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The Indifference Real Option Valuation Model and Its Applications

by

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Abstract

This paper investigates the indifference real option valuation model and applies the model to three cases in real business: the Collinsville plant, the car park, and the NaS Battery. The primary characteristic of the model is that it takes an investor's risk aversion level into consideration to value a real option. The model is applicable in two investment scenarios. One is when an investor holds a portfolio of tradable assets and cash, and the other is an investor deposits his entire fund in a bank to earn interest. By comparing the results of the case studies obtained by the indifference real option valuation model and the risk neutral approach, this paper discussed the advantages and limitations of the indifference real option valuation model.

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "The Indifference Real Option Valuation Model and Its Applications" submitted by Huichun Yu in partial fulfillment of the requirements for the degree of Master of Business Administration

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Dedication

My parents, Quanshun Yu and Xiumei Zhang

My supervisor, Dr. Robert J. Elliott and his wife, Ann Elliott

TABLE OF CONTENTS

List of Tables

Chapter One

Introduction

Given investment valuation information, decision makers wish to make investment decisions and choose optimally. Investment decision is characterized by uncertainty, irreversibility, and timing, (Dixit and Pindyck, 1994, p.3); these influence an investor's choices of the investment valuation method.

Traditional investment theories include the net present value (NPV) approach, Jorgenson's user cost of capital approach, and Tobin's q. These three approaches were founded on the NPV rule, (Dixit and Pindyck, 1994, p.4). Under the NPV principle, cash flows are discounted at the risk adjusted discount rate. An investor will choose to invest in a project if the NPV of the cash inflows is larger than the initial cost. The NPV principle is applicable if the investment is reversible. When an investor has the option to delay the investment (Arya, Fellingham and Glover, 1998, Pindyck, 1991), the flaw of the rule appears. For instance, the NPV rule ignores the option value embedded in the project, (Ross, 1995).

Myers (1977) introduced the concept of a "real option" which viewed real assets as financial call options. A real option is "the right, but not the obligation, to take an action at a predetermined cost, called the exercise price, for a predetermined period of time" (Copeland and Antikarov, 2001, p.5). Consequently, real option approach takes flexibility into account. Flexibility is a source of option value. Thus, when the investment is irreversible, the real option valuation approach is more applicable than the traditional NPV principle. There are several real option valuation methods. For instance, with the Black-Scholes model (Black and Scholes, 1973), an investor can value a real option like a European call or put option. A further example is the Market Asset Disclaimer (MAD) approach proposed by Copeland and Antikarov (2001). Under the MAD approach, the value of a real option is determined by a risk-neutral binominal model. The market value of a project is the NPV of its cash flows without any flexibility (Copeland and Antikarov, 2001). A real option can also be valued using the indifference pricing approach and the valuation method developed by van der Hoek and Elliott (2006). The real option valuation has numerous applications such as valuing strategic decisions, real estate, and research and development projects.

The aim of this study is to investigate the indifference real option valuation model which is founded on the exponential utility function and the indifference pricing approach developed by van der Hoek and Elliott (2006). The model in thesis and van der Hoek and Elliott (2006)'s work have two primary differences. The first one is the model in this thesis considers two states, while van der Hoek and Elliott (2006)'s model takes four states into account. The second difference between the two models is that this thesis takes two investment scenarios into consideration, which makes it more applicable in practice.

This valuation model is applied to three real cases: the Collinsville plant, the car park, and the NaS Battery. The real options embedded in these three cases are also valued by the risk neutral approach. By comparing the results given by the indifference valuation model and the risk neutral approach, we investigate the advantages and limitations of the indifference valuation model.

The thesis is organized as follows. In section 2, we review the literature on investment and real option related issues. Section 3 describes the model used in this paper. In section 4, we describe three case studies on real options using the indifference valuation model and the risk neutral approach. In this section, we discuss the advantages and disadvantages of the indifference valuation model by comparing the results given by the two approaches. In the final section, we summarize the results.

Chapter Two

Literature Review

2.1 Chapter Overview

In this chapter, prior research on investment decision making and real option is discussed.

Section 2.2 introduces the concept of a real option. The term "real option" was introduced by Myers (1977). A real option, defined by Copeland and Antikarov, (2001), is "the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting, or abandoning) at a predetermined cost, called the exercise price, for a predetermined period of time---the life of the option", (Copeland and Antikarov, 2001, p. 5). It has been suggested that the real option valuation approach is more useful than the traditional NPV rule when target projects are irreversible, (Pindyck, 1991, Arya, Fellingham and Glover, 1998).

Section 2.3 reviews literature on real option valuation models. Several real option valuation models are discussed, such as the Black-Scholes model, (Black and Scholes, 1973), the Market Asset Disclaimer approach, (Copeland and Antikarov, 2001), the indifference pricing approach based on behavioral analysis, (van der Hoek and Elliott, 2006).

Section 2.4 reviews literature on how real option valuation has been used in practice. Anderson (2000) and Krychowski and Quélin (2010) have written papers on real option analysis in decision making process. Williams (1991), and Hui and Leung (2002)'s research on real option focuses on investments in real estate. Manfred, Thorsten and Randolf (1999), and Huchzermeier and Loch (2001) investigate the influences of real option analysis on research and development projects.

2.2 Introduction of a real option

Investment is to pay a cost now with the expectation of favorable future benefits. For instance, governments build infrastructures, firms spend money on research and development projects and individuals devote time to maintain a relationship. Investors wish to determine optimal investment decisions. Dixit and Pindyck (1994) identified three characteristics of most investment decisions: "uncertainty, irreversibility, and timing", (Dixit and Pindyck, 1994, p.3). Investors' investment decisions are determined by the interactions of these three characteristics which also influence their choices of decision making tools. For instance, it is inappropriate to use the traditional net present value (NPV) rule in investment decision making when target projects are irreversible, (Pindyck, 1991, Arya, Fellingham and Glover, 1998). In addition to irreversibility, investors are provided with the option to delay the investment to optimal timing. Therefore, real options are worth investigating in order to enhance decision making in these irreversible projects.

. The term "real option" was introduced by Myers (1977). He suggested that corporate assets can be viewed as financial call options. Real options are "opportunities to purchase real assets on possibly favorable terms", the value of which depends on the corporate investment in the future, (Myers, 1977). The definition of a real option, defined by Copeland and Antikarov (2001) is that "a real option is the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting, or abandoning) at a predetermined cost, called the exercise price, for a predetermined period of time---the life of the option", (Copeland and Antikarov, 2001, p.5). The philosophy of a real option is that flexibility has value, (Trigeorgis, 1993, Copeland and Keenan, 1998). According to the type of flexibility a real option offers, Copeland and Antikarov (2001) classified real options into four types: simple options, switching options, compound options and rainbow options.

Researchers have made attempts to compare the real option valuation approach and other investment decision-making tools, (Majd and Pindyck, 1987, Copeland and Keenan, 1998). They have suggested that the traditional NPV rule will mislead investors when there is a high level of uncertainty or investors have the opportunity to delay the investment for more information and, in these situations, the real option valuation approach is more useful.

2.3 Real option valuation methods

There are several real option valuation methods. We now discuss the Black-Scholes model, the market asset disclaimer (MAD) approach, the indifference pricing approach, and a comprehensive approach. As these valuations methods are founded on different assumptions, they are viable in different situations.

2.3.1 The Black-Scholes model

Under the assumption that options are fairly priced, and there is no arbitrage opportunity by obtaining a portfolio of options and stock, Black and Scholes (1973) developed theoretical valuation models for European call and put options. Under seven strict assumptions, the value of European options is determined by exercise prices, stock prices, time, volatility, and the risk-free interest rate.

The Black-Scholes model was derived to value financial options. Several attempts have been made to apply the Black-Sholes model to real option valuation, (Luehrman, 1998, Campbell, 2002). Luehrman (1998) and Campbell (2002) both viewed an investment opportunity as a call option. Luehrman (1998) fitted it onto the Black-Scholes model and proposed two new variables, "NPVq and cumulative volatility", to capture the option value. Campbell (2002) concluded that the "cum-dividend American call option" should only be early exercised if the NPV is larger than the interests earned by deferring the investment.

2.3.2 The Market Asset Disclaimer (MAD) approach

Copeland and Antikarov (2001) studied the market asset disclaimer approach. They proposed two assumptions: (1) The MAD assumption: a project has a best unbiased estimate for its market value is its intrinsic NPV, (Copeland and Antikarov, 2001, p. 84). (2) Cash flows in the discounted cash flow model follow a random walk with an unchanged volatility, (Copeland and Antikarov, 2001, p. 219).

Using the MAD approach, the value of a real option is determined by a risk-neutral binominal model, the inputs of which include the net present values of the project in different uncertainty scenarios. Copeland and Antikarov (2001) suggested that the Black-Scholes model and the MAD approach share the same philosophy, which is to replicate the project. The difference between these two approaches is that the Black-Scholes model is built on Itô calculus, while the MAD approach uses an algebraic method to estimate the option value, (Copeland and Antikarov, 2001, p.110).

2.3.3 The indifference pricing approach

Morgenstern and von Neumann (1953) investigated decision making of rational decision makers under uncertainty situations using behavioral analysis. Their results could be illustrated by a lottery example. They suggested that, with known ranking of an investor's expected utility based on the lottery payoff, they could develop a utility function. As it has been assumed that an investor wants to maximize his expected utility, the investor will prefer the option which provides a higher expected utility than the other.

Henderson and Hobson (2004) pointed out that perfect replication, as assumed in the Black-Scholes model and the MAD approach, does not always exist, due to "non-tradable assets", "transaction costs" and "portfolio constrains", (Henderson and Hobson, 2004). In this case, investors could make investment decisions by maximizing their expected utility. They suggested that a utility function could be either "the positive real line", such as the power utility, or "the whole real line", such as the exponential utility, (Henderson and Hobson, 2004). The utility indifference price is defined as that at which investors are indifferent between giving up a claim, and paying some money in order to obtain the claim at time T. They proposed models in discrete and continuous time settings. The advantages of the indifference pricing approach include risk consideration and "economic justification", (Henderson and Hobson, 2004).

2.3.4 A comprehensive approach

van der Hoek and Elliott (2006) suggested that the risk-neutral pricing approach is applicable to tradable assets, the payoff of which could be replicated by a twin security. Non-tradable assets and non-hedgeable risks are usually priced with the indifference pricing approach. They used the indifference pricing approach to value real options, and developed a comprehensive real option valuation method

In this approach, the initial wealth is invested in cash and a tradable asset. In a discrete time model, as the goal is to maximize an investor's expected utility at time T=1, van der Hoek and Elliott (2006) calculated the optimal amount of tradable asset held by the investor by setting the derivative of the expected utility expression to zero. In this way, they determined the value of initial wealth.

van der Hoek and Elliott (2006) extended the above calculation to a "general attainable claim" written on a non-tradable asset, (van der Hoek and Elliott, 2006, p. 216). According to the indifference pricing approach, the seller of the claim is indifferent between paying nothing and giving up the claim at T= 0, and obtaining some compensation at T=0 in order to sell the claim at T=1. The compensation at T=0 is defined as "the indifference asking price", (van der Hoek and Elliott, 2006, p. 216). The indifference biding price could be determined in the same way. As the claim is assumed to be replicated by cash and a tradable asset, they calculated the optimal amount of the tradable asset. Thus, the indifference prices could be determined.

2.4 Applications

Real option valuation has various applications in investment decision making. It could be applied to the valuation of research and development projects, real estate, and strategic decision making.

2.4.1 Strategic decision making

Anderson (2000) built a bridge between real option valuation and strategic decision making. He designed a dual option framework to analyze strategic decision making processes. An abandonment option is embedded in the initial development state which can be estimated based on a "simple compound option", (Anderson, 2000). Fixed asset commitments have continuous deferral options embedded in them. The abandon approach and the deferral approach support the strategic option development and exercise, respectively. It is suggested that, by assuming a "stochastic development in the state variable", the options could be valued with the risk-neutral approach when a complete market exists and with the dynamic programming approach when the market is incomplete (Anderson, 2000).

Krychowski and Quélin (2010) investigated how real option analysis is applied to the strategic investment decision making in a mobile telecommunication company, Mobitel. The "late-entry scenario" of the 3G investment is viewed as a deferral option and is valued with the Black-Scholes model. Compared with the traditional NPV approach, real option analysis allowed Mobitel to determine the value to wait for more information and other options, and to consider both risks and benefits, (Krychowski and Quélin, 2010). It is suggested that real option analysis could be well applied to specific strategic investment decision making, such as "market entry and high-tech investment projects", (Krychowski and Quélin, 2010).

2.4.2 Real estate

Williams (1991) pointed out that undeveloped real estate has options embedded in it, such as option to determine "the date and the density level of the development", and the option to "abandon the development", (Williams, 1991). He quantitatively analyzed the optimal exercise policies of the real options embedded in the undeveloped real estate project, and determined the optimal timing to develop and abandon the project.

In their case study of Hong Kong Disneyland, Hui and Leung (2002) applied the option pricing theory to valuation of the project. There are five kinds of options embedded in the Disneyland project, "the option to switch use, to expand and to defer, and strategic options in the competitive environment", (Hui and Leung, 2002). They used the binomial risk-neutral option valuation model to value Hong Kong Disneyland with different options embedded in it. It is suggested that the value of the project will be improved when the option to switch, to expand and to defer are taken into consideration and the value would be significantly increased when strategic options in the competitive environment are involved in the valuation.

2.4.3 Research and development projects

Manfred, Thorsten and Randolf (1999) investigated real option valuation and characteristics of input variables in pharmaceutical research and development projects. They suggested that the Geske model, compared with the Black-Scholes model and the binomial model, is the most appropriate method to value a research and development (R&D) project. They viewed the investment in a pharmaceutical R&D project as a compound option including a call option for investment in "testing an identified substance" and another one lies in the investment in "production capacity and market introduction", (Manfred, Thorsten and Randolf, 1999).

Huchzermeier and Loch (2001) identified five specific R&D uncertainties, "market payoffs, project budgets, product performance, market requirements, and project schedules". They suggested that dynamic programming is an appropriate valuation model to value R&D projects because it does not require replication. A main contribution of their paper is to connect the five influential factors to the real option valuation model.

Chapter Three

Mathematical Modeling

3.1 Chapter Overview

In this chapter, the indifference real option valuation model is investigated. It draws upon work within real option theories and specifically the work done by van der Hoek and Elliott (2006) where they combined the exponential utility function and the certainty equivalence approach to value a claim written on a non-tradable asset.

Section 3.2 investigates the theoretical framework of the indifference valuation model. Due to “non-tradable assets”, “transaction costs” and “portfolio constrains”, (Henderson and Hobson, 2004), perfect replicating usually does not exist. It makes the replicating approach such as the risk neutral method less practical. Morgenstern and von Neumann (1953) provided a behavioral analysis method to analyze an investor’s behavior. Henderson and Hobson (2004) suggested that the exponential utility function and the indifference pricing are worth investigating in real option valuation.

Section 3.3 introduces an expected utility function. In this thesis, the exponential utility function is used to model an investor’s investment behavior. An investor usually wants to maximize his expected utility. Thus, the amount of the initial wealth invested in a tradable asset is determined by equating the derivative of the expected exponential function to zero. The optimal amount enables an investor to achieve his maximum expected utility.

Section 3.4 investigates the indifference pricing approach. In the two-period discrete time model, with the indifference price, an investor will be indifferent between giving up the investment opportunity at T=0 and keeping it until T=1 at a cost. The indifference pricing model considers two states at T=1. It is required that the tradable security involved in the model has the same probabilities with the non-tradable asset with a real option embedded in it to move to the two states at T=1.

Section 3.5 discusses another valuation scenario when an investor chooses to deposit his entire fund in a bank to earn interest. In this scenario, he will obtain a constant expected utility. In this section, the indifference valuation model is derived to value a real option for this scenario.

3.2 Theoretical background

In section 2.3.3, we reviewed Morgenstern and von Neumann (1953) and Henderson and Hobson (2004)'s works which can be viewed as the theoretical foundation of the valuation model discussed in the following part.

It was pointed by Henderson and Hobson (2004) that the exponential utility function and the indifference pricing are worth investigating in real option valuation because the replicating methods such as the market asset disclaimer (MAD) approach introduced by Copeland and Antikarov (2001) are not always viable. As we reviewed in section 2.3.2, the MAD approach used the intrinsic net project value as the best unbiased estimate of its market value, (Copeland and Antikarov, 2001, p.84). To make a perfect replicating portfolio, the project has to be divisible, which is usually not possible in practice. Thus, such a perfect replicating portfolio does not always exist. Henderson and Hobson (2004) identified three reasons that explain why a perfect replicating portfolio usually does not exist: "non-tradable assets", "transaction costs" and "portfolio constrains". Particularly, the underlying asset of a real option is usually non-tradable. For example, when a firm is investing in a research and development project, it will not sell it even if it has turned out a failure because of technological pertinence and technical security. As reviewed in section 2.3.3, Morgenstern and von Neumann (1953) provided a useful behavioral analysis framework to analyze an investor's behavior, the utility function. The basic philosophy of Morgenstern and von Neumann (1953)'s utility function is that an investor usually prefers an option which provides him with a higher expected utility.

A real option valuation model is investigated in Chapter 3. It draws upon work within real option theories and specifically the work done by van der Hoek and Elliott (2006) where they combined the exponential utility function and the certainty equivalence approach to value a claim written on a non-tradable asset. The reason I am using their logic is because their approach takes an investor's risk aversion level into consideration and does not make strict assumptions on the tradable asset involved in the model. According to Henderson and Hobson (2004), risk consideration is a crucial advantage of the indifference pricing approach. Thus, it is more applicable in practice. Following their logic, I have determined my model in the following sections. The model in thesis and van der Hoek and Elliott (2006)'s work have two primary differences. The first one is the model in this thesis considers two states, while van der Hoek and Elliott (2006)'s model takes four states into account. The second difference between the two models is that this thesis takes two investment scenarios into consideration, which makes it more applicable in practice. The model in the thesis targets to analyze cases in which the value of the non-tradable asset with a real option embedded in it has two states at T=1.

Another thing worth mentioning is that according to Markowitz (1952), an investor could not always obtain the maximum utility as well as the minimum risk at the same time. To select a desirable portfolio, he needs to achieve a balance between returns and risks. That is to say, if two investment options have the same risk aversion level, an investor would choose to the one which provides the investor with a higher expected utility. With a known distribution of payoffs, an investment option will have a constant variance of returns. In this circumstance, the primary criteria to choose an optimal option is to maximize an investor's expected utility.

3.3 The exponential utility function

Utility functions have different forms, such as the exponential function, the logarithmic function and the linear function, (Wikipedia: Utility). In this thesis, we shall use the exponential utility function to model investment behavior.

That is, we consider:

$$U(x) = -\exp(-\gamma x), (\gamma > 0). \tag{2.1}$$

This expected utility function U(x) is “increasing and concave”, (van der Hoek and Elliott, 2006, p.214).

Suppose that an investor’s initial investment portfolio is worth X at time T=0. Some of X can be invested in a tradable security S, such as a stock, or the investor can also deposit cash in a bank to earn at the interest rate r.

Suppose that at T=0, the investor holds Z shares of S and cash C.

Then,

$$X = ZS_0 + C.$$

Assume that at time T=1, the tradable security had two states, λ_1 andλ_2. Suppose that S(λ_1) = S_1^+ and S(λ_2) = S_1^-. $S_1^+ \neq S_1^-$. Suppose that there is no dividend paid during the n-year time period.

The interest rate is r with R=r+1.

In an efficient market, arbitrage opportunities should not exist, or, at least, should not exist for very long time. Under this assumption, $S_1^+ \neq S_1^-$, unless $S_1^+ = S_1^- = RS_0$. Otherwise, there will be arbitrage opportunity.

To see this, suppose that $S_1^+ = S_1^- < RS_0$ or $S_1^+ < S_1^- < RS_0$ or $S_1^- < S_1^+ < RS_0$, an investor can short sell S_0 at T=0 and invest it in the stock market. At T=1, the net payoff of the investment is $RS_0 > S_1$. As $RS_0 - S_1^- > 0$ and $RS_0 - S_1^+ > 0$, Then the investor gains a positive net payoff.

Suppose that $S_1^+ = S_1^- > RS_0$ or $S_1^+ > S_1^- > RS_0$ or $S_1^- > S_1^+ > RS_0$, an investor can short sell S_0 at T=0 and deposit it in a bank to earn at the interest rate. At T=1, the net payoff of the investment is $S_1 - RS_0$. As $S_1^- - RS_0 > 0$ and $S_1^+ - RS_0 > 0$; Then the investor gains a positive net payoff.

In both cases, the investor can obtain a positive net payoff without any initial cost. This violates the no arbitrage assumption.

Then at T=1, the value of the initial wealth is:

$$X_1 = ZS_1 + CR,$$

So

$$X_1 = XR + Z(S_1 - S_0R). \tag{2.2}$$

The investor's expected utility at T=1 is:

$$E[U(XR + Z(S_1 - S_0R))] = E[-\exp(-\gamma(XR + Z(S_1 - S_0R))]. \tag{2.3}$$

As the investor's goal is to maximize his expected utility and the only unknown variable in this equation is Z, the investor should determine the number of stocks, Z, he bought at T=0 in order to maximize this expected utility.

S has been assumed to have a "up and down" state at T=1. Assume that at T=1, the probability for S to increase to S_1^+ is p, and the probability of S to decrease to S_1^- is 1-p. Thus, the above equation (2.3) becomes,

$$p[-\exp(-\gamma(XR + Z(S_1^+ - S_0R))] + (1-p)[-\exp(-\gamma(XR + Z(S_1^- - S_0R))].$$

We calculate the derivative of the above function with respect to Z and equate it to zero.

Write $y = -\gamma XR - \gamma Z(S_1^+ - RS_0)$ and $w = -\gamma XR - \gamma Z(S_1^- - RS_0)$.

Then,

$$E[U(X)] = -pe^y - (1-p)e^w.$$

So $$(E[U(X)])' = -pe^y y' - (1-p)e^w w' = 0.$$

This gives $$e^{\gamma Z(S_1^- - S_1^+)} = -\frac{(1-p)(S_1^- - RS_0)}{p(S_1^+ - RS_0)}.$$

So $$Z^* = -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1}\ln[-\frac{(1-p)(S_1^- - RS_0)}{p(S_1^+ - RS_0)}]. \tag{2.4}$$

It has been demonstrated that $S_1^+ = S_1^-$ and $S_1^- < RS_0 < S_1^+$. That is, $(S_1^- - RS_0)(S_1^+ . - RS_0) < 0$.

Thus, $-\frac{(1-p)(S_1^- - RS_0)}{p(S_1^+ - RS_0)} > 0$. As the independent variable in the logarithm function is defined as positive, it makes sense to calculate the natural logarithm of $-\frac{(1-p)(S_1^- - RS_0)}{p(S_1^+ - RS_0)}$.

Write $\pi = \frac{(RS_0 - S_1^-)}{(S_1^+ - S_1^-)}$.

Then,

$$Z^* = -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1}\ln[\frac{(1-p)\pi}{p(1-\pi)}].$$

$Z \in [-\infty, +\infty]$, and Z* is the unique value which makes the derivative of the expected utility function equal to zero.

When $Z^* < -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1}\ln[\frac{(1-p)\pi}{p(1-\pi)}]$,

$$(E[U(X)])' > 0.$$

When $Z^* > -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1}\ln[\frac{(1-p)\pi}{p(1-\pi)}]$

$$(E[U(X)])' < 0.$$

Thus, Z* is the optimal amount of S held by an investor. It maximizes the investor's expected utility at T=1. Clearly for the value Z*, the expected utility function takes its maximum value.

Substituting Z* back into (2.3), we obtain,

$$E[U(XR + Z(S_1 - S_0R))]_{\max}$$

$$= -\exp(-\gamma XR)p^{\pi}(1-p)^{1-\pi}[(\frac{\pi}{1-\pi})^{1-\pi} + (\frac{1-\pi}{\pi})^{\pi}].$$

$$= -\exp(-\gamma XR)\frac{p^{\pi}(1-p)^{1-\pi}}{\pi^{\pi}(1-\pi)^{1-\pi}}.$$

That is:

$$E[U(XR+Z(S_n-S_0R))]_{\max}= -\exp(-\gamma XR)\frac{p^{\pi}(1-p)^{1-\pi}}{\pi^{\pi}(1-\pi)^{1-\pi}}.$$

Thus,

$$V(X)=-\exp(-\gamma XR)\frac{p^{\pi}(1-p)^{1-\pi}}{\pi^{\pi}(1-\pi)^{1-\pi}}. \qquad (2.5)$$

That is, with initial wealth X at T=0, an investor can buy $-\frac{1}{\gamma}(S_1^+-S_1^-)^{-1}\ln[\frac{(1-p)\pi}{p(1-\pi)}]$ shares of stock and deposit the remaining cash in a bank to earn at the interest free rate. In this way, he can maximize his expected utility at T=1, and obtain $-\exp(-\gamma XR)\frac{p^{\pi}(1-p)^{1-\pi}}{\pi^{\pi}(1-\pi)^{1-\pi}}$.

To determine the maximum expected utility, it has been assumed that the tradable asset has a known distribution of returns. Thus, the initial investment portfolio has a constant variance and, suggested by Markowitz (1952), the same risk aversion level. From this perspective, it is reasonable to target at the maximum expected utility so as to make investment decision.

3.4 Indifference valuation modeling

As discussed above, the replicating approaches, such as the MAD approach and the risk neutral valuation method, can be used to value a real option if a perfect replicating portfolio exits. However, the underlying asset of a real option is usually a non-tradable asset, such as real estate, and a research and development project. Due to "market incompleteness", "portfolio constrains", (Henderson and Hobson, 2004), and "non-hedgeable risks", (van der Hoek and Elliott, 2006, p. 214), it is inappropriate to use financial option valuation methods to value a real option related to a non-tradable underlying asset. The indifference pricing approach might then be used to model investors' decision making, (Henderson and Hobson, 2004, van der Hoek and Elliott, 2006, p. 214).

Suppose there are a non-tradable asset Y and a real option G written on Y. G(Y) represents the value of Y with a real option embedded in it. Suppose that at time T=1, Y has two states, μ_1 and μ_2. Accordingly, G also has two states at T=1, g_1 and g_2. That is,

$$G_{1,i} = g_i = G(Y_{\mu i}). \tag{3.1}$$

As discussed above, an investor wishes to maximize his expected utility at T=1. Suppose that the investor's initial investment portfolio is worth X which can be invested by buying Z shares of stock S and depositing an amount C of cash. There is no dividend paid during the time period.

Without the real option, his maximal expected utility has been found to be the amount (2.5). Including the real option, we write the maximum expected utility as:

$$V_G(X) = \max_Z E[U(X_1 - G_1)]. \tag{3.2}$$

where G_1 is the value of non-tradable asset with the real option at T=1.

The indifference asking price of the real option should make the seller indifferent between receiving money v_a at time T=0 so as to sell the asset at T=1, and giving up the deal at T=0. Thus, v_a should satisfy:

$$V(X) = V_G(X + v_a). \tag{3.3}$$

Similarly, the indifference biding price of the real option will make the buyer indifferent between paying the money v_b at time T=0 so as to buy the asset at T=1, and giving up the deal at T=0. Thus, v_b should satisfy:

$$V(X) = V_{-G}(X - v_b). \tag{3.4}$$

With the real option, the expected utility is:

$$E[-\exp[-\gamma(XR + Z(S_1 - RS_0) - G_1)]]. \tag{3.5}$$

Assume that at T=1, the probabilities for G to go to an up state, g_1, and a down state, g_2, are p_1 and p_2, respectively. Assume that when the tradable asset S moves to the up state λ_1, G will move to the state g_1. Accordingly, when the tradable asset S moves to the down state λ_2, G will move to the state g_2. However, for S, the upstate λ_1 means that its value increases and the down state λ_2 mean that its value decreases at T=1, while for G, the states, g_1 and g_2, do not have similarly specific meanings. Thus, the expected value of the utility function becomes:

$$E[-\exp[-\gamma(XR+Z(S_1-RS_0)-G_1)]]=$$

$$-p_1[\exp[-\gamma(XR+Z(S_1^+-RS_0)-g_1)]]-p_2[\exp[-\gamma(XR+Z(S_1^--RS_0)-g_2)]].$$

Write $u=-\gamma(XR+Z(S_1^+-RS_0)-g_1)$, $w=-\gamma(XR+Z(S_1^--RS_0)-g_2)$.

Then , equating the derivative of the above function with respect to Z to zero:

$$(E[U(X)])'=(-\gamma)[-p_1e^u(S_1^+-RS_0)-p_2e^w(S_1^--RS_0)]=0.$$

So

$$\frac{e^u p_1}{e^w p_2}=\frac{RS_0-S_1^-}{S_1^+-RS_0}.$$

Then,

$$\frac{e^u p_1}{e^w p_2}=\frac{RS_0-S_1^-}{S_1^+-RS_0}.$$

That is,

$$e^{-\gamma Z(S_1^+-S_1^-)}\cdot\frac{e^{\gamma g_1}p_1}{e^{\gamma g_2}p_2}=\frac{RS_0-S_1^-}{S_1^+-RS_0}.$$

The optimal value of Z is then:

$$Z^{**} = -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1} \ln[\frac{(RS_0 - S_1^-)(p_2 \exp(\gamma g_2))}{(S_1^+ - RS_0)(p_1 \exp(\gamma g_1))}].$$

Writing $\pi = \frac{(RS_0 - S_1^-)}{(S_1^+ - RS_0)}$.

$$Z^{**} = -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1} \ln[\frac{\pi(p_2 \exp(\gamma g_2))}{(1-\pi)(p_1 \exp(\gamma g_1))}]. \tag{3.6}$$

Clearly, $Z \in [-\infty, +\infty]$, and Z** is the unique value which makes the derivative of the expected utility function equal to zero.

When $Z^{**} < -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1} \ln[\frac{\pi(p_2 \exp(\gamma g_2))}{(1-\pi)(p_1 \exp(\gamma g_1))}]$,

$$(E[U(X)])' > 0.$$

When $Z^{**} > -\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1} \ln[\frac{\pi(p_2 \exp(\gamma g_2))}{(1-\pi)(p_1 \exp(\gamma g_1))}]$.

$$(E[U(X)])' < 0.$$

Thus, Z** is the optimal amount of S which should be held by an investor. Z** maximizes the investor's expected utility at T=1.

Substituting Z** into (3.3), we obtain:

$$V_G(X) = -\exp(-\gamma XR)E[\exp(-\gamma Z^{**}(S_1 - RS_0) + \gamma G_1)].$$

That is,

$$\begin{aligned} &V_G(X) = \\ &-\exp(-\gamma XR)[\exp(-\gamma Z^{**}(S_1^+ - RS_0)(p_1 \exp(\gamma g_1)) \\ &+ \exp(-\gamma Z^{**}(S_1^- - RS_0)(p_2 \exp(\gamma g_2))]. \end{aligned}$$

So
$$V_G(X) = -\exp(-\gamma XR)\frac{(p_1 \exp(\gamma g_1))^{\pi}}{\pi^{\pi}} \cdot \frac{(p_2 \exp(\gamma g_2))^{1-\pi}}{(1-\pi)^{1-\pi}}.$$

Then,

$$V_G(X) = -\exp(-\gamma XR)\frac{(p_1 \exp(\gamma g_1))^{\pi}}{\pi^{\pi}} \cdot \frac{(p_2 \exp(\gamma g_2))^{1-\pi}}{(1-\pi)^{1-\pi}}. \tag{3.7}$$

When G=0,

$$V_G(X) = V(X) = -\exp(-\gamma XR)\frac{p_1^{\ \pi} p_2^{\ 1-\pi}}{\pi^{\pi}(1-\pi)^{1-\pi}}.$$

We require,

$$V_G(X + v_a) = V(X).$$

Then,

$$V_G(X + v_a) = -\exp(-\gamma(X + v_a)R)\frac{(p_1 \exp(\gamma g_1))^{\pi}}{\pi^{\pi}} \frac{p_2(\exp(\gamma g_2))^{1-\pi}}{(1-\pi)^{1-\pi}}.$$

Suppose that $p = p_1$ and $1 - p = p_2$.

Then,

$$\exp(-\gamma v_a R) = [\frac{p_1}{p_1 \exp(\gamma g_1)}]^{\pi} [\frac{p_2}{p_2 \exp(\gamma g_2)}]^{1-\pi}.$$

We find:

$$v_a = -\frac{1}{\gamma R}[\pi \ln \frac{1}{\exp(\gamma g_1)} + (1-\pi)\ln \frac{1}{\exp(\gamma g_2)}].$$

$$= \frac{1}{R}[\pi(g_1) + (1-\pi)(g_2)].$$

So the indifference asking price of G at T=0 is:

$$v_a(G) = \frac{1}{R}[\pi(g_1) + (1-\pi)(g_2)]. \tag{3.8}$$

Similarly, we can show the indifference biding price of G at T=0 is:

$$v_b(G) = -\frac{1}{R}[\pi(-g_1) + (1-\pi)(-g_2)]. \tag{3.9}$$

One thing worth mentioning here is that by solving the equation $V_G(X + v_a) = V(X)$, the investor's risk aversion measurement γ, as a common factor, is divided out. Thus, it is not included in the indifference ask and biding price equations.

In this valuation scenario, given the real option, an investor will adjust his investment portfolio. He will invest in $-\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1}\ln[\frac{(RS_0 - S_1^-)(p_2\exp(\gamma g_2))}{(S_1^+ - RS_0)(p_1\exp(\gamma g_1))}]$ shares instead of $-\frac{1}{\gamma}(S_1^+ - S_1^-)^{-1}\ln[\frac{(1-p)\pi}{p(1-\pi)}]$ shares of the tradable asset. Additionally, with known returns of the tradable asset and its relationship with the value of real option, it might be possible to use the replicating approach to replicate the payoffs of the real option. However, the two approaches will give different results because they have different theoretical background. The replicating approach is built on the no arbitrage rule while the indifference valuation model is founded on the behavioral analysis and indifference pricing rule. The replicating approach is suboptimal because, due to non-tradable underlying assets and market incompleteness, a perfect replicating portfolio usually does not exist and other methods such as the indifference valuation model which is in accordance with no arbitrage rule will give some different results.

3.5 Another valuation scenario

In section 3.3 and 3.4, we have assumed that an investor invests in a tradable asset and deposits the remaining cash in a bank to earn interest. There is another possible scenario that the investor chooses to deposit all his wealth into a bank. Under this condition, the investor will have a constant expected utility at T=1. The indifference asking and biding prices could be determined in the same way. Suppose that the decision making point is n years away from T=0.

When deposit was the only investment method, equation (2.3) became:

$$E[U(X)] = E[-\exp(-\gamma(XR^n))].$$

With G at T=1, the expected utility also equals to:

$$E[U(X)] = E[-\exp(-\gamma((X+v_a)R^n - G_1)].$$

The difference asking price could be obtained by equating the above two equations:

$$E[U(X)] = E[-\exp(-\gamma(XR^n))] = E[-\exp[-\gamma((X+v_a)R^n - G_1)].$$

Similarly, the difference biding price could be obtained by:

$$E[U(X)] = E[-\exp(-\gamma(XR^n))] = E[-\exp[-\gamma((X-v_a)R^n + G_1)].$$

Thus, the indifferent asking price is:

$$v_a(G) = \frac{1}{\gamma R^n} \ln[p \exp(\gamma g_1) + (1-p)\exp(\gamma g_2)]. \tag{4.1}$$

The indifference biding price is:

$$v_b(G) = -\frac{1}{\gamma R^n} \ln[p \exp(-\gamma g_1) + (1-p)\exp(-\gamma g_2)]. \tag{4.2}$$

When n=1, the indifference asking price of G at T=0 is:

$$v_a(G) = \frac{1}{\gamma R} \ln[p \exp(\gamma g_1) + (1-p)\exp(\gamma g_2)]. \tag{4.3}$$

When n=1, the indifference bidding price of G at T=0 is:

$$v_b(G) = -\frac{1}{\gamma R} \ln[p \exp(-\gamma g_1) + (1-p)\exp(-\gamma g_2)]. \tag{4.4}$$

That is, (4.3) and (4.4) is special situation that an investor has to decide whether or not to exercise a real option at time T=1.

Chapter Four

Applications

4.1 Chapter Overview

In this chapter, the real option valuation model investigated in Chapter 3 and the risk neutral approach are applied to three cases: the Collinsville plant, the car park and the NaS battery project. These three cases are adapted from real business cases, but transaction details have been modified. Based on the numerical results of the case studies, Chapter 4 investigates the advantages and disadvantages of the indifference real option valuation model.

In the Collinsville plant case discussed in section 4.2, a real option was embedded in the Collinsville plant which was not publicly traded and therefore, did not have a fair market value. The real option had a deferral option feature. In this case, it is assumed that the American Chemical Corporation held a portfolio of tradable assets. Thus, the indifference valuation model (3.8) could be applied to the case.

In the car park case discussed in section 4.3, there was an expansion option embedded in the car park construction project. In this case, the car parking firm did not invest in any tradable security and its owner deposited all the cash in the bank to earn interest. Thus, the indifference valuation model (4.2) is used to do the case analysis.

In the NaS battery project case discussed in section 4.4, there was an abandonment option embedded in the NaS battery project. In this case, the decision making point was two years away from time T=0. There were multiple sources of uncertainty. It was assumed that cash flows generated by the project followed the stochastic process and the Cox-Ross-Rubinstein model was used to describe the process. Like the car park case, cash was the only tradable asset for the investor. Thus, the value of the abandonment option was determined by the indifference valuation model (4.2).

4.2 Case Study One: The Collinsville Plant

4.2.1 Case Description[1]

In 1979, the Collinsville plant was owned by the American Chemical Corporation and mainly produced sodium chlorate with a production capacity of 40,000 tons per year. Sodium chlorate was mainly used to bleach pulp in the paper and pulp industries. It was a raw material used to produce chloride dioxide, a bleaching agent in these industries.

Due to strategic and regulatory concerns, the American Chemical Corporation was seeking a buyer for the Collinsville plant. In 1979, Dixon provided the American Chemical Corporation with an offer which gave it an option to sell the plant for $20 million in 1980. Dixon produced specialty chemicals such as sulfuric acid and liquid sulfur dioxide. These chemicals were also sold to paper and pulp industries. The acquisition of the Collinsville plant would complete its product portfolio. The synergy effect mainly arose in the reduction of sales expenses because the two firms shared some big clients. To obtain the real option, the American Chemical Corporation must pay Dixon $150,000 in 1979.

The American Chemical Corporation had a weighted average cost of capital (WACC) was 14%. Other chemical corporations which purely produced sodium chlorate, Brunswick and Southern, had an equity Bata value (β_e) of 1.1 and 1.2, respectively. Their debt ratios were 15% and 21%, respectively. The target debt ratios of the American Chemical Corporation and Dixon were 35%. The American Chemical

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[1] Harvard Business School. (2005). *American Chemical corporation.*

Corporation had a Standard & Poor's bond rating of Aa, which indicated that it could borrow at a rate of 11%. Suppose that the interest rate of the long-term Treasury bill is 8.5%. The tax rate was 48%.

It was expected on average, the price of sodium chlorate would increase by 8% per year. The lifetime of the plant was 10 years from 1980. Its projected financial statements across its lifetime are shown in Table 1 and Table 2.

As Dixon provided the American Chemical Corporation with an offer which gave it the option to sell the plant for $20 million in 1980, it was expected that based on the 1979 estimation, the value of the Collinsville plant in 1980 would either increase by 30% at a probability of 75% or decrease by 20% at a probability of 25%. The anticipated decrease was due to potential increase in price of power and labor cost.

The American Chemical Corporation was also an active trader in the financial market. It held a portfolio of stocks and bonds. According to the financial advisor's prediction, the value of the portfolio would increase by 50% at a probability of 75% or decease by 10% at a probability of 25% in one year's time. It also deposited some cash in a bank to earn interest.

Managers at the American Chemical Corporation held stock options which were designed to encourage managers to maximize shareholders' value. It was supposed that the management team at the American Chemical Corporation had an exponential utility preference and their average risk aversion level was 1.5 measured by the risk parameter in the exponential utility function.

As the American Chemical Corporation had to determine the biding value provided by potential bidders, it needed to decide the value of the real option provided by Dixion so as to choose optimally.

Table 1 Projected Balance Sheet for the Collinsville Plant (000s)

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Assets	-									
Accounts receivable	$1,328	$1,680	$1,976	$2,136	$2,303	$2,303	$2,303	$2,303	$2,303	$2,303
Inventories	598	756	889	961	1,036	1,036	1,036	1,036	1,036	1,036
Net property, plant & equipment	10,025	9,440	8,840	8,230	7,560	6830	6040	5190	4280	3310
Total	$11,951	$11,876	$11,705	$11,327	$10,899	$10,169	$9,379	$8,529	$7,619	$6,649
Liabilities										
Accounts payable	730	924	1,087	1,175	1,267	1,267	1,267	1,267	1,267	1,267
Net assets	$11,221	$10,952	$10,618	$10,152	$9,632	$8,902	$8,112	$7,262	$6,352	$5,382

Table 2 Projected Income Statement for the Collinsville Plant (000s)

				For the Years Ended December 31						
1987 1988 1989	1980	1981	1982	1983	1984	1985	1986			
Revenue										
Sales-tons	32,000	35,000	38,000	38,000	38,000	38,000	38,000	38,000	38,000	38,000
Average prince/ton	$415	$448	$484	$523	$565	$610	$659	$711	$768	$830
Sales:$000	$13,280	$15,687	$18,394	$19,866	$21,455	$23,171	$25,025	$27,027	$29,189	$31,524
Manufacturing costs										
Variable-Power	$6,304	$7,060	$7,908	$8,857	$9,919	$11,110	$12,443	$13,936	$15,608	$17,481
-Graphite	645	791	875	940	992	1,042	1,092	1,142	1,192	1,242
-Salt&Other	1,285	1,621	1,753	1,836	1,956	2,056	2,156	2,256	2,356	2,456
Total Variable	$8,234	$9,472	$10,536	$11,633	$12,867	$14,208	$15,691	$17,334	$19,156	$21,179
Fixed-Labor	1,180	1,297	1,427	1,580	1,738	1,888	2,038	2,188	2,338	2,488
-Maintenance	256	277	299	322	354	374	394	414	434	454
-Other	1,154	1,148	1,179	1,113	1,153	1,153	1,153	1,153	1,153	1,153
Total Fixed	$2,590	$2,722	$2,905	$3,015	$3,245	$3,415	$3,585	$3,755	$3,925	$4,095
Total manufacturing costs	$10,824	$12,194	$13,441	$14,648	$16,112	$17,623	$19,276	$21,089	$23,081	$25,274
Other Charges										
Selling	$112	$125	$138	$152	$168	$173	$178	$183	$188	$193
R&D	451	478	508	543	591	631	671	711	751	791
Depreciation	$1,060	$1,110	$1,160	$1,210	$1,270	$1,330	$1,390	$1,450	$1,510	$1,570
Total	$1,623	$1,713	$1,806	$1,905	$2,029	$2,134	$2,239	$2,344	$2,449	$2,554
Operating Profit	$833	$1,780	$3,147	$3,313	$3,313	$3,415	$3,510	$3,594	$3,659	$3,696

4.2.2 Case Analysis

In this case, the Collinsville plant was a company-owned plant and did not have a fair market value. Thus, the net present value of estimated free cash flows generated from its normal operations would be used to measure its value.

$$NPV = \sum_{t=0}^{n} \frac{FCFF_t}{(1+WACC)^t}.$$

Free cash flows generated from its normal operations could be calculated according to data in Table 1 and Table 2. Table 3 presented the summary of the calculation of the free cash flows generated from the Collinsville plant's normal operation.

Table 3 Free cash flows generated from normal operation (000s)

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Operating Profit	$833	$2,893	$4,513	$4,803	$4,887	-$1,475	-$2,685	-$4,038	-$5,554	-$7,252
Plus:Depreciation	$1,060	$1,110	$1,160	$1,210	$1,270	$1,330	$1,390	$1,450	$1,510	$1,570
Minus: Capital Expenditure	$485	$525	$560	$600	$600	$600	$600	$600	$600	$600
Minus:Investment in Working Capital	$204	-$316	-$266	-$144	-$150	$0	$0	$0	$0	$0
Plus:Working Capital Recovery										$2,072
Plus: Tax Benefit from PPE writeoff										$0
FCFF	$1,204	$3,794	$5,379	$5,557	$5,707	-$745	-$1,895	-$3,188	-$4,644	-$4,210

To determine the NPV, another crucial issue is to determine an appropriate WACC which correctly reflect shareholders' required rate of return given the plant's risk aversion level. In this case, it is inappropriate to use the American Chemical Corporation's WACC as the discount factor because the corporation's WACC was the average one among all of its subsidiaries. It reflected the average risk aversion level of the corporation. Thus, we could use the other two chemical corporations which purely produced sodium chlorate, Brunswick and Southern, as two comparables of the Collinsville plant because all these three firms specialized in the sodium chlorate sector.

From case description, we learn that Brunswick and Southern had an equity Bata value of 1.1 and 1.2, respectively and their debt ratios were 15% and 21%. Equity Bata value is influenced not only by the specific risk in the industry but also by the capital structure. Thus, we should calculate the unlevered Bata (β_u) which is only affected by the industrial risk.

$$\beta_u = \frac{\mathrm{E}}{E + B(1-T)} \beta_e.$$

Thus, Brunswick had an unlevered β_u of 1.0 and Southern had an unlevered β_u of 1.05. Then, the unlevered β_u of the Collinsville plant could be obtained by taking the arithmetic mean of the two. It was 1.025. According to the case description, the target debt ratios of American Chemical Corporation and Dixon were 35%. Therefore, the unlevered β_e of the Collinsville plant was 1.312.

The cost of equity is:

$$E(R_E) = r_f + \beta(E(R_M) - r_f).$$

That is, assuming that the market premium was 7%, for the Collinsville plant, shareholders of the American Chemical Corporation had a required rate of return 17.68%.

The WACC is:

$$WACC = \frac{E}{V} R_D(1-T) + \frac{E}{V} R_E.$$

That is, the WACC of the Collinsville plant was 13.5%.

Then, we could calculate the intrinsic value of the Collinsville plant.

$$NPV = \sum_{t=0}^{10} \frac{FCFF_t}{(1+13.5\%)^t} = \$19,665,000.$$

Thus, in 1979, the value of the Collinsville plant, measured by its NPV was $19,665,000.

According to the case description, it was expected that based on the 1979 estimation, the value of the Collinsville plant in 1980 would either increase by 30% at a probability of 75% or decrease by 20% at a probability of 25%. The anticipated decrease was due to potential increase in price of power and labor cost. In 1980, the value of the Collinsville plant would either move to an up state of $25,564,500 or a down state of $15,732,000.

Let V_1 represent the value of the Collinsville plant in 1980 if American Chemical Corporation continued to operate the plant. Let X represent the value of price provided by Dixon to purchase the plant, which was $20 million.

Thus, in 1980, the value of the Collinsville plant was:

$$G_1 = \max[V_1, X].$$

That is, in the upstate, G_1 is $25,564,500 and in the down state, G_2 is $20,000,000.

For the board of directors at American Chemical Corporation, the value of the plant with the real option embedded in it in 1979 was:

$$G_0 = v_b(G_1).$$

According to the case description, the American Chemical Corporation was also an active trader in the financial market. It held a portfolio of stocks and bonds. According to the financial advisor's prediction, the value of the portfolio would increase by 50% at a probability of 75% or decease by 10% at a probability of 25% in one year's time. Thus, it could choose to invest the "initial" wealth in the investment and deposit the remaining in the bank to earn interest.

That is,

$$\pi = \frac{RS_0 - S_1^-}{S_1^+ - S_1^-} = 0.31.$$

Thus, in 1979, the value of the Collinsville plant with the real option embedded in it was:

$$v_b(G) = -\frac{1}{R}[\pi(-g_1) + (1-\pi)(-g_2)].$$

$$=\$20,023,037$$

As we have already calculated that its intrinsic NPV was $19,665,000, the value of the real option provided by Dixon was $358,037. As the American Chemical Corporation needed to pay Dixon $150,000 to get the real option, the net payoff of the offer provided by Dixon was worth $208,037.

4.2.3 The risk neutral approach

With a real option embedded in a non-tradable asset, an investor can hedge the asset with the real option. The risk neutral valuation approach is a discounting approach which discounts payoffs of the hedging portfolio at the risk free rate. To make the hedging portfolio totally riskless, an investor will choose an appropriate hedge ratio to make the payoffs of the hedging portfolio in the up and down states equal (Copeland and Antikarov, 2001). As discussed in section 2.3.2, Copeland and Antikarov (2001) made a market disclaimer assumption that a project has a best unbiased estimate for its market value is its intrinsic NPV, (Copeland and Antikarov, 2001, p. 84). Under this assumption, the real option embedded in a non-tradable asset could be valued using the risk neutral method, (Copeland and Antikarov, 2001, p. 95).

In the real neutral approach,

$$(Y_0 - bN_0)(1+r) = uY_0 - bN_u = dY_0 - bN_d.$$

where r is the risk free rate, Y_0 is the value of the asset at T=0, N_0 is the value of the option payoff at T=0, u is the up state factor, d is the down state factor, N_u is the value of the option payoff in the up state at T=1, N_d is the value of the option payoff in the down date at T=1, and b is the hedging ratio.

The hedging ratio is:

$$b = \frac{(u-d)Y_0}{N_u - N_d}.$$

Thus, the value of the underlying asset with a real option embedded in it is:

$$N_0 = \frac{1}{1+r}[\frac{1+r-d}{u-d}N_u + \frac{u-1-r}{u-d}N_d].$$

where $\frac{1+r-d}{u-d}$ is called the risk neutral probability and N_0 is the risk neutral price.

Thus, we can use this approach to analyze the Collinsville Plant case and compare the result with the one obtained by the indifference approach discussed in section 4.2.2.

It was expected that based on the 1979 estimation, the value of the Collinsville plant in 1980 would either increase by 30% with a probability of 75% or decrease by 20% with a probability of 25%. We have already figured out that in 1980, the value of the Collinsville plant would either move to an up state of \$25,564,500 or a down state of \$15,732,000.

That is, in 1980, the option payoff at the upstate was:

$$N_u = \$25{,}564{,}500.$$

In 1980, the option payoff at the downstate was:

$$N_d = \$20{,}000{,}000.$$

In this case, the hedging ratio is:

$$b = \frac{(u-d)Y_0}{N_u - N_d} = 1.77.$$

That is, by taking short position of 1.77 shares of the hedging portfolio, the investment in the Collinsville plant would be riskless. Whatever the up state or the down state the value of the plant moved to in 1980, the American Chemical Corporation would obtain a constant rate of return.

Thus, in 1979, the value of the Collinsville plant with an option embedded in it was:

$$N_0 = \frac{1}{1+r}[\frac{1+r-d}{u-d}N_u + \frac{u-1-r}{u-d}N_d].$$

$$=\$21,356,465$$

As we have already calculated that its intrinsic NPV was \$19,665,000, the value of the real option provided by Dixon was \$1,691,465. The American Chemical Corporation needed to pay Dixon \$150,000 to get the real option, the net payoff of the offer provided by Dixon determined by the risk neutral approach was worth \$1,541,465.

4.2.4 Discussion

In section 4.2.2 and 4.2.3, we used the indifference valuation approach and the risk neutral valuation method to analyze the Collinsville plant case and determine the value of the real option embedded in it.

First of all, when the initial wealth is invested in a tradable asset and the remaining cash is deposited in a bank to earn interest, the indifference valuation model has a risk neutral structure as the risk neutral valuation method.

The indifference valuation model is shown below:

$$v_b(G) = -\frac{1}{R}[\pi(-g_1) + (1-\pi)(-g_2)].$$

The risk neutral valuation equation is:

$$N_0 = \frac{1}{1+r}[\frac{1+r-d}{u-d}N_u + \frac{u-1-r}{u-d}N_d].$$

However, the two approaches achieved different computational results regarding to the Collinsville plant case. According to the indifference valuation model, the real option embedded in the plant was \$358,037. The real option value obtained by the risk neutral valuation method was \$1,691,465. The difference lies in the different philosophies the two approaches built on.

The indifference valuation approach is built on the behavioral rule which is to maximize an investor's expected utility, and the certainty equivalence theory which allows an investor to be indifferent between two scenarios. Thus, it has assumed the existence of an initial investment portfolio held by the investor. The valuation is influenced by the payoffs of the tradable asset included in the investment portfolio. That is why the "risk neutral probability" at the indifference valuation model is $\pi = \frac{RS_0 - S_1^-}{S_1^+ - S_1^-}$.

Unlike the indifference valuation approach, the risk neutral valuation method is built on the market disclaimer assumption which used the intrinsic net present value of the project as the best unbiased estimate of its market value, (Copeland and Antikarov, 2001, p. 84). Thus, the risk neutral probability in this model only depends on the up state and down state factors of the plant's value, which is $b = \frac{(u-d)Y_0}{N_u - N_d}$.

In the Collinsville plant case,

$$\pi = \frac{RS_0 - S_1^-}{S_1^+ - S_1^-} = 0.31.$$

and the risk neutral probability in the risk neutral valuation model is:

$$\frac{1+r-d}{u-d} = 0.57.$$

The difference between these two factors has resulted in the difference between the real option values obtained by the two methods.

Secondly, the risk neutral approach has several advantages: (1) The risk neutral approach limits the valuation in the non-tradable asset and the real option embedded in it and does not involve other tradable assets. Thus, it is not necessary to deal with comparable return and risk issues; (2) The risk neutral probability remains constant at each decision making point as long as value of the project has constant up and down movement factors. This makes the calculation easier than the indifference valuation portfolio approach; (3) As this approach discounts at the risk free rate, there is no need to determine the risk adjusted discount rate.

However, the risk neutral approach has one crucial limitation. It is that this approach assumes that the non-tradable asset hedged by a real option is divisible and tradable. In the model, if an investor holds one share of a non-tradable asset, to hedge the portfolio, he will need to take a short position of $b = \dfrac{(u-d)Y_0}{N_u - N_d}$ shares of a real option. For one thing, as the asset is non-tradable and transaction costs might be high, the short position is not attainable in practice. For another thing, when the hedging ratio $b = \dfrac{(u-d)Y_0}{N_u - N_d}$ is not an integral number, the real option has to be divisible so as to make the hedging portfolio riskless. In practice, it is usually impossible to split a project. Therefore, a perfect hedging portfolio of a real asset does not exist. Under this condition, it is not practically reasonable to analyze cases with the risk neutral approach.

In the Collinsville plant case, there are two states and a tradable security with known payoffs. It is therefore possible to use the risk neutral approach to value the real option. However, the indifference valuation approach is also applicable in this situation for two reasons. The first is that the indifference valuation approach deals with the issue how an investor invests his existing portfolio. By choosing an appropriate amount to invest in the stock, the American Chemical Corporation would maximize its shareholders' expected utility. The risk neutral approach on the other hand requires the

American Chemical Corporation to construct a new investment portfolio to make the entire portfolio riskless. In practice, there are costs to rearrange its investment portfolio, especially when the firm has several similar projects. A second reason is that the risk neutral approach is based on the no-arbitrage assumption. "Transaction costs" and "holding costs" often prevent an investor from fully earning the arbitrage profit, (Pontiff, 2006). For real options, the underlying asset is usually a non-tradable asset; so the costs related to arbitrage tend to be high because of the illiquidity of the asset and the opportunity cost arising when there are several firms bidding for the project and the company chooses the real option one. Thus, even in a complete market, the risk neutral valuation approach might not be the optimal method to value a real option.

Thirdly, as we can see from the section 4.2.3 case analysis, the risk parameter has been divided out and does not influence the real option valuation. However, the risk investor's risk aversion level still influences investors' investment decision making. Take the Collinsville plant case for example. As stated in the case description, the American Chemical Corporation was an active trader in the financial market and its managers had a relatively low risk aversion level measured by the risk parameter 1.5 in the exponential utility function. As these managers had the incentive to maximize shareholders' value, they tended to achieve a balance between their own risk aversion level and the shareholders' risk aversion level. Unlike managers, shareholders were able to diversify their investment portfolios. According to Henderson (2008) and Billett and Liu (2008), managers are usually more risk averse than well diversified shareholders. Thus, shareholders might have a lower risk aversion level than managers at the

American Chemical Corporation. Suppose that stock options effectively motivated managers to behave according to shareholders' interest. They would more likely to take higher risk when making investment decisions. For instance, if another bidder provided the American Chemical Corporation with the bidding that the biding price was written on some stock index, the shareholders at the American Chemical Corporation might prefer this firm's real option instead of Dixon's if the potential rate of return was higher. Therefore, managers would not choose Dixon's bidding.

Fourthly, risk consideration is one crucial advantage of the indifference valuation approach. Compared with the risk neutral approach, another advantage of the indifference valuation approach is that it is not developed based on the creation of a "perfect" hedging portfolio. However, it has its own limitation. To build the valuation model, it has been assumed that the probabilities of the tradable asset are the same as those of the non-tradable claim to move to the two states at T=1.

In the Collinsville plant case, the American Chemical Corporation held a portfolio of tradable assets and cash. The tradable portfolio and the net present value of the Collinsville plant had the same probabilities, 75% and 25%, to move to their "up and down" states, respectively. Under this condition, the model (3.8) and (3.9) could be directly applied the real option valuation. Suppose that the probabilities of movements of either the tradable portfolio or the net present value of the Collinsville plant were changed, the indifference pricing model (3.8) and (3.9) would no longer directly applicable.

Finally, in section 2.4.1, we reviewed Anderson (2000)'s and Krychowski and

Quélin (2010)'s work on how real option analysis was applied to strategic decision making. Both of them have suggested that value is created when a real option allows an investor to delay the investment so as to more information. In the Collinsville plant case, the real option provided by Dixon had a deferral feature because it allowed the managers at the American Chemical Corporation to delay their decision making to one year. Thus, this indicates that in 1979, even if a bidder bided for the Collinsville plant at the same price with the value determined by the real option analysis in section 4.2.2, the American Chemical Corporation might still prefer Dixon's bidding as it could generate more information during one year's time.

In addition, as suggested by Krychowski and Quélin (2010), real option analysis takes both benefits and risk in to account, which was viewed as a primary advantage of real option analysis. The indifference valuation model takes an investor's subjective risk aversion level into consideration, which, according to Krychowski and Quélin (2010), could be its main advantage over the risk neutral method.

4.3 Case Study Two: Car Park

4.3.1 Case Description[2]

Sun Parking was a parking garage firm located in Union City. The company had ten years' history and was specialized in the gar parking industry. Particularly, its car parks were located at central urban areas. In 1999, due to business expansion, Wilson, the CEO of Sun Parking, decided to build a new four-floor car park to satisfy increasing demand at central Union City.

Sun Parking expected that it would take the firm one year to construct the four-floor car park. The new car park would be put into use in 2000. Several architectural firms had provided Sun Parking with their construction proposals. One of those architectural firms, Nice Way, gave a proposal that would allow Sun Parking to choose to expand the four-floor garage to a six-floor one in 2001 as the architects at Nice Way had worked out a flexible architectural structure. The construction of two new floors would last one year and would not influence the operation of the existing car park. Due to the unique construction design, Nice Way charged a higher construction fee which was $3,000,000.

The proposed car park had 200 spaces for 200 cars per floor. According to Nice Way's construction proposal, the pre-cast construction fee for the new car park was $16,000 per space and would increase by 10% for each floor above the ground one. The lifetime of the car par was 20 years. The expansion would cost Sun parking $900,000 in 2001. In addition, it would cost the company $3.6 million per year to lease the land.

The marking department did a marketing research and prepared a marketing report.

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[2] de Neufville, R., Scholtes,S. and Wang, T. (2006).Valuing Real Options by Spreadsheet: Parking Garage Case Example. Journal of Infrastructure Systems, 12(2), 107-111.

According the marking report, each space used was able to earn $10,000 per year with an annual average operating expense of $2,000. The annual demand for parking spaces was about 550 in 2000. The future demand for parking spaces was uncertain. However, according to previous experience, the marketing department at the company had estimated that the demand would increase either by an average rate of 2% per year with a probability of 10% or an average rate of 6% per year with a probability of 90%. Instead of uncertain demand for car parking places in central Union City, maintaining fees were also unknown. It depended on the architectural condition. Thus, the operating fee might be either increase or decrease. After prediction, in 2001, in the former scenario, the value of the car part would decrease by 5%; while in the latter scenario, the value of the car part would increase by 20%. More marketing information would be obtained after the car park was put into use.

The parking garage company had a weighted average cost of capital (WACC) of 12%. The long-term Treasury bond rate was 7%. Sun Parking was able to borrow at 11%. It had a debt ratio of 20%.

Sun Parking was sole proprietorship. Wilson was a conservative investor and did not invest in any securities. He preferred to deposit all of cash in a bank to earn interest. It was assumed that Wilson had an exponential utility preference. As a conservative investor, he had a relatively high risk aversion level. Measured by the risk parameter in the exponential utility function, his risk aversion level was 3.

Wilson needed to choose one architectural firm to do the construction. Therefore, he had to evaluation each construction proposal by comparing both benefits and costs.

For the one given by Nice Way, Wilson needed to determine the value of the proposed car park with the expansion option embedded in it. He also had to determine the value of the expansion option.

4.3.2 Case Analysis

In this case, the architectural firm, Nice Way, gave Sun Parking a construction proposal which allowed allow Sun Parking to choose to expand the four-floor garage to a six-floor one in 2001. The expansion opportunity came from a flexible architectural structure figured out by architects at Nice Way. Thus, based on Nice Way's construction proposal, the value of the new car park comprised of its intrinsic value, the net present value generate from the car park's operation, and the value of the expansion option embedded in the construction project.

To use the indifference pricing approach to value the expansion option, we should first determine its expected NPV.

According to the case description, the marketing department at the company had estimated that the demand would increase either by an average of 2% per year with a probability of 30% or an average of 6% per year with a probability of 70%. Thus, on average, the demand for car parking spaces would increase by 4.8% per year. In addition, we also learn that the proposed car park had 200 spaces for 200 cars per floor. Each space used was able to earn $10,000 per year with an annual average operating expense of $2,000. The pre-cast construction fee for the new car park was $16,000 per space and would increase by 10% for each floor above the ground one. The lifetime of the car par was 20 years. The average operation fee was $2,000 per space and it would cost the company $3.6 million per year to lease the land. Thus, based on the above information, we could project cash flows generated by the car park. Table 4 shows the projected cash flows.

Table 4 Projected cash flows by the new car park

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Demand		550	576	604	633	663	695	729	764	800
Capacity		800	800	800	800	800	800	800	800	800
Revenue		5,500,000	5,764,000	6,040,672	6,330,624	6,634,494	6,952,950	7,286,692	7,636,453	8,003,002
Cost:										
Lease Fee	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000
Construction Expense	3,000,000									
Operating Expense		1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000
Total cost	6,600,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000
Cash Flow	(6,600,000)	300,000	564,000	840,672	1,130,624	1,434,494	1,752,950	2,086,692	2,436,453	2,803,002

Table 4 (continued) Projected cash flows by the new car park

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Demand	839	879	921	965	1012	1060	1111	1165	1220	1279	1340
Capacity	800	800	800	800	800	800	800	800	800	800	800
Revenue	8,000,000	8,000,000	8,000,000	8,000,000	8,000,000	8,000,000	8,000,000	8,000,000	8,000,000	8,000,000	8,000,000
Cost:											
Lease Fee	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000	3,600,000
Construction Expense											
Operating Expense	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000	1,600,000
Total cost	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000	5,200,000
Cash Flow	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000

The NPV of the new car park was:

$$NPV = \sum_{t=0}^{20} \frac{\text{Cash flows}_t}{(1+\text{WACC})^t}.$$

. In this case, Sun Parking had a weighted average cost of capital (WACC) of 12%. As we know, Sun Parking had ten years' history and was specialized in the gar parking industry. Particularly, its car parks were located at central urban areas. Therefore, we would like to use the firm's WACC to discount the cash flows generated by the new car park because, according to the case description, the firm's WACC reflected Sun Parking shareholders' required rate of return toward the car parking business, especially its specialized business in the car parking industry which focused on the central urban areas.

That is, the value of the car park in 1999 was $6,070,539. In 2000, the value of the car park would either decease by 5% to $5,767,012, or increase by 20% to$7,284,647.

As stated in the case description, the proposed car park had 200 spaces for 200 cars per floor. That is, its car parking spaces would increase by 50% if the four-floor car park was expanded to a six-floor one in 2001. However, the cash flows generated from the car park's operation was not linearly related to the number of car parking spaces. The increase in the cash flows should be smaller than that in the car parking spaces. Thus, it is reasonable to assume that, in 2000, the value of the new car park would increase by 15% if Sun Parking chose to expand the car park because the CEO of Sun Parking was a conservative investor indicated by the risk parameter value of 3. He would not exaggerate the potential increase in revenues.

That is, if the demand would increase by an average rate of 2% per year, in 2000, the value of the car park would be $5,767,012. The NPV of the new car park in 2000 would increase by 15%, to $6,632,064 if Sun Parking chose to expand the car park. If the demand would increase by an average rate of 6% per year, in 2000, the value of the car park would be $7,284,647. The NPV of the new car park in 2000 would increase by 15%, to $8,377,344 if Sun Parking chose to expand the car park.

According to the case description, it would cost Sun Parking 0.9 million dollars to do the expansion. In the former scenario, the net payoff of the expansion would be -$34,948 which meant that the expansion would destroy the value of the park. Thus, Sun Parking would choose not to expand the car park. In the latter scenario, the net payoff of the expansion would be $192,697. Thus, the value of the car park was increased due to the expansion. In this scenario, Sun Parking would choose to expand the car park.

In this case, the CEO of Sun Parking was a conservative investor and did not invest in any securities. He preferred to deposit all of cash in a bank to earn interest. Thus, this is the scenario discussed in section 3.5. That is, an investor chooses to deposit all his wealth into a bank. Thus, we should use the valuation model (4.2) to do the real option valuation.

The model used to value the new car park with an expansion option embedded in it turns out to be:

$$v_b = -\frac{1}{\gamma R}\ln[p\exp(-\gamma g_1) + (1-p)\exp(-\gamma g_2)].$$

In this case, the demand for car parking would increase either by an average rate of 2% per year with a probability of 10% or an average rate of 6% per year with a probability of 90%. Thus, p=0.9.

As discussed above, if the demand for car parking would increase either by an average rate of 6% per year, Sun Parking would choose to expand the car park and the value of the car park was $7,477,344.

$$g_1 = \$7,477,344.$$

As discussed above, if the demand for car parking would increase either by an average rate of 2% per year, Sun Parking would choose not to expand the car park and the value of the car park was $5,767,012.

$$g_2 = \$5,767,012.$$

Thus, the value of the new car park with an expansion option embedded in it was:

$$v_b = \$6,090,901.$$

That is, the value of the new car park with an expansion option embedded in it was $6,090,901. As its NPV without flexibility was $6,070,539. The value of the real option was $20,362.

4.3.3 The risk neutral approach

As discussed in section 4.2.3, the car park case could also been analyzed with the risk neutral approach.

According to the case description, if the demand would increase by an average rate of 2% per year, Sun Parking would choose not to expand the car park and the value of the car park would be \$5,767,012. In the other scenario, if the demand would increase by an average rate of 6% per year, Sun Parking would choose to expand the car park, and the value of the car park turned out to be \$7,477,344.

In this case, the hedging ratio is:

$$b = \frac{(u-d)Y_0}{N_u - N_d} = 0.89.$$

That is, by taking a short position of 0.89 shares of the hedging portfolio, the investment in the car park would be riskless. Whatever the up state or the down state the value of the new car park moved to in 1999, Sun Parking would obtain a constant rate of return.

The risk neutral probability was:

$$\frac{1+r-d}{u-d} = 0.48.$$

Thus, in 1999, the value of the car park with an expansion option embedded in the car park project was

$$N_0 = \frac{1}{1+r}[\frac{1+r-d}{u-d}N_u + \frac{u-1-r}{u-d}N_d].$$

$$=\$6,156,983.$$

The value of the expansion option determined by the risk neutral approach was $86,444.

4.3.4 Discussion

In section 4.3.2 and 4.3.3, we have used the revised indifference valuation model and the risk neutral approach to value the expansion option embedded in the car park project owned by Sun Parking. We would like to discuss the case analysis from the following four aspects.

First of all, in section 4.3.2, the revised indifference valuation model has been used to do the valuation because according to the case description, the only tradable asset for Sun Parking was cash. There are primarily three differences between the original valuation model (3.8) and (3.9), and the revised model (4.1) and (4.2).

The first one is that the two models are used in different scenarios. The original model (3.8) and (3.9) is used when an investor invests in a portfolio of tradable assets and cash. The Collinsville plant case is an example of this scenario. The revised model (4.1) and (4.2) is used when an investor deposits all of his wealth into bank in order to earn interest. In section 4.3, we have analyzed the car park case which belongs to this category.

The second difference between the original valuation model and the revised model lies in the utility level an investor has. When using the model (3.8) and (3.9) to do the real option valuation, an investor will be able to maximize his expected utility by determining the optimal amount of tradable asset included in the investment portfolio. The formula (2.5) is the maximum expected utility an investor could obtain at time T=1:

$$V(X) = -\exp(-\gamma X R'')\frac{p^{\pi}(1-p)^{1-\pi}}{\pi^{\pi}(1-\pi)^{1-\pi}}.$$

Unlike model (3.8) and (3.9), the model (4.1) and (4.2) allows the investor to obtain a constant expected utility level which is shown below:

$$V(X) = -\exp(-\gamma X R^{n}).$$

The third difference lies in the risk parameter in the exponential utility function. As we discussed in section 4.2.4, the risk parameter in the exponential utility function is not included in the original model (3.8) and (3.9). Thus, even if the risk aversion level of shareholders at the American Chemical Corporation would influence their decision making, it did not influence the real option value determined by the indifference valuation approach. Unlike the original model, the revised model (4.1) and (4.2) includes the risk parameter in it. Therefore, it could directly influence Sun Park's CEO Wilson's opinion on the expansion option provided by the architectural firm Nice Way. In addition, the risk parameter also influences the model structures. Take the indifference asking price for example. Model (3.8) has a risk-neutral structure:

$$v_a(G) = \frac{1}{R}[\pi(g_1) + (1-\pi)(g_2)].$$

While model (4.1) has a different structure:

$$v_a(G) = \frac{1}{\gamma R}[p\exp(\gamma g_1) + (1-p)\exp(\gamma g_2)].$$

Secondly, in this case, the investor has been assumed to be a conservative investor with the risk parameter of 3. We would like to do a rough sensitivity analysis to see how the changes of the risk parameters in the exponential utility function would influence the real option valuation.

In section 4.3.2, we have figured out that given the risk parameter of 3, the value of the expansion option embedded in the car park project would be $20,362.

Suppose that the CEO of Sun Parking changed his risk aversion level measured by the risk parameter in the exponential utility function from 0.2 to 5.4, the value of the expansion option embedded in the car park is shown in Table 5.

Table 5 Sensitivity analysis of the car par case

	0.2	0.6	1	1.4	1.8	2.2	2.6
Option value	730,831	661,085	568,394	456,176	334,774	216,731	110,832

Table 5 (continued) Sensitivity analysis of the car par case

	3	3.4	3.8	4.2	4.6	5	5.4
Option value	20,362	(55,162)	(117,814)	(169,954)	(213,692)	(250,743)	(282,451)

From the above calculation, we can see that when the value of the risk parameter changed from 0.2 to 5.4, the value of the expansion option changed from $730,831 to -$282,451. The changes were resulted from the investor's attitude toward risk.

When the investor is more conservative, he would think more about the negative sides of the expansion option. For example, Sun Parking might be facing default risk if Wilson finally chose to let Nice Way construct the new car park. If Nice Way broke the deal in 2000 and did not to do the expansion, Sun Parking would not obtain the value of flexibility suggested by the construction proposal. In addition, Sun Parking had to pay a higher construction fee in 1999. Therefore, due to this concern, if CEO Wilson were more conservative, he might choose a construction proposal which did not embed any real option in it, instead of the one given by Nice Way. In this way, he would be sure about how much profit Sun Parking would earn by constructing the new car park. Therefore, with an exponential utility preference and a higher risk aversion level, Wilson would view the expansion option as negative if he used the indifference valuation model to value the option.

When the investor was less conservative, he would think more about the positive sides of the expansion option despite of potential higher risk such as potential default risk. A less conservative investor wishes to pursue higher profit. For example, with a risk parameter of 1 in the exponential utility function, Sun Parking's CEO would attach importance to the value of flexibility in Nice Way's construction proposal. He would choose Nice Way to do the construction as long as its construction proposal provided Sun Parking with a potentially higher rate of return. Therefore, if he used the indifference valuation model to value the expansion option, he would attach a higher value to the option when he was less risk aversion.

Therefore, an investor's risk aversion level could significantly influence his attitude toward a real option. That is why we say that it's the primary advantage of the indifference valuation approach to take an investor's subjective risk aversion level into consideration in the real option valuation.

The indifference valuation model highly depends on the assumption of the investor's risk aversion level. Given this approach to value a real option, we may obtain quite different values if we make different assumption on the risk parameter in the utility function. Illustrated by a lottery example, Morgenstern and von Neumann (1953) suggested that, with known ranking of an investor's expected utility based on the lottery outcome, they could develop a utility function. In the car park case, we could obtain an accurate exponential utility function only if we are given Wilson's expected utility based on the pay off of the expansion option. This is a difficult practice in real business. Therefore, the indifferent pricing model (4.1) and (4.2) is significantly influenced by accuracy of the estimated risk parameter.

Thirdly, in section 4.3.4, we have computationally shown what a replicating portfolio is like and how a perfect replicating portfolio could be constructed.

In the car park case, given the risk neutral valuation method to value the expansion option, Sun Parking it had to take a short position of 0.89 shares of the hedging portfolio in order to create a perfect hedging portfolio. This was only theoretically practical because, as discussed in section 4.2.4, the car park project was not divisible in practice. Section 4.3.4 provides a numerical evidence for the limitation.

As suggested by Copeland and Antikarov (2001), the market asset disclaimer assumption was built on the hypothesis that there exists a tradable security which has highly correlated distribution of "rate of return" with the underlying non-tradable asset, (Copeland and Antikarov, 2001, p.95). That is, the tradable security and the non-tradable should have similar risk and return characteristics. Markowitz (1952) used the variance of returns to measure risk and used the mean of returns to measure return.

Therefore, the two comparables were supposed to have proportional means and variances of return. The risk described by Markowitz (1952) is influence both by the systematic risk and the firm-specific risk.

We have presented in both the Collinsville plant and the car park cases that the systematic risk characteristics of the project are reflected in the discount factor, WACC. WACC is influenced by the capital structure, the tax rate and the systematic risk. Thus, it is crucial to determine the appropriate systematic risk character in order to value the project correctly. The Collinsville plant and the car park cases are two different situations. In the Collinsville plant case, the firm's WACC reflected its shareholders' required rate return towards the average systematic risk among all the industries it had operations in. It was inappropriate to use it to value this specific plant. In the car park case, Sun Parking specialized in the car parking industry. The firm's WACC reflects its shareholders' required rate return towards the systematic risk in this particular market. Thus, this WACC has been directly used in the car park valuation in section 4.3.2.

Fourthly, in section 2.4.2, we have reviewed Williams (1991)'s, and Hui and Leung (2002)'s work on how real option valuation has been applied to the investment in real estate. Williams (1991) pointed out that undeveloped real estate has options embedded in it, such as options to determine "the date and density level of the development", and the option to "abandon the development". The car park case provided another example of Williams (1991)'s argument that undeveloped real estate might also have an expansion option embedded in it. Hui and Leung (2002) had a similar finding that an expansion option is also embedded in the Hong Kong Disney land project. They suggested that the value of the project will be improved when the options to switch use, to expand and to defer are taken into consideration, (Hui and Leung, 2002). The real option valuation analysis with both the indifference pricing model and the risk neutral valuation approach has supported Hui and Leung (2002)'s conclusion that the value of a project is increased due to an expansion option. However, the car park case also indicates that this conclusion might depend on which valuation method is used. As discussed above, if the CEO of Sun Parking had a high risk aversion level, the expansion option would destroy the value of the car park project.

4.4 Case Study Three: NaS Battery Project

4.4.1 Case Description[3]

In 2004, Nixon Bus Company received a proposal from a new technology firm, New Sight. The proposal concerned the launch of a NaS energy storage project, the key component of which was a NaS battery. A NaS Battery was made of sodium (Na) and sulfur (S), and had a high energetic density. This kind of battery was usually used to store energy and support the electric grid. It could significantly improve the energetic efficiency and reduce cost.

According to New Sight's proposal, the NaS battery would replace the gas turbines used by Nixon at that time. Its main advantage was to store energy, which would allow Nixon's gas compressor station to continue working at peak hours. Nixon would benefit from the NaS project by saving labor and reducing demand charges. The construction of the NaS energy storage project would take two years and would come into use in 2006. As part of the promotion plan, the new technology firm offered to buy back the NaS battery at $500,000 in 2006 if Nixon chose to stop launching the project.

It would cost Nixon $500,000 in 2004 and $ 300,000 in 2005 to construct the project. The project had a lifetime of 11 years. After launching the project, Nixon would reduce demand charges by $41,000 per year and annually save labor costs by $220,000. The annual cost to charge batteries was $35,000. The annual operation fee of the project was $10,000. Nixon would need to overhaul the NaS battery at a cost of $130,000 every

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[3] M, Kishinevsky, Y., Sliker, G., Monaco, J.D., Zwillenberg, M., Lalier, J., Schainker, R. and Eckroad, S. (2006). Real Options Analysis for Emerging Energy Technologies. Retrieved January 5, 2011 from http://www.highenergyconsulting.com/pdf/EmergingTechnologies2006.pdf

8 years.

Towards the new technology project, shareholders at Nixon had a weighted average cost of capital (WACC) of 11%. Suppose that the risk free interest rate was 5%, and interest was compounded yearly. Nixon had a debt structure of 30% and could borrow at a rate of 10%.

The value of the NaS project was influenced by the natural gas prices, labor costs and operation efficiency. As gas turbines used natural gas, the economic benefit to launch the project would increase if the price of natural gas increased during the project's lifetime. Due to the uncertainty of natural gas prices and the potential reduction of operation costs, it was estimated that the projected cash flows would move randomly with a volatility of 30%. The period interval was 2 years.

Decision makers at Nixon Bus Company are a group of young engineers. They are also the owners of the company. They were seeking high pace of development of the company. Thus, they prefer to deposit the firm's available fund in a bank to earn the interest. It was assumed that the decision makers at the Nixon Bus Company had an exponential utility preference. They had a moderate risk aversion level with an average risk parameter of 1.2 measured in the exponential utility function.

Nixon Bus Company would like to update its technology in order to improve its operation effectiveness. With limited available fund, it had to choose a new technology which offered a high benefit. Decision makers at Nixon Bus Company needed to determine whether to accept the proposal, and invest in the NaS energy storage project. To make the decision, they should first determine the value of the new NaS project.

4.4.2 Case Analysis

In this case, a new technology firm, New Sight, gave Nixon Bus Company a proposal concerning the launch of a NaS energy storage project. There was an abandonment option embedded in the project. Thus, the value of the NaS energy storage project comprised its intrinsic value and the abandonment option value. Unlike the Collinsville plant, discussed in section 4.2, and the car park case, discussed in section 4.3, the decision making point in this case was two years away from time T=0. This case had a similar feature with the car park case that the decision makers deposit the firm's available fund in a bank to earn the interest. Thus, we can use the model (4.2) to do the real option valuation, shown below:

$$v_b(G) = -\frac{1}{\gamma R^n}\ln[p\exp(-\gamma g_1) + (1-p)\exp(-\gamma g_2)].$$

Table 6 shows the projected cash flows generated by the new NaS battery project.

Table 6 Projected cash flows generated by the new Nas battery project (000s)

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Demand charges reduction			41	41	41	41	41	41	41	41	41	41	41
Labor savings			220	220	220	220	220	220	220	220	220	220	220
Cash inflows	0	0	261	261	261	261	261	261	261	261	261	261	261
Construction Cost	500	600											
Overhaul fee										130			
Operation Expenses			10	10	10	10	10	10	10	10	10	10	10
Cost to charge batteries			35	35	35	35	35	35	35	35	35	35	35
Cash outflows	500	300	45	45	45	45	45	45	45	175	45	45	45
Net cash flows	(500)	(300)	216	216	216	216	216	216	216	86	216	216	216

According to the case description, shareholders at Nixon had a weighted average cost of capital (WACC) of 11% towards the new technology project. Therefore, we would use 11% as the discount factor to obtain the NPV. That is, the value of the NaS battery project was $477,534 in 2004.

In section 4.4.1, it was estimated that the projected cash flows would move randomly with a volatility of 30%. The period interval was 2 years. We could use a binomial lattice to illustrate the movement of the value of the new project from 2004 to 2006. The Cox-Ross-Rubinstein model could be used to describe the stochastic process of the cash flows and find the parameters.

That is, in the Cox-Ross-Rubinstein model, $\sigma = 30\%$, $\Delta t = 2$.

$$u = \exp(\sigma\sqrt{\Delta t}) = 1.53.$$

$$d = \frac{1}{u} = 0.65.$$

Thus, in 2006 the value of the NaS project would either increase by a factor of 1.53 to $730,627, or decrease by a factor of 0.65 to $310,397. According to Compeland and and Antikarov (2001), the probability of the value of the NaS project to move to the up state could be determined by:

$$V_{2004}\exp(WACC \cdot \sqrt{\Delta t}) = pV_{2006}^{+} + (1-p)V_{2006}^{-}.$$

Thus, the probability was:

$$p = 0.41.$$

Thus, in 2006 the value of the NaS project would either increase by a factor of 1.53 to $730,627 at a probability of 41%, or decrease by a factor of 0.65 to $310,397 at a probability of 59%.

When the value of project moved to the up state in 2006, Nixon Bus Company would choose not to abandon the project.

$$g_1 = \$730,627.$$

When the value of project moved to the down state in 2006, Nixon Bus Company would choose to abandon the project.

$$g_2 = \$500,000.$$

It has been assumed that the risk free interest rate was 5%, and interest was compounded yearly. Additionally, the decision makers at the Nixon Bus Company had an exponential utility preference. They had a moderate risk aversion level with a risk parameter of 1.2 measured in the exponential utility function.

In this case, n=2. Thus, the value of the NaS project with an abandonment project embedded in it was:

$$v_b(G) = -\frac{1}{\gamma R^n}\ln[p\exp(-\gamma g_1) + (1-p)\exp(-\gamma g_2)].$$

=$532,414.

Thus, the value of the abandonment option was $54,880.

4.4.3 The risk neutral approach

In section 4.4.3, we would like to use the risk neutral approach to do the valuation for the abandonment option. Unlike the Collinsville plant case, discussed in section 4.2, and the car park case, discussed in section 4.3, the decision making point in this case was two years away from time T=0.

Suppose that it is at T=1 when an investor could choose whether to exercise a real option. Interest is compounded yearly. The real neutral approach becomes:

$$(Y_0 - bN_0)(1+r)^n = uY_0 - bN_u = dY_0 - bN_d.$$

where r is the risk free rate, Y_0 is the value of the asset at T=0, N_0 is the value of the option payoff at T=0, u is the up state factor, d is the down state factor, N_u is the value of the option payoff in the up state at T=1, N_d is the value of the option payoff in the down date at T=1, and b is the hedging ratio.

The hedging ratio is:

$$b = \frac{(u-d)Y_0}{N_u - N_d}.$$

Thus, the value of the underlying asset with a real option embedded in it is:

$$N_0 = \frac{1}{(1+r)^n}[\frac{((1+r)^n - d)}{u-d}N_u + \frac{(u-(1+r)^n)}{u-d}N_d.$$

where $\frac{((1+r)^n - d)}{u-d}$ is called the risk neutral probability and N_0 is the risk neutral price.

In this case, the hedging ratio is

$$b = \frac{(u-d)Y_0}{N_u - N_d} = 1.82.$$

The risk neutral probability is:

$$\frac{((1+r)^n - d)}{u-d} = 0.51.$$

Therefore, determined by the risk neutral approach, the value of the NaS project with an abandonment option embedded in it was $560,200. The value of the abandonment option was $82,666.

4.4.4 Discussion

In section 4.4, we used the indifference valuation approach and a risk neutral approach to analyze the NaS battery project. We would like to discuss the case analysis from the following aspects.

Firstly, the NaS battery project case is different from the Collinsville plant case, discussed in section 4.2, and the car park case, discussed in section 4.3, that the decision making point in this case was two years instead of one year away from time T=0. Another difference lies in the option embedded in the project. In this case, there was an abandonment option embedded in the NaS battery project, while the option embedded in the Collinsville plant had a deferral feature and an expansion option was embedded in the car park case.

It has been shown that how exercise time makes different impacts on the indifference valuation model (4.2) and the risk neutral approach.

In indifference valuation model (4.2):

$$v_b(G) = -\frac{1}{\gamma R^n} \ln[p \exp(-\gamma g_1) + (1-p)\exp(-\gamma g_2)].$$

Therefore, in the difference valuation model, exercise time makes an impact on the valuation by influencing the discount factor R^n. When cash is the only tradable asset, the indifference biding price has a negative relationship with the exercise time, given that other variables remain unchanged.

Similarly, when an investor's initial investment portfolio consists of a tradable asset and cash. He could use formula (3.8) and (3.9) to do the real option valuation. Take (3.8) for example. If the time length between T=0 and T=1 is n years and interest is compounded annually, the asking price becomes:

$$v_a(G) = \frac{1}{R^n}[\pi(g_1) + (1-\pi)(g_2)].$$

where π equals $\frac{R^n S_0 - S_1^-}{S_1^+ - S_1^-}$.

The exercise time n influences both the discount factor R^n and the "risk neutral probability" $\frac{R^n S_0 - S_1^-}{S_1^+ - S_1^-}$. The risk neutral probability has a positive relationship with the exercise time n. However, the relationship between the exercise time and the value is ambiguous.

As discussed in section 4.4.3, the value of the underlying asset with a real option embedded in it is:

$$N_0 = \frac{1}{(1+r)^n}[\frac{((1+r)^n - d)}{u-d}N_u + \frac{(u-(1+r)^n)}{u-d}N_d.$$

In the risk neutral approach, in addition to the discount factor $(1+r)^n$, exercise time also influences the risk neutral probability $\frac{((1+r)^n - d)}{u-d}$. The risk neutral probability has a positive relationship with the exercise time n. However, the relationship between the exercise time and the value is ambiguous.

Secondly, the three cases, the Collinsville plant case, discussed in section 4.2, the car park case, discussed in section 4.3, and the NaS Battery, discussed in section 4.4, have a similar characteristic. It is that the investors' risk aversion levels measured by the risk parameter in the exponential utility function were given in case descriptions. In practice, it is usually necessary to determine an investor's risk aversion level. van der Hoek and Elliott (2006) provided a method to infer an investor's risk perception at a particular decision making node, (van der Hoek and Elliott, 2006, p.222). Their method targets the indifference valuation model (4.4). The philosophy is to back out the risk parameter.

Given that:

$$V_0 = -\frac{1}{\gamma R}\ln[p\exp(-\gamma V_1^+) + (1-p)\exp(-\gamma V_1^-)].$$

The above formula was adapted from van der Hoek and Elliott (2006)'s work, (van der Hoek and Elliott, 2006, p. 222). Thus, with known V_0, R, V_1^+, V_1^- and p, we can back out γ from the above formula.

We will use the NaS battery case as an example to illustrate this method. In this case, it was estimated that the projected cash flows would move randomly with a volatility of 30%. Thus, we used the Cox-Ross-Rubinstein model to describe the stochastic process of the cash flows and find the parameters, R, V_{2006}^+, V_{2006}^-, and p. Then, according to van der Hoek and Elliott (2006), the risk parameter could be backed out.

This method might help an investor to infer his risk parameter at each decision making node in practice. However, it has one crucial drawback which is to use objective calculation to determine subjective risk aversion level. This violates Morgenstern and von Neumann (1953)'s argument.

As reviewed in section 2.3.3, Morgenstern and von Neumann (1953)'s utility function belongs to behavioral analysis framework. The risk parameter could be determined by ranking an investor's expected utility on the lottery payoff, (Morgenstern and von Neumann, 1953). Morgenstern and von Neumann (1953)'s approach indicated that the expected utility function should be determined by the subjective perception toward the investment payoff. It depends on it is who that makes the investment decision. On the contrary, van der Hoek and Elliott (2006)'s approach is drawn on objective investment information. It would be no matter who is the decision maker. Thus, it will destroy the value of the indifference valuation model that is founded on the behavioral analysis.

Secondly, in this case, decision makers at Nixon Bus Company have been assumed to have a risk parameter of 1.2 in their exponential utility preference. We would like to do a rough sensitivity analysis to see how the changes of the risk parameters would influence the real option valuation.

In section 4.4.2, we have figured out that given the risk parameter of 1.2, the value of the expansion option embedded in the NaS battery project would be $54,880.

Suppose that decision makers at Nixon Bus Company changed their risk aversion level measured by the risk parameter in the exponential utility function from 0.2 to 3.2, the value of the abandonment option turned out to be:

Table 7 Sensitivity analysis of the NaS battery project case

	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6
Option Value	60,583	59,426	58,277	57,136	56,004	54,880	53,766	52,662

Table 7 (continued) Sensitivity analysis of the NaS battery project case

	1.8	2	2.2	2.4	2.6	2.8	3	3.2
Option Value	51,568	50,485	49,414	48,353	47,304	46,268	45,243	44,232

From the above calculation, we can see that when the value of the risk parameter increase from 0.2 to 3.2, the value of the expansion option decrease from \$60,583 to \$44,232. It supports the finding in section 4.3.4 that when an investor is more conservative, he will attach importance of the negative sides of a real option, such as the default risk. Thus, with a higher risk parameter, the indifference valuation approach will give a relatively lower value. On the contrary, when an investor is less conservative, he will attach importance of the positive sides of a real option, such as a higher rate of return. Thus, with a higher risk parameter, the indifference valuation approach will give a relatively higher value

Finally, in section 2.4.3, we have reviewed literature on how research and development (R&D) projects are influenced by real option valuation. The NaS battery project had a R&D feature because it was to develop a new technology in order to meet internal needs of the firm to improve operational effectiveness. In addition, the research work was done by the external firm, New Sight. Huchzermeier and Loch (2001) suggested that dynamic programming is an appropriate valuation model to value R&D projects because it does not require replication. In addition to the dynamic programming approach, the indifference valuation model is another real option valuation model that does not need to do a replication. We have calculated in section 4.4.3 that the hedging ratio is in the NaS battery project was 1.82. It provides another computational example that a perfect replicating portfolio usually requires that the underlying asset has to be divisible and tradable which violate the reality in the NaS battery project. From this perspective, Nixon Bus Company should use the indifference valuation model instead of the risk neutral to do the valuation. Discussed in section 4.4.2, the value of the NaS battery project was $532,414.

Chapter Five

Summary and Conclusion

Real option valuation methods have been widely studied. This thesis investigates the indifference valuation model which can be used in two difference scenarios. Model (3.8) an (3.9) is applicable when an investor invests his initial wealth in a portfolio of tradable assets and case. Model (4.1) and (4.2) is applicable when an investor deposits his entire fund in a bank to earn interest. It draws upon work within real option theories and specifically the work done by van der Hoek and Elliott (2006) where they combined the exponential utility function and the certainty equivalence approach to value a claim written on a non-tradable asset. Unlike van der Hoek and Elliott (2006)'s work, the model in this thesis supposes that the value of a non-tradable asset moves to two states instead of four states. More importantly, this thesis takes two investment scenarios into consideration, which makes it more applicable. Thus, it targets to analyze cases in which the value of the non-tradable asset with a real option embedded in it move to two states at T=1 under both investment scenarios.

The indifference valuation model has been applied to three cases: the Collinsville plant, the car park, and the NaS battery. The real options in these three cases are also valued by the risk neutral approach. By comparing the numerical results from the two methods, we investigate the advantages and disadvantages of the indifference pricing model.

The indifference valuation model has two primary advantages.

The first is that it takes an investor's risk aversion level into consideration. An investor's risk aversion level usually influences his investment decision making. Thus, the risk parameter in the utility function is supposed to be able to influence the real option value, according to which an investor makes the ultimate decision. By doing a rough sensitivity analysis in section 4.3.4 and 4.4.4, we find that in both the car park, and the NaS battery cases, the value of a real option determined by the indifference valuation model has a negative relationship with value of the risk parameter in the exponential function. That is, a more conservative investor will attach importance to the negative sides of a real option such as potential default risk; while a less conservative investor will value the positive sides of a real option. That is how the risk aversion level influences real option valuation and an investor's decision making.

The second is that model does not require the existence of a perfect replicating portfolio. Henderson and Hobson (2004) suggested that due to "non-tradable assets", "transaction costs" and "portfolio constrains", a perfect replicating portfolio usually does not exist. With computational results obtained from the three case studies, we find that under in the incomplete market, the market disclaimer assumption, (Copeland and Antikarov, 2001, p.84), a perfect replicating portfolio in the risk neutral approach would exist only if the underlying asset is divisible. This makes the market disclaimer assumption and the risk neutral approach theoretically viable; but has crucial limitations in practice.

The indifference valuation model has two disadvantages.

The first disadvantage of the model is that model (3.8) and (3.9) still relies on the financial market. In the scenario when model (3.8) and (3.9) is applicable to, the initial wealth is invested in a tradable asset and cash. It is required that the payoffs of the tradable asset are known. In addition, the tradable asset and the non-tradable asset with a real option embedded it have the same probabilities to move to the up and down states at time T=1. This limits the application of the model because, in practice, such a tradable asset does not always exist even though the assumption behind it is less strict than the assumption behind a twin security.

The second disadvantage of the indifference valuation model is that it is difficult to infer an investor's risk aversion level. van der Hoek and Elliott (2006) proposed a method concerning model (4.2) to infer an investor's risk aver level; but it is to use objective calculation to determine subject risk parameter. It violates the philosophy of the risk parameter. According to Morgenstern and von Neumann (1953), the risk parameter could be determined by ranking an investor's expected utility on the lottery outcome. This shines light on how the risk parameter might be inferred. One limitation of the three case studies is that the risk parameters were all given in case descriptions. Thus, this issue has been left for future research

Three issues have been left for future research.

To begin with, this thesis does not concern the determination of an investor's risk aversion level. Future research which investigates the exponential utility function and the indifference valuation model could focus on how the risk parameter in the exponential utility function could be accurately inferred. It will make the behavioral analysis approach in real option valuation more applicable.

Secondly, the model uses the exponential utility function to measure an investor's utility preference. Other utility functions, such as a quasi-linear utility function, could be used. In this way, when an investor prefers a utility function other the exponential one, this model will again be applicable.

Thirdly, in the three case studies, the real options are simple options. For instance, the real option in the Collinsville plant case had a deferral option feature. The car park case had an expansion option embedded in the car park project and the real option in the NaS battery project was an abandonment option. Future research could use the model to analyze switching options, compound options and rainbow options. In addition, as reviewed in section 2.4, projects in real business are sometimes more complex than the three cases in this thesis. These complex projects usually have several option embedded in them. Thus, future research could use the indifference valuation model to analyze more complex cases in real business.

Endnotes

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