2017-05

Making Meaning of Periodic Functions through Body Movements

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MAKING MEANING OF PERIODIC FUNCTIONS THROUGH BODY MOVEMENTS

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We report high school students' meaning making process of the sinusoidal function when engaged in a dynamic activity based on body movements. Using a distance sensor connected to graphing software, students were asked to reproduce a sinusoidal function. We explain how technology facilitated students' understanding of this function and its parameters. We also report students' difficulties in attempting to generate this graph, and the implications for understanding periodic phenomena, commonly studied and applied in science and engineering.

Keywords: STEM; Sinusoidal Function; High school; Technology

RESEARCH PROBLEM

Trigonometric functions represent one of the most difficult topics in senior high school mathematics courses. These functions are essential for modelling and studying periodic phenomena in physics, biology, and chemistry and have important applications for science, engineering, and technology. Although scholars agreed that the initial stages in learning trigonometric functions present several difficulties (e, g. Blacket & Tall, 1991; Brown, 2006; Buendía-Abalos & Cordero-Osorio, 2005; Kendal & Stacey, 1996; Martínez-Ortega, Mejía-Velasco & Martínez, 2016; Moore, 2009; Weber, 2005), Moore (2010) noticed that mathematics education research on these types of functions are scarce. Within trigonometric functions, *sinusoidal functions* (i.e. variations of sine

2017. In P. Preciado Babb, L. Yeworiew, & S. Sabbaghan (Eds.). *Selected Proceedings of the IDEAS Conference: Leading Educational Change*, pp. 159-168. Calgary, Canada: Werklund School of Education, University of Calgary.

and cosine) deserve special attention: with them, we can easily determine the other trigonometric functions. In this paper, we analyze and report students' process of meaning making when introduced to this type of function through a dynamic activity based on body movements.

The strategy consisted of generating a graph using a distance sensor connected to graphing software. While this approach has been commonly reported in the literature (e.g. Bazzini, 2002), we have found no reference for the particular case of trigonometric functions. We address the following research questions in this paper: (1) What meanings do the students make of the sinusoidal function when they generate and study graphics of uniform bodily movements? (2) In what manner does the use of digital tools influence the meanings of the sinusoidal functions?

Our goal is to report and analyze the arguments used by students to justify, explain, and verify conjectures when trying to generate a sinusoidal function using the distance sensor and their body movements.

THEORETICAL PERSPECTIVE

Contrary to the Platonist perspective that assumes mathematical objects are independent from humans, we followed an embodied cognition perspective (e.g. Bazzini, 2002), which places an emphasis on the bodily experiences when learning mathematics: The embodied experiences underpin the grounding metaphors that shape the individual's development of mathematical concepts. Moreno Armella (2014) also argued in favour of this perspective, claiming that it is more desirable for didactics to take an approach that considers the existence of mathematical objects closer to human activity because when students have to access mathematical objects, they do it only with their bodies and the symbolic models.

When approaching functions in a mathematics course, very often the teacher introduces a phenomenon that can be studied through a particular function. We reverse this process for pedagogical purposes: Students were asked to produce body movements that would result in a particular type of graph. In this process, the images generated on the screen ”interplay with body experiences and reasoning schemas and are translated from bodily experience to mathematical concepts” (Bazzani, 2002, p. 268).

RESEARCH DESIGN

The activities, conducted in 3 sessions of 2 hours each, were led by the first author with a group of ten students in their first year of senior high school (equivalent to Grade 10 in Canada), who had not been introduced to sinusoidal functions before. For each session, an activity guide was designed, which included students' written responses and sketching of graphs. The activities were developed through collaborative learning, scientific debate and self-reflection, as described by Hitt and González-Martín (2014).

The purpose of the first session was for students to identify and link the way movement can be represented in a Cartesian graph. With the support of a movement sensor, the graph was produced in real time on the computer screen. Students were requested to walk toward the sensor and away from it. It was expected that participants would identify characteristics linked to the movement in the generated graphs. Then, students were asked to draft, with paper and pencil, the graph corresponding to the action of going from home to school. Later, students drafted the same movement, but this time for the whole week. In this last part of the activity, students were expected to start identifying characteristics of the periodic movement.

In the second session students were shown a sinusoidal-type graph and were required to make the necessary moves in front of the distance sensor to reproduce the graph. The goal was for students to identify that the graph corresponds to a periodic movement.

In the third session students were introduced to the parameters in the formal expression of the sinusoidal function $f(t) = A\sin(Bt + H) + V$. Students were expected to describe the effects of varying the parameters in the graph of the function.

The data for this qualitative study consisted of the products generated by participants—i.e. written responses and sketches of graphs—and the video recording of each session. Each video was split into small segments (one to three minutes). Selected segments were transcribed for further analysis. Only the first author had access to the raw data. Selected excerpts and graphs produced by students are included in this paper to showcase students' processes of meaning making, as well as some of their struggles to generate the sinusoidal function. Our choice for the examples was informed by feedback from a presentation to a broader audience (Martínez-Ortega, Mejía-Velasco & Martínez, 2016) and multiple discussions with the second author.

Findings

In the first session, students generated graphs corresponding to walking toward and away from the sensor: The independent variable corresponds to time and the dependent variable to the distance. These graphs corresponded to lines, as shown in Figures 1 and 2.

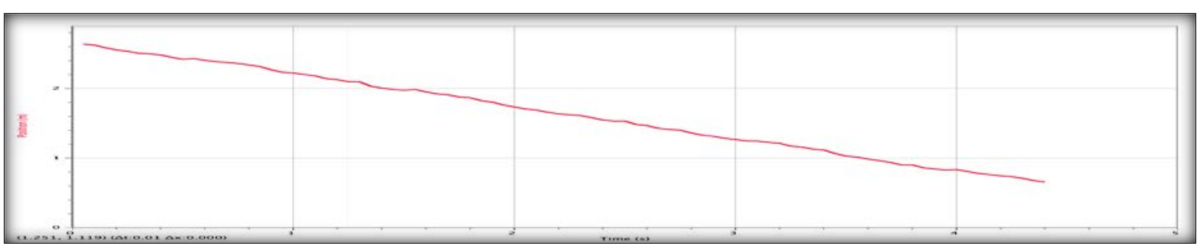

Figure 1: Graph generated by Alma when walking toward the sensor.

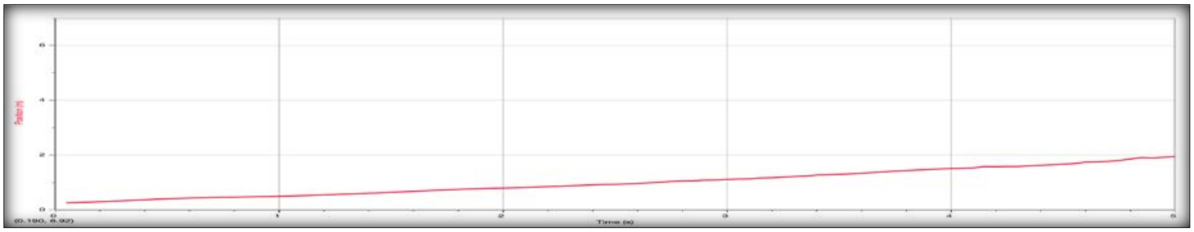

Figure 2: Graph generated by Kevin when walking away from the sensor.

Students did not immediately identify that the graphs represented distance as a function of time. It was necessary that students repeated the action of walking in front of the sensor to realize that when they approached the sensor, the graph was a line with a negative slope, whereas, when they walked away from it, the graph was a line with a positive slope. Then students were asked to sketch, using time and distance as variables, the graph corresponding to the action of going from their home ('casa' in Spanish) to school ('escuela' in Spanish) and then back to home after spending some time in classes at school. An example of this sketch was Alma's graph, which explicitly related the sketch with the previous activity. In this example, she set a new reference point, the school, as shown in Figure 3.

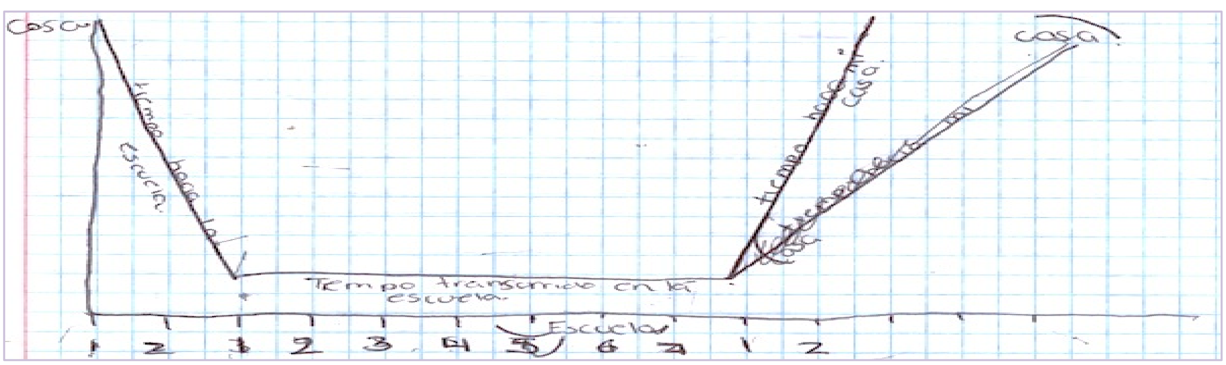

Figure 3: Graph generated by Alma for a whole trip from home to school to home.

According to Alma, she tried to make a symmetric graphic, she cancelled the first segment that represented back to home and made a similar segment of going to school (Figure 3).

In the second session Alma related the graph generated through the sensor with the action of going back and forth from home to school during one week. As she mentioned, "it is the same action" (Figure 4). In this activity, students started to identify characteristics of the periodic movement, which they referred to as "repetitive." For instance, Alma established that she takes two hours commuting from home to school and two hours returning home.

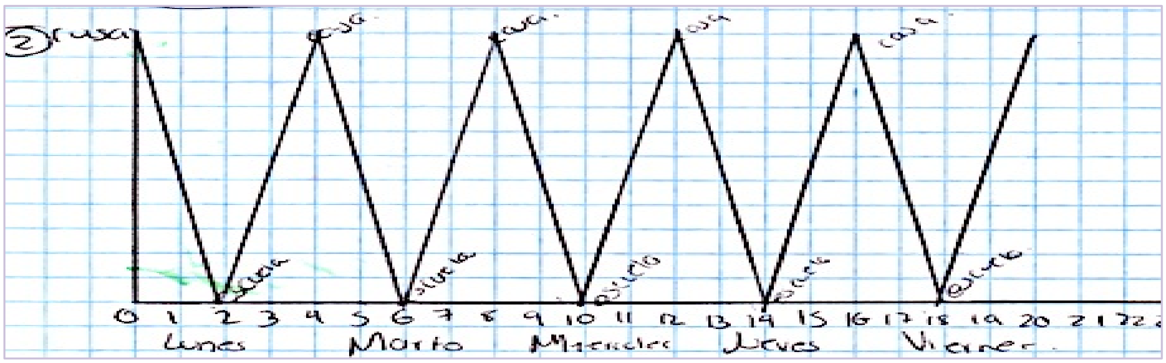

Figure 4: Graph generated from Alma after five trips.

Although there was evidence that students understood the relationship between time and distance in the graphs they generated, there were struggles when they attempted to generate the sinusoidal function in the second session. Students walked in front of the distance sensor in different manners; for instance, one of them walked rising their knees, or as some of them described, "walking as a drunk man." They also tried to relate the type of the graph with the speed they were walking.

After several attempts, Carlos thought about walking "making the same movement, that is going back and forth" (Carlos). This student, standing in front of the sensor, moved two steps toward the sensor and two steps backwards.

When asked by the researcher why did he think that the movement should be like that, Carlos answered: "Because if one gets closer (toward the sensor), it was going downwards (\); as one gets farther (from the sensor), it was going upwards (/)." After making this movement, Carlos concluded that this type of graph is generated by periodic movements. He asked whether the required graph could be generated moving the hand back and forth in front of the distance sensor and repeating this several times. After being invited to try, Carlos verified his conjecture in front of the sensor.

In the third session students were introduced to a formal expression for the sinusoidal function, and some students made explicit connections between some of the general parameters and associated movements. After introducing the sinusoidal function, the researcher discussed with the students some characteristics of this type of functions; the main feature stressed was its repetition (periodicity). Students identified the oscillations with the steps (forth and back); the period (P), time involved in a step; the amplitude (A), size of the step, vertical displacement (V), distance from the person to the sensor. Samantha commented that "the farther from the sensor, the higher the graph goes." The teacher explained the horizontal displacement (H) and introduced a mathematical expression for this type of function: $f(t) = A\sin(Bt + H) + V$

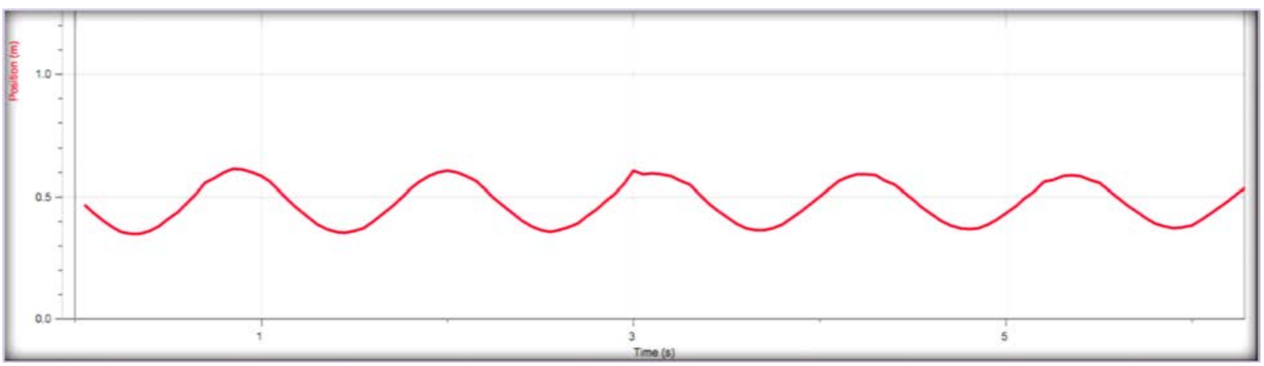

Figure 5: $f(t) = 0.12sin(5.5t - 0.46) + 0.47$

The values for the parameters in the previous graph (Figure 5) were determined by two students (Samantha and Carlos); except for the horizontal displacement, Mariano calculated the values for the rest of the parameters; the other students only calculated the amplitude and vertical displacement.

CONCLUSION

In this paper we reported students' meaning, making processes when attempting to generate a sinusoidal function through their body movements. Very often, sinusoidal functions are used to model and study phenomena and teachers assume students interpret properly the connection between the graph of this function and the phenomena. However, we observed that even when students already understood the parameters and the effects of moving back and forth from the distance sensor, the connection to the periodic function represented difficulties. It was necessary to realise that the movement that was described by a sinusoidal function was periodic.

We believe that both the action of walking uniformly and the way the graph was generated supported students to make sense of the periodicity of sinusoidal function. The use of the sensor facilitated the generation, reproduction, and most importantly the discussion and analysis of the harmonic phenomena. The way in which the graphs were generated allowed students to identify and assign meaning to the signs and features that represent the parameters of sinusoidal functions.

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