988) test. Each test has the null hypothesis that a unit root is present. This seems limited in power, since we can only conclude that the series is stationary, if it rejects the null, or that we fail to reject the null. The regression being evaluated in log levels is:

$$y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{j=1}^{K} \delta_j y_{t-j} + \varepsilon_t \tag{4.1}$$

where $y_t$ is our variable of interest, and $t$ is our trend term. It is also necessary to test the first difference of the log levels, in order to conclude the series is of order I(1). That is:

$$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{j=1}^{K} \delta_j \Delta y_{t-j} + \varepsilon_t \tag{4.2}$$

The Weighted Symmetric (WS) test is becoming more widely used since it has been shown to have somewhat more power than the augmented Dickey-Fuller (ADF) test. The

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[5] Source: http://research.stlouisfed.org/fred2/series/GS3M/22

Phillips-Perron (PP) test is a generalization of the ADF test under non-Gaussian errors.

The results for the unit root tests are dependant on the number of lags chosen, since they are used to control for serial correlation. The optimal number of lags ($K$) is chosen by imposing the Akaike Information Criterion (AIC) plus two rule, formulated in TSP 4.5. A variable for the mean and trend were also included. The F-statistic is calculated for the joint test of $\beta = 0$ and $\rho = 1$.

In log levels, the large p-values indicate that each variable fails to reject the null hypothesis that the series follows a random walk, when including drift and trend in the regression, as seen in Table 4.1.

Once the first differences of the log levels are taken, every series, with the exception of the price level, can be concluded as stationary at the 5% level, see Table 4.1. For the consumer price index, we still fail to reject the null that a unit root is present in the Weighted-Symmetric and Dickey-Fuller tests. In contrast, the Phillips-Perron statistic does show that the price level is first-difference stationary. Using these statistical results, we can conclude that each of the series that we are interested in evaluating does contain at least one unit root.

**TABLE 4.1: Statistical Results for Unit Roots in the Monetary Aggregates**

| | LOG LEVEL | | | | | | FIRST DIFFERENCE LOG LEVELS | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | WS | | ADF | | PP | | WS | | ADF | | PP | |
| Series | Lags | p-value | Lags | p-value | Lags | p-value | Lags | p-value | Lags | p-value | Lags | p-value |
| SS1 | 3 | 0.982 | 5 | 0.969 | 5 | 0.986 | 2 | 0.000 | 4 | 0.000 | 4 | 0.000 |
| DIV1 | 4 | 0.990 | 6 | 0.987 | 6 | 0.995 | 5 | 0.000 | 5 | 0.000 | 5 | 0.000 |
| CE1 | 8 | 0.197 | 8 | 0.256 | 8 | 0.235 | 5 | 0.000 | 5 | 0.000 | 5 | 0.000 |
| SS2 | 2 | 0.999 | 2 | 0.997 | 2 | 0.999 | 2 | 0.000 | 2 | 0.000 | 2 | 0.000 |
| DIV2 | 16 | 1.000 | 5 | 0.988 | 5 | 0.999 | 4 | 0.000 | 4 | 0.000 | 4 | 0.000 |
| CE2 | 4 | 0.325 | 4 | 0.465 | 4 | 0.360 | 16 | 0.000 | 16 | 0.000 | 16 | 0.000 |
| SS3 | 3 | 0.999 | 5 | 0.995 | 5 | 0.999 | 4 | 0.000 | 4 | 0.000 | 4 | 0.000 |
| DIV3 | 3 | 0.998 | 5 | 0.995 | 5 | 0.999 | 4 | 0.000 | 4 | 0.000 | 4 | 0.000 |
| CE3 | 5 | 0.997 | 5 | 0.992 | 5 | 0.998 | 4 | 0.000 | 4 | 0.000 | 4 | 0.000 |
| CPI | 16 | 1.000 | 16 | 0.813 | 16 | 0.985 | 15 | 0.483 | 15 | 0.407 | 15 | 0.000 |
| NOMY | 14 | 1.000 | 7 | 0.992 | 7 | 0.994 | 13 | 0.000 | 6 | 0.000 | 6 | 0.000 |
| IPI | 14 | 0.938 | 5 | 0.111 | 5 | 0.529 | 8 | 0.000 | 4 | 0.000 | 4 | 0.000 |

*Cointegration Tests*

For further causality models to be formulated, we also need to discern whether cointegration is present between the series as well. The Engle-Granger (1987) two-step method applies the null hypothesis of no cointegration. Recall, that in order to conclude cointegration, we must use both variables of interest as the dependent variable. The two regressions are denoted as:

$$m_t = \alpha + \beta t + \delta x_t + \varepsilon_t \tag{4.3}$$

$$x_t = \alpha + \beta t + \delta m_t + \varepsilon_t \tag{4.4}$$

where $m$ is one of our nine monetary aggregates, and $x$ denotes one of the other variables we are interested in, such as the price level, real GDP, or nominal GDP.

We want to test whether the residuals of the above regression are stationary, in order to conclude cointegration of the two variables. If we fail to reject the null hypothesis of a non-stationary error term, and therefore no cointegration, the p-value associated will exceed the 5% significance.

Table 4.2 reports the p-values associated with each bivariate regression. The column headings are the dependent variable in each regression. Again, to correct for serial correlation, the optimal number of augmenting lags were chosen using the AIC plus two rule.

The large p-values show that we fail to reject the no cointegration hypothesis, with the exception of SS1 and the price level, which appear to cointegrate. The only alteration to subsequent testing will be that we must incorporate the lagged error term from the cointegrating regression of SS1 and the price level, in their respective equations. This error correction method will need to be implemented in Chapter five, when analyzing the common and codependent features of the variables. Using the non-stationary and cointegration results thus far, we can now continue with the single-equation Granger causality tests.

**TABLE 4.2: Cointegration Results: Money and Prices, Nominal GDP, and Real GDP**

| | Money and Prices | | | | Money and Nominal GDP | | | | Money and Real GDP | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | lags | M | lags | CPI | lags | M | lags | $GDP_{nom}$ | lags | M | lags | $GDP_{real}$ |
| SS1 | 3 | 0.005 | 3 | 0.005 | 4 | 0.201 | 4 | 0.329 | 5 | 0.985 | 7 | 0.207 |
| DIV1 | 3 | 0.376 | 3 | 0.352 | 5 | 0.599 | 5 | 0.683 | 6 | 0.994 | 7 | 0.245 |
| CE1 | 14 | 0.023 | 8 | 0.430 | 14 | 0.163 | 5 | 0.963 | 6 | 0.562 | 2 | 0.565 |
| SS2 | 2 | 0.842 | 8 | 0.530 | 5 | 0.105 | 5 | 0.112 | 2 | 0.998 | 7 | 0.254 |
| DIV2 | 10 | 0.775 | 16 | 0.518 | 5 | 0.218 | 5 | 0.265 | 5 | 0.996 | 7 | 0.269 |
| CE2 | 4 | 0.290 | 4 | 0.869 | 14 | 0.076 | 13 | 0.323 | 4 | 0.667 | 7 | 0.270 |
| SS3 | 3 | 0.356 | 3 | 0.249 | 5 | 0.747 | 5 | 0.752 | 6 | 0.997 | 7 | 0.255 |
| DIV3 | 3 | 0.347 | 3 | 0.241 | 5 | 0.711 | 5 | 0.716 | 6 | 0.997 | 7 | 0.254 |
| CE3 | 5 | 0.239 | 3 | 0.216 | 5 | 0.773 | 5 | 0.809 | 3 | 0.995 | 7 | 0.259 |

*Granger Causality Tests*

Granger (1969) introduced causality in order to observe the interaction between money and the central bank's objective of choice. In this paper, we are particularly interested in the relationship between the price level, nominal output, or real output. The results from his test allow policy-makers to utilize money as a more accurate intermediate target.

The equation used in this section is as follows:

$$\Delta x_t = \alpha + \sum_{i=1}^{K_1} \beta \Delta x_{t-i} + \sum_{j=1}^{K_2} \delta_j \Delta m_{t-j} + \varepsilon_t \tag{4.5}$$

where $\Delta x_t$ is the inflation rate, the growth rate of nominal GDP, or the growth rate of real GDP. The null hypothesis is that money does not cause prices, nominal GDP, or real GDP. The optimal lag length, from one through 12, was chosen by implementing the AIC plus two rule once again.

The first step involves estimating equation 4.5 using OLS to calculate the unrestricted sum of squared residuals. Then, run the restricted model, where all the $\delta_j$'s equal zero, to obtain the restricted sum of squared residuals. These values are required to calculate our F-statistic, used in testing our null hypothesis of no causality:

$$F = \frac{(SSR_r - SSR_u) / K_2}{SSR_u / (T - K_1 - K_2 - 1)} \tag{4.6}$$

In comparing the p-values from Table 4.3, we can see the null that money does not cause prices is rejected for all nine of the aggregates. Similarly, in terms of nominal or real

GDP, we also conclude there does exist a causal relationship with all three classes of money, using any of the aggregation techniques. These causality conclusions allow policy-makers to conduct their business according to their respective target of interest, using any monetary aggregate as its intermediate objective. However, this single equation method may fail to capture all of the effects of various macroeconomic variables in the financial system. Thus, the next section explores the multi-equation VAR method used to incorporate all the variables of interest: prices, money, interest rate, and real income, into one system.

**TABLE 4.3: Statistical Results for Granger Causality Tests**

| | Money to CPI | | Money to $GDP_{nom}$ | | Money to $GDP_{real}$ | |
|---|---|---|---|---|---|---|
| Series | AIC lags | p-value | AIC lags | p-value | AIC lags | p-value |
| SS1 | (12,1) | 0.001 | (3,2) | 0.000 | (5,12) | 0.000 |
| DIV1 | (12,3) | 0.001 | (3,1) | 0.000 | (7,1) | 0.000 |
| CE1 | (12,4) | 0.000 | (3,1) | 0.000 | (12,1) | 0.000 |
| SS2 | (12,1) | 0.000 | (3,1) | 0.000 | (5,1) | 0.000 |
| DIV2 | (12,1) | 0.005 | (3,1) | 0.000 | (5,1) | 0.000 |
| CE2 | (12,6) | 0.021 | (3,1) | 0.000 | (7,1) | 0.000 |
| SS3 | (12,8) | 0.009 | (3,1) | 0.000 | (7,1) | 0.000 |
| DIV3 | (12,8) | 0.008 | (3,1) | 0.000 | (7,1) | 0.000 |
| CE3 | (12,8) | 0.002 | (3,1) | 0.000 | (7,1) | 0.000 |

Note: Lags are listed as (dependent variable lags, monetary aggregate lags)

*The Vector Autoregression Framework*

Instead of using a single equation, as in the previous section, the vector autoregression (VAR) method is a multi-equation model in which each variable is jointly determined. If the variables are cointegrated, the vector error correction method (VECM) must be applied. The error correction term is the lagged residual term from the respective cointegrating regression. However, since our variables are not cointegrated, with the exception of SS1 and prices, we will only need to apply the VAR methodology.

We will focus on Sims' (1992) four-variable VAR, including the interest rate (TBILL), log level of the monetary aggregate (M), the logged consumer price index (CPI), and the logged real GDP (Y). The VAR method is implemented because we are uncertain of the correct model specification, and the system of variables may interact differently than theory would tell us. Each of the four variables of interest is treated as endogenous, and this framework examines the lagging interactions among the series. Since we are using monthly data, we set a uniform lag length at 12 months. In Table 4.4, the variable in each column indicates the causality to the variable in each row. The p-values reported are for the null hypothesis that no causality is present.

The question answered in the previous section was in regards to the causation of money on prices, nominal output, and real output. However, each causal relationship in the system will impact the monetary decisions made by the central bank, depending which type of operating target they have implemented. If the target is inflation, as is the case in the United States and Canada, the causation on prices may be most important. Conversely, if the interest rate is targeted, perhaps the variables affecting the Treasury bill rate will have more significance. Alternatively, perhaps the growth rate of money is the objective of the central bank. For these reasons, each causal relationship will be discussed below.

In terms of influencing output, the null that the interest rate does not cause real output is rejected for each form of monetary aggregate at the 5% significance level. Conversely, the null hypothesis is not rejected in the case of output causing the interest rate, with the exception of using the CE1 aggregate version. These results seem to indicate that a one-way causal relationship does exist between the 91-day Treasury bill rate and output.

The no causality effect of prices on real output cannot be rejected in every form of monetary aggregate, and the same results hold for the opposite direction.

At the 5% significance level, the no causality hypothesis is failed to be rejected in terms of the effect of money on real output, with the exception of CE2. In similar form, we fail to reject the null that output does not cause money, in every case, excluding CE3, which does reject at the 5% level. In terms of the money and output relationship, reverse causation does not appear to be in effect using almost every version of the monetary aggregate. This is not consistent with the quantity theory of money, but is consistent with most of Serletis' (2001) results.

In terms of prices and the interest rate, it appears that the interest rate causes prices using each form of the aggregate, but prices fail to cause the interest rate using SS1, CE1, SS3, and CE3.

The relationship between money and the interest rate or prices is also not completely clear cut, the results vary according to which version of the monetary aggregate used. For instance, prices cause money in every case but SS1 and DIV1, while money causes prices in each case but SS2 and DIV2. For the interest rate relationship, money causes the interest rate in each form of the aggregate with the exception of SS1, DIV2 and CE3, while interest rates cause money in each case except SS1, DIV1, SS2, and DIV2.

In Table 4.4, the forecast error variance decompositions are also reported. This is the percentage of forecast variance after 60 months, for each variable in the rows, explained by the variable in each column. After a five year period, variance in real output is not largely affected by shock in the money supply. However, changes in the interest rate or price level do explain a high percentage of the variance in output. In terms of variance in the forecast of the 91-day Treasury bill rate, it appears that output and monetary shocks will have a greater impact than shocks in the price level. The percentage of forecast variance in the price level, aside from internal shocks, is most greatly influenced by the money supply, after a five-year period.

The impulse response functions, which show how each variable responds to shocks in the monetary aggregates, are illustrated in Figures 4.1 through 4.12.

In the next section, we will examine if common features, or cyclical behaviour, is present within our macroeconomic variables. We are specifically interested whether there exists comovement in the business cycle relationship between money and output or prices.

**TABLE 4.4: Results From VAR Model {TBILL, M, CPI, $GDP_{real}$}**

| | Marginal significance levels | | | | Forecast error variance (60 month horizon) | | | |
|---|---|---|---|---|---|---|---|---|
| | TBILL | M | CPI | $GDP_{real}$ | TBILL | M | CPI | $GDP_{real}$ |
| TBILL | 0.000 | 0.542 | 0.436 | 0.007 | 52.883 | 11.730 | 8.184 | 27.203 |
| SS1 | 0.644 | 0.000 | 0.079 | 0.063 | 19.317 | 36.541 | 25.919 | 18.222 |
| CPI | 0.000 | 0.010 | 0.000 | 0.568 | 12.812 | 19.913 | 48.403 | 18.873 |
| REALY | 0.000 | 0.201 | 0.246 | 0.000 | 18.318 | 3.207 | 32.758 | 45.717 |
| | | | | | | | | |
| TBILL | 0.000 | 0.001 | 0.037 | 0.008 | 44.229 | 20.723 | 5.429 | 29.619 |
| DIV1 | 0.207 | 0.000 | 0.085 | 0.935 | 11.518 | 58.565 | 5.626 | 24.291 |
| CPI | 0.000 | 0.041 | 0.000 | 0.558 | 7.494 | 34.696 | 38.810 | 19.000 |
| REALY | 0.000 | 0.202 | 0.480 | 0.000 | 11.589 | 3.763 | 25.913 | 58.735 |
| | | | | | | | | |
| TBILL | 0.000 | 0.000 | 0.083 | 0.139 | 51.998 | 6.463 | 17.519 | 24.020 |
| CE1 | 0.000 | 0.000 | 0.036 | 0.214 | 31.138 | 30.474 | 24.414 | 13.974 |
| CPI | 0.000 | 0.001 | 0.000 | 0.353 | 14.877 | 3.820 | 66.178 | 15.125 |
| REALY | 0.000 | 0.397 | 0.448 | 0.000 | 16.029 | 12.828 | 27.316 | 43.827 |
| | | | | | | | | |
| TBILL | 0.000 | 0.008 | 0.033 | 0.013 | 51.091 | 23.864 | 3.242 | 21.803 |
| SS2 | 0.980 | 0.000 | 0.024 | 0.288 | 2.751 | 87.976 | 4.489 | 4.783 |
| CPI | 0.000 | 0.818 | 0.000 | 0.160 | 12.383 | 31.792 | 44.376 | 11.448 |
| REALY | 0.000 | 0.622 | 0.359 | 0.000 | 17.586 | 11.259 | 28.460 | 42.695 |
| | | | | | | | | |
| TBILL | 0.000 | 0.117 | 0.001 | 0.021 | 52.485 | 27.838 | 1.659 | 18.018 |
| DIV2 | 0.075 | 0.000 | 0.005 | 0.184 | 0.821 | 92.395 | 0.212 | 6.573 |
| CPI | 0.000 | 0.335 | 0.000 | 0.110 | 14.792 | 53.345 | 28.801 | 3.062 |
| REALY | 0.000 | 0.088 | 0.817 | 0.000 | 19.170 | 12.032 | 26.146 | 42.652 |
| | | | | | | | | |
| TBILL | 0.000 | 0.009 | 0.015 | 0.024 | 51.500 | 11.853 | 7.462 | 29.186 |
| CE2 | 0.009 | 0.000 | 0.015 | 0.160 | 8.662 | 64.407 | 14.806 | 12.124 |
| CPI | 0.000 | 0.000 | 0.000 | 0.367 | 10.169 | 17.587 | 55.653 | 16.591 |
| REALY | 0.000 | 0.032 | 0.433 | 0.000 | 19.855 | 2.079 | 29.682 | 48.384 |
| | | | | | | | | |
| TBILL | 0.000 | 0.008 | 0.054 | 0.004 | 28.400 | 38.187 | 4.274 | 29.139 |
| SS3 | 0.000 | 0.000 | 0.006 | 0.139 | 0.356 | 69.313 | 21.920 | 8.411 |
| CPI | 0.000 | 0.032 | 0.000 | 0.255 | 2.175 | 34.147 | 43.373 | 20.306 |
| REALY | 0.000 | 0.315 | 0.097 | 0.000 | 14.278 | 2.628 | 25.189 | 57.905 |
| | | | | | | | | |
| TBILL | 0.000 | 0.010 | 0.047 | 0.004 | 29.397 | 37.846 | 4.157 | 28.600 |
| DIV3 | 0.000 | 0.000 | 0.005 | 0.104 | 0.532 | 68.676 | 22.761 | 8.031 |
| CPI | 0.000 | 0.038 | 0.000 | 0.228 | 2.488 | 33.241 | 44.339 | 19.931 |
| REALY | 0.000 | 0.211 | 0.090 | 0.000 | 15.356 | 2.236 | 25.415 | 56.993 |
| | | | | | | | | |
| TBILL | 0.000 | 0.199 | 0.168 | 0.011 | 26.010 | 42.566 | 7.059 | 24.365 |
| CE3 | 0.000 | 0.000 | 0.027 | 0.033 | 1.513 | 61.978 | 30.628 | 5.880 |
| CPI | 0.000 | 0.029 | 0.000 | 0.383 | 2.996 | 31.888 | 47.212 | 17.904 |
| REALY | 0.000 | 0.067 | 0.168 | 0.000 | 13.324 | 3.286 | 25.084 | 58.305 |

**Figure 4.1: Response of TBILL to shocks from SS1, DIV1, and CE1, respectively**

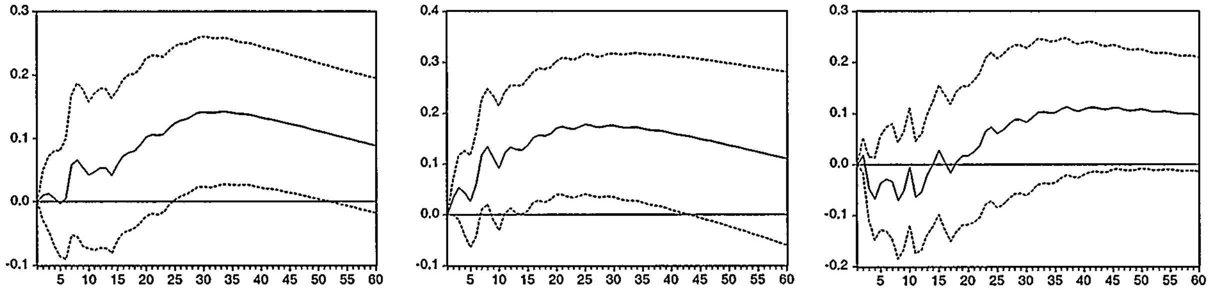



**Figure 4.2: Response of SS1, DIV1, and CE1 from shocks in themselves**

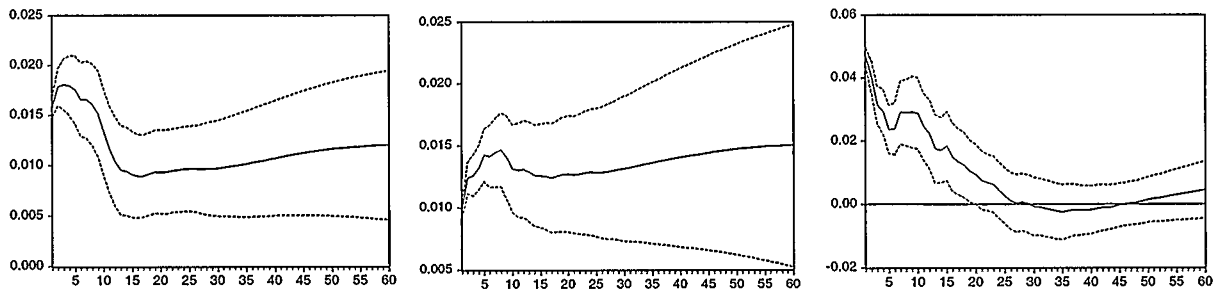



**Figure 4.3: Response of CPI from shocks in SS1, DIV1, and CE1 respectively**

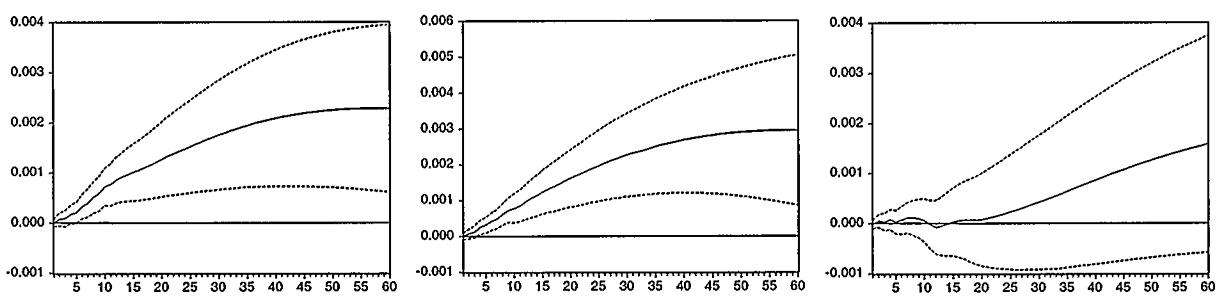



**Figure 4.4: Response of $GDP_{real}$ from shocks in SS1, DIV1, and CE1 respectively**

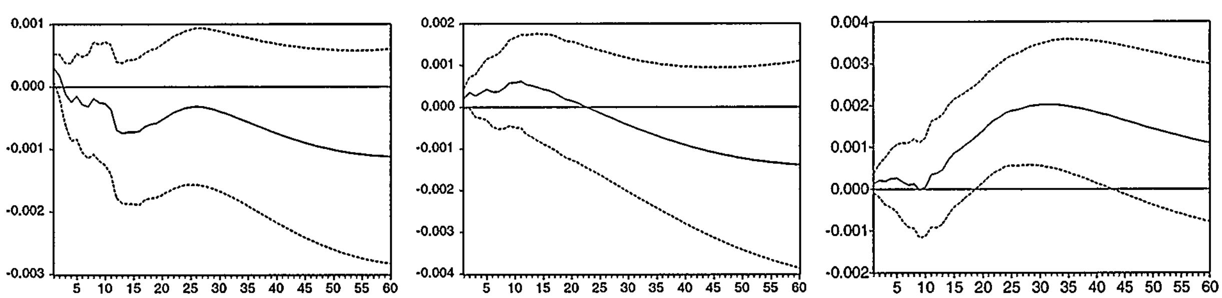


**Figure 4.5: Response of TBILL to shocks from SS2, DIV2, and CE2, respectively**

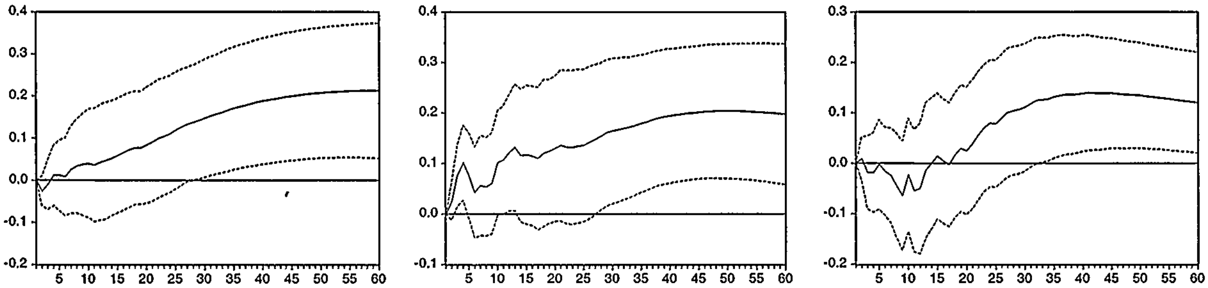



**Figure 4.6: Response of SS2, DIV2, and CE2 to shocks in themselves**

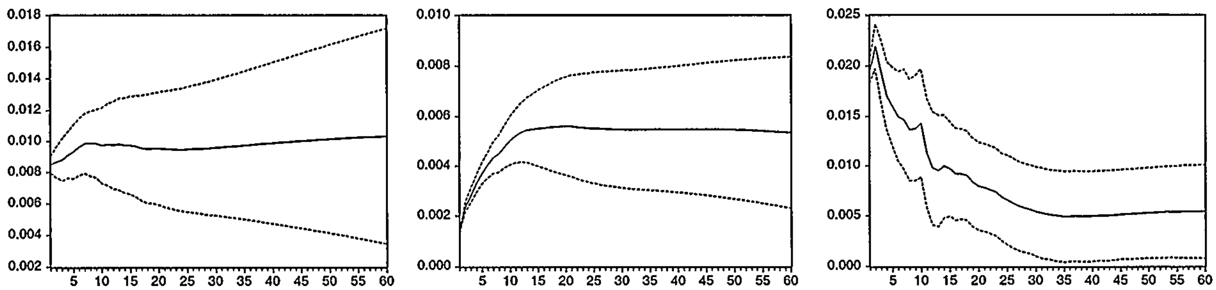



**Figure 4.7: Response of CPI to shocks from SS2, DIV2, and CE2, respectively**

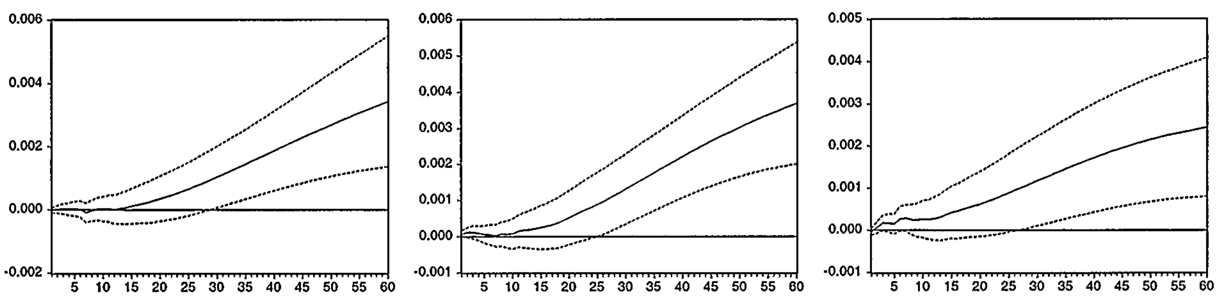



**Figure 4.8: Response of $GDP_{real}$ to shocks from SS2, DIV2, and CE2, respectively**

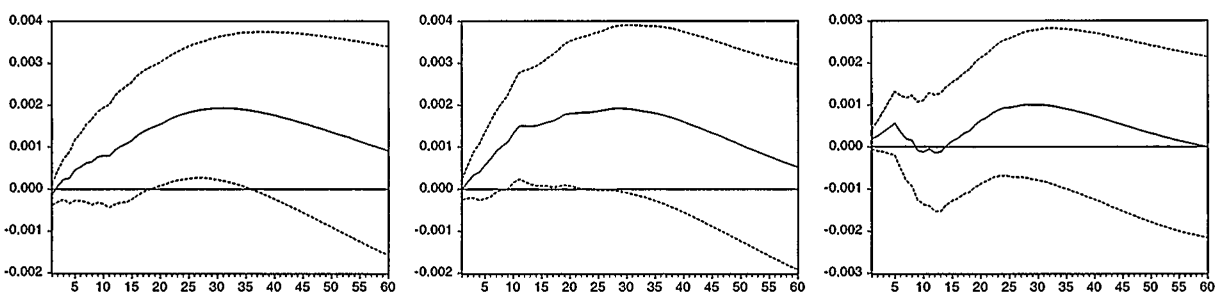


**Figure 4.9: Response of TBILL to shocks from SS3, DIV3, and CE3, respectively**

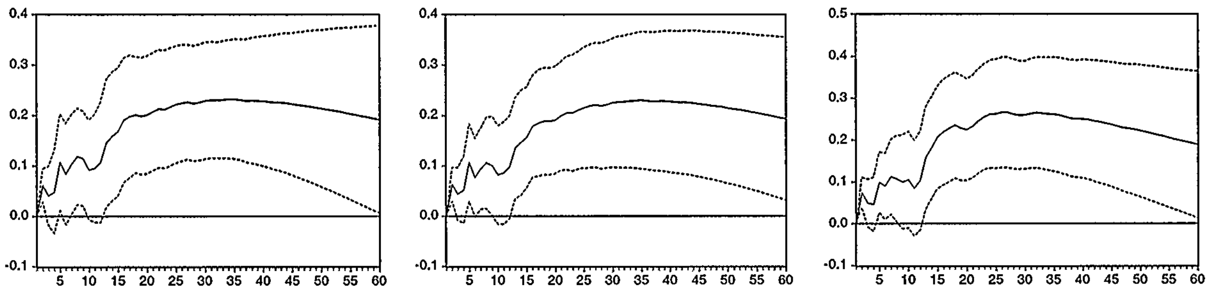



**Figure 4.10: Response of SS3, DIV3, and CE3 to shocks in themselves**

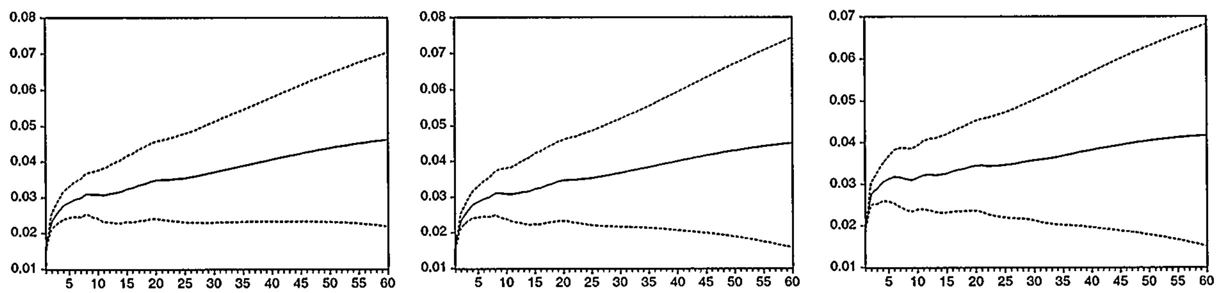



**Figure 4.11: Response of CPI to shocks from SS3, DIV3, and CE3, respectively**

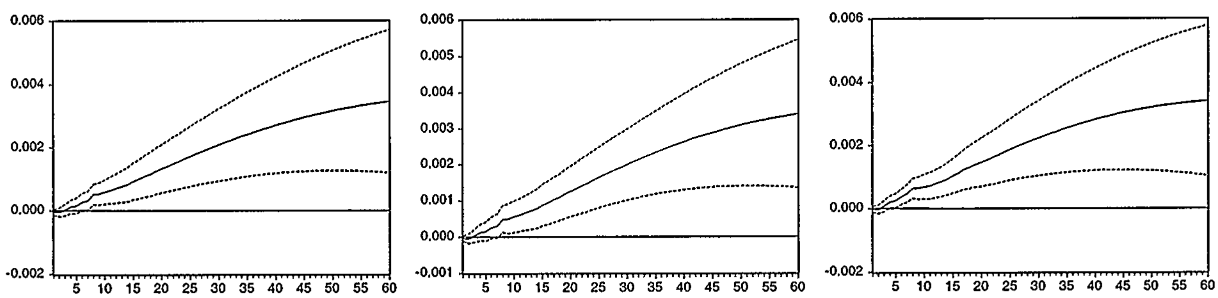
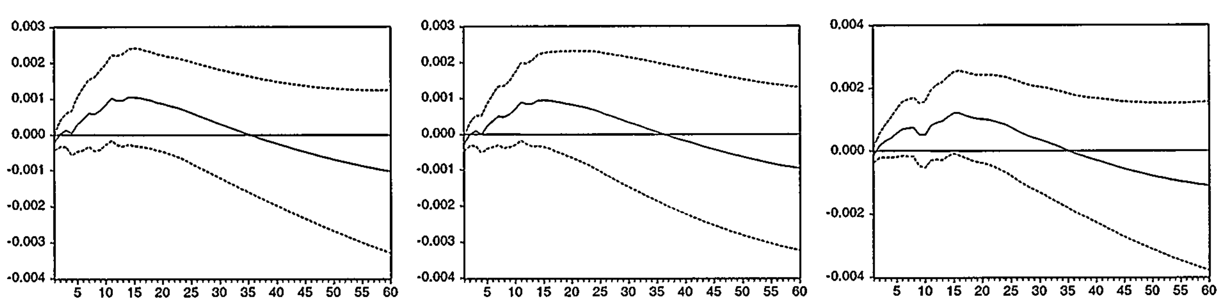



**Figure 4.12: Response of $GDP_{real}$ to shocks from SS3, DIV3, and CE3, respectively**



# CHAPTER FIVE: COMMON AND CODEPENDENT CYCLES

## *The Introduction of Common Trends and Cycles*

In the previous chapter, we examined the monetary aggregates and the price level, nominal output, and real output for common stochastic trends, or cointegration. As indicated earlier, cointegration exists when a linear combination of two non-stationary variables is stationary. Similarly, a 'common feature' between two series is represented when each variable contains a certain common feature, but a linear combination of the two variables does not. The common feature present in the error structure among series is denoted as serial correlation, and is discussed in the next section.

The testing for common trends and cycles within time series data was developed by Engle and Granger (1987) which was furthered by Engle and Kozicki (1993). However the common feature literature was actually initiated by the study of detrending a series. Beveridge and Nelson (1981) introduced the method of actually removing the non-stationary portion of a series. Stock and Watson (1988) furthered this literature when they defined the 'common trend representation' as the decomposition of a variable into a trend, cyclical, and noise component. It follows:

$$y_t = \tau_{yt} + c_{yt} + \varepsilon_{yt} \tag{5.1}$$

$$x_t = \alpha\tau_{yt} + c_{xt} + \varepsilon_{xt} \tag{5.2}$$

where, $\alpha$ denotes the relative factor between the two trends, $\tau_{it}$ the trend, $c_{it}$ the cyclical, and $\varepsilon_{it}$ the noise component of variable $i$ at time $t$.

After detrending the series, we have:

$$\Delta y_t = c_{yt} + \varepsilon_{yt} \tag{5.3}$$

$$\Delta x_t = \beta c_{yt} + \varepsilon_{xt} \tag{5.4}$$

where $\beta$ denotes the relative factor between the two cyclical portions.

Engle and Kozicki (1993) along with Vahid and Engle (1993) realized that this decomposition into the various components enabled them to test for the existence of common cycles. However, they first examined whether each series contained a serial correlation feature. If the first differences of a set of non-stationary variables are serially correlated, the second step of Engle and Kozicki's (1993) common cycle tests is to check if there exists a linear combination that does not possess the common feature. For instance, suppose you can rewrite equations 5.3 and 5.4, as the linear combination:

$$\begin{aligned} \Delta z_t &= c_{yt} + \varepsilon_{yt} - \mu(\beta c_{yt} + \varepsilon_{xt}) \\ &= c_{yt} - \mu\beta c_{yt} + \varepsilon_{yt} - \mu\varepsilon_{xt} \end{aligned} \tag{5.5}$$

In the above regression, if this new variable $\Delta z_t$ does not possess the common feature that is present within the $x$ and $y$ variables, we can conclude that a common cycle is present. However, according to Engle and Kozicki (1993) there may be a common cycle present, but it may not be exact. Vahid and Engle (1997) further analyzed the existence of an unsynchronized common cycle between two variables. This cycle is denoted as 'codependence', and will be discussed in the last section. This can be interpreted as the linear combination of two variables, which contain common features, that has a lower moving average order. Expanding equation 5.5, we can be observe a lag and lead representation as:

$$\begin{aligned}\Delta z_t &= c_{y,t-k} + \varepsilon_{y,t-k} - \mu(\beta c_{y,t-k} + \varepsilon_{xt}) \\ &= c_{y,t-k} - \mu\beta c_{y,t-k} + \varepsilon_{y,t-k} - \mu\varepsilon_{xt}\end{aligned} \tag{5.6}$$

Therefore, the null hypothesis is that the linear combination ($\Delta z_t$) does exhibit codependence in one of the unsynchronized terms. We will test the lag and lead terms, up to a maximum of 12, for our monthly data.

In the following sections, we first discuss the presence of serial correlation and common cycles, followed by a test for codependence.

### *Serial Correlation Features*

If non-stationary variables are first-differenced, the test for common cycles must first begin with a test for a common serial correlation feature within each series. Serial correlation exists within a variable when the innovation term associated with a given time period is correlated with the error terms in other time periods. We are specifically interested in whether two time series share a common trend, or serial correlation feature. Therefore, we must first test each monetary aggregate and the price level, nominal output, and real output to see if a serial correlation feature is present in each of the series. We proceed with the test for serial correlation using the VAR methodology, as introduced by Engle and Kozicki (1993) and Vahid and Engle (1993). Each variable is jointly determined using the following equations:

$$\Delta m_t = \alpha_1 + \beta_1 \Delta m_{t-1} + \delta_1 \Delta x_{t-1} + \gamma_1 \hat{\varepsilon}_{t-1} + \zeta_{mt} \tag{5.7}$$

$$\Delta x_t = \alpha_2 + \beta_2 \Delta m_{t-1} + \delta_2 \Delta x_{t-1} + \gamma_2 \hat{\varepsilon}_{t-1} + \zeta_{xt} \tag{5.8}$$

where $m_t$ is the logged monetary aggregate and $x_t$ is the logged price level, nominal output, or real output. This is a test of whether past changes are important in determining the current value. Note that we have only included the lagged error term ($\hat{\varepsilon}_{t-1}$) in the serial correlation test of SS1 with the price level, since it was found in the previous chapter that they are cointegrated.

In the table below, the null hypothesis of "no serial correlation" is tested using a Lagrange multiplier (LM) statistic of the number of observations multiplied by the proportion of explained variation, or $N \times R^2$. This follows a chi-squared distribution, with three degrees of freedom. For our regressions, the critical value is 7.81 at the five percent significance level. As can be seen in the table below, each test statistic exceeds our critical value, which allows us to reject the null and conclude that a serial correlation feature is indeed present within each of our bivariate models. This differs from Serletis (2001) who found that only eight of his 15 bivariate models between the monetary aggregates and prices had a serial correlation feature present. With regards to output, 11 of his 15 aggregates contained a serial correlation feature in money and nominal or real output, in contrast to our results where each bivariate model contains serial correlation. This presence of a serial correlation feature among the first-differenced series allows us to continue with the test for common cycles within the series.

**TABLE 5.1: Serial Correlation for Money and CPI, $GDP_{nom}$, and $GDP_{real}$**

| | Dependent Variable | | Dependent Variable | | Dependent Variable | |
|---|---|---|---|---|---|---|
| Series | Money | CPI | Money | $GDP_{nom}$ | Money | $GDP_{real}$ |
| SS1 | 65.598 | 245.702 | 62.841 | 101.980 | 60.230 | 122.715 |
| DIV1 | 83.889 | 249.261 | 75.768 | 93.644 | 72.075 | 104.448 |
| CE1 | 46.566 | 256.025 | 58.754 | 93.161 | 68.935 | 109.960 |
| SS2 | 16.044 | 248.282 | 21.448 | 90.515 | 18.267 | 103.916 |
| DIV2 | 281.599 | 244.393 | 288.471 | 89.680 | 288.598 | 102.127 |
| CE2 | 60.395 | 248.132 | 60.148 | 89.354 | 60.441 | 108.491 |
| SS3 | 165.086 | 251.063 | 158.935 | 92.293 | 159.584 | 105.680 |
| DIV3 | 159.235 | 251.234 | 156.379 | 93.045 | 156.234 | 106.739 |
| CE3 | 130.862 | 252.813 | 123.313 | 101.564 | 125.510 | 112.653 |

*Common Cycles*

Empirically, once we have found that a serial correlation feature is present, we can continue with our test for common cycles. Formally, our regression takes the form,

$$\Delta m_t = \theta_0 + \theta_1 \Delta x_t + u_t \tag{5.9}$$

Engle and Kozicki (1993), along with Vahid and Engle (1993), propose the use of two-stage least squares (2SLS) and the limited information maximum-likelihood (LIML) methods, where the lagged variables $\Delta m_{t-1}$, $\Delta x_{t-1}$, and $\Delta \hat{\varepsilon}_{t-1}$ are used as instruments. Again, the lagged error term from the cointegrating regression ($\Delta \hat{\varepsilon}_{t-1}$) is only included in the test for a common cycle between SS1 and the price level. This test for common features has the identical properties as the test for validity of the instruments, as shown in the previous literature, specifically by Engle and Kozicki (1993). The null hypothesis is that all the instruments are indeed valid, or a common cycle is present. The LM test statistics for both the 2SLS and LIML methods, calculated as $N$ x $R^2$, are formulated from the following regression:

$$\hat{u}_t = \delta_0 + \delta_1 \Delta m_{t-1} + \delta_2 \Delta x_{t-1} + \delta_3 \hat{\varepsilon}_{t-1} + \zeta_t \tag{5.10}$$

These LM statistics, along with the estimated coefficient ($\theta_1$) and its test statistic for common features between money and prices, nominal output, and real output can be found on Tables 5.2, 5.3, and 5.4 respectively. The critical value for this chi-squared random variable is 5.99 at the five percent significance level, using two degrees of freedom. As seen in the tables below, each LM statistic exceeds the critical value in each monetary aggregate version, with the exception of CE1 and SS2 with the price level, nominal output, and real output. Therefore, we can conclude that there does not exist a common cycle between money and our other key variables for seven of our nine monetary aggregate forms. This result is comparative with the results from Serletis (2001), where it was found that some, but not all of the monetary aggregates shared common cycles with the key macroeconomic variables. However, the cycle between money and prices or output may not be exact, and we need to proceed with a test for the existence of unsynchronized cycles, or codependence.

**TABLE 5.2: Common Cycle Tests for Money and Prices**

| | | Dependent Variable | | |
|---|---|---|---|---|
| | | IV Test | | LIML Test |
| Series | Statistics | $\Delta m_t$ | $\Delta p_t$ | $\Delta m_t$ |
| SS1 | $\phi_1$ | 1.772 | 0.064 | 1.976 |
| | t-stat | 2.651 | 3.461 | 1.874 |
| | LM stat | 24.746 | 93.756 | 24.639 |
| DIV1 | $\phi_1$ | 1.951 | 0.098 | 2.262 |
| | t-stat | 3.213 | 4.832 | 3.347 |
| | LM stat | 43.879 | 99.210 | 43.681 |
| CE1 | $\phi_1$ | 0.128 | 0.007 | 0.127 |
| | t-stat | 0.040 | 0.382 | 0.039 |
| | LM stat | 1.664 | 153.893 | 1.664 |
| SS2 | $\phi_1$ | 0.849 | 0.851 | 0.853 |
| | t-stat | 1.779 | 2.080 | 1.782 |
| | LM stat | 1.217 | 1.665 | 1.217 |
| DIV2 | $\phi_1$ | 0.428 | 0.122 | 5.927 |
| | t-stat | 3.555 | 2.859 | 3.216 |
| | LM stat | 251.615 | 162.723 | 161.904 |
| CE2 | $\phi_1$ | 1.772 | 0.104 | 1.867 |
| | t-stat | 1.454 | 2.920 | 1.502 |
| | LM stat | 9.384 | 37.846 | 9.378 |
| SS3 | $\phi_1$ | 7.175 | 0.040 | 11.911 |
| | t-stat | 6.386 | 6.756 | 7.108 |
| | LM stat | 101.003 | 113.044 | 92.516 |
| DIV3 | $\phi_1$ | 7.469 | 0.042 | 11.828 |
| | t-stat | 6.659 | 69.984 | 7.253 |
| | LM stat | 93.753 | 105.983 | 86.207 |
| CE3 | $\phi_1$ | 7.652 | 0.042 | 10.752 |
| | t-stat | 5.913 | 6.732 | 6.480 |
| | LM stat | 74.362 | 96.392 | 70.939 |

Note: At the 5% level, critical value for LM statistic is 7.81.

**TABLE 5.3: Common Cycle Tests for Money and $GDP_{nom}$**

| | | Dependent Variable | | |
|---|---|---|---|---|
| | | IV Test | | LIML Test |
| Series | Statistics | $\Delta m_t$ | $\Delta(py)_t$ | $\Delta m_t$ |
| SS1 | $\phi_1$ | 1.809 | 0.138 | 2.558 |
| | t-stat | 2.985 | 3.654 | 3.310 |
| | LM stat | 26.804 | 40.186 | 25.821 |
| DIV1 | $\phi_1$ | 0.245 | 0.036 | 0.364 |
| | t-stat | 0.679 | 0.789 | 0.545 |
| | LM stat | 52.578 | 70.877 | 52.549 |
| CE1 | $\phi_1$ | -7.144 | -0.140 | -7.144 |
| | t-stat | -3.663 | -3.665 | -3.663 |
| | LM stat | 0.020 | 0.020 | 0.020 |
| SS2 | $\phi_1$ | 0.684 | 1.318 | 0.689 |
| | t-stat | 2.395 | 2.502 | 2.404 |
| | LM stat | 0.634 | 0.692 | 0.634 |
| DIV2 | $\phi_1$ | 0.556 | 0.432 | 2.058 |
| | t-stat | 6.350 | 4.242 | 4.472 |
| | LM stat | 127.972 | 57.123 | 56.904 |
| CE2 | $\phi_1$ | -1.154 | -0.186 | -1.282 |
| | t-stat | -1.594 | -2.660 | -1.661 |
| | LM stat | 9.322 | 25.954 | 9.295 |
| SS3 | $\phi_1$ | 2.928 | 0.049 | 12.515 |
| | t-stat | 4.078 | 3.191 | 4.067 |
| | LM stat | 99.777 | 61.111 | 58.649 |
| DIV3 | $\phi_1$ | 3.199 | 0.055 | 11.269 |
| | t-stat | 4.347 | 3.506 | 4.415 |
| | LM stat | 89.376 | 58.143 | 55.214 |
| CE3 | $\phi_1$ | 4.025 | 0.072 | 8.664 |
| | t-stat | 4.884 | 4.465 | 5.162 |
| | LM stat | 59.114 | 49.391 | 44.933 |

Note: At the 5% level, critical value for LM statistic is 7.81.

**TABLE 5.4: Common Cycle Tests for Money and $GDP_{real}$**

| | | Dependent Variable | | |
|---|---|---|---|---|
| | | IV Test | | LIML Test |
| Series | Statistics | $\Delta m_t$ | $\Delta y_t$ | $\Delta m_t$ |
| SS1 | $\phi_1$ | 1.272 | 0.111 | 1.749 |
| | t-stat | 2.251 | 3.110 | 2.509 |
| | LM stat | 30.782 | 58.949 | 30.322 |
| DIV1 | $\phi_1$ | -0.359 | -0.055 | -1.022 |
| | t-stat | -1.017 | -1.244 | -1.802 |
| | LM stat | 51.353 | 76.879 | 49.990 |
| CE1 | $\phi_1$ | -7.148 | -0.140 | -7.148 |
| | t-stat | -3.805 | -3.807 | -3.805 |
| | LM stat | 0.024 | 0.024 | 0.024 |
| SS2 | $\phi_1$ | 0.497 | 1.452 | 0.505 |
| | t-stat | 1.780 | 2.081 | 1.815 |
| | LM stat | 1.257 | 1.681 | 1.256 |
| DIV2 | $\phi_1$ | 0.419 | 0.340 | 2.464 |
| | t-stat | 5.377 | 3.430 | 3.762 |
| | LM stat | 174.328 | 70.956 | 70.734 |
| CE2 | $\phi_1$ | -1.561 | -0.245 | -1.692 |
| | t-stat | -2.225 | -3.144 | -2.300 |
| | LM stat | 8.022 | 16.017 | 7.990 |
| SS3 | $\phi_1$ | 0.726 | 0.012 | 25.018 |
| | t-stat | 1.124 | 0.850 | 1.962 |
| | LM stat | 140.014 | 80.048 | 78.372 |
| DIV3 | $\phi_1$ | 0.975 | 0.017 | 19.108 |
| | t-stat | 1.485 | 1.150 | 2.469 |
| | LM stat | 131.789 | 79.031 | 76.376 |
| CE3 | $\phi_1$ | 1.991 | 0.035 | 10.124 |
| | t-stat | 2.632 | 2.367 | 3.737 |
| | LM stat | 91.522 | 73.992 | 67.114 |

Note: At the 5% level, critical value for LM statistic is 7.81.

***Codependent Cycles***

Codependence exists between two variables when a linear combination of them has a lower moving average order than others. This is important since the paper by Engle and Kozicki (1993) failed to acknowledge that unsynchronized common cycles might be present within the data, even when exact common cycles are not. Thus, the codependent cycle tests will only be run on the variables that have rejected a common cycle. In our case, they include each monetary aggregate except SS2 and CE1. As indicated earlier, the representation for the lag and lead model is used for our codependence test. These codependence tests will examine the significance of 12 monthly monetary lags and leads with prices, nominal output, and real output.

As seen in Tables 5.5, 5.6, and 5.7, the null of codependent cycles being present in the data is rejected with each of the monetary aggregates tested, using a critical value of 5.99 at the five percent significance level. This leads us to conclude that the relationship between money and prices, nominal output, and real output is not cyclical, with the exception of CE1 and SS2 with each macroeconomic variable. Although causation may be present, common features may not.

**TABLE 5.5: Codependent Cycle Tests between Money and Prices**

Cycle between $\Delta m_{t+k}$ and $\Delta y_t$ when k=

| Series | -12 | -11 | -10 | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SS1 | 23.51 | 24.25 | 21.94 | 25.47 | 21.05 | 23.91 | 20.21 | 24.14 | 19.73 | 24.63 | 21.43 | 27.00 |
| DIV1 | 36.35 | 35.07 | 39.73 | 36.01 | 37.75 | 38.36 | 31.22 | 35.68 | 31.57 | 37.57 | 33.53 | 41.46 |
| DIV2 | 144.05 | 151.78 | 156.00 | 155.04 | 150.09 | 161.89 | 158.79 | 163.26 | 164.15 | 162.00 | 163.25 | 162.73 |
| CE2 | 9.96 | 10.23 | 10.18 | 10.27 | 10.30 | 10.25 | 10.34 | 10.29 | 10.81 | 10.39 | 10.88 | 10.50 |
| SS3 | 87.49 | 89.77 | 89.11 | 97.14 | 66.45 | 104.23 | 62.87 | 82.36 | 70.30 | 78.00 | 85.90 | 86.11 |
| DIV3 | 84.05 | 88.45 | 86.30 | 94.96 | 64.39 | 102.40 | 61.16 | 82.25 | 68.90 | 75.41 | 85.30 | 82.92 |
| CE3 | 69.59 | 74.94 | 72.71 | 81.05 | 59.04 | 78.92 | 51.82 | 63.56 | 48.80 | 56.99 | 67.32 | 61.56 |

| Series | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SS1 | 26.59 | 24.00 | 24.50 | 24.02 | 20.03 | 21.79 | 21.65 | 20.66 | 24.03 | 23.93 | 35.62 | 44.89 |
| DIV1 | 44.98 | 42.48 | 44.05 | 41.69 | 37.90 | 34.76 | 36.98 | 34.27 | 39.96 | 38.06 | 44.23 | 41.26 |
| DIV2 | 158.10 | 162.24 | 160.37 | 161.19 | 164.61 | 165.77 | 165.49 | 166.26 | 165.26 | 165.52 | 164.72 | 165.27 |
| CE2 | 10.05 | 9.78 | 7.31 | 11.29 | 10.37 | 11.12 | 10.73 | 10.89 | 9.36 | 10.90 | 10.58 | 8.75 |
| SS3 | 94.47 | 99.05 | 98.97 | 88.32 | 100.11 | 85.92 | 87.84 | 79.94 | 81.85 | 69.41 | 82.72 | 69.60 |
| DIV3 | 91.66 | 94.68 | 92.45 | 86.69 | 96.33 | 87.72 | 85.70 | 80.51 | 80.72 | 69.40 | 78.37 | 67.31 |
| CE3 | 76.62 | 80.39 | 70.14 | 89.54 | 64.69 | 80.17 | 68.74 | 67.98 | 68.16 | 56.84 | 76.43 | 56.58 |

Note: At the 5% level, critical value for LM statistic is 5.99.

**TABLE 5.6: Codependent Cycle Tests between Money and Nominal GDP**

| | Cycle between $\Delta m_{t+k}$ and $\Delta y_t$ when k= | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Series | -12 | -11 | -10 | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 |
| SS1 | 31.06 | 28.81 | 22.24 | 20.29 | 28.04 | 24.89 | 28.09 | 27.83 | 27.60 | 27.68 | 28.11 | 30.58 |
| DIV1 | 50.85 | 42.04 | 52.79 | 53.99 | 53.56 | 53.94 | 47.34 | 45.77 | 46.98 | 52.17 | 42.48 | 52.12 |
| DIV2 | 68.28 | 64.84 | 68.41 | 62.62 | 59.36 | 60.81 | 59.93 | 66.04 | 66.77 | 64.62 | 65.59 | 67.07 |
| CE2 | 11.20 | 10.22 | 9.41 | 9.63 | 9.96 | 7.56 | 10.17 | 10.63 | 10.17 | 10.53 | 10.09 | 10.05 |
| SS3 | 65.21 | 63.41 | 68.97 | 62.29 | 58.78 | 60.03 | 56.87 | 58.86 | 62.30 | 57.27 | 60.24 | 71.26 |
| DIV3 | 63.67 | 64.30 | 69.02 | 62.52 | 58.83 | 61.27 | 57.02 | 60.44 | 63.18 | 57.36 | 60.32 | 71.74 |
| CE3 | 52.63 | 59.72 | 69.69 | 66.63 | 58.46 | 64.70 | 60.60 | 64.64 | 58.27 | 52.61 | 51.40 | 67.21 |
| Series | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| SS1 | 28.71 | 19.55 | 26.78 | 22.49 | 28.06 | 27.19 | 30.17 | 24.40 | 23.86 | 22.53 | 38.78 | 54.67 |
| DIV1 | 52.34 | 47.42 | 52.04 | 50.83 | 47.81 | 44.17 | 44.01 | 45.84 | 45.48 | 43.81 | 49.32 | 50.05 |
| DIV2 | 62.01 | 57.70 | 55.30 | 58.95 | 60.44 | 61.39 | 62.30 | 60.61 | 62.99 | 66.27 | 56.57 | 66.67 |
| CE2 | 10.57 | 8.16 | 10.38 | 8.27 | 10.60 | 9.72 | 10.63 | 9.52 | 9.60 | 8.00 | 9.12 | 11.56 |
| SS3 | 69.40 | 67.89 | 60.66 | 61.26 | 67.66 | 66.83 | 68.48 | 68.70 | 68.46 | 69.13 | 65.51 | 63.92 |
| DIV3 | 70.45 | 66.36 | 59.11 | 59.38 | 67.15 | 66.39 | 67.93 | 68.72 | 67.97 | 69.47 | 62.19 | 64.86 |
| CE3 | 63.45 | 67.99 | 57.07 | 64.09 | 65.41 | 68.75 | 61.83 | 68.78 | 66.34 | 69.95 | 64.66 | 68.51 |

Note: At the 5% level, critical value for LM statistic is 5.99.

**TABLE 5.7: Codependent Cycle Tests between Money and Real GDP**

Cycle between $\Delta m_{t+k}$ and $\Delta y_t$ when k=

| Series | -12 | -11 | -10 | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SS1 | 26.71 | 27.38 | 15.85 | 16.67 | 25.47 | 20.75 | 24.67 | 27.42 | 23.04 | 24.63 | 26.32 | 31.60 |
| DIV1 | 52.05 | 54.06 | 42.58 | 44.71 | 47.53 | 46.69 | 50.46 | 54.22 | 52.89 | 51.47 | 54.59 | 53.76 |
| DIV2 | 75.45 | 70.86 | 79.46 | 73.68 | 72.02 | 72.25 | 71.33 | 77.22 | 77.16 | 75.38 | 76.98 | 79.38 |
| CE2 | 11.28 | 10.22 | 9.15 | 9.83 | 9.97 | 7.42 | 9.76 | 10.78 | 9.95 | 10.64 | 10.12 | 9.92 |
| SS3 | 75.75 | 69.74 | 75.64 | 72.73 | 71.69 | 73.41 | 71.39 | 78.96 | 77.54 | 77.53 | 79.16 | 75.77 |
| DIV3 | 76.22 | 69.16 | 75.00 | 71.92 | 71.50 | 73.16 | 71.23 | 79.37 | 77.07 | 77.62 | 79.20 | 73.30 |
| CE3 | 74.20 | 71.30 | 69.05 | 68.79 | 71.67 | 60.08 | 66.13 | 77.50 | 77.28 | 76.93 | 76.37 | 77.14 |

| Series | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SS1 | 28.34 | 24.34 | 26.12 | 19.31 | 25.29 | 23.25 | 28.91 | 26.51 | 25.71 | 24.83 | 38.00 | 47.39 |
| DIV1 | 52.40 | 51.15 | 48.91 | 48.99 | 50.30 | 50.70 | 51.36 | 49.36 | 51.11 | 50.19 | 45.23 | 46.20 |
| DIV2 | 77.87 | 73.11 | 70.70 | 73.88 | 74.25 | 73.95 | 73.92 | 71.08 | 74.90 | 77.85 | 67.61 | 78.69 |
| CE2 | 975.00 | 7.08 | 10.03 | 8.50 | 10.32 | 9.42 | 10.52 | 9.21 | 10.45 | 7.76 | 9.59 | 10.84 |
| SS3 | 82.97 | 82.08 | 80.87 | 81.68 | 81.75 | 82.57 | 80.27 | 77.34 | 78.65 | 73.77 | 80.60 | 81.49 |
| DIV3 | 82.57 | 82.42 | 80.56 | 81.09 | 81.85 | 82.95 | 80.72 | 78.06 | 79.31 | 72.30 | 81.27 | 81.30 |
| CE3 | 83.29 | 81.02 | 80.98 | 81.50 | 79.32 | 79.54 | 82.01 | 75.05 | 78.50 | 67.01 | 81.22 | 73.71 |

Note: At the 5% level, critical value for LM statistic is 5.99.

## CHAPTER SIX: CONCLUSION

This paper, through the use of Bayesian classification, is re-clustering the monetary asset data in the United States in an aim to capture the shared movements throughout the 44-year sample period. Typically, the monetary assets grouped by the Federal Reserve are classified by their properties as a medium of exchange, applying their user costs as proxies. Our classification results in three distinct groups of monetary assets, in comparison to the six nested levels of aggregation published by the Federal Reserve. We are specifically interested in determining the link between our new classes of money and some key variables such as prices, output, and the interest rate. After a new classification was complete, we construct the three versions of monetary aggregates that have been widely used in the past: the simple sum, divisia, and the currency equivalent indexes.

After completing the tests for time series properties of the data, we observe several parallels with the previous literature, as well as some new results emerging. As in the earlier articles, unit roots are still present within the aggregates, and there does not appear to be cointegration between money and nominal output or real output. In terms of the price level and our simple sum version of class one (SS1), the no cointegration null hypothesis was rejected at the 5% level, but not at the 10% level.

In terms of causality results, Barnett et al. (1992) reported that none of their fifteen monetary aggregates appear to cause inflation using either the single equation or the VAR method, while our results are mixed. In the single equation technique, the null of no causality from money to prices is rejected for all of our aggregates. The VAR method reported that money causes prices using the CE1, SS2, DIV2, CE2, SS3, DIV3, and CE3 aggregate versions. This is a significant consequence specifically if the central bank is using an inflation target. Using this information, it appears that some our new classes of money can be used to influence the economy in such a way that an objective of a stable inflation rate can be reached. Similarly, the single equation framework seems to support the money to real output causal theory for all aggregates, which can also be useful to stimulate the economy.

In reviewing the common features chapter, we do see that although serial correlation common features exist in each series, they do not exhibit a common cycle except for CE1 and SS2 with each of our macroeconomic variables of interest. In examining the existence of unsynchronized cycles, or codependence, we also fail to show evidence of any shared features. Although these results seem somewhat discouraging, the results from the VAR model can still be implemented in policymaking.

Therefore, this paper is an attempt to further the macroeconomic view of 'money', using statistical cluster analysis to hopefully achieve a variable that has a more transparent relationship with prices and output. The results regarding the causality and forecasting power of the new monetary aggregates are important for attaining the Federal Reserve's future targets. If the central bank implements inflation targeting, we demonstrate that our new monetary aggregates, with the exception of SS2 and DIV2, support the causality theory of money to prices in the single or multi-equation methods. This differs significantly from previous literature on the Federal Reserve's monetary asset groupings. Thus, we illustrate that it is possible to utilize Bayesian statistical foundations to re-cluster the data in a more numerical (and therefore influential) way, rather than based on shared liquidity properties as in M1 or M2 currently published by the Federal Reserve.

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