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Market Power in Electricity Markets

by

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Abstract

Electricity markets exhibit unique price dynamics not found elsewhere in other commodity markets. Characteristics such as limited storability, highly inelastic demand, and physical laws requiring continuous production to match consumption cause erratic price dynamics and short periods of extremely high prices known as *spikes*. We create a diffusion model under the assumption that a participating firm controls sufficient production capacity can significantly increase the spot price by bidding according to a strategy defined by a *decision curve*. We then extend Barlow's diffusion model to incorporate the decision curve and show results.

Acknowledgements

I would like to thank my supervisor, Dr. Antony Ware for his guidance and limitless patience when my mathematical ability was lacking; Drs. Yousry Elsabrouty and Shawn McCann who, without their help with many mathematical concepts, I may very well have chosen a different subject as my major; and finally Drs. Kristine Bauer and Wenyuan Liao for years of mentorship and support.

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Chapter 1

Introduction

Following the successful deregulation of other markets such as telecommunications and the airline industry, Canada and the United States began deregulating their energy sectors in the 1990s. Prior to deregulation, electricity prices were strictly regulated and the sector operated as a natural monopoly. These monopolies operated under a *cost-of-service* basis, whereby the regulator both approves new investments and sets the price. These prices were set to a level sufficient to cover all operating expenses and guarantee a fair rate of return on the investment of new generation capacity. Under this regulatory framework, a utility must prove that new generation capacity was necessary and, if approved, the additional capacity will be included in the rates customers are required to pay once built. However, the risk of investment falls on the consumers and taxpayers instead of the investors; there is little to no incentive to cancel the production of additional generation capacity once approved.

The Alberta electricity industry was dominated by three vertically integrated utilities: TransAlta, Alberta Power (ATCO) and Edmonton Power (later became EPCOR in 1996). These companies each received approval from the regulator to

construct new generation capacity during a period of economic growth. After receiving approval, but prior to construction, energy prices and the Alberta economy collapsed. Under normal circumstances, this event would cause the cancellation of these construction projects. However, with the return on investment guaranteed by the regulator, construction proceeded as scheduled. The last of these plants were completed in the 1980s and created an expensive energy surplus [33].

Deregulation has historically created market conditions ripe for abuse by firms capable of unilaterally influencing market prices (see Section 1.4). Indeed, the California electricity market is perhaps the most notorious example of mismanaging electricity deregulation. California required parties to offer their electricity through a third-party exchange, the California Power Exchange. The market structure was designed to prevent withholding capacity to influence prices; however, market manipulation persisted despite their efforts. Licensing requirements discouraged new investments and, as electricity consumption increased, this created opportunities for market manipulation by the few remaining firms. By exporting electricity out of California, this export would then be recorded by the California market and prices would rise due to an expected shortage; the same firms would then import the same amount of electricity back into the market at the new inflated price. Further exasperating the issue, these imports congested power lines preventing other imports from entering the market, raising electricity prices dramatically. This event would later be known as the *California energy crisis*, see [31].

Following the aftermath of the California energy crisis, market designers are now particularly interested in limiting forms of market power which weaken or eliminate competitive restraints imposed by rival firms while still incentivizing innovation, see [11]. In theory, competition should drive offers to short-run marginal cost. However, during a market's infancy before deregulation, firms may lack sufficient competition and may use their large offer control to manipulate markets as was the

case in California. Although market power comes in many forms, we henceforth limit its scope to one of particular interest: the ability for a firm to profitably move market prices away by manipulating the available supply in the market, see Section 1.4.

1.1 Market Organization

Several potential market designs are commonly discussed when considering market deregulation (see, e.g., [33]) and [39, pp. 273–275]). We will focus our attention on design used in Alberta, the mandatory power pool (or an *energy-only market*). In an energy-only market, all available electricity generation must be bid into the pool in price-quantity pairs (e.g., 15 MW at $25/MWh). From these bids, a graph of the available supply at a given price is created and its inverse is called the *merit order curve*, *bid stack*, or *power stack* (see Figure 1.2). A *system operator* then dispatches units of electricity in *merit order* (ascending order of price) until the generation matches the demand. To facilitate the trading of electricity, an electricity market typically offers two available markets: the pool and futures markets. The *market pool* (or *power pool*) usually consists of electricity traded to be used on a short term basis. This market is usually comprised of a *day-ahead* market used for the majority of energy demands for the day and an *adjustment* market to ensure sufficient power is dispatched to accommodate unplanned events such as mechanical failure or adjust for less predictable sources of power generation such as wind and solar.

In contrast, the *futures* and *forward* market facilitates trading electricity on longer time horizons through the exchange of *derivative* contracts or financial instruments. Additionally, market participants may form bilateral agreements for the delivery of electricity independent of these markets.

In Alberta, the *Alberta Electric Systems Operator* (AESO) operates a day-ahead market and an ancillary services market consistenting of many services taking the role of an adjustment market. In the day-ahead market, initial price-quantity pairs are bid into the market up to seven days ahead from the time of delivery (in hours) up until the *gate close* at noon of the previous day to dispatch. At the gate close, market participants are free to change their price-quantity pairs or change their production capacity with a valid *acceptable operating reason* (AOR). Two hours until settlement, prices can no longer be changed, but it is still possible to change quantity and capacity with an AOR; see Figure 1.1. In the absence of an AOR, a firm may be forced to bid their potential capacity into the market and settle the shortfall in the markets. In the ancillary markets AESO operates the following services of which it is the sole purchaser: the Operating Reserve, Transmission Must-Run, Black Start, and Load Shed Service for imports.

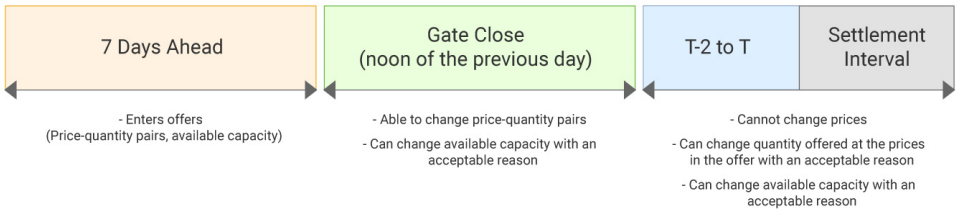

Figure 1.1: Timeline of the bidding process in Alberta from AESO [3]

These ancillary markets are not the focus of the work to follow, but we briefly describe their function for the sake of describing a complete picture of market. For further information, we refer the reader to the AESO website, `www.aeso.ca` for Alberta markets and [20] in general.

In the *Operating Reserve* market, unexpected variation in supply or demand is resolved by either reducing production from a *curtailable load*, production that can be decreased at a moment's notice, or increasing production from *unloaded capacity*, reserve capacity which is synchronized to the grid and can be produced immediately,

(also known as *spinning reserve*).

In the *Transmission Must-Run* service, producers are required to generate a minimum specified output when transmission capacity is insufficient to maintain system reliability.

In the *Black Start* service, for generators incapable of self-starting, they may require electricity from an outside source. Generators capable self-starting are contracted to re-energize the transmission system in the event of an outage.

In the *Load Shed Service for imports*, consumers are contracted to quickly reduce their consumption load in response to sudden losses in imported electricity from *interties* (transmission facilities used to connect two electric systems for import and export) in the event of a *contingency* (an event causing the loss of a system element such as the loss of a transmission line).

We conclude this section by describing an important element of the Alberta market, the *Balancing Pool*. The Balancing Pool was created by the Government of Alberta to manage various financial accounts arising from the transition to a competitive market and maintaining obligations related to Power Purchase Arrangements (PPAs) and participate in regulatory, dispute resolution and other proceedings under the *Electric Utilities Act* [35]. *Power Purchase Arrangements* were created by the Government of Alberta were created to separate ownership of generation assets from the power those assets generated. A purchaser of a PPA would retain the rights to bid the power generated from an asset into the market by compensating for the owner's fixed and variable costs. These PPAs were auctioned off to buyers in 2000 to introduce new market participants and mitigate the market power incumbent utilities would have had in the market.

Not all PPAs were successfully auctioned off in 2000 (in fact, only eight of the twelve available PPAs were sold) and the Balancing Pool further functions as a holder of unsold PPAs to manage them commercially until another market

participant purchases the PPA at a fair price[1]. As such, the Balancing Pool can control a significant amount of offer control as we will later see in Section 3.1. For further information on the history of PPAs, we direct the reader to [5].

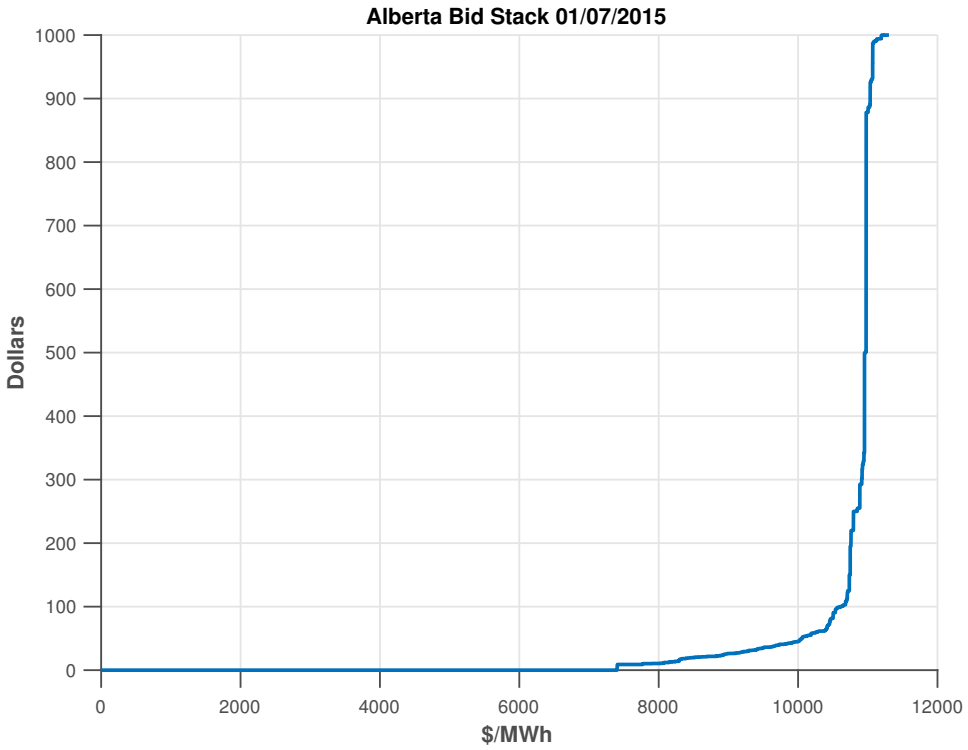

Figure 1.2: A representative bid stack in Alberta on 01/07/2015.

1.2 Market Participants

To better understand the market dynamics, we distinguish between three categories of market participants: producers, consumers and retailers, these participants interact the market in different ways.

[1]The Electric Utilities Act outlines an exception - the Balancing Pool always holds hydro PPAs. The Balancing Pool otherwise aims to divest itself of any PPAs whenever the Balancing Pool believes it will receive fair market value in a competitive sale

Consumers constitutes any end-users of electricity. Household consumers purchase power fromo retailers at fixed rates. As such, these consumers are insensitive to the spot price and will not respond to price changes. Other onsumers may be larger industrial consumers that may participate directly in the power pool, sign bilateral contracts with producers, or purchase from retailers. These consumers use large amounts of electricity and are sensitive to fluctuations in the spot price. As such, these consumers may hedge their positions in the futures market, or participate in ancillary services such as the Load Shed Service for imports.

Retailers purchase electricity directly on the power pool, futures markets or through bilateral contracts with producers to supply electricity to their clients that usually do not directly participate in electricity markets. Most consumers purchase electricity through retailers at a uniform tariff and, as a result, proper forecasting and hedging are of particular importance for retailers to maximize their profit during periods when electricity is high. We will generally regard consumers and retailers as members of the same category of market participants and omit large industrial consumers.

Producers own and operate the system elements that generate electricity. System elements are categorized as *dispatchable* and *nondispatchable* sources. Dispatchable sources can freely dispatch electricity at the request of the system operator at a specified volume or turned off (e.g., hydroelectricity and natural gas). By contrast, nondispatchable sources cannot be directly controlled due to factors outside of the operator's control and as such, cannot be turned on and off to meet demand (e.g., solar and wind energy).

These two types of producers also differ in market strategy: dispatchable sources may participate in other markets to provide on-demand energy such as in the operating reserve market. By contrast, nondispatchable sources are more limited in their ability to participate in these markets and must participate in the balancing

market to cover variations in production settled in the pool market.

The behaviour of these participants informs some of the basic assumptions we will use for our modelling ahead. In general, consumers and the retailers that supply them are price insensitive. As such, authors typically relegate industrial consumers to a footnote and use the assumption that the short-term demand side is nearly perfectly inelastic. On the producer side, dispatchable sources contribute to the bulk of the variation in the power stack. Nondispatchable energy providers typically act as price takers and bid in at the minimum. These nondispatchable sources make up the *base load* supply. As such, we treat nearly all the price variation as a consequence of demand moving sufficiently beyond the current available nondispatchable capacity.

1.3 Deregulation in Alberta

Previously, energy markets were either regulated in the case of natural gas and electricity or controlled by producers in the case of coal and crude oil. Deregulation in tandem with privatization naturally created energy markets. Under a regulated framework, the energy sector was vertically integrated with prices set by the regulator. A company with reliable access to supplies need only provide adequate service and manage the regulatory oversight process to be successful [20, p. 22]. In a deregulated framework, prices are no longer set by the regulator and instead follow the usual economic principle of supply and demand. The reliance on markets to both obtain energy commodity supplies and to find outlets for production requires additional risk management capabilities. This paved the way for considerable activity in derivative markets.

The Alberta market presents some obstacles toward a competitive market: the market is relatively small and demand is heavily concentrated. This leads to both

swings in demand and little room for outside competition. As such Alberta is not fully deregulated and market participants are subject to the *Fair, Efficient, and Open Competition Regulation* (FEOC), see [34]. This regulation sets rules such as requiring all generation capacity to be offered into the power pool (Section 2 (f)), forbidding the offering of electricity from a generating unit for the purpose of creating or increasing congestion (Section 3 (i)) as was the case in the California Energy Crisis, or the preferential sharing of market records not made available to the public (Section 3.1). Notably, This regulation has been put to the test when TransAlta was fined $56MM in a 2015 Alberta Utilities Commission decision (see Section 1.5).

Of particular interest for what is to come, the Market Surveillance Administrator (MSA) has tolerated economic withholding (see Section 1.4). The MSA has repeatedly stated that, in the context of an energy-only market, this is a necessity to give investors sufficient confidence that investments will be profitable and thus encourage new investment to maintain an adequate supply, see [6, 24, 27]. This may appear as a strange justification for economic withholding; however, as [7] notes, in early 2017 the Balancing Pool had offer control of more than 20% of Alberta's installed capacity (and has since risen significantly at the time of this writing). Because the Balancing Pool generally offered into the pool near variable cost, this has had an effect of depressing prices below what it might have otherwise settled at if the Balancing Pool were behaving like a competitive firm. These bidding practices are a concern to other generators as the Balancing Pool suppresses the price below what would otherwise be the competitive equilibrium price. In summary, the MSA must both balance opposing forces such as the Balancing Pool supressing prices below the competitive level which as a consequence, stifles investment, versus market power raising prices beyond competitive levels and hurting the consumers.

The FEOC curtails the effect of economic withholding by limiting participants

to no more than 30% offer control of the total maximum capability of generation units in Alberta. However, we will later see that such a large proportion of the market is not necessary for opportunities to exercise market power.[2]

1.4 Market Power

There is no formal definition of market power; we loosely define it as "the ability of a single firm to profitably move market prices away from the competitive level" where *competitive level* refers to the clearing price had all participants offered their energy into the market at variable cost [26].

Under the above definition, we are interested in the effects of market power through *physical withholding* or *economic withholding*. In the case of economic withholding, the available supply is offered at a sufficiently high price beyond the supplier's marginal cost and opportunity costs to ensure it is not called upon to increase the system marginal price. In the case of physical withholding, the commodity is not offered into the market at all. In Alberta, market participants are required to offer all of their available supply into the market. As such, economic withholding is the main form of market power of interest. Physical withholding generally does not occur except with a valid operating reason such as shutting down a generation facility for maintenance.

To illustrate the potential effect of market power, we present a simple example.

Example. Consider an energy market with five generation facilities. The generation capacity and marginal cost for each facility are presented in Table 1.3.

Suppose every firm is a price taker in the market and bids their generation capacity at marginal cost (we assume no fixed costs). We form the bid stack and

[2] As [6] notes, economic withholding may be disappearing in the future as Alberta transitions to include a capacity market. However, a presentation by AESO [2] still expects opportunities for market power 5% of the time.

Generation Facility	Capacity (MWh)	Marginal Cost ($/MWh)
A	10	0
B	15	25
C	15	50
D	10	75
E	10	95

Table 1.3: Example generation facilities and with varying generation capacity and marginal costs.

for a given level of demand, set the *system marginal price* to be the intersection between the bid stack and the (assumed inelastic and thus horizontal) demand curve. Each unit of electricity will be sold at this price in this settlement period. Set the demand to 17 MWh and a corresponding system marginal price of $25. In this case, generation facility A is called on to deliver 10 MWh and receives $250 and generation facility B is called on to deliver 7 MWh and receives $175, see Figure 1.4.

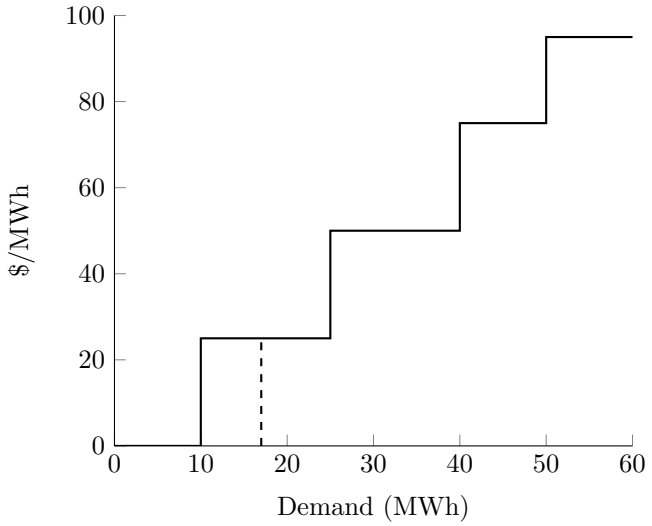

Figure 1.4: The bid stack with a demand of 17 MWh and system marginal price of $25.

Now instead consider the same situation except a firm owns both generation facilities A and B. In this case, suppose the firm exercises market power by bidding facility B's generation capacity at \$100/MWh. This effectively shifts the supply curve to the left and raises the system marginal price from \$25/MWh to \$50/MWh, see Figure 1.5.

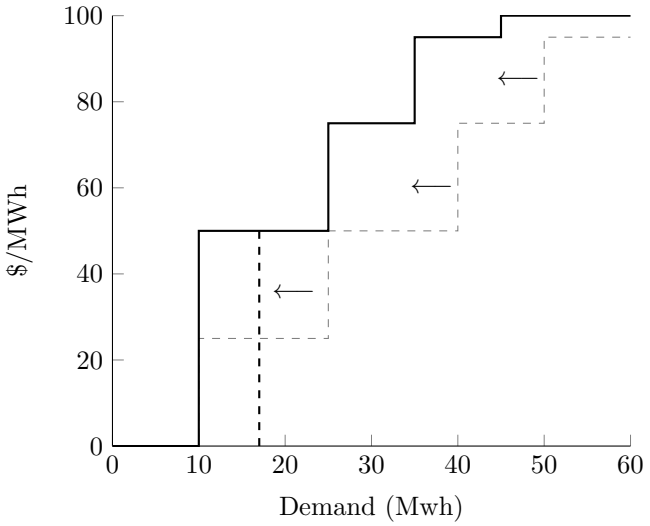

Figure 1.5: The effect on the power stack when generation facility B's capacity is bid in at \$100/MWh instead of at the marginal cost (\$25/MWh).

In the absence of market power, the firm previously made $10 \times 25 = \$250$ in net profit. Economically withholding generation capacity at facility B increases the net profit to $10 \times 50 = \$500$. If the agent could precisely forecast the market demand to be 17 MWh, they could instead bid 9 MWh of facility B's capacity at the usual marginal cost and economically withheld the remaining 6 MWh. This would be just enough to set the system marginal price to \$50 and our agent would produce 16 MWh and make $10 \times 50 + 9 \times 25 = \$750$ in net profit.

In both cases, the agent was able to double and triple their net profits by exercising market power. Even just following a simple strategy, withholding everything

from facility B, the agent was able to double their net profit.

Example. In our previous example, we illustrate the ability of a firm to raise its profits by modifying its bidding strategy based on the level of demand. We now extend the above to show that it is possible to raise the expected payoff of the firm by exercising market power.

We return to the same scenario described by Table 1.3 and Figure 1.4. Suppose at time $t = 1$ all the firms bid their generation capacity at marginal cost and 17 MWh of demand. The firm controlling generation facility A and B believes the demand at time $t = 2$ will rise to either 22 MWh with probability 0.8 or 37 MWh with probability 0.2. From Figure 1.6 below, we see that this change in demand will either leave the system marginal price unchanged or raise it to \$50/MWh.

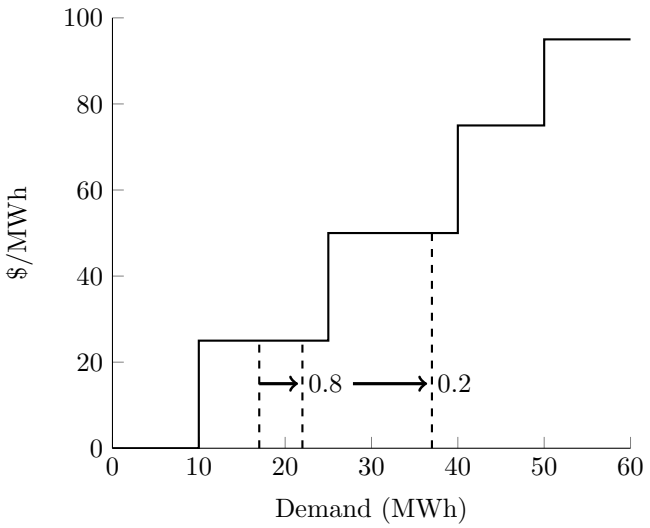

Figure 1.6: The bid stack with a demand of 17 MWh moving to either 22 MWh with probability 0.2 or 37 MWh with probability 0.8.

If the firm does not alter its bidding strategy, their expected net revenue is thus:

$$\mathbb{E}\left[\text{Net Revenue}\right] = (10 \times 25)0.8 + (10 \times 50 + 15 \times 25)0.2 = \$375.$$

However, suppose the firm notices that if the demand at time $t = 2$ was 37 MWh, there would only be 14 MWh separating the system marginal price from moving up to \$95/MWh. The firm thus decides to change their bidding strategy and bids all the generation capacity at facility B in at \$100/MWh as in Figure 1.5. Their expected net revenue becomes

$$\mathbb{E}[\text{Net Revenue}] = (10 \times 50)0.8 + (10 \times 95)0.2 = \$590,$$

an approximately 53% increase in expected net revenue through exercising market power. Although this may seem extreme, the MSA [27, Fig. 2.12] provides an example of economic withholding contributing to raising prices to over \$400/MWh.

We conclude our example with some observations.

1. Economically withholding 15 MWh of generation capacity increased profits regardless of the demand at time $t = 2$. In most situations, exercising market power comes at some risk to the firm. We choose to present this example to illustrate the influence a small amount of economic withholding can have across all outcomes.

2. In the above example, nothing prevents the firms from colluding. If the demand were 17 MWh as in the first example, our firm would receive \$425 or \$250 in net revenue. If every firm mutually decided to bid in near a market cap (say \$100/MWh). Revenue could then be magnified and shared amongst the firms. In reality, market forces such as new firms entering the market, and regulations punishing cooperative bidding prevent these *argumentum ad absurdum* outcomes. For more information on market participant behaviour, see [11, 24].

3. In reality, a firm will not have access to the bid stack. That is, the bids of all competing firms are generally unpublished until some time has passed. At the time of this writing, the AESO's most recently published bid stack is six months old.

1.5 Electricity Characteristics

Electricity markets are subject to many unique characteristics that are not present in other markets. In this section we discuss some of these characteristics that affect the price dynamics.

Electricity production is constrained by limited production throughput and economically inefficient storage. With few exceptions such as hydroelectricity, a generating asset cannot store its production input (e.g., oil or natural gas) or store its output to respond to sudden increases in demand. As a consequence, firms are generally incapable of responding to positive demand shocks and prices can rise significantly in the presence of market power or an unplanned production shortfall. This effect is further magnified due to a lack of commodity substitutes and the general price inelasticity or consumers discussed in Section 1.2. The inability to store electricity further leads to a breakdown of the usual relationship between spot and futures prices in other commodities. Eydeland and Geman [14] note that the usual no-arbitrage argument used to establish a cash-and-carry relation does not hold in the case of power because it is generally not possible to store electricity until the expiration of a futures contract.

Limits on transmission lines further limit the ability to respond to changes in supply or demand as well as create regions where a firm may act as a monopoly. Consider two firms serving two regions connected by transmission lines as in Figure 1.7. Suppose the transmission lines have a combined limit of 30 MWh, firm

A supplies the red (square) cluster, and the firm B supplies the blue (circle cluster). Further, suppose that firm B's generation asset has higher costs and supplies electricity to an area with large demand. Usually, firm A's lower costs would help maintain a competitive market. However, whenever demand exceeds 30 MWh, firm B has a monopoly on the remaining demand in their cluster. As Borenstein and

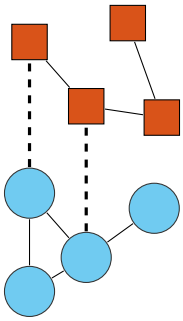

Figure 1.7: An electric grid supplies by two firms, one firm is connected to the red (square) cluster while the other is connected to the blue (circle) cluster. Two transmission lines (dashed) connect the clusters.

Bushnell [9] explains, knowing that potential imports into a region are limited by capacity constraints on transmission lines may reduce the output level a market participant would choose if there were no limits on transmission capacity. As discussed in the introduction, this constraint is precisely one of the causes of the California energy crisis.

Transmission lines can also be intentionally congested with scheduled exports to prevent other firms from entering a market. According to an MSA proceedings [25], in November 2010, the MSA observed unusually high levels of scheduled exports on the BC intertie. At the same time, a market participant raised concern that it was being blocked by TransAlta from importing capacity to Alberta on the BC intertie as a result of these exports. In the agreed statement of facts, TransAlta admitted that this would likely restrict or prevent other market participants from importing

capacity and did not dispute that it expected to derive an economic benefit.

Over short time horizons, the price of electricity is almost perfectly inelastic; there are usually no substitutes for electricity and most consumers purchase from a retailer at fixed prices. Price sensitivity is usually restricted to major industrial consumers that can react to periods of high prices by changing their consumption patterns or by participate in the Load Shed Service for imports service. This demand elasticity appears in many models and simplifies the relationship between supply and demand by modelling demand as a vertical line.

Depending on the geographic region, prices show strong daily and seasonal effects. Daily prices are strpmg;y influenced by consumers waking up (resp. going to sleep) and increasing (resp. decreasing) consumption. Daily high and low prices are typically separated into two time series: we follow the convention that *off-peak* hours for electricity delivered in the hours between 22:00–06:00 and *peak* hours for electricity delivered from 06:00–22:00. Similarly, prices are strongly influenced by weather effects and therefore follow a seasonal pattern with increased consumption during the summer (resp. winter) months for air conditioning (resp. heating). Price cyclicility can be further correlated to the day of week, holidays, and even major sporting events. According to the MSA [28], TransAlta took particular advantage of seasonal effects in 2010 and 2011 by intentionally scheduling maintenance during peak periods of demand during cold winter nights. As a result of these allegations (among others, see Section 3.1) and the substantial financial benefit, TransAlta was fined an administrative penalty of $51.9MM and $4.3MM in MSA costs, see [29].

A final central feature is the presence of *spikes* characterized by a large upward change in the spot price followed by a steep downward drop. As seen in Figure 1.8, price spikes are a common fixture in the Alberta market. There is a limit to these price spikes as Alberta has a price cap of $1000/MWh. This is in contrast to the California Energy Crisis where the price was uncapped on the spot market. Utilities

were required by law to provide electricity to the consumers, purchasing energy at uncapped spot prices, but consumers were shielded by government price caps; this asymmetry in price caps led to the bankruptcy of Pacific Gas and Electric Company.

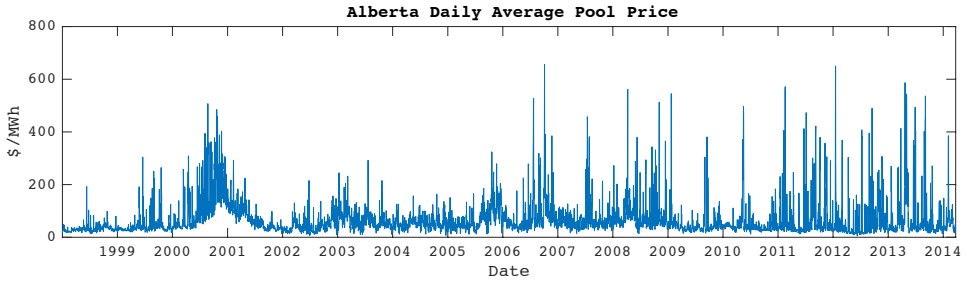

Figure 1.8: Daily average pool price in Alberta from 1998 to 2014.

Geman [17] notes that price spikes often occur during periods characterized by disruption in transmission, generation outages or weather. The short term demand inelasticity is typically characterized by society's need for consistent continuous power to properly function. This fact, in conjunction with the additional storage and production capacity limitations outlined in this section above, means sudden changes in energy production or energy consumption require an immediate response and cannot be smoothened out by means seen in other markets to prevent price spikes.

Deng [13] explains that price jumps and spikes are most prominent in commodity markets without significant storage capacity and demand exhibits low elasticity of which, electricity markets are considered the perfect example. As seen in Figure 1.2, the bid stack features many kinks at various capacity levels and the steep upward slopes at these transitions. Price spikes are a feature of the bid stack and primarily occur at kinks.

Chapter 2

Pricing Models

2.1 A Review of Pricing Models

In this section, we review a brief history of pricing models used in electricity markets. Electricity price modelling has been broadly studied using a variety of different methods. We restrict our discussion to SDE based pricing models and direct the reader to [38] for a review of other approaches.

Electricity models differ from most other commodities due to its unique characteristics; electricity is not economically storable, demand is highly susceptible to weather effects, and Kirchoff's laws necessitate a constant equilibrium between production and consumption. These characteristics cause unique price dynamics such as the ubiquitous price spikes that the classical literature beginning with Black and Scholes, and Merton's respective seminal papers are unsuited to capture.

The first tractable structural model begins with Barlow's model [8]. This class of models depends on basic economic principles using supply and demand curves and their corresponding market clearing price to determine the price dynamics. Barlow begins with these curves and creates a spot price process by applying the inverse

Box–Cox transform,

$$f_\alpha(x) = \begin{cases} (1+\alpha x)^{\frac{1}{\alpha}} & \alpha \neq 0 \\ e^x & \alpha = 0 \end{cases}$$

to an *Ornstein–Uhlenbeck* (OU) stochastic process satisfying,

$$\mathrm{d}X_t = -\lambda(X_t - \mu)\,\mathrm{d}\,t + \sigma\,\mathrm{d}W_t.$$

The OU process simulates a vertical (highly inelastic) demand curve and the inverse Box–Cox transform mimics the power stack (see Figure 1.2). The equilibrium spot price is then $S_t = f_\alpha(X_t)$. Barlow calibrates this model to Alberta and California energy markets and exhibits a spot price process with spikes relying only on the steepness of the the Box–Cox transform and no additional jump process. Our model is based on Barlow's and his model will be discussed in detail later.

Kanamura and Ōhashi [21] followed a similar approach Barlow's model and instead replaces the Box–Cox Transform with another power stack function. They define a hockey-stick shaped supply curve

$$P_t = f(S_t) = \begin{cases} \alpha_1 + \beta_1 D_t & D_t \leq z - s \\ a + bD_t + cD_t^2 & D_t \in (z-s, z+s) \\ \alpha_2 + \beta_2 D_t & D_t \geq z + s \end{cases}$$

where z is the midpoint of a quadratic function connecting two linear parts supply curve, see Figure 2.1. This was then combined with an OU process adjusted for a seasonal component, $X_t = D_t - \overline{D}_t$, where D_t is the demand process and $\overline{D}_t$ is the average of historical demand on day t. This model was calibrated to the PJM market and it was able to generate price spikes with a comparable frequency to historical data.

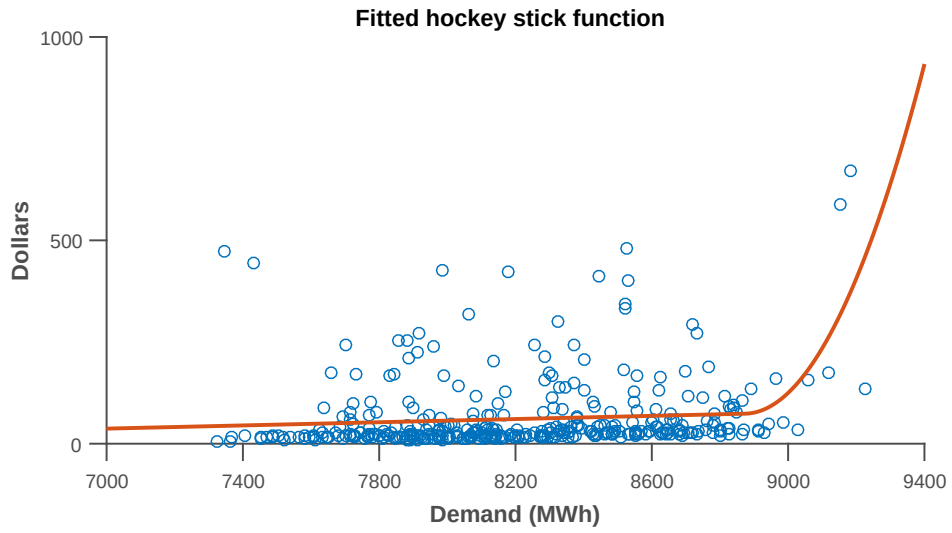

Figure 2.1: Kanamura and Ōhashi's hockey stick model fitted to Alberta's daily average price and demand from 1998 to 2015 detrended for seasonality.

In contrast to the structural models, *reduced-form* models do not seek to capture hourly forecasts, but rather focus on capturing the main features of daily electricity prices with an eye toward derivative pricing and risk management. Reduced-form models tend to abstract away the underlying economic principles in favour of mathematical tractability.

Lucia and Schwartz [23] argued that single-factor models imply that changes in spot prices and in forward and future prices are perfectedly correlated. That is, an increase (resp. decrease) in spot price corresponds to an increase (resp. decrease) in the forward or future price contrary to the data. Following the idea of Schwartz and Smith [36], this model is uses a mean reverting component and a long-term

equilibrium price level writing

$$\begin{cases} \mathrm{d}X_t = -\kappa X_t \,\mathrm{d}t + \sigma_X \,\mathrm{d}W_t \\ \mathrm{d}Y_t = \mu \,\mathrm{d}t + \sigma_Y \,\mathrm{d}\tilde{W}_t \\ \mathrm{d}W_X \,\mathrm{d}W_Y = \rho \,\mathrm{d}t \\ S_t = \exp(f(t) + X_t + Y_t) \end{cases}$$

where $f(t)$ is a deterministic function capturing seasonality. This model has the advantage that the spot and forward prices are lognormal and Black–Scholes-like formulae can be derived for option prices. However this model fails to adequately capture electricity spikes and as Lucia and Schwartz concludes, incorporating jumps is an important extension to the model. Since then many authors have incorporated jumps to their model, see, e.g., [12].

Incorporating sudden price jumps are an obvious approach to modelling spikes. Merton first introduced *jump-diffusion models* in [32] to model the effect of new information on a stock price. This is described by the SDE,

$$dS_t/S_t = (\mu - \lambda\overline{\kappa})dt + \sigma dW_t + \kappa dq_t,$$

where μ is the instantaneous expected return on the stock; σ^2 the instantaneous return conditioned on no new information from the Poisson process; q_t the Poisson process presumed to be independent of W_t; λ the arrival rate of each Poisson event; and k the magnitude of the random jump with mean $\overline{\kappa}$. The magnitude parameter k is modelled so that $\ln(1+k)$ is normally distributed with mean γ and variance δ^2 and $\overline{k} = \mathbb{E}(k) = \exp(\gamma + \delta^2/2) - 1$. This model combines the usual geometric Brownian motion (GBM) with a jump component governed by the dq_t term. Note that when the arrival rate $\lambda = 0$, this reduces to a usual GBM as the jumps will

never occur.

Cartea and Figueroa [12] introduced a jump component to an OU process and used it to study the English and Welsh electricity markets and concluded that the model addressed the peculiarities of those markets. However as [17] argues, the problem remains to then bring electricity prices back down following a jump. Merton's model uses a single parameter for both the jump and the diffusion process; following a spike, the rate of mean reversion should be very high and much lower otherwise, see [30]. It is clear that a simple constant mean reversion parameter is not enough to capture 'normal' price dynamics and control price spikes. Although Merton's model has shown promise in English and Welsh markets, the peculiarities of each market may require additional modifications. Geman and Roncoroni [17] proposed an alternative jump-diffusion model by adding a parameter that fixes the direction of a jumps as the price crosses a threshold to positive results, i.e, upward jumps are used to reach price spikes and downward jumps are used to return to previous price levels.

A critique of jump-diffusion models is that they cannot generate adequate prices in the presence of *spike clustering*, i.e., the underlying Poisson process generates spikes with too much regularity and cannot easily model consecutive spikes often seen in the market data (such as a period of high prices due to a generator outage). In *Markov regime-switching* models, the regime-switching mechanism naturally addresses this criticism. In such a model, we represent the spot price process by n separate states or regimes with different underlying stochastic processes X_t^i, $i = 1, 2, \ldots n$. The switching mechanism between the regimes is governed by an unobservable random variable. As [38] notes, most regime-switching models require only two or three regimes to capture electricity price dynamics; these models typically use a base regime, a "spike" regime, and possibly an intermediary regime.

Kholodnyi [22] proposed a simple relationship between two regimes in his model;

the price simply jumps from X_t to λX_t for some constant λ when there is a regime switch. Deng [13] considered a two-factor, two-state model by defining a spike regime to be the base regime shifted by an exponentially distributed random variable to determine the magnitude of the spike. Huisman and Mahieu [19] proposed a three-state model where the initial jump regime is immediately followed by a reversing regime and the base regime with probability 1. However, by fixing the Markov chain in this manner, the model does not allow for consecutive high prices. This put the value of such a transition matrix into question; it is not obvious what advantage Huisman and Mahieu's model has over jump-diffusion models if they cannot produce appropriate spike clustering.

Anderson and Davidson [4] took a different approach to regime-switching. They modelled both demand and generating capacity separately, including submodels for outages and load. The ratio of the two, denoted by α, determines the probability of a spike via a thresholding function,

$$\epsilon(t) = \frac{1}{2}(\tanh(22(\alpha(t) - 0.85))) + \frac{1}{2}.$$

The value ϵ is then compared at each time step with a uniform $U[0,1]$ random variable to determine whether the price is drawn from a "low" or "high" spot price distribution. From PJM data, they concluded that the model performs well when modelling seasons where planned outages are known and provide a direct link to the implications of a firm's maintenance and repair strategy. However, this model requires historical load data, a list of power plants operating in the market and their corresponding capacity which may not always be available.

We conclude with a brief description of omitted models out of the scope of this brief review. We follow [38] and classify the remaining models as *multi-agent*, *statistical*, and *computational intelligence* based models. Agent simulation models are

based on a game-theoretic approach by defining actors in the system and form a system of equations to be solved to find an equilibrium. As the name implies, these models seek to simulate agents interacting with each other and construct a price process by matching supply and demand. Statistical models approach modelling through direct application of statistical techniques such as regression and other implementations of common econometric models. Finally, computational intelligence models implement some notion of learning or training such as the use of neural networks. We direct the reader to [38] for a thorough discussion of these remaining models.

2.2 Barlow's Model - Preliminaries

Financial markets, usually begin with a geometric Brownian motion to model an asset. A stochastic process is said to be a *geometric Brownian motion* if it satisfies the following stochastic differential equation (SDE):

$$\begin{cases} \mathrm{d}X_t = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t \\ S_t = e^{X_t}, \end{cases}$$

where W denotes a standard one-dimensional Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and S the price of the asset. This SDE is often used to model asset prices as it enjoys two crucial properties: asset prices are nonnegative prices and the process is *Markovian*, or dependent only on the current state and independent of the past. However, as Barlow [8] notes, this model fails to capture power prices without a mechanism to capture the typical mean reverting behaviour that is characteristic of electricity markets.

To capture this mean reversion, we use the simplest stochastic process with this

property, the Ornstein–Uhlenbeck OU(λ, μ, σ) process defined by the SDE,

$$\begin{cases} \mathrm{d}X_t = -\lambda(X_t - \mu)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t \\ S_t = X_t \end{cases}$$

where $\lambda > 0$ is the time scale that X_t reverts to a mean value a and σ the volatility of the Brownian motion. The model's inherent mean reversion is clear: if X_t is larger than (resp. lesser than) a, then $-\lambda(X_t - a) < 0$ (resp. $-\lambda(X_t - a) > 0$) pushing the stochastic process back towards a. As Barlow [8] shows, this model does not provide a good fit for historical data and does not produce a satisfactory link between the spot and forward prices. This does not come entirely as a surprise as an OU process still admits negative values[1] and mean reversion naturally overly simplifies long term price dynamics. Assume the forward price at time t of the spot price at time T is given by

$$F(t,T) = \mathbb{E}_{\mathbb{Q}}[X_T|\mathcal{F}_t]$$

where $\mathbb{Q}$ denotes parameters under the risk neutral measure and $\mathcal{F}_t$ the filtration adapted to X_t. It can be readily seen by applying Itô's formula to $f(x_T, T) = x_T e^{\lambda T}$ (see Appendix A.1) that

$$F(t,T) = \left(\mu^{\mathbb{Q}}(1 - e^{-\lambda^{\mathbb{Q}}(T-t)}) + X_t e^{-\lambda^{\mathbb{Q}}(T-t)}\right). \tag{2.1}$$

Barlow argues that historical data tends to estimate a value of 50–180 for λ and for values of $T - t$ as small as one month, spot prices have very little effect on forward prices due to the rapid decay of $e^{-\lambda(T-t)}$. As such, fitting large values of λ is the

[1] In some cases, negative prices naturally occur to incentivize additional consumption to remove excess generation from the grid.

primary disadvantage to using such a model.[2]

2.3 A Structural Model for Electricity Prices

In this section, we describe the construction of Barlow's model from [8]. We later adapt it to show the effect of the same price process after including a market power adjustment.

Consider power at a single location without imports and define

$$u_t(x) = \text{supply at time } t \text{ if the price is \$x},$$
$$d_t(x) = \text{demand at time } t \text{ if the price is \$x}.$$

We say the price of power S_t at time t is the unique value satisfying

$$u_t(S_t) = d_t(S_t).$$

Setting $u_t(x) = g(x)$ to be a deterministic supply function independent of t and by assuming demand to be inelastic, i.e., d_t independent of u_t, we set $d_t(x) = D_t$ for some stochastic process D_t. From these assumptions, it then follows that

$$D_t = d_t(S_t) = u_t(S_t) = g(S_t).$$

If we choose an invertible g, we may thus write $S_t = g^{-1}(D_t)$. For the sake of this discussion, we begin with g[3] of the form

$$g(x) = a_0 - b_0 x^\alpha, \quad \alpha < 0.$$

[2]Such a large λ should not come as a surprise as the mean reversion speed serves as the primary force to return to ordinary levels after a spike.

[3]In principle, we may choose any g provided it satisfies certain regularity conditions to ensure existence and uniqueness of our associated SDE. See, e.g., [16].

Note that $g(x)$ is bounded from above by a_0 to reflect the producer's maximum production capacity. We then write

$$g^{-1}(y) = \left(\frac{a_0 - y}{b_0}\right)^{1/\alpha}$$

and restrict $y < a_0$ to ensure the supply does not exceed the maximum a_0. Recall that in Alberta energy markets there is a price cap of \$1000/MWh; a candidate model must be able to reflect this cap. We simply capture this property with a piecewise function so that all prices exceeding \$1000/MWh are set to \$1000/MWh as required. Therefore we arrive at the following model:

$$S_t = \begin{cases} \left(\frac{a_0 - D_t}{b_0}\right)^{1/\alpha}, & D_t < a_0 - \epsilon_0 b_0, \\ e_0^{1/\alpha}, & D_t \geq a_0 - \epsilon_0 b_0 \end{cases}$$

where the price cap is represented by $\epsilon_0^{1/\alpha}$.

We model the demand D_t by an $\mathrm{OU}(\lambda, a_1, \sigma_1)$ process, that is D_t satisfies the stochastic differential equation,

$$dD_t = -\lambda(D_t - a_1)dt + \sigma_1 dW_t.$$

It is convenient to consider the transformation $D_t = a_1 - \sigma_1 Y_t$[4], where $Y_t = \mathrm{OU}(\lambda, 0, 1)$ so that

$$S_t = \begin{cases} \left(\frac{a_0 - a_1 + \sigma_1 Y_t}{b_0}\right)^{1/\alpha}, & Y_t > \frac{a_0 - a_1 \epsilon_0 b_0}{\sigma_1}, \\ \epsilon_0^{1/\alpha}, & Y_t \leq \frac{a_0 - a_1 \epsilon_0 b_0}{\sigma_1}. \end{cases}$$

[4]In fact, any affine transformation of an OU process with constant shape parameters is again an OU process, see Appendix A.2 for details.

Next, set

$$\frac{a_0 - a_1 + \sigma_1 Y_t}{b_0} = 1 + \alpha X_t$$

where

$$X_t = \frac{a_0 - a_1 - b_0 + \sigma_1 Y_t}{\alpha b_0}$$

is an $\mathrm{OU}(\lambda, (a_0 - a_1 - b_0)/\alpha b_0, \sigma_1/\alpha b_0)$ process. Then, substituting this in,

$$S_t = \begin{cases} (1 + \alpha X_t)^{1/\alpha}, & 1 + \alpha X_t > \epsilon_0 \\ \epsilon_0^{1/\alpha}, & 1 + \alpha X_t \leq \epsilon_0. \end{cases}$$

To extend the model from $\alpha < 0$ to $\alpha \in \mathbb{R}$, define an invertible function

$$f_\alpha(x) = (1 + \alpha x)^{1/\alpha}, \quad f_0(x) = e^x \tag{2.2}$$

so that the pricing model becomes

$$S_t = \begin{cases} f_\alpha(X_t), & 1 + \alpha X_t > \epsilon_0, \\ \epsilon_0^{1/\alpha}, & 1 + \alpha X_t \leq \epsilon_0. \end{cases}$$

Where X_t satisfies the differential equation

$$\mathrm{d}X_t = -\lambda(X_t - \mu)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t. \tag{2.3}$$

We call S_t a *nonlinear Ornstein–Uhlenbeck process* or a $\mathrm{NLOU}(\lambda, \mu, \sigma, \alpha, \epsilon_0)$. When $\alpha = 1$, this is simply an OU process. For $\alpha = 0$, S_t satisfies the log-linear model

$$d\ln(S_t) = \lambda(\mu - \ln(S_t))dt + \sigma dW_t.$$

For $\alpha < 0$, the function $f_\alpha(X_t)$ grows faster than an exponential and generates more

pronounced price spikes than the log-linear case; as Barlow finds, in most almost all cases $\alpha << 0$. This is a strong case for using an NLOU process compared to the usual linear, or log-linear models. Weber [37] warns that such transforms methods represent only a crude representation of the power stack (c.f., Figure 1.2 and Figure 2.3) and for longer price horizons, will yield an unsuitable stationary time series since the mean reversion limits price uncertainty. As such, caution should be exercised while using a single factor model when λ is too large.

2.3.1 A Word On Notation

In what follows, we will adhere to the following conventions. The function $f_\alpha(x)$ always refers to (2.2) used to map the OU process X to the spot price S and $g_\alpha(x) := f_\alpha^{-1}(x)$ its inverse. For convenience we may suppress the subscript α and write $f(x)$ or $g(x)$ instead. We will write ϵ_c to denote the spot price cap instead of ϵ_0^α. We set $\epsilon_c = \$1000$ and use the two values interchangably for notational compactness for consistency with the Alberta electricity market's price cap; in general ϵ_c may be any nonnegative real number.

Note that unlike S_t, the image of f and g are unbounded from above and thus the values $\min(f_\alpha(x), \epsilon_c)$ and $\min(g_\alpha(x), \epsilon_c)$ are of particular importance. In what follows, we will generally assume the price does not exceed the cap or go below the price floor so that $f_\alpha(x) = \min(f_\alpha(x), \epsilon_c)^+$ and $g_\alpha(x) = \min(g_\alpha(x), \epsilon_c)^+$ where the superscript $+$ denotes the maximum of $\min(\cdot, \cdot)$ and 0. Redefining f and g in this manner will be useful later to extend the domain of f and g to all of $\mathbb{R}$ in later integrals.

We write $\phi(x; \mu, \sigma)$ for the Gaussian pdf with mean μ and variance σ^2 at the point x and $\Phi(x; \mu, \sigma)$ for the Gaussian cdf with mean μ and variance σ^2 at the

point x. That is,

$$\phi(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2},$$
$$\Phi(x;\mu,\sigma) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right].$$

where erf denotes the error function. When μ, σ are suppressed, $\phi(x)$ and $\Phi(x)$ will denote the standard Gaussian pdf and cdf, respectively.

Finally, note that an OU process $\mathrm{OU}(\lambda, \mu, \sigma)$, we denote the mean reversion level and drift by μ and σ, this is not to be confused with the conditional transition density which is normal with mean μ_1 and standard deviation σ_1.

2.3.2 Parameter Calibration

Here we continue Barlow's description and, use the method described by Box and Cox [10] to calibrate the NLOU process by first calibrating λ, a, σ holding α constant, and finding α afterward. Set $\Delta_t = 1/365$ and make observations $S_k = \hat{S}_{k\Delta_t}$ for $t \in [0, T]$. Now, since $Y_t = g(S_t)$ is an OU process, it has a continuous time solution

$$Y_t = Y_0 e^{-\lambda t} + a(1 - e^{-\lambda t}) + \sigma \int_0^t e^{-\lambda(t-s)} dW_s \tag{2.4}$$

provided $S_t < 1000$. Discretizing the continuous time solution, our sampled process $Y_k = g(\hat{S}_k)$ can be described by an AR(1) process

$$Y_k = \varphi Y_{k-1} + c + \sqrt{\eta}\varepsilon_k \tag{2.5}$$

where $\varepsilon_k \sim \mathcal{N}(0, 1)$ are i.i.d. standard normal random variables and

$$\varphi = e^{-\lambda\Delta_t}, \quad \eta = \frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda\Delta_t}), \quad c = a(1 - e^{-\lambda\Delta_t}). \tag{2.6}$$

where this change of variables can be seen by inspection from (2.4) Rearranging (2.5)

$$\sqrt{\eta}\varepsilon_k = Y_k - \varphi Y_{k-1} - c$$

it is clear the transition density from Y_{k-1} to Y_k is

$$q(x,y) = \frac{1}{\sqrt{2\pi\eta}} e^{-\frac{1}{2\eta}(y-x\varphi-c)^2}$$

and so the transition density of the observations $\hat{S}_k$ is

$$\begin{aligned} p(x,y) &= q(g(x),g(y))|(g(y))'| \\ &= |(g(y))'|\frac{1}{\sqrt{2\pi\eta}} e^{-\frac{1}{2\eta}(g(y)-g(x)\varphi-c)^2}. \end{aligned}$$

For the conditional likelihood function of $\hat{S}$,

$$\mathcal{L}(\hat{S}_1,\ldots,\hat{S}_n|\hat{S}_0) = \prod_{k=1}^{n} p(\hat{S_{k-1}}, \hat{S}_k)$$

the corresponding log-likelihood is

$$\begin{aligned} \log(\mathcal{L}) = \sum_{k=1}^{n} \log|(g(S_k))'| - \frac{n}{2}\log(2\pi\eta) \\ - \frac{1}{2\eta}\sum_{k=1}^{n}(g(\hat{S}_k) - \varphi g(\hat{S}_{k-1}) - c)^2. \end{aligned} \tag{2.7}$$

Fixing α and minimizing $\log(\mathcal{L})$ over φ, c, η is straightforward. Indeed from the first order conditions,

$$\begin{aligned}
\frac{\partial \log(\mathcal{L})}{\partial c} = 0 &\implies \hat{c} = \frac{1}{n}\sum_{k=1}^{n}\left(g(\hat{S}_k) - \hat{\varphi} g(\hat{S}_{k-1})\right) \\
\frac{\partial \log(\mathcal{L})}{\partial \varphi} = 0 &\implies \hat{\varphi} = \frac{1}{\sum_{k=1}^{n} g(\hat{S}_{k-1})^2}\sum_{k=1}^{n}\left(g(\hat{S}_k)g(\hat{S}_{k-1}) - \hat{c}\, g(\hat{S}_{k-1}\right) \\
\frac{\partial \log(\mathcal{L})}{\partial \eta} = 0 &\implies \hat{\eta} = \frac{1}{n}\sum_{k=1}^{n}(g(\hat{S}_k) - \hat{\varphi} g(\hat{S}_{k-1}) - c)^2.
\end{aligned}$$

Next $\hat{c}$ and $\hat{\varphi}$ can be expressed as the solution to the matrix equation:

$$\begin{bmatrix} 1 & \frac{1}{n} g(\hat{S}_{k-1}) \\ \sum_{k=1}^{n} g(\hat{S}_{k-1}) & \sum_{k=1}^{n} g(\hat{S}_{k-1})^2 \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{\varphi} \end{bmatrix} = \begin{bmatrix} \frac{1}{n}\sum_{k=1}^{n} g(\hat{S}_k) \\ \sum_{k=1}^{n} g(\hat{S}_k) g(\hat{S}_{k-1}) \end{bmatrix}$$

Substituting these solutions into (3.12), we obtain

$$\log(\mathcal{L}(\hat{S}|\alpha)) = \sum_{i=1}^{n} \log((g(\hat{S}_k))') - \frac{n}{2}\log(\hat{\eta}) - n\left(\frac{1}{2} + \log\left(\sqrt{2\pi}\right)\right).$$

This can then be minimized over α using numerical methods. In our case, we used MATLAB's `patternsearch`.

Using historical price data from AESO, we calibrate using historical price data from 1999 to 2014. The data was preprocessed by taking the daily averages to reduce the impact of daily cycles and remove inconvenient factors such as duplicate time periods due to daylight savings. The estimated parameters are presented in Table 2.2. Experimentation with other time periods containing price spikes[5] produced similar parameters.

Using these parameters, spot prices are simulated in Figure 2.3. In our expe-

[5] In 2017, the Balancing Pool had offer control of more than 20% of Alberta's installed capacity, their bidding style suppressed price spikes for some time. These spikes have since recently returned, see Section 3.1.

a	λ	σ	α
2.103 ± 0.748	133.331 ± 0.004	1.872 ± 0.019	-0.362 ± 0.110

Table 2.2: Parameter estimates using AESO data from 1999 to 2014 using daily averages (5923 data points) and the standard error at 95% confidence.

rience, the simulations produce better results by using the recursive scheme from (2.4) (as opposed to say, implementing Euler–Maruyama). This is not entirely a surprise as the parameters were calibrated using that model. For further information on simulating stochastic processes, we direct the reader to [18] and Appendix A.3.

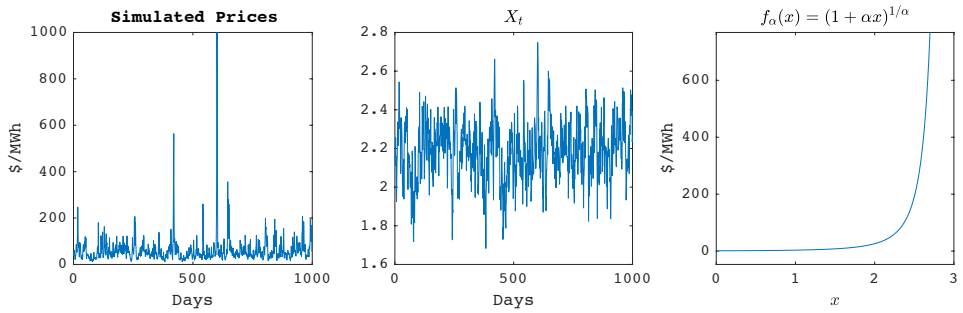

Figure 2.3: A simulated S_t, the underlying process X_t and a representative power stack $f_\alpha(x) = (1 + \alpha x)^{1/\alpha}$ using the values in Table 2.2.

The effect of the steep power stack function $f_\alpha(x)$ is evident; the small peak in X_t is enough to take us well beyond $1000 if it was not for the price cap. The large mean reversion parameter λ is responsible for the sharp return to the mean following a price spike.

Chapter 3

Market Power

3.1 Market Power in the Diffusion Model

In this section, we extend Barlow's model to model the effects of exercising market power through economic withholding. To be the best of our knowledge, there is no research adapting an SDE to incorporate market power at this time. However, models do exist that consider generating unit outages and operate on a similar principle to how we shift the bid stack in Section 1.4. In these models, the generating unit outage removes their bid from the stack entirely and differs from our approach that simply moves the available supply to the price cap, see e.g., [15, Chapter 7]. In what follows we assume that other firms do not alter their bidding strategies in response. Further, we index time steps by the variable k and assume a single time step beginning at time t_{k-1} and ending at time t_k. That is, the firm will make a decision to be resolved at time t_k based on the observable information at time t_{k-1}. Decision times t_k are separated by a constant time step Δt and slightly abusing notation, we write $X_k = X_{k\Delta t}$

For values of S_k below the price cap of \$1000/MWh $= \epsilon_c$, it is helpful to instead

work directly on the underlying OU process $X_k \sim \mathrm{OU}(\lambda, \mu, \sigma)$.

We model the effect of economic withholding on electricity prices by shifting the demand curve to the right as seen in Figure. 1.5. With this in mind, we shift X_t by introducing a C representing a reduction in supply available in the market. This shift increases the spot price from $f_\alpha(x)$ to $f_\alpha(x + C)$. Assume firms bid their available production capacity at variable cost and suppose the firm exercising market power has no fixed or variable costs.

Define

$$\mathrm{xmin} \stackrel{\mathrm{def}}{=} g(0.01), \quad \mathrm{xmax} \stackrel{\mathrm{def}}{=} g(1000),$$

and

$$\tau_\alpha \stackrel{\mathrm{def}}{=} \mathrm{xmax} - \mathrm{xmin}\,.$$

The value τ_α should be interpreted as the amount X_k must change to move the spot price S_k from \$0.01 to \$1000 and serves as a scaling constant for given alpha. In this sense, a firm capable of shifting X_k to $X_k + \tau_\alpha$ possesses sufficient market power to be able to always move the prices to the price cap. We model a firm's ability to exercise market power under the map

$$X_k \mapsto X_k + \tau_\alpha \theta p \tag{3.1}$$

where $\theta \in [0, 1]$ scales the amount of market power an agent has and p the fraction of energy controlled by the firm withheld from the market. That is, when $p = 0$, the agent offers everything into the market at the usual price and no market power is exercised. In general, θ could be changed by a firm by creating or buying additional production capacity, but we will treat it as a fixed constant here. To exercise market power, the firm simply has to choose a level p to withhold and the price will increase to $f_\alpha(X_k + \theta\tau_\alpha p)$. Conditioning on the initial value $X_{k-1} = x_{k-1}$, at each time t

the firm will thus select a value p to maximize its cash flow.

Before we proceed, we first recall some basic facts of OU processes. The transition density of an $\mathrm{OU}(\lambda, \mu, \sigma)$ process conditioned on $X_{k-1} = x_{k-1}$ follows a Gaussian distribution with mean and variance

$$\mathbb{E}[X_k | X_{k-1} = x_{k-1}] = \mu + (x_0 - \mu)e^{-\lambda \Delta t} \tag{3.2}$$

and

$$\mathbb{V}[X_k | X_{k-1} = x_{k-1}] = \frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda \Delta t}).$$

See Appendix A.1, (A.1) for more detail.

Let

$$\mu_1 = \mu + (x_{k-1} - \mu)e^{-\lambda \Delta t}, \quad \sigma_1 = \frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda \Delta t}).$$

We can then write the expected spot price with some care[1]

$$\mathbb{E}[S_k | X_{k-1} = x_{k-1}] = \int_{\mathbb{R}} \min(f_\alpha(x), 1000)\phi(x; \mu_1, \sigma_1)\, \mathrm{d}x,$$

and after the change of variables $x = \sigma_1 z + \mu_1$ to rescale ϕ to a standard Gaussian,

$$\mathbb{E}[S_k | X_{k-1} = x_{k-1}] = \int_{\mathbb{R}} \min(f_\alpha(\sigma_1 z + \mu_1), 1000)\phi(z)\, \mathrm{d}z.$$

To incorporate market power, the map $X_k \mapsto X_k + \tau_\alpha \theta p$ is equivalent to substituting the mean μ_1 with $\mu_{p,\theta} = \mu_1 + \tau_\alpha \theta p$, see Theorem 1. For convenience later, we define

$$\Lambda_p(z) = \min(f_\alpha(\sigma_1 z + \mu_{p,\theta}), 1000)$$

[1] This integral is generally not integrable over $\mathbb{R}$ as the natural domain of f is incompatible with the support of ϕ. Integrals over $\mathbb{R}$ of f will be fixed by setting $f_\alpha(x) = 1000$ for values of x outside of the domain such that $x > g(1000)$ and 0 otherwise. See Section 2.3.1.

For each p, the expected spot price S_k is then

$$\mathbb{E}[S_k|X_{k-1} = x_{k-1}] = \int_{\mathbb{R}} \Lambda_p(z)\phi(z)\,\mathrm{d}z.$$

Let M be the total production supply for the firm. We model the expected payoff P_t for the firm in two extreme cases; these cases will be distinguished with a subscript in the expectation. In the first case, the firm exercises economic withholding and bids Mp units at the price cap. We assume that whenever f_α exceeds the price cap of \$1000, all M units of production are dispatched[2] and settled at \$1000. Hence

$$\begin{aligned}\mathbb{E}[\Lambda_{p,1}|X_{k-1} = x_{k-1}] &\stackrel{\text{def}}{=} \mathbb{E}[P_k|X_{k-1} = x_{k-1}] \qquad (3.3)\\ &= M(1-p)\int_{f_\alpha \le 1000} \Lambda_p\phi(z)dz + 1000M\int_{f_\alpha > 1000}\phi(z)dz,\end{aligned}$$

where we append a subscript i to $\Lambda_{p,i}$ in (3.3) to distinguish the models that follow. When the spot price reaches the price cap, f_α is no longer invertible; a firm cannot measure how much of the remaining $(1-p)$ units will be called on. The above equation represents the extreme case of a maximal payout.

In the second case, the agent has either exercised physical withholding and the remaining Mp units are not available, or the firm is called to produce no additional electricity if the spot price reaches the price cap. Regardless of the prevailing spot price, the agent will sell at most Mp units. That is,

$$\begin{aligned}\mathbb{E}[\Lambda_{p,2}|X_{k-1} = x_{k-1}] &= \mathbb{E}[P_t|X_{k-1} = x_{k-1}] \qquad (3.4)\\ &= M(1-p)\int_{\mathbb{R}} \Lambda_p(z)\phi(z)dz. \qquad (3.5)\end{aligned}$$

[2]In the event equally priced offers (such as at the price cap), dispatch in Alberta is ordered using a pro rata methodology not captured here. For more information, see [1, Section 202.3].

Equations (3.3) and (3.4) are cast as the following optimization problems:

$$\begin{aligned} \arg\max_{p} \quad & \mathbb{E}[\Lambda_{p,1}|X_{k-1} = x_{k-1}] \\ \text{subject to} \quad & 0 \leq p \leq 1 \end{aligned} \tag{3.6}$$

and

$$\begin{aligned} \arg\max_{p} \quad & \mathbb{E}[\Lambda_{p,2}|X_{k-1} = x_{k-1}] \\ \text{subject to} \quad & 0 \leq p \leq 1. \end{aligned} \tag{3.7}$$

For each $x_0 \in X_0$, we find the optimum p and collect the set of p as a function of x_{k-1} in what we will call a *decision curve*, see Figure 3.1 and 3.2. Formally, for a given payoff function i, we define the decision curve by

$$DC(x) = \arg\max_p \mathbb{E}[\Lambda_{p,i}|X_{k-1} = x_{k-1}]. \tag{3.8}$$

A firm then exercises market power using the decision curve through a straightforward procedure. First invert the spot price S_{k-1} to return the underlying OU process X_{k-1}, then choose a value p from the decision curve.

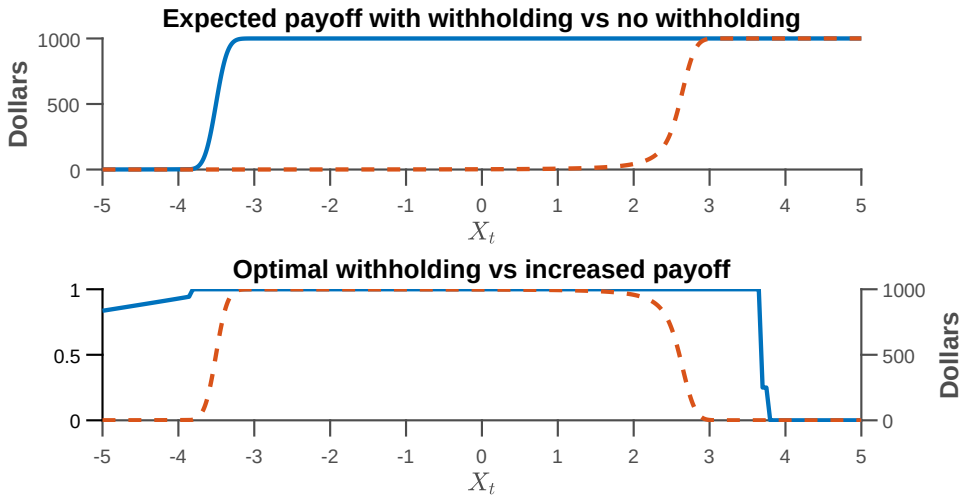

Figure 3.1: Top: The expected payoff (under (3.6)) while exercising optimal market power versus the case with no market power. Bottom: A comparison of the withholding amounts versus the expected payoff. These values are generated using the parameters from Table 2.2 and $\theta = 0.3$

It is not always possible to invert S_{k-1} if the price cap is attained; in most practical cases there will be no problem as spikes are short lived. Figure 3.1 displays the decision curve and the corresponding increase in expected payoff. It is clear that allowing a firm to dispatch all their unsold load upon reaching the price cap is an unreasonable model. Nevertheless, it is a useful terminal case to bound further models from above. As seen here, a large θ can incentivize a firm to economically withdraw from the market to consistently drive prices to extreme values. Recall from Section , at one point the Balancing Pool possessed more than 20% of Alberta's installed capacity and the FEOC limits participants to no more than 30% of offer control. For these reasons, investigating the value of $\theta = 0.3$ is not as unreasonable as Figure 3.1 suggests.

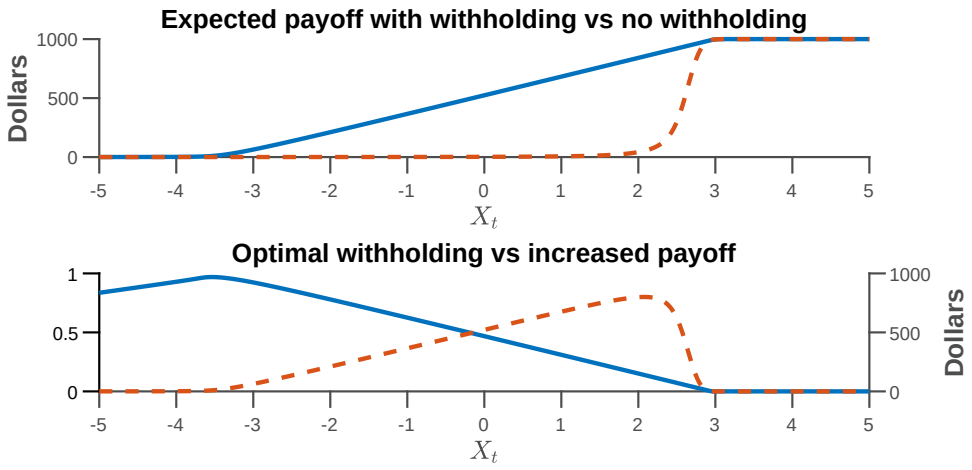

Figure 3.2: Top: The expected payoff (under (3.6)) while exercising optimal market power versus the case with no market power. Bottom: A comparison of the withholding amounts versus the expected payoff. These values are generated using the parameters from Table 2.2 and $\theta = 0.3$

Figure 3.2 considers the other extreme case where no additional revenue is gained from exercising market power. We note that even when the price of electricity is around \$1/MWh ($X_{k-1} < 1$), it is surprisingly profitable for our firm to withhold large amounts of generation capacity in hopes of raising the price despite receiving no additional revenue from the economically withheld units if prices reach the cap. The presence of a financial incentive to exercise market power even at extremely low prices is supportive of the unpredictable nature of real electricity market. Withholding even a large amount of production capacity sufficiently penalize the firm when the price is small compared to the potential payoff in the event of a supply shock.

3.2 Modeling Payoffs at the Price Cap

In this section, we generalize the market power model in Section 3.1 to more accurately describe payoffs above the price cap. In the previous two scenarios, a firm either did not produce any additional units if demand pushed the firm to the price cap, or produced all of its withheld capacity. We propose a model that supposes that our firm will always be called upon to produce just enough additional electricity to clear the market.

To make this clearer, suppose at time t_k, there are 100 MW/h of electricity bid into the market and 150 MW/h of demand. Our firm controls 80 MW/h of production capacity, and bid 60 MW/h at the price cap. In the previous section, the firm observes the spot price reaching the price cap, but would not know that demand exceeds supply by 50 MW/h. The firm would then behave by either assuming there was insufficient demand for them to be called to produce any additional capacity at the cap or by assuming there would be sufficient demand to dispatch all of their withheld capacity. Here, the firm will assume that they are called to produce precisely the remaining 50 MW/h units necessary to clear the market.

From Section 3.1, withholding p units corresponds to a shift in the OU process from X_k to $X_k + \tau_\alpha \theta p$. This assures that whenever $X_k = g_\alpha(\epsilon_c) + \tau_\alpha \theta p$, there is sufficient demand to generate all of its withheld electricity. That is, if given p fixed, $S_k = \epsilon_c$, $X_k < g_\alpha(\epsilon_c) + \tau_\alpha \theta p$, at most $g_\alpha(\epsilon_c) + \tau_\alpha \theta p - X_k$ units of additional power can be sold.

We assume that no other firm will be called to meet the shortfall and thus our firm receives a $1000(1-p)$ in revenue for its $1-p$ additional units[3]. Therefore, as

[3]We've assumed that only our firm is exercising market power and as such, no other firms have bid in any units at ϵ_c. This can, however, be generalized to multiple firms by allowing our firm only proportion of the additional units.

a function of p, we generate an additional $\epsilon_c p$ dollars whenever

$$X_k = g_\alpha(\epsilon_c) + \tau_\alpha \theta p. \tag{3.9}$$

Using (3.9) we can rewrite the additional revenue $\epsilon_c p$ in terms of x,

$$\epsilon_c \frac{x - g_\alpha(\epsilon_c)}{\tau_\alpha \theta}.$$

Finally, by varying $p \in [0, 1]$, we can see that this x satisfies

$$g_\alpha(\epsilon_c) \leq x \leq g_\alpha(\epsilon_c) + \tau_\alpha \theta p.$$

The expected payoff is thus

$$\begin{aligned}
\mathbb{E}[\Lambda_{p,3} | X_{k-1} = x_{k-1}] &= \mathbb{E}[P_t | X_{k-1} = x_{k-1}] \\
&= M(1-p) \int_{-\infty}^{\infty} f_\alpha(x) \phi(x; \mu_1, \sigma_1) \, \mathrm{d}x \\
&\quad + \frac{M\epsilon_c}{\tau_\alpha \theta} \int_{g_\alpha(\epsilon_c)}^{g_\alpha(\epsilon_c) + \tau_\alpha \theta p} ((x - g_\alpha(\epsilon_c))) \phi(x; \mu_1, \sigma_1) \, \mathrm{d}x \\
&\quad + M\epsilon_c p \int_{g_\alpha(\epsilon_c) + \tau_\alpha \theta p}^{\infty} \phi(x; \mu_1, \sigma_1) \, \mathrm{d}x.
\end{aligned}$$

This expression has the following interpretation. The first integral represents the payoff of the bids the firm made below the price cap. The second integral represents the additional payoffs from a firm producing up to p additional units of electricity. Finally the third integral represents the additional payoffs when demand exceeds the firms production capacity.

The second and third integrals can be calculated analytically in terms of ϕ and Φ. The third integral is the most straight-forward. Under the change of variables

$x = \mu_1 + \sigma_1 z$, the integral becomes

$$I_3 = \int_{\frac{g_\alpha(\epsilon_c)+\tau_\alpha\theta p-\mu_1}{\sigma_1}}^{\infty} \phi(z)\,\mathrm{d}z = \Phi\left(\frac{\mu_1 - g_\alpha(\epsilon_c) - \tau_\alpha\theta p}{\sigma_1}\right).$$

For the second integral, under the change of variables $x = \mu_1 + \sigma_1 z$

$$I_2 = \frac{\epsilon_c}{\tau_\alpha\theta}\int_{g_\alpha(\epsilon_c)}^{g_\alpha(\epsilon_c)+\tau_\alpha\theta p} \frac{x - g_\alpha(\epsilon_c)}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\,\mathrm{d}z.$$

Set

$$z_0 = \frac{g_\alpha(\epsilon_c) - \mu_1}{\sigma}$$

so that

$$\begin{aligned} x - g_\alpha(\epsilon_c) &= \mu_1 + \sigma_1 z - g_\alpha(\epsilon_c) \\ &= \sigma_1 z - (g_\alpha(\epsilon_c) - \mu_1) \\ &= \sigma_1(z - z_0), \end{aligned}$$

and

$$\mathbb{E}\left[\frac{X_k - g_\alpha(\epsilon_c)}{\tau_\alpha\theta}\right] = \frac{\epsilon_c\sigma_1}{\tau_\alpha\theta}\int_{z_0}^{z_0+\frac{\tau_\alpha\theta p}{\sigma_1}} \frac{(z - z_0)}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\,\mathrm{d}z.$$

This integral can be evaluated as

$$\begin{aligned} I_2 = \frac{\epsilon_c\sigma_1}{\tau_\alpha\theta}\Bigg[&\phi(z_0) - \phi\left(z_0 + \frac{\tau_\alpha\theta p}{\sigma_1}\right) \\ &- z_0\left[\Phi\left(z_0 + \frac{\tau_\alpha\theta p}{\sigma_1}\right) - \Phi(z_0)\right]. \end{aligned}$$

Finally, the first integral I_1 must be computed numerically; in our case we successfully used MATLAB's `quadgk` for all numerical integrals. These integrals are not without their own challenges, we direct the reader to Appendix for more detail.

The expected payoff function gives rise to the following optimization problem.

$$\begin{aligned} \arg\max_{p} \quad & \mathbb{E}[\Lambda_{p,3} | X_{k-1} = x_{k-1}] \\ \text{subject to} \quad & 0 \leq p \leq 1. \end{aligned} \tag{3.10}$$

In Figure 3.3, we compare the firms payoffs with and without withholding. For values of $X_k < -1$ the firm's optimal withholding amount is quite large despite a low spot price of less than \$0.40. At these values, the price is so low that the firm can match their payoffs and simultaneously increasing the possibility that a supply shock will drastically increase the price. Figure 3.4 compares the three decision curves

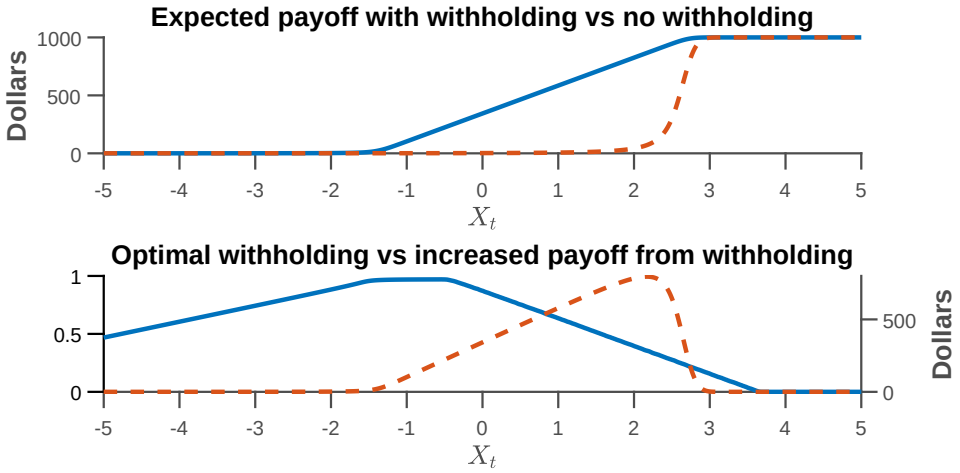

Figure 3.3: Top: The expected payoff with witholding (blue) compared to the expected payoff without any withholding. Bottom: A comparison of the withholding amount and the additional revenue from doing so. ($\theta = 0.3$)

defined by optimization problems (3.6), (3.7), (3.10). The three curves follow the same path until additional revenue at the price cap separate them. It is clear that case (3.6) is too generous, without adequate supply from other competitng firms, our firm will essentially leave the market and drive prices towards the cap.

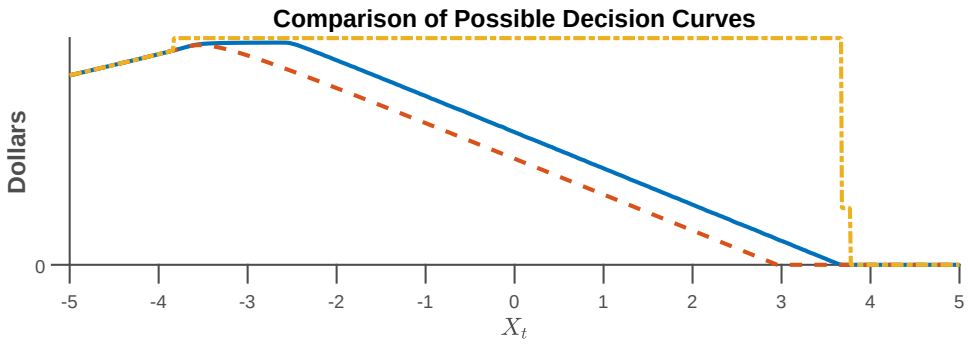

Figure 3.4: The three decision curves corresponding to (3.6) (Yellow, double dashed), (3.7) (Red, dashed), (3.10) (Blue, solid).

3.3 A Model Incorporating Market Power

In the previous sections, we discussed the case when a firm would increase its expected revenue by strategically withholding under Barlow's model. We now extend Barlow's construction to include our notion of a decision curve.

Consider a slightly different hypothesis from Barlow's model. As before, we index time by a parameter k and set Δ_t to be a constant interval between decision points t_{k-1} and t_k. Suppose the underlying process X_k is under the influence of the decision curve from (3.8) and hence X_k does not follow an OU process. For every S_k under the map g, suppose that the process X_k is instead composed of the sum of two components: an OU process and a market power term. For set of observations $\hat{S_k}$, $k = 0 \dots n$, we define $\hat{X}_k$ under the following recursive definition:

$$\begin{cases} \hat{X}_0 = g(S_0), \\ \hat{X}_k = g(S_k) - \tau_\alpha \theta \, \mathrm{DC}(\hat{X}_{k-1}). \end{cases} \tag{3.11}$$

Having subtracted off the market power term, we suppose $\hat{X}_k$ the resulting $\hat{X}_k$ is an OU process.

As in 2.3.2, we will use maximum likelihood estimation to determine the OU parameters. The procedure is nearly identical and some preliminary steps will not be repeated here. We then form a set of observations $Y_k = g(S_{k\Delta_t})$. Next we construct our adjusted sample process $\hat{Y}_k$ free of market power according to (3.11) and define the transition density from $\hat{Y}_{k-1}$ to $\hat{Y}_k$ to be

$$q(x,y) = \frac{1}{\sqrt{2\pi\eta}} e^{-\frac{1}{2\eta}(x-y\varphi-c)^2}.$$

For the transition density in S_t, we will follow the usual method of differentiating the cdf. However, unlike the previous case, the sampled process $Y_k = g(S_{k\Delta_t})$ is not an OU process, i.e., the transition density will not be normal. This will require a further adjustment to construct the $\hat{Y}$ as follows:

$$\begin{aligned}
\frac{d}{ds}\mathbb{P}[S_k < s|\mathcal{F}_{k-1}] &= \frac{d}{ds}\mathbb{P}[g(S_t) < g(s)|\mathcal{F}_{k-1}] \\
&= \frac{d}{ds}\mathbb{P}[Y_t < g(s)|\mathcal{F}_{k-1}] \\
&= \frac{d}{ds}\mathbb{P}[\hat{Y}_k < g(s) - \tau_\alpha\theta\,\mathrm{DC}(\hat{Y}_{k-1})|\mathcal{F}_{k-1}] \\
&= \frac{d}{ds}\int_{-\infty}^{g(s)-\tau_\alpha\theta\,\mathrm{DC}(\hat{Y}_{k-1})} p_{\hat{Y}_k}(y)\mathrm{d}y \\
&= |g(s)'|\, p_{\hat{Y}_k}\left(g(s) - \tau_\alpha\theta\,\mathrm{DC}(\hat{Y}_{k-1})\right)
\end{aligned}$$

where $p_{\hat{Y}_k} = q(x,y)$ is the conditional density of $\hat{Y}_k$, is normal with mean $\varphi\hat{Y}_{k-1}$ and variance θ (holding the second argument constant).

Therefore, then have the conditional transition density

$$p(x,y) = |g(x)'|q(h(x),h(y)),$$

where $h(\cdot)$ describes the action of (3.11), i.e., h maps prices to the market-power-free

OU process (Note that under $\mathcal{F}_{t-1}$, $h'(s) = g'(s)$). The log-likelihood is thus

$$\begin{aligned}\mathcal{L}(\hat{S}_0, \dots \hat{S}_n) &= \sum_{k=1}^{n} \log \left| g(\hat{S}_k)' \right| - \frac{n}{2} \log(2\pi\eta) \\ &\quad - \frac{1}{2\eta} \sum_{k=1}^{n} \left(\hat{Y}_k - \tau_\alpha \theta \, \mathrm{DC}(\hat{Y}_{k-1}) - \varphi \hat{Y}_{k-1} - c \right)^2 \\ &= \sum_{k=1}^{n} |\hat{S}_k^{\alpha-1}| - \frac{n}{2} \log(2\pi\eta) \\ &\quad - \frac{1}{2\eta} \sum_{k=1}^{n} \left(\hat{Y}_k - \tau_\alpha \theta \, \mathrm{DC}(\hat{Y}_{k-1}) - \varphi \hat{Y}_{k-1} - c \right)^2 \end{aligned} \tag{3.12}$$

It is straight forward to calculate the first order conditions and construct the corresponding system of equations

$$\begin{bmatrix} n & \sum_{k=1}^{n} \hat{Y}_{k-1} \\ \sum_{k=1}^{n} \hat{Y}_{k-1} & \sum_{k=1}^{n} \hat{Y}_{k-1}^2 \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{\varphi} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} \hat{Y}_k \\ \sum_{k=1}^{n} \hat{Y}_k \hat{Y}_{k-1}, \end{bmatrix} \tag{3.13}$$

and

$$\hat{\eta} = \frac{1}{n} \sum_{k=1}^{n} (\hat{Y}_k - \hat{\varphi} \hat{Y}_{k-1} - \hat{c})^2. \tag{3.14}$$

After solving for $\hat{c}, \hat{\varphi}, \hat{\eta}$, we find the OU parameters a, λ, σ using the change of variables (2.6). All that remains is to substitute $\hat{c}$, $\hat{\varphi}$, and $\hat{\eta}$ into the log likelihood and maximize

$$\mathcal{L}(\alpha) = \sum_{k=1}^{n} \log(\hat{S}_i^{\,\alpha-1}) - \frac{n}{2} \log(\hat{\eta}) - n \left(\frac{1}{2} + \log \left(\sqrt{2\pi} \right) \right).$$

Note that it suffices to consider

$$\mathcal{L}^*(\alpha) = \alpha \sum_{k=1}^{n} \log(\hat{S}_k) - n \log \left(\sum_{k=1}^{n} \hat{Y}_k - \hat{\varphi} \hat{Y}_{k-1} - \hat{c} \right). \tag{3.15}$$

As before, we use MATLAB's `patternsearch` to maximize $\mathcal{L}$ as a function of α.

Having found $\hat{\alpha}, \hat{\mu}, \hat{\sigma}, \hat{\lambda}$, we iteratively repeat the process outlined in this section given our updated parameters until $\hat{\alpha}, \hat{\mu}, \hat{\sigma}, \hat{\lambda}$ converges to a desired tolerance. We then conclude by updating the decision curve DC. This procedure is summarized in Algorithm 3.1.

Algorithm 3.1 Iterative Log-Likelihood

procedure ITERATIVELL(α, μ, σ, λ, DC, $\{S\}_{k=0}^{n}$)
  for each $S_i \in \{S\}_{k=0}^{n}$ **do**
    $X_0 \leftarrow g(S_0)$ ▷ Initial value
    $X_k \leftarrow g(S_k) - \tau_\alpha \, \mathrm{DC}(g(S_{k-1})$ ▷ Recursively define X_k
  end for
  $(\hat{\mu}, \hat{\sigma}, \hat{\lambda}) \leftarrow$ Solution to system of equations (3.13), (3.14)
  $\hat{\alpha} \leftarrow$ Mnimize $-\mathcal{L}$ (3.15) in α ▷ We used `patternsearch` here
  if $\max\{(|\mu - \hat{\mu}|, |\sigma - \hat{\sigma}|, |\lambda - \hat{\lambda}|), |\alpha - \hat{\alpha}|\} < TOL$ **then**
    DC $\leftarrow$ Update DC with parameters $(\hat{\mu}, \hat{\sigma}, \hat{\lambda}, \hat{\alpha})$
    Return $\hat{\mu}, \hat{\sigma}, \hat{\lambda}, \hat{\alpha}$, DC
  else
    $(\mu, \sigma, \lambda, \alpha) \leftarrow (\hat{\mu}, \hat{\sigma}, \hat{\lambda}, \hat{\alpha})$ ▷ Update parameters
    DC $\leftarrow$ Update DC with parameters $(\mu, \sigma, \lambda, \alpha)$
    Go to 2
  end if
end procedure

Running Algorithm 3.1 using parameters initialized from Table 2.2 and Alberta spot price data from 1999 to 2014 rapidly converges to the NLOU parameters in Table 3.5. The value of $\alpha > 0$ is of particular interest; when $\alpha > 0$, the power stack function f is no longer steep enough to generate price spikes on its own. In Figure 3.6, it is apparent that X_k takes on a maximum value of just over 4, and $f_\alpha(4) \approx \$40$ yet the simulated prices reach nearly \$400. Satisfying price spikes would be vanishingly improbable in Barlow's model with a positive α, but the decision curve can produce three spikes in this case.

Unfortunately, λ is still very large. As we discussed in Section 2.2, large λ makes this model unsuitable for forward prices. Other approaches were explored to create a model with a smaller λ. However, a firm exercising market power is not incentivized

μ	λ	σ	α
0.1954 ± 2.854	135.583 ± 2.935	10.8106 ± 0.036	0.0505 ± 0.280

Table 3.5: Parameter estimates using Algorithm 3.1. These estimates use AESO data from 1999 to 2014 with daily averages (5923 data points). The standard error estimates were calculated at 95% confidence in the row below at 95% confidence.

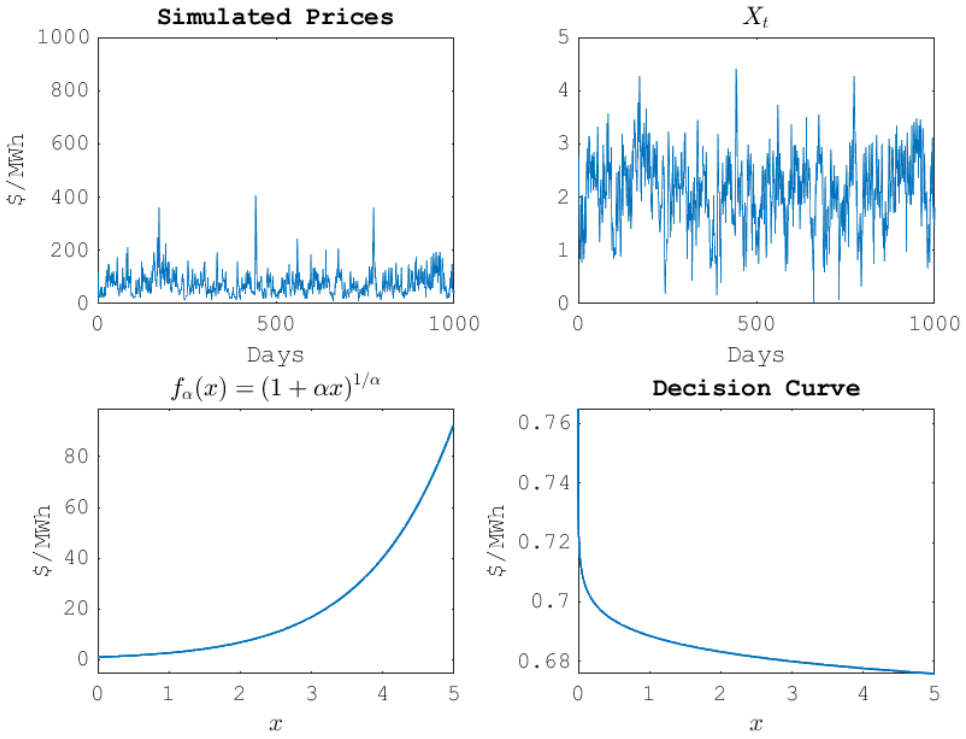

Figure 3.6: Simulations using the parameters generated by Algorithm 3.1.

to withhold a lot when prices are already high. Without a large amount withheld via the decision curve, the underlying OU process must have gotten large on its own; the only force that can return it to the mean in these cases is λ. Nevertheless, it is not difficult to envision a situation where the power stack matches the one found in Figure 3.6. In these cases, it is possible to preserve a close representation of the power stack function and generate the spikes via the decision curve.

So far, we have always found μ, σ, λ analytically and then numerically finding

α. A natural alternative would be to instead simultaneously calculate $\alpha, \mu, \sigma, \lambda$ simultaneously in one step. Algorithm 3.2 is arguably simpler:

Algorithm 3.2 One Step Log-Likelihood

procedure OneStep(α, μ, σ, λ, DC, $\{S\}_{k=0}^{n}$)
  for each $S_i \in \{S\}_{k=0}^{n}$ **do**
    $X_0 \leftarrow g(S_0)$ ▷ Initial value
    $X_k \leftarrow g(S_k) - \tau_\alpha \, \mathrm{DC}(g(S_{k-1})$ ▷ Recursively define X_k
  end for
  $\alpha_0, \mu_0, \sigma_0, \lambda_0 \leftarrow$ Initial guess ▷ Used `patternsearch`
  $\hat{\alpha}, \hat{\mu}, \hat{\sigma}, \hat{\lambda} \leftarrow$ Minimize $-\mathcal{L}$ (3.12) using guess ▷ Used `fmincon`
  if $\max\{(|\mu - \hat{\mu}|, |\sigma - \hat{\sigma}|, |\lambda - \hat{\lambda}|), |\alpha - \hat{\alpha}|\} < TOL$ **then**
    DC $\leftarrow$ Update DC with parameters $(\hat{\mu}, \hat{\sigma}, \hat{\lambda}, \hat{\alpha})$
    Return $\hat{\mu}, \hat{\sigma}, \hat{\lambda}, \hat{\alpha}$, DC
  else
    $(\mu, \sigma, \lambda, \alpha) \leftarrow (\hat{\mu}, \hat{\sigma}, \hat{\lambda}, \hat{\alpha})$ ▷ Update parameters
    DC $\leftarrow$ Update DC with parameters $(\mu, \sigma, \lambda, \alpha)$
    Go to 2
  end if
end procedure

In the spirit of leaving the parameter calibration approach in Section 2.3.2. Algorithm 3.1 instead calibrates its initial parameters using MATLAB's `fmincon` with an initial guess supplied by the *genetic algorithm*, `ga`. The parameters are presented in Table 3.7.

μ	λ	σ	α
0.9542	96.1918	0.2302	-1
-0.0685 ± 1.8264	146.3274 ± 0.1415	5.2837 ± 0.0188	-0.1386 ± 0.0029

Table 3.7: Initial guess for NLOU parameters supplied by `patternsearch` in the first row and in the second row the parameter estimates for Algorithm 3.2. These estimates use AESO data from 1999 to 2014 with daily averages (5923 data points). The standard error estimates were calculated at 95% confidence in the row below at 95% confidence.

The parameters are similar to the previous case except for the crucial sign of α is now negative. We simulate prices using these parameters and present the results in Figure 3.8; note that these plots use the same Brownian increments dW_t

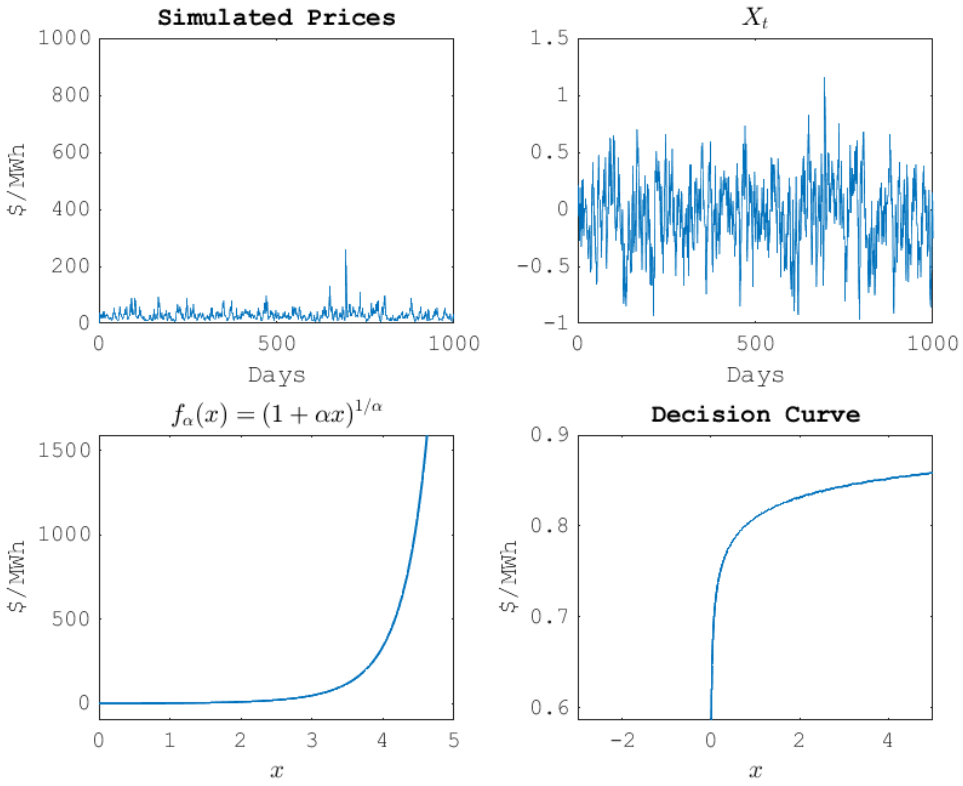

Figure 3.8: Simulations using the parameters generated by Algorithm 3.2.

as those from Figure 3.6; all the differences between these two processes come from the OU parameters. From the perspective of matching Alberta's power stack, (cf. Figure 1.2) this is a more promising result. However, in this case, it is not clear there are advantages over Barlow's approach as long as a smaller λ remains out of reach. Indeed, a two-factor model seems necessary as discussed by Lucia and Schwartz earlier.

3.4 Concluding remarks

We extended Barlow's diffusion model and showed that a decision curve can be used to control price spikes. In the models current form, the decision curve does not influence prices for large X_t since firms are incentivized to sell while prices are high. As such, a large λ remains necessary to revert extreme prices. Nevertheless, we have shown in a comparison of Figure 3.6 and Figure 3.8 that the decision curve can directly generate reasonable price spikes.

In this work we considered a single firm exercising market power. An obvious extension is to consider two or more noncooperative actors with separate decision curves. Perhaps an industrial firm purchasing electricity on the spot market with some flexibility to lower their consumption in response to high prices to share the role of reducing prices following a spike with λ.

Appendix A

A.1 Calculating of $F(t,T) = \mathbb{E}[X_T|\mathcal{F}_t]$

This calculation can be reduced to finding X_t (i.e., solving the SDE) and taking its expectation. We recall the stochastic differential equation defining the OU process X_t:

$$\mathrm{d}X_t = -\lambda(X_t - a)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t.$$

Consider the function $f(x_t, t) = x_t e^{\lambda t}$. By an application of Itô's formula, we see

$$\begin{aligned}\mathrm{d}f(x_t,t) &= \frac{\partial f}{\partial t}\,\mathrm{d}t + \frac{\partial f}{\partial x_t}\,\mathrm{d}x_t + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\,\mathrm{d}x_t^2 \\ &= \lambda x_t e^{\lambda t}\,\mathrm{d}t + e^{\lambda t}\,\mathrm{d}x_t.\end{aligned}$$

By using the relation for $\mathrm{d}X_t$, it follows

$$\begin{aligned}\mathrm{d}f(X_t,t) &= \lambda X_t e^{\lambda t}\,\mathrm{d}t + e^{\lambda t}\,\mathrm{d}X_t \\ &= \lambda X_t e^{\lambda t}\,\mathrm{d}t + e^{\lambda t}(-\lambda(X_t - a)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t) \\ &= e^{\lambda t} a\lambda\,\mathrm{d}t + \sigma e^{\lambda t}\,\mathrm{d}W_t.\end{aligned}$$

Integrating by both sides from T to t, we get

$$\begin{aligned} X_T e^{\lambda T} - X_t e^{\lambda t} &= \int_t^T e^{\lambda s} a\lambda \, \mathrm{d}s + \int_t^T \sigma e^{\lambda s} \, \mathrm{d}W_s. \\ &= a(e^{\lambda T} - e^{\lambda t}) + \int_t^T \sigma e^{\lambda s} \, \mathrm{d}W_s \end{aligned}$$

and

$$X_T = X_t e^{-\lambda(T-t)} + a(1 - e^{-\lambda(T-t)}) + \int_t^T \sigma e^{-\lambda(T-s)} \, \mathrm{d}W_s. \tag{A.1}$$

From this representation, it is straight forward to calculate

$$\begin{aligned} \mathbb{E}[X_T|\mathcal{F}_t] &= \mathbb{E}\left[X_t e^{-\lambda(T-t)} + a(1 - e^{-\lambda(T-t)}) + \int_t^T \sigma e^{-\lambda(T-s)} \, \mathrm{d}W_s | \mathcal{F}_t \right] \\ &= X_t e^{-\lambda(T-t)} + a(1 - e^{-\lambda(T-t)}) + \mathbb{E}\left[\int_t^T \sigma e^{-\lambda(T-s)} \, \mathrm{d}W_s | \mathcal{F}_t \right] \\ &= X_t e^{-\lambda(T-t)} + a(1 - e^{-\lambda(T-t)}) \end{aligned}$$

where the last equality follows from the property of Wiener integrals, $\int_0^t g(s) \, \mathrm{d}W_s \sim \mathcal{N}(0, g(s)^2)$. It is helpful to note that the variance can be easily obtained:

$$\mathbb{V}[X_T|\mathcal{F}_t] = \mathbb{E}\left[\left(\int_t^T \sigma e^{-\lambda(T-s)} \, \mathrm{d}W_s \right)^2 | \mathcal{F}_t \right]$$

and by Itô's isometry,

$$\begin{aligned} \mathbb{E}\left[\left(\int_t^T \sigma e^{-\lambda(T-s)} \, \mathrm{d}W_s \right)^2 | \mathcal{F}_t \right] &= \mathbb{E}\left[\int_t^T \sigma^2 e^{-2\lambda(T-s)} \, \mathrm{d}s \right] \\ &= \frac{\sigma^2}{2\lambda} \left(1 - e^{-2\lambda(T-t)} \right). \end{aligned}$$

A.2 Affine Combinations of an OU Process

It is often useful to take affine transformations of an OU process X_t. We present details of this calculation here. Consider the OU process $\mathrm{OU}(\lambda, a_1, \sigma_1)$ defined by

$$\mathrm{d}D_t = -\lambda(D_t - a_1)\,\mathrm{d}t + \sigma_1\,\mathrm{d}W_t.$$

We will examine the affine transformation $\alpha D_t + \beta$ for arbitrary constants α, β. With $f(x,t) = \alpha x + \beta$, an application of Itô's formula yields

$$\begin{aligned}\mathrm{d}f(x,t) &= \frac{\partial f}{\partial t}\,\mathrm{d}t + \frac{\partial f}{\partial x_t}\,\mathrm{d}x_t + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\,\mathrm{d}x_t^2 \\ &= \alpha\,\mathrm{d}x\end{aligned}$$

and so

$$\begin{aligned}\mathrm{d}f(D_t,t) &= \mathrm{d}(\alpha D_t + \beta) \\ &= -\alpha\lambda(D_t - a_1)\,\mathrm{d}t + \sigma_1\alpha\,\mathrm{d}W_t \\ &= -\lambda(\alpha D_t + \beta - \beta - \alpha a_1)\,\mathrm{d}t + \sigma_1\alpha\,\mathrm{d}W_t \\ &= -\lambda((\alpha D_t + \beta) - (\beta + \alpha a_1)\,\mathrm{d}t + \sigma_1\alpha\,\mathrm{d}W_t.\end{aligned}$$

That is we arrive at the following result:

Theorem 1. *If $D_t \sim \mathrm{OU}(\lambda, a_1, \sigma_1)$ then $\alpha D_1 + \beta \sim \mathrm{OU}(\lambda, \beta + \alpha a_1, \sigma_1\alpha)$ for any constants $\lambda, a_1, \sigma_1, \alpha, \beta$.*

In particular, when $Y_t \sim \mathrm{OU}(\lambda, 0, 1)$ we have $a_1 - \sigma_1 Y_t \sim \mathrm{OU}(\lambda, a_1, \sigma_1) \sim D_t$.

A.2.1 Efficient Computation of $\mathbb{E}_{p,3}$

Numerical integration of $\mathbb{E}_{p,3}$ can be challenging due to the presence of $f\phi$ in the integrand. The function f takes on relatively small values and the Gaussian decays rapidly in regions away its centre. As such, adaptive methods such as `quadgk` may have trouble locating regions where the integrand is nonzero. Thus it is helpful to locate the maximum of $f\phi$ analytically and integrate in a neighbourhood of this point. Although the techniques used in finding the maximum are an elementary, we present the details here.

For an integral

$$I = \phi(z) f(\mu + z\sigma),$$

recall that the domain of f depends on the sign of α:

$$\begin{cases} \alpha < 0 & [-\infty, g(1000)] \\ \alpha > 0 & [-\frac{1}{\alpha}, g(1000)]. \end{cases}$$

Its derivative is

$$\begin{aligned} I' &= -z\phi(z) f(\mu + z\sigma) + \frac{\phi(z) f(\mu + z\sigma)\sigma}{1 + \alpha(\mu + z\sigma)} \\ &\quad \phi(z) f(\mu + z\sigma) \left[\frac{\sigma}{1 + \alpha(\mu + z\sigma)} - z \right] \\ &= \frac{\phi(z) f(\mu + z\sigma)}{1 + \alpha(\mu + z\sigma)} \left[\sigma - z(1 + \alpha(\mu + z\sigma)) \right]. \end{aligned}$$

The critical points thus lie at point satisfying the quadratic:

$$\sigma - z(1 + \alpha\mu) - z^2 \alpha\sigma = 0$$

or

$$z^2 + z\frac{(1+\alpha\mu)}{\alpha\sigma} - \frac{1}{\alpha} = 0.$$

The roots of this equation are

$$z = -\frac{(1+\alpha\mu)}{2\alpha\sigma} \pm \frac{1}{2}\sqrt{\frac{(1+\alpha\mu)^2}{\alpha^2\sigma^2} + \frac{4}{\alpha}} = \frac{1}{2}\frac{(1+\alpha\mu)}{\alpha\sigma}\left[-1 \pm \sqrt{1 + \frac{4\alpha\sigma^2}{(1+\alpha\mu)^2}}\right]$$

At this point, it is straightforward to find the maximum value: either one of these roots or the end points will be the global maxima. However, it is helpful to continue examining the sign of the discriminant. Indeed,

$$1 + \frac{4\alpha\sigma^2}{(1+\alpha\mu)^2} = 0 \implies (1+\alpha\mu)^2 + 4\alpha\sigma^2 = 1 + 2\alpha\mu + \alpha^2\mu^2 + 4\alpha\sigma^2 = 0$$

or

$$\alpha^2 + \alpha\left(\frac{2}{\mu} + \frac{4\sigma^2}{\mu^2}\right) + \frac{1}{\mu^2} = 0.$$

The roots of this equation in α are

$$\alpha = -\frac{1}{2}\left(\frac{2}{\mu} + \frac{4\sigma^2}{\mu^2}\right) \pm \frac{1}{2}\sqrt{\left(\frac{2}{\mu} + \frac{4\sigma^2}{\mu^2}\right) - \frac{4}{\mu^2}}.$$

The root is real whenever

$$\left(\frac{2}{\mu} + \frac{4\sigma^2}{\mu^2}\right)^2 - \frac{4}{\mu^2} = \frac{16\sigma^2}{\mu^3} + \frac{16\sigma^4}{\mu^4} > 0$$

or whenever $\mu + \sigma^2 > 0$. In most cases, this criterion will hold and the integrand will have critical points.

A.3 Simulating Stochastic Processes

The *Euler–Maruyama* method is analogous to the Euler method for ordinary differential equations. Consider the Ornstein–Uhlenbeck stochastic differential equation

$$\mathrm{d}X_t = -\lambda(X_t - a)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$$

where W_t is a Brownian motion or *Weiner process*. This can be discretized by

$$X_{n+1} = X_n - \lambda(X_n - \mu)\Delta t + \sigma\Delta W_n$$

where ΔW_n are iid Wiener increments with zero mean and variance Δt. That is $\Delta W_n = [W_{t+h} - W_t] \sim \Delta t\mathcal{N}(0,1)$, where the time step $\Delta t = h$. Thus given an initial value $X_0 = x_0$, one only needs to sample normal random variates to generate sample paths.

As seen in Appendix A.1, an OU process has an analytic solution for its associated SDE described by (A.1). This can then be simulated by

numerically approximating the stochastic integral by

$$\int_0^T \sigma e^{-\lambda(T-s)}\,\mathrm{d}W_s = \sum_{n=0}^{N-1}\int_{t_n}^{t_{n+1}} \sigma e^{-\lambda(t_{n+1}-s)}\,\mathrm{d}W_s \approx \sigma\sum_{n=0}^{N-1} e^{-\lambda(\Delta t)}\Delta W_n.$$

where $t_0 = 0$ and $t_{n+1} - t_n = T/N = \Delta t$.

A final method we will discuss uses the fact that an OU process X_t conditioned on an initial value $X_0 = x_0$ is normal with mean

$$\mathbb{E}[X_t|X_0 = x_0] = X_0 e^{-\lambda t} + a(1 - e^{-\lambda t})$$

and variance

$$\mathbb{V}[V_t|X_0 = x_0] = \frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda t}).$$

It follows that we can simply simulate X_t by

$$X_t = x_0 e^{-\lambda t} + a(1 - e^{-\lambda t}) + \varepsilon\sqrt{\frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda t})}$$

where ε is a standard normal random variable. A sample path can then be created using the scheme

$$\begin{cases} X_0 = x_0, \\ X_{n+1} = x_n e^{-\lambda \Delta t} + a(1 - e^{-\lambda \Delta t}) + \varepsilon\sqrt{\frac{\sigma^2}{2\lambda}(1 - e^{-2\lambda \Delta t})}. \end{cases}$$

A.4 An alternative formulation

Here we present an equivalent perspective to the one found in Section 3.1. From this perspective, we introduce a function π that changes the withholding amounts as we integrate over the domain. We include this perspective because it may be easier to implement generalizations of the decision curve model. In Section 3.1, we introduced the notation $\Lambda_{p,i}$, $i = 1, 2, 3$ to distinguish different potential payoffs. Here, the choice of function π determines the payoffs a firm will receive of which the $\Lambda_{p,i}$, $i = 1, 2, 3$ from (3.6), (3.7), and (3.10) are special cases of π. For illustrative purposes, we choose π so that the expectation is equivalent to (3.10).

We redefine our function $f_\alpha(x)$ to

$$f_\alpha(x) \stackrel{\text{def}}{=} \begin{cases} \min(1000, (1 + \alpha x)^{1/\alpha}), & \text{if } 1 + \alpha x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

We do this to extend the domain of f to all of $\mathbb{R}$ to assist when integrating f against

the support of the normal density. This modified f takes the value of 0 whenever the spot price S_t is negative and 1000 if S_t exceeds the price cap. We further set for notational compactness,

$$\text{xmax} \stackrel{\text{def}}{=} \frac{1000^{\alpha} - 1}{\alpha} = x_*.$$

Here X_* is the smallest value such that $f_\alpha(x_*) = 1000$. Note that $x_* > 0$ for all $\alpha \in \mathbb{R}$. Then if $\alpha \leq 0$,

$$f_\alpha(x) = (1 + \alpha x)^{\frac{1}{\alpha}}, \qquad -\infty < x < x_*$$

and if $\alpha > 0$,

$$f_\alpha(x) = (1 + \alpha x)^{\frac{1}{\alpha}}, \qquad -\frac{1}{\alpha} < x < x_*.$$

Recall that the conditional transition density of an OU process satisfies $X_{n+1}|X_n = x_n \sim \mathcal{N}(\mu_n, \sigma^2)$ and $S_n = f_\alpha(x_n)$. The amount of power supplied by our firm is a function π of p and x times $M\theta$, i.e., $M\theta\pi(x,p)$ where M is the maximum supply the agent controls and $\theta \in [0,1]$ a measure of market power. Then whenever $x = x_* + M\tau\theta p$ we will supply an additional p units of power and from this equation, we write

$$p = \frac{x - x_*}{\xi}, \qquad \xi = M\tau\theta p$$

$$\pi(p,x) = \begin{cases} 1 - p & x < x_* \\ 1 - p + \frac{x - x_*}{\xi} & x_* \leq x < x_* + Mp\theta\tau = x_* + \xi p \\ 1, & x > x_* + \xi p. \end{cases}$$

The revenue is then $M\theta\pi(p,x)f_\alpha(x)$, and the expected revenue is

$$\mathbb{E}(P_t) = \frac{M\theta}{\sigma} \int_{\mathbb{R}} f_\alpha(x)\pi(p,x)\phi\left(\frac{\mu_n + \xi p - x}{\sigma}\right) \mathrm{d}x$$

To calculate this, we separate it into three integrals:

$$\begin{aligned}
\mathbb{E}(P_t) = &\frac{M\theta(1-p)}{\sigma} \int_{-\infty}^{x_*} f_\alpha(x)\phi\left(\frac{\mu_n + \xi p - x}{\sigma}\right) \mathrm{d}x \\
&+ \frac{1000M\theta}{\sigma} \int_{x_*}^{x_*+\xi p} \left[\frac{x - x_*}{\xi} - p\right] \phi\left(\frac{\mu_n + \xi p - x}{\sigma}\right) \mathrm{d}x \\
&+ \frac{1000M\theta}{\sigma} \int_{x_*}^{\infty} \phi\left(\frac{\mu_n + \xi p - x}{\sigma}\right) \mathrm{d}x. \\
= &M\theta[I_1 + I_2 + I_3]
\end{aligned}$$

We will label these integrals I_1, I_2, and I_3 respectively.

The calculation of I_3 is straight forward, under the change of variables $\sigma z = x - \xi p - \mu_n$,

$$I_3 = \frac{1}{\sigma} \int_{x_*+\xi p}^{\infty} \phi\left(\frac{\mu_n + \xi p - x}{\sigma}\right) \mathrm{d}x = \int_{\frac{x_*+\mu_n}{\sigma}}^{\infty} \phi(z)\, \mathrm{d}z = \Phi\left(-\frac{x_* + \mu_n}{\sigma}\right).$$

Next for I_2, under the same change of variables,

$$\frac{1}{\sigma} \int_{x_*}^{x_*+\xi p} \left[\frac{x - x_*}{\xi} - p\right] \phi\left(\frac{\mu_n + \xi p - x}{\sigma}\right) \mathrm{d}x = \int_{\frac{x_*-\xi p-\mu_n}{\sigma}}^{\frac{x_*-\mu_n}{\sigma}} \frac{\sigma z + \mu_n - x_*}{\xi} \phi\left(z\right) \mathrm{d}z$$

which evaluates to

$$\frac{\mu_n - x_*}{\xi} \left[\Phi\left(\frac{x_* - \mu_n}{\sigma}\right) - \Phi\left(\frac{x_* - \xi p - \mu_n}{\sigma}\right)\right] - \frac{\sigma}{\xi} \left[\phi\left(\frac{x_* - \mu_n}{\sigma}\right) - \phi\left(\frac{x_* - \xi p - \mu_n}{\sigma}\right)\right].$$

Finally, I_1 has three cases depending on the value of α:

$$I_1 = \begin{cases} \frac{1}{\sigma}\int_{-\infty}^{x_*}(1+\alpha x)^{\frac{1}{\alpha}}\phi\left(\frac{\mu_n+\xi p-x}{\sigma}\right)\mathrm{d}x, & \alpha<0 \\ \frac{1}{\sigma}\int_{-1/\alpha}^{x_*}(1+\alpha x)^{\frac{1}{\alpha}}\phi\left(\frac{\mu_n+\xi p-x}{\sigma}\right)\mathrm{d}x, & \alpha>0 \\ \frac{1}{\sigma}\int_{-\infty}^{x_*}e^{x}\phi\left(\frac{\mu_n+\xi p-x}{\sigma}\right)\mathrm{d}x, & \alpha=0. \end{cases}$$

Case 1: $\alpha < 0$. Under the change of variables $(1+\alpha x) = e^{\alpha y}$, the integral becomes

$$\frac{1}{\sigma}\int_{-\infty}^{\frac{\ln(1+\alpha x_*)}{\alpha}} e^{y}e^{(1+\alpha)y}\phi\left(\frac{\mu_n+\xi p-\frac{e^{\alpha y}-1}{\alpha}}{\sigma}\right)\mathrm{d}y$$

We further change variables under $z = \ln(1+\alpha x_*)/\alpha - y$ and note that $\exp(z+y) = 1000$ to get

$$\frac{1000(1+\alpha x_*)}{\sigma}\int_0^{\infty} e^{-(1+\alpha z)}\phi\left(\frac{\mu_n+\xi p-\frac{(1+\alpha x_*)e^{-\alpha z}-1}{\alpha}}{\sigma}\right)\mathrm{d}z.$$

Case 2: $\alpha > 0$. Note that the integrand is the same as in the first case. As such the only thing that differs is the lower limit of integration $-1/\alpha$. However, it is easy to verify that even the limits of integration are unchanged in this case.

Case 3: $\alpha = 0$. In this case, use the change of variables $\sigma z = x - \xi p - \mu_n$ and write

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x_*} e^{x}e^{-\frac{1}{2}\left(\frac{\mu_n+\xi p-x}{\sigma}\right)^2}\mathrm{d}x = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{x_*-\xi p-\mu_n}{\sigma}} e^{\sigma z+\xi p+\mu_n}e^{\frac{z^2}{2}}\,\mathrm{d}z$$

It is a routine exercise in completing the square to get

$$\frac{e^{\xi p+\mu_n+\frac{\sigma^2}{2}}}{\sqrt{2\pi}}\int_{-\infty}^{\frac{x_*-\xi p-\mu_n}{\sigma}} \phi(z-\sigma)\,\mathrm{d}z = e^{\xi p+\mu_n+\frac{\sigma^2}{2}}\Phi\left(\frac{x_*-\xi p+\mu_n+\sigma^2}{\sigma}\right).$$

In the above model, I_2 and I_3 have explicit forms that are easily calculated. However I_1 requires numerical methods to evaluate. Due to the asymptotic be-

haviour of the normal pdf, values outside of a certain range will be very close to zero and in our experience, numerical routines may struggle to find the region where the integral is "interesting" without help by analytically finding the maximum of the integrand. In this subsection, we will find the maximum of the integrand for I_1 when $\alpha \neq 0$. Here, the integrand is

$$f(z) = e^{-(1+\alpha)z}\phi\left(\underbrace{\frac{\mu_n + \xi p + \frac{1}{\alpha}}{\sigma}}_{A} - \underbrace{\frac{(1+\alpha x_*}{\alpha\sigma}}_{B} e^{-\alpha z}\right)$$

and the derivative is thus

$$f'(z) = \left(-(1+\alpha) - \alpha B e^{-\alpha z}(A - Be^{-\alpha z})\right) f(z).$$

Under the change of variables $\gamma = e^{-\alpha z}$, the roots of the derivative are determined by a quadratic equation in γ:

$$-(1+\alpha) - \alpha AB\gamma + \alpha B^2\gamma^2 = 0.$$

Since $\gamma = e^{-\alpha x} \geq 0$ for all x, it suffices to consider only the postive roots.

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