A 2-D Signal Processing Model to Predict the Effect of Mutual Coupling on Array Factor

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***Abstract*—A semi-analytical method for modeling the effects of electromagnetic mutual coupling in uniform linear array (ULA) of N antennas is proposed. The coupling is described as a two-dimensional (2-D) spatiotemporal transfer function derived from S-parameter measurements. The proposed 2-D transfer function enables prediction of the distortions in array factor due to coupling, and thereby enable the potential design of coupling-compensation algorithms. The method is verified with simulations in the 1.5-2.0 GHz range on both an $N = 7$ element ULA using CST Microwave Studio using 50 Ω terminations and a $N = 3$ element ULA in FEKO but with non-50 Ω impedance obtained from measurements of a CMOS low noise amplifier (LNA). Coupling effect on array factor of delay-sum type beamformer was examined. The proposed model matches within an error of 4-12% and 4-10% with respect to the results from two full-wave electromagnetic simulators CST Microwave Studio and FEKO, respectively, in the frequency range 1.75-2 GHz.**

***Index Terms*—Mutual coupling, antenna arrays, beamforming.**

I. INTRODUCTION

MUTUAL coupling (MC) is an unavoidable electromagnetic effect present in antenna arrays, which results due to the near-field photonic interaction of antennas located spatially close to one another [1]–[4]. Mutual coupling leads to an undesirable increase in the system noise figure [5]–[7], distorted array patterns [6], and reduced capacity in communications systems [8]. MC results in active reflection coefficients at the terminating low noise amplifiers (LNAs), which has resulted in simultaneous noise and power matching for a single RF beam at a particular frequency [7], [9], [10]. To achieve optimal performance, the effects of MC need to be considered during the design of the array processing algorithm [1], [11], [12]. Well-known methods for analyzing the effects of MC on far-field array pattern are based on statistical algorithms, which exhibit high computational intensity [13]. In [14], element patterns were predicted using a one-time far field measurement of the E-field. Full-wave computational electromagnetic models for a large array of antennas take into account MC between elements. However, such models are extremely high in computational complexity.

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In this work, we use multidimensional signal processing (MDSP) and S parameters to predict the effect of MC on array factor. We derive a two dimensional (2-D) spatio-temporal transfer function to model the MC effect on a uniform linear array (ULA) of antennas. The proposed transfer function takes the S-parameters (either measured or simulated) of the ULA as the input to predict the distortion in the array factor due the effect of MC. Three assumptions are made in this work: i) an array prototype and its S-parameters are available from either full-wave simulations or measurements, ii) the array prototype can be a sub-array of a larger array iii) the single element pattern (in isolation) is obtained using either a full-wave simulation or measurement and is available.

We will use the following notations. Spatially discrete temporally continuous (i.e. mixed) 2-D signal space is the pair $(n_x, ct) \in \mathbb{Z} \times \mathbb{R}$, where n_x is the ULA antenna index, $c \approx 3 \times 10^8$ is speed of light and t is time. The corresponding 2-D mixed transform domain is $(z_x, s_{ct}) \in \mathbb{C}^2$, where z_x is the spatial **z**-transform variable and s_{ct} is the normalized temporal Laplace transform variable. For the 2-D signal $a\,(n_x, ct)$ the 1-D and 2-D transforms are defined as $\mathcal{T}_{ct}\{a(n_x, ct)\} \overset{1-D}{\Leftrightarrow} \mathcal{A}(n_x, s_{ct})$ and $\mathcal{T}_{x,ct}\{a(n_x, ct)\} \overset{2-D}{\Leftrightarrow} A(z_x, s_{ct})$, respectively. Also, $[\mathbf{A}]_{i,j}$ represents the ij^{th} element of a matrix $\mathbf{A}$, $\mathbf{I}_N$ is the $N \times N$ identity matrix, and $||z|| = \sqrt{z\bar{z}}$, for $z \in \mathbb{C}$ is the complex modulus.

II. MUTUAL COUPLING 2-D TRANSFER FUNCTION

Consider a ULA of N antennas with a subsequent array processing algorithm. The total array response can be represented by a 2-D mixed transform domain transfer function as $T_E\,(z_x, s_{ct}) = A_E\,(z_x, s_{ct})\, A_B\,(z_x, s_{ct})\, H_C\,(z_x, s_{ct})$, where $A_E\,(z_x, s_{ct})$ models the radiation pattern of a single antenna in isolation, $A_B\,(z_x, s_{ct})$ models the array factor produced by the underlying array processing algorithm [15], [16] and $H_C\,(z_x, s_{ct})$ is the proposed transfer function that models the effect of MC. Here, $H_C\,(z_x, s_{ct})$ represents the multiplicative MC effect in the 2-D transform domain and is defined as

$$H_C(z_x, s_{ct}) = \int_0^{\infty} \sum_{n_x=0}^{N_x-1} h_C(n_x, ct) z_x^{-n_x} e^{-s_{ct}ct} cdt \quad (1)$$

where $h_C(n_x, ct)$ is the 2-D impulse response that represents the coupling between the ULA elements. Explicit knowledge of $H_C\,(z_x, s_{ct})$ allows the prediction of distortions in the total array response and thereby embed potential compensation schemes into the array processing algorithm $A_B\,(z_x, s_{ct})$. In

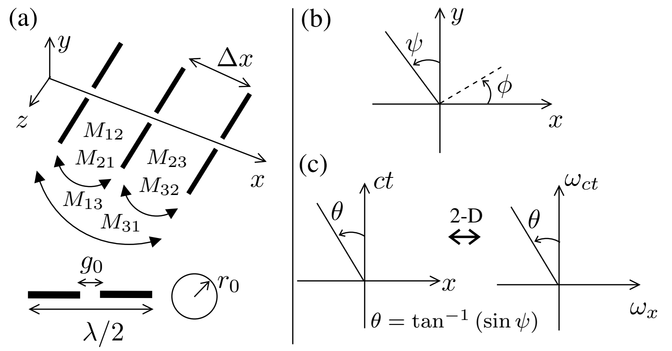

Fig. 1. (a) A ULA of $N = 3$, $\lambda/2$ dipole antennas placed along the x-axis, where the MC is represented by M_{ij}, $i, j = 1, 2, 3$, $r_0 \approx 15\ mm$, $g_0 \approx 0.375\ mm$ and $\Delta x = \lambda/2 \approx 73.89\ mm$. (b) Spatial angle convention on the $x-y$ plane, where ϕ is the azimuth angle and ψ is the spatial direction of arrival of the impinging EM waves. (c) Angle convention in (x, ct) and 2-D frequency domain (ω_x, ω_{ct}).

subsequent sections we derive $H_C(z_x, s_{ct})$ using the measured/simulated S-parameters of the ULA prototype.

A. Relationship Between Mutual Coupling and S-Parameters

Typically, each antenna in the ULA is connected to a low noise aimplifer (LNA) via a transmission line segment. For simplicity, we assume N identical LNAs corresponding voltage $v_{l,n}(ct) \Leftrightarrow V_{l,n}(s_{ct})$.

Let Z_A be the impedance seen when looking out of the antenna ports (into the LNA) and the characteristic impedance of the transmission line be Z_0. For a single antenna, $V_{l,n}(s_{ct}) = Z_A V_{i,n}(s_{ct})/(Z_M + Z_A)$ where Z_M is the measured mutual impedance, and $v_{i,n}(ct) \Leftrightarrow V_{i,n}(s_{ct})$ is the voltage induced onto the antenna. We can express this relationship for the entire array in terms of measurable S-parameters, characteristic, and amplifier impedance as [3], [7], [17]

$$\mathbf{v}_l = \mathbf{Z}_A \left(\mathbf{Z}_M + \mathbf{Z}_A\right)^{-1} \mathbf{v}_i \tag{2}$$

where $\mathbf{v}_l = [V_{l,1} \ \cdots \ V_{l,N}]^T \in \mathbb{C}^N$ and $\mathbf{v}_i = [V_{i,1} \ \cdots \ V_{i,N}]^T \in \mathbb{C}^N$, $\mathbf{Z}_M = (\mathbf{I}_N - \mathbf{S})^{-1}(\mathbf{I}_N + \mathbf{S})\mathbf{Z}_0$, $\mathbf{Z}_A = Z_A \mathbf{I}_N$, and $\mathbf{Z}_0 = Z_0 \mathbf{I}_N$.

We then define the MC matrix $\mathbf{M}_N \triangleq \mathbf{Z}_A \left(\mathbf{Z}_M + \mathbf{Z}_A\right)^{-1}$ with elements consisting of Laplace functions $[\mathbf{M}_N]_{i,j}(s_{ct}) \in \mathbb{C}$ that have corresponding impulse responses in the temporal domain denoted as $[\mathbf{m}_N]_{i,j}(ct)$. These impulse responses enable MC to be represented by a matrix of 1-D linear filters.

B. Mutual Coupling for an N-Element ULA

As shown in Fig. 1 (a), we consider a ULA of vertically-polarized thick-dipole elements located on the x-axis, where Δx is the inter-element spacing (i.e. the spatial sampling period). With respect to some reference element in the ULA, each element can be indexed as $n_x \pm m$ for $m \in \mathbb{Z}$, where $m = 0$ for the reference element. This convention allows us to utilize the MC matrix in (2) to write the MC transfer function.

In general, for an N element array where N is odd and the center as the reference element we have

$$H_C(z_x, s_{ct}) = [\mathbf{M}_N]_{\eta,\eta}(s_{ct}) + \sum_{n=1}^{\eta-1} \left([\mathbf{M}_N]_{\eta,\eta-n}(s_{ct})\, z_x^{n} + [\mathbf{M}_N]_{\eta,\eta+n}(s_{ct})\, z_x^{-n} \right) \tag{3}$$

where $\eta = \frac{1}{2}(N+1)$.

The 2-D frequency response of the MC transfer function is obtained by computing (3) on the unit circle $z_x = e^{j\omega_x}$ and on the imaginary axis $s_{ct} = j\omega_{ct}$, where ω_k is the normalized frequency variable corresponding to dimension $k \in \{x, ct\}$. However, for comparison purposes with the full-wave electromagnetic simulations it is required to compute the polar response corresponding to $H_C\left(e^{j\omega_x}, j\omega_{ct}\right)$ as follows.

As shown in Fig. 1 (b) and (c), the angle ψ measured from the array broadside in $(x, y) \in \mathbb{R}^2$ corresponds to angle $\theta = \tan^{-1}(\sin\psi)$ in the $(x, ct) \in \mathbb{R}^2$ and $(\omega_x, \omega_{ct}) \in \mathbb{R}^2$ domains [15], [18]. Therefore, along the spatial direction ψ, in the 2-D frequency domain we have the relationship $\omega_x = -\omega_{ct}\tan\theta = -\omega_{ct}\sin\psi$ [15], [19]. Thus, the polar response of the MC transfer function can be computed at a particular temporal frequency ω_{ct} as a function of the spatial angle ψ as $H_C\left(e^{-j\sin\psi\omega_{ct}}, j\omega_{ct}\right)$. To be consistent with the convention used by the method-of-moments (MoM) based EM simulation using both CST microwave studio and FEKO software the angle ϕ should be used, where $\phi = \pi/2 + \psi$.

C. Symmetric Coupling for ULA

For a ULA of N elements, where N is odd, by symmetry, we can assume that $[\mathbf{M}_N]_{\eta,\eta-n}(s_{ct}) \approx [\mathbf{M}_N]_{\eta,\eta+n}(s_{ct})$. Validity of this assumption is confirmed by simulations in a subsequent section in the paper. In general, for the center element of an N (odd) element ULA we have

$$H_C(j\omega_{ct}, \phi) = k_0(s_{ct}) + 2\sum_{n=1}^{\eta} k_n(s_{ct}) \cos\left(n\omega_{ct}\cos\phi\right), \tag{4}$$

where $k_0(s_{ct}) = [\mathbf{M}_N]_{\eta,\eta}(s_{ct})$ and $k_n(s_{ct}) = [\mathbf{M}_N]_{\eta,\eta-n}(s_{ct}) \approx [\mathbf{M}_N]_{\eta,\eta+n}(s_{ct})$.

In the spatial domain, the symmetry in MC manifests as a phase shift of π in the ϕ direction between a given outer element and its corresponding element on the other side of the center element in the ULA.

III. Antenna-Array Full-Wave Simulations

We employ the computational electromagnetic tools CST Microwave Studio and FEKO to simulate an N-element ULA and use the results to validate the field patterns predicted by the proposed MC transfer function for both $Z_A = 50\ \Omega$ and $Z_A \neq 50\ \Omega$. The ULA configuration is shown in Fig. 1 (a) where half-wave length dipole antennas of radius r_0 and port gap g_0 are assumed [3]. The S-parameter matrix $\mathbf{S}$ can be found using the transient domain solver, by which the MC transfer function $H_C(z_x, s_{ct})$ can be computed. Note that in a practical setting, the S-parameters could be measured from the fabricated antenna array with a network analyzer. The element pattern at temporal frequency f_0 [Hz] can be found by the 2-D transform domain transfer function of a single element $A_E(z_x, s_{ct})$ by computing $A_E(e^{j2\pi f_0 \sin\psi}, j2\pi f_{c0})$ where ψ is the spatial angle. Computational electromagnetic simulations at N_s discrete frequency points $f_{c0}, k = k\Delta f/c, k = 1, 2, 3, ..., N_s - 1$ produce N_s complex element patterns which can be used to approximate $A_E(e^{j\omega_x}, j\omega_{ct})$ in the MATLAB environment.

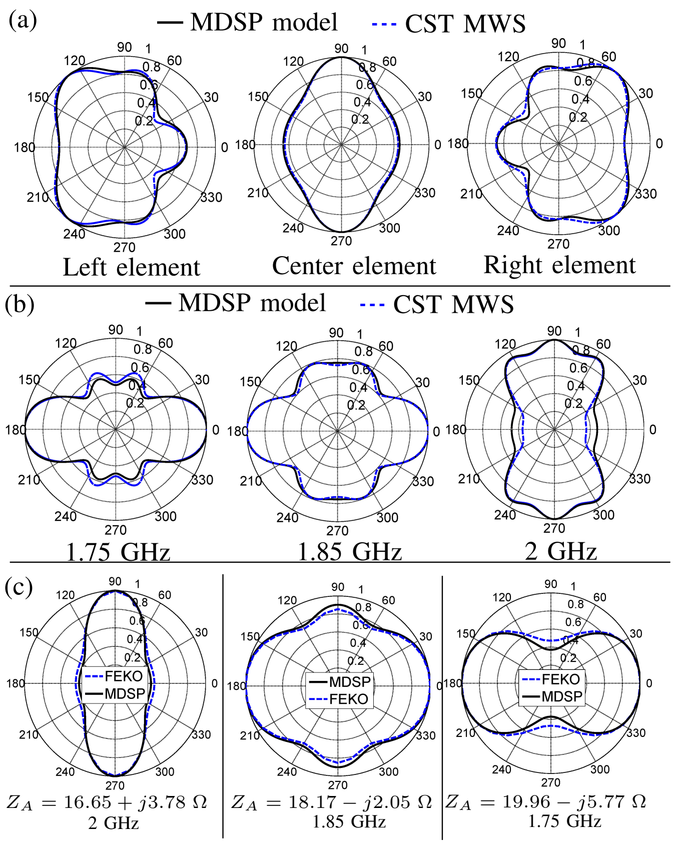

Fig. 2. (a) Case 1: Coupling effect for $N = 3$ ULA with $\Delta x = \lambda/2$ spacing at 2 GHz computed for left, center and right elements. (b) Case 2: Normalized magnitude of the MC transfer function $||H_C(e^{-j\omega_{ct}\sin\psi}, j\omega_{ct})||$ for an $N = 7$ element ULA with $\Delta x = \lambda/2$ spacing at three different frequencies. (c) Case 3: Normalized $||H_C(e^{-j\omega_{ct}\sin\psi}, j\omega_{ct})||$ on the center element in a $N = 3$ element ULA of dipole elements with $\Delta x = \lambda/2$ obtained from both proposed model and FEKO simulations at 2 GHz, 1.85 GHz and 1.75 GHz. Simulations assume an LNA connected to antenna ports.

A. Validation of the MC model

Note that, by considering a ULA of dipole elements, we are able to assume omni-directional radiation patterns on the $x-y$ plane, leading to $A_E(e^{j\omega_x}, j\omega_{ct}) \equiv 1$. Therefore, on the $x-y$ plane, the far-field radiation patterns observed in the full-wave EM simulators are due to the effect of MC only and can be directly compared with the radiation patterns predicted by the proposed model. We provide three simulation cases to validate the proposed MC model.

Case 1: *3-Element Dipole ULA, 2 GHz, Multi-Elements, 50 Ω LNA:* We predict the far-field beam pattern of each element in a $N = 3$ element ULA with $\lambda/2$ spacing. We assume dipole antennas with all far-field patterns computed in the horizontal (i.e. $x-y$) plane. All 3 elements are considered at the operational frequency of 2 GHz. We also assume that a perfectly impedance matched (i.e. 50 Ω) LNA is connected to each antenna. The far-field patterns predicted by the proposed MC model for the left. central and right elements are compared with the radiation patterns obtained by CST Microwave Studio. These patterns are shown in Fig. 2 (a), where it is observed that the radiation pattern predicted by the S-parameters at the antenna ports in combination with the proposed MC model agrees well with the results obtained by full-wave simulations in CST Microwave Studio.The symmetry of the patterns for the center and outer elements should be noted and compared to the phase shift discussion in Section. II-C.

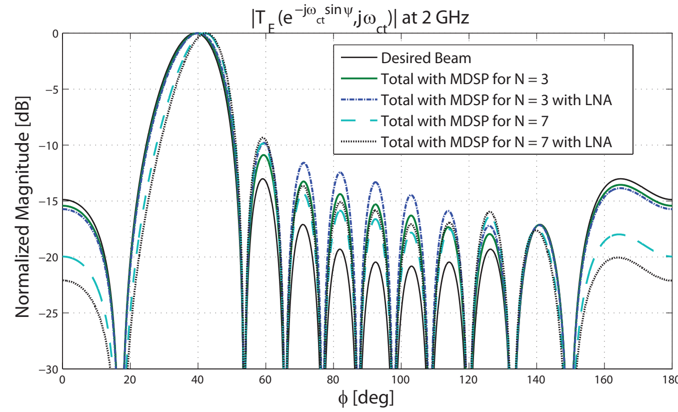

Fig. 3. Predicted effect on the total array pattern including the array factor of the delay-sum beamformer and MC effect. We compute the total array pattern at 2 GHz by setting $z_{ct} = j\omega_{ct}$ and $z_x = e^{-j\omega_{ct}\sin\psi}$ in $T_E(z_x, s_{ct})$. We only consider the MC significant from the nearest elements to the center of the 11 element ULA for both $N = 3$ and $N = 7$.

Case 2: *7-Element Dipole ULA, Multi-Frequency, 50Ω LNA:* We compute the far-field radiation pattern of the *central element* in an $N = 7$ element ULA of dipole antennas, computed in the $x-y$ plane. To demonstrate the broadband behavior of the model, the patterns are computed at three different frequencies 1.75 GHz, 1.85 GHz, and 2.0 GHz. The far-field patterns obtained using the S-parameters at the antenna ports in combination with the proposed MC model is provided in Fig. 2 (b) and is observed to be in good agreement (within an error of 12%) with the far-field patterns obtained directly via full-wave simulation in CST Microwave Studio.

Case 3: *3-Element Dipole ULA, Multi-frequency, CMOS LNA:* We test the model when the antennas are terminated by a practical LNA having frequency dependent load impedance. The measured S-parameters from a custom CMOS LNA integrated circuit we had previously fabricated in 90 nm CMOS [17] was used in place of the ideal 50 Ω terminations. Fig. 2 (c) shows the far-field pattern at the center of the $N = 3$ element dipole ULA, computed at three frequencies. The measured S-parameters of the ULA in conjunction with the proposed MC model clearly provides a good approximation (within an error of 4-10%) of the far-field pattern when compared to the full-wave simulated patterns from FEKO, when $Z_A \neq 50\ \Omega$. For comparison purposes, FEKO was selected for this case, because CST Microwave Studio does not allow the termination of the ULA with frequency dependent impedances.

B. Predicted Effect on Beamforming

We use the proposed MC model to predict distortions in the array factor of a beamformer due to MC. We use the classical weighted delay-and-sum beamformer given by

$$A_B(j\omega_{ct}, \phi, \alpha) = \frac{\sin[\frac{N\omega_{ct}}{2}(\cos\phi - \cos\alpha)]}{\sin[\frac{\omega_{ct}}{2}(\cos\phi - \cos\alpha)]}, \quad (5)$$

where $0 \leq \alpha \leq \pi$ is the main lobe direction. We compute the response of the beamformer with and without the effect of MC (i.e. $A_B(j\omega_{ct}, \phi, \alpha)$ and $A_B(j\omega_{ct}, \phi, \alpha)H_C(e^{-j\omega_{ct}\sin\psi}, j\omega_{ct})$, respectively) at 2 GHz

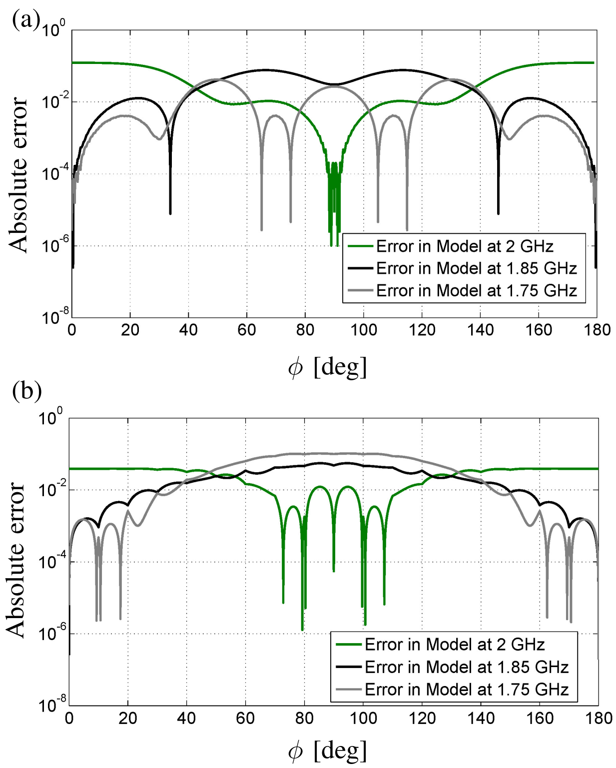

Fig. 4. Absolute error between the MC model $H_C(e^{-j\omega_{ct}\sin\psi}, j\omega_{ct})$ and the full-wave EM simulations for (a) $N = 7$ ULA with $Z_A = 50\ \Omega$ and (b) $N = 3$ ULA including the impedance of the LNA.

for $N = 7$ and $N = 3$ element cases. Note that we use an 11-element beamformer, but we are only considering the MC from the elements closest to the center for $N = 3$ and $N = 7$. This is a fair assumption as the effect of MC from distant elements decays quite rapidly [4]. Fig. 3 shows the distortion in the main and side lobes due to the effect of MC, where it is observed that the desired main lobe direction has changed in the worst case from $\alpha = 39.27^o$ to $\alpha' = 44.7^o$ and an increase in power of the side lobes with a worst case of ≈ 7.17 dB

C. Error in the MC Model

We quantify the pattern differences in Fig. 2 using the error between full wave simulations and the MDSP model as shown in Fig. 4 (a).

For $N = 7$ the MC model has a maximum error relative to the CST Microwave Studio simulations at 2 GHz of 12%, at 1.85 GHz of 7.6%, and at 1.75 GHz of 4.2%. As shown in Fig. 4 (b), the MC model for $N = 3$ has a maximum error relative to the FEKO simulations at 2 GHz of 3.9%, at 1.85 GHz of 5.5%, and at 1.75 GHz of 10.1%.

IV. Conclusions

We proposed a semi-analytical method using the concepts of MDSP and S-parameters to derive a 2-D spatiotemporal transfer function, that models the effect of MC in a ULA of antennas. The proposed model matches with the results obtained from the full-wave EM simulation within a maximum error of 12%, thereby validating the hypothesis that measured S-parameters can be used in a semi-analytical MDSP framework to accurately predict the effect MC towards the array pattern.

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