THE UNIVERSITY OF CALGARY

# Attenuation of the effect of harmonic distortion on synthetic Vibroseis data using an "exact" wave-shaping filtering method.

by
Johanne Lortie

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "Attenuation of the effect of harmonic distortion on synthetic Vibroseis data using an "exact" wave-shaping filtering method" submitted by Johanne Lortie in partial fulfillment of the requirements for the degree of Master of Science.

F A Cook

Supervisor, Dr. F. A. Cook
Dept. of Geology and Geophysics

E S Krebes

Dr. E. S. Krebes
Dept. of Geology and Geophysics

Dr. R. R. Stewart
Dept. of Geology and Geophysics

P Lancaster

Dr. P. Lancaster
Dept. of Mathematics and Statistics

May 13, 1988

## ABSTRACT

When acquiring Vibroseis data in the field, differences between the shape of the pilot and recorded sweep may be observed. Part of these differences are attributed to the distortion of the pilot sweep at the source. A specific type of distortion affecting the pilot sweep results from the non-linear effects in the vibrator hydraulics and the non-linear reaction of the ground to the force exerted. According to Seriff and Kim (1970), these non-linear processes have the effect of generating a series of waveforms whose frequencies are harmonics of those of the pilot sweep. The distortion resulting from the superposition of these harmonics on the pilot sweep is known as harmonic distortion.

The effect of harmonic distortion on Vibroseis data is to generate a series of wavetrains which can interfere with seismic reflections and consequently alter the quality of the data. In order to attenuate this effect, several methods were designed to reduce harmonic distortion at the source. Although efficient, these methods have not always been used and some data are affected.

The effect of harmonic distortion may be attenuated during the processing step using a filtering method. The filtering method chosen for this specific problem is the "exact" wave-shaping filtering method designed by Mereu (1976). This technique, entirely defined in the time domain, presents the interesting advantage of producing an "exact" output in the zone of interest. Modifications to the design of the filter were brought to keep the size of the wave-shaping filter within reasonable limits. The filtering technique was applied on sweeps of different lengths and the results obtained were very encouraging.

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# INTRODUCTION

The purpose of this thesis is to study a filtering method designed by Mereu (1976), called the "exact" wave-shaping filtering technique. This method, entirely defined in the time domain, involves a process by which zeros are gradually inserted between the non-zero terms of a symmetric time series. Mereu (1976), (1977), (1978) showed that the technique is efficient at defining good quality filters under the condition that the Z-transform of the given wavelet has no roots lying on the unit circle.

In the present study, an attempt is made to use this filtering technique to attenuate the effect of harmonic distortion on Vibroseis data. Harmonic distortion is the distortion of the pilot sweep caused by the superposition of its harmonics. The effect of harmonic distortion on a correlated Vibroseis trace is to add a series of wavetrains which may interfere with some reflections. Because these wavetrains can seriously affect the quality of the data, many authors such as Seriff and Kim (1970), Cunningham (1979), Eisner (1974) and Rietsch (1981) have described the precautions to take and/or the methods to use to avoid interference with the data in the zone of interest. Although these methods are efficient, they have not always been used in the field. Consequently some Vibroseis data are affected by correlation noise. Cook (1984) gives an example of seismic data affected by correlation noise. In the present study, it is suggested using the "exact" wave-shaping filtering technique to attenuate the effect of harmonic distortion.

Many different filtering methods have been designed to attenuate the noise in seismic data. In the time domain, two methods are commonly used: the inverse wave-shaping filtering method and the Wiener or least-squares filtering method. In the first method, the filter is obtained by polynomial factorization and division of the desired wavelet by the given wavelet. In the second method, the filter is defined by minimizing the error between the actual and desired output. Many authors, for example Robinson and Treitel (1967), Claerbout and Robinson (1964), and Treitel and Robinson (1964), have studied the applications and limitations of these two methods. Mereu (1976) compared the results obtained with his method to the results obtained using the Wiener filtering method. His conclusion is that his filtering technique gives an "exact" output in the zone of interest while Wiener's filtering technique gives an optimum output in a least-squares sense over the same zone of interest.

In Mereu's filtering technique, the filter is generally stable provided the Z-transform of the given wavelet has no roots lying on the unit circle. Because an efficient filter giving an exact output can theoretically be obtained, Mereu's filtering technique was chosen to attenuate the effect of harmonic distortion on Vibroseis data.

Early in this study, a limitation of the "exact" wave-shaping filtering method was observed. Effectively, the length of the filter tends to become very long and can even overcome the working space available on a computer. In such a case, no good quality filter can be obtained. However, it was realized that the significant portion of the filter is usually contained in a restricted time interval. The original computer program included in Mereu (1978) was then modified to allow the truncation of the filter during the calculation of its coefficients. This new truncation technique, referred to as the "alternate technique", is very efficient and is recommended when using long input wavelets.

Some examples of wave-shaping filters specifically defined to attenuate the effect of harmonic distortion on Vibroseis data are given. These examples include some "exact" wave-shaping filters obtained using the original program written by Mereu (1978) and some truncated wave-shaping filters obtained using the modified version of the program.

Chapter 1 reviews the "exact" wave-shaping filtering method and Chapter 2 covers its stability characteristics. The structure of the computer program written by Mereu (1978) is described in Chapter 3. Also included is the definition of the control parameters and their effect on the quality of the filter. Chapter 4 presents two truncation techniques: the standard technique defined by Mereu (1976) and the newly developed "alternate technique." The precautions one should take when truncating the filter are also given. Finally, the "exact" wave-shaping filtering technique is applied to the problem of harmonic distortion in Chapter 5. Two approaches are presented. In the first approach, wave-shaping filters are defined for attenuating the harmonic distortion present on uncorrelated data. In the second approach, wave-shaping filters are defined for attenuating the correlation noise that affects correlated data. For both approaches, computer generated sweeps were used.

# CHAPTER 1

# THE EXACT WAVE-SHAPING FILTERING METHOD

## 1.1 INTRODUCTION

This chapter reviews the exact wave-shaping filtering method designed by R. F. Mereu (1976).

The basic procedure involves a process by which zeros are inserted between each of the non-zero terms of a symmetric time series. This zero insertion process is described in section 1.2. The exact wave-shaping filter is defined in section 1.3 and the sources of error affecting this technique are discussed in section 1.4.

## 1.2 BASIC PROCEDURE - THE ZERO INSERTION PROCESS

The basic procedure used by Mereu (1976) for the design of the exact wave-shaping filter involves a zero insertion process derived from the following property:

> "When the signs of alternate terms of a symmetric time series are reversed and the newly created time series is convolved with the original series, the resultant time series will have alternate values exactly equal to zero" (Mereu, 1976, p. 660).

Figure 1.1A illustrates the two consecutive operations involved in this process. The first operation consists of reversing the signs of the alternate values of a symmetric time series. The second operation is the convolution of the new time series, B(t), with the original one, A(t). As shown, the resulting time series, R(t) has its alternate values equal to zero.

When these consecutive operations are repeated, with only the signs of the alternate non-zero terms reversed, the resultant time series contains more zeros. This is shown in Figure 1.1B where the two operations are applied on the time series, R(t), defined in part A. The new resultant time series, R'(t), now contains three zeros between the non-zero terms.

The number of zeros inserted between the non-zero terms increases each time these two consecutive operations are repeated. Figure 1.2 shows that the number of inserted zeros, $N_Z$, increases exponentially as the number of repetitions increases. This is expressed by

"$N_Z = 2^N - 1$ (1.1)

where N is the number of repetitions" (Mereu, 1976, p.660)

This zero insertion process constitutes the basic procedure used by Mereu for the design of an exact wave-shaping filtering method.

A_

GIVEN SYMMETRIC TIME SERIES

A(t) : (0.3,1.09,0.3)

SIGN REVERSAL OF ALTERNATE TERMS

B(t) : (−0.3,1.09,−0.3)

CONVOLUTION OF A(t) WITH B(t)

R(t) : (−0.09,0,1.0,0,−0.09)

B_

NEW SYMMETRIC TIME SERIES

R(t) : (−0.09,0,1.0,0,−0.09)

SIGN REVERSAL OF ALTERNATE
NON-ZERO TERMS

C(t) : (0.09,0,1.0,0,0.09)

CONVOLUTION OF R(t) WITH C(t)

R′(t) : (−0.008,0,0,0,0.98,0,0,0,−0.008)

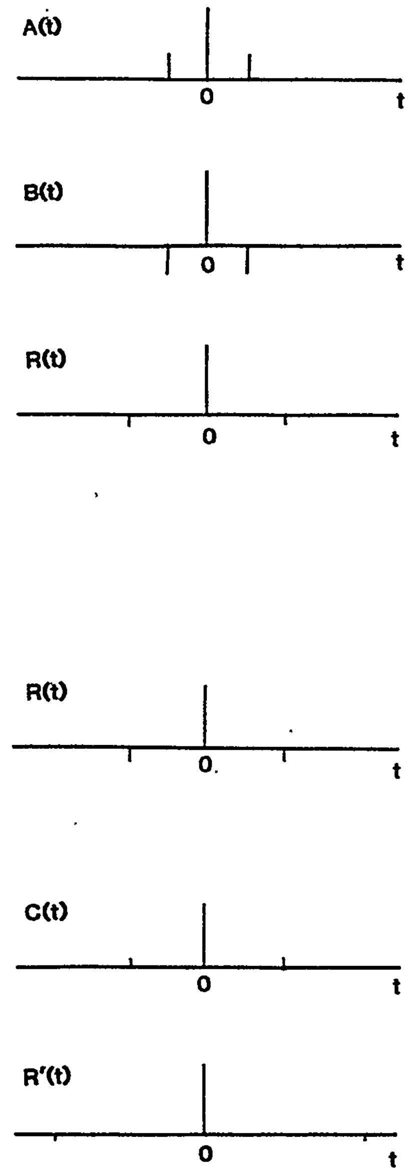



Figure 1.1 Examples showing the two operations involved in the zero insertion process

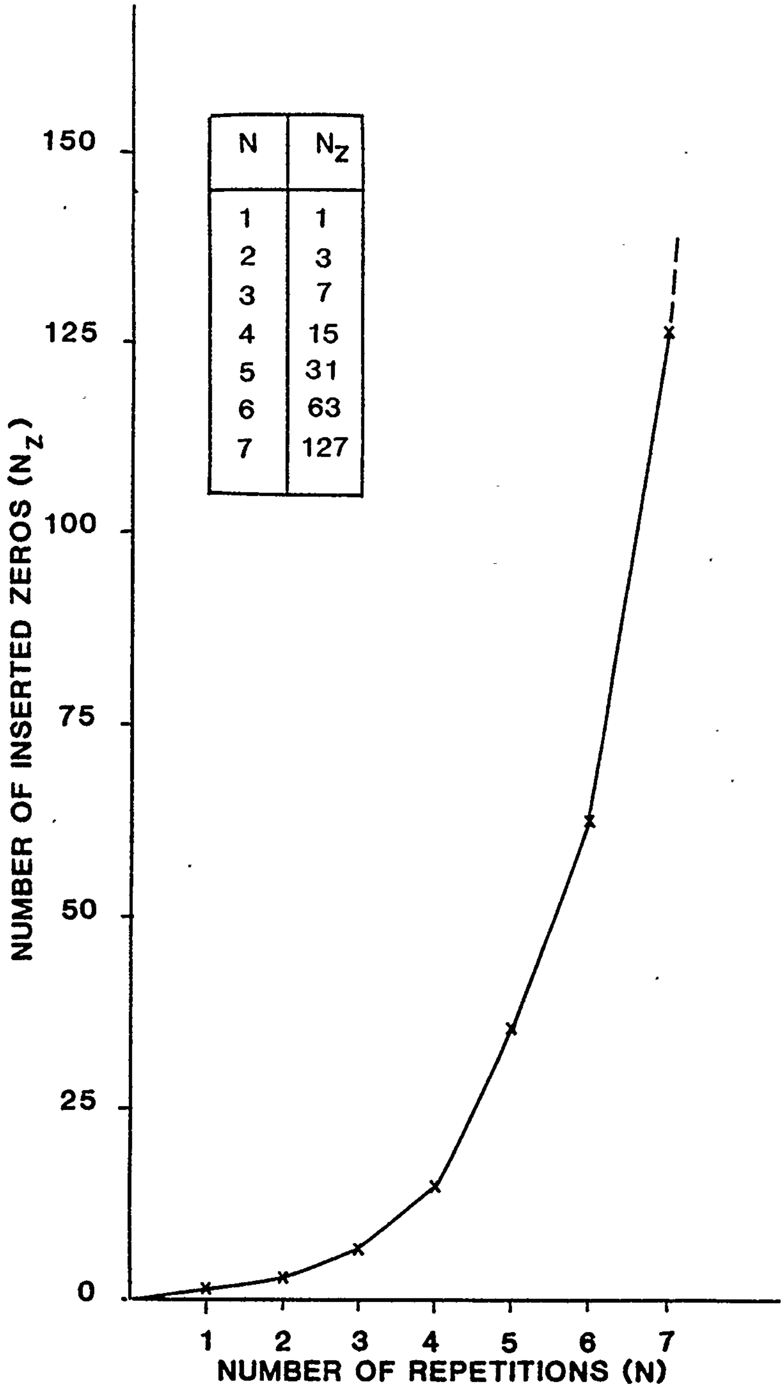



Figure 1.2 Graph showing the relationship between the number of inserted zeros and the number of repetitions of the zero insertion process

## 1.3 DEFINITION OF THE FILTER

This section reviews the exact wave-shaping filter as defined by Mereu (1976, pgs. 660-663).

Mereu utilized the zero insertion process described in section 1.2 to design an exact wave-shaping filter, F, such that

$$W * F = D \qquad (1.2)$$

where W is the given wavelet
D is the desired wavelet
* is the convolution operator

He defined the filter F by the following formula:

$$F = F_0 * F_1 * F_2 * F_3 * \dots F_N * D \qquad (1.3)$$

where $F_0$ is the wavelet W, reversed in time

$F_1 = (W*F_0)$ with signs of alternate terms changed

$F_2 = (W*F_0*F_1)$ with signs of alternate non-zero terms changed

$F_3 = (W*F_0*F_1*F_2)$ with signs of alternate non-zero terms changed

$F_N = (W*F_0*\dots F_{N-1})$ with signs of alternate non-zero terms changed

N is the number of subfilters.

When the filter F is applied to the given wavelet, W, the following time series are produced:

$$W*F_0 = \emptyset_{WW} \tag{1.4}$$

= autocorrelation of the given wavelet

This corresponds to the symmetric time series required for the zero insertion process.

$$W*F_0*F_1 = [(W*F_0)] * [(W*F_0) \text{ with signs of alternate terms changed}] \tag{1.5}$$

= a time series in which the non-zero terms are separated by one zero.

$$W*F_0*F_1*F_2 = [(W*F_0*F_1)] * [(W*F_0*F_1) \text{ with signs of alternate non-zero terms changed}]. \tag{1.6}$$

= a time series in which the non-zero terms are separated by three zeros.

o
o
o

$$W*F_0*F_1*\ldots F_N = [(W*F_0*F_1*\ldots F_{N-1})] * [W*F_0*F_1*\ldots F_{N-1}) \text{ with signs of alternate non-zero terms changed}] \tag{1.7}$$

= a time series in which the non-zero terms are separated by $2^N-1$ zeros.

and, finally

$$W*F_0*F_1*...F_N*D = W*F \qquad (1.8)$$

$$= \text{actual output.}$$

For convenience, Mereu (1976) grouped all the terms defining the filter F into the three following components:

$$F = F_0 * G_N *D \qquad (1.9)$$
$$\text{where } G_N = F_1 * F_2 * F_3 *... F_N$$

Because the subfilters $F_1$, $F_2$,...$F_N$ are derived from a symmetric time series, they are symmetric and the result of their convolution is also a symmetric time series. For this reason, Mereu called the component $G_N$ the symmetric filter.

As the wave-shaping filter results from the convolution of terms whose length increases exponentially ($F_1$, $F_2$,...$F_N$), the length of F increases very rapidly. In order to shorten the length of F Mereu (1976) suggests dropping the insignificant terms of the subfilters and symmetric filter. As shown later, the off-centered subfilter coefficients generally decrease faster than the magnitude of the central term. Because of this, dropping the very small magnitude coefficients relative to the central term has the effect of gradually shortening the number of non-zero coefficients in each subfilter. Eventually, only one term remains in the last subfilter. The effect of dropping the insignificant subfilter and symmetric filter coefficients on the wave-shaping filter quality will be discussed in more detail in Section 3.4.2. In general, when the limit of significance for the subfilter coefficients is properly chosen, the errors introduced by dropping the very small coefficients is negligible.

Figure 1.3 illustrates an example of the successive time series produced when the filter F is applied to the given wavelet W. In this example, the given input is a 21 point wavelet (LW = 21) and the desired output is a 9 point wavelet (LD = 9) centered at zero time. The series of diagrams shows part of the evolution of the subfilters, symmetric filter, wave-shaping filter and actual output as N increases from 1 to 8. The number of coefficients composing each of these time series is indicated on the right-hand side. It shows how the symmetric filter and wave-shaping filter become gradually longer as N increases. The sequence of diagrams also shows how the actual output develops towards an isolated desired wavelet.

In the example of Figure 1.3, all subfilter and symmetric filter coefficients of amplitude smaller than $10^{-12}$ were dropped from the calculations (after normalization about their respective central terms). This is why the 8th subfilter is composed of a single term ($LF_8 = 1$). Table 1.1 shows how the length of the subfilters, symmetric filter and wave-shaping filter are modified by dropping all subfilter and symmetric filter coefficients of magnitude smaller than $10^{-12}$. After the 8th subfilter, the symmetric filter in case B contains 151 points. If the

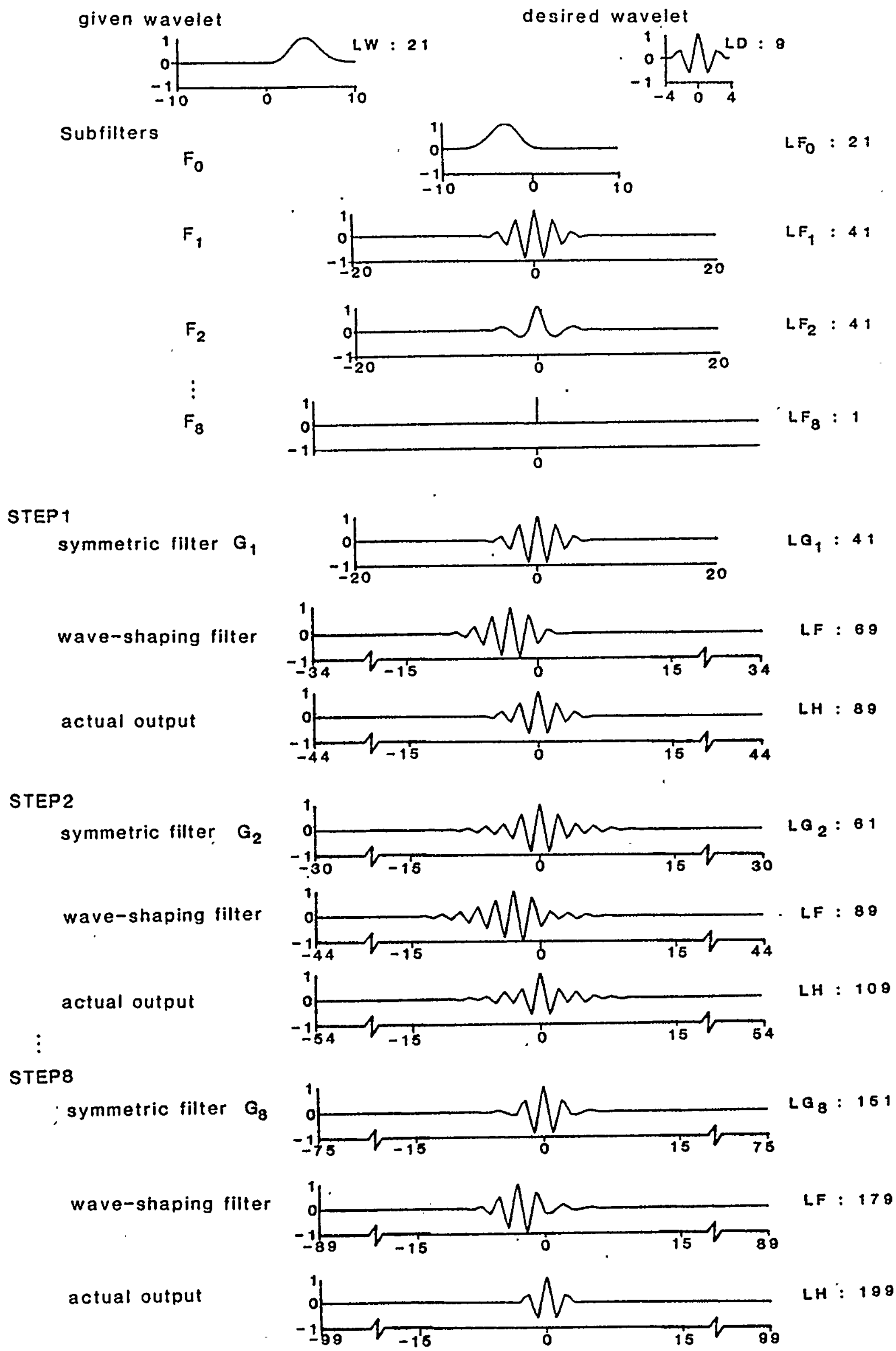



Figure 1.3 Series of diagrams showing part of the evolution of the subfilters, wave-shaping filter and actual output as N increases

**Length of the given wavelet : 21 points**

**Length of the desired wavelet : 9 points**

| | CASE A : Limit of significance : 0 | | CASE B : Limit of significance : $10^{-12}$ | |
|---|---|---|---|---|
| subfilter | number of non-zero coefficients | subfilter length (#points) | number of non-zero coefficients | subfilter length (#points) |
| $F_0$ | 21 | 21 | 21 | 21 |
| $F_1$ | 41 | 41 | 41 | 41 |
| $F_2$ | 41 | 81 | 21 | 41 |
| $F_3$ | 41 | 161 | 17 | 65 |
| $F_4$ | 41 | 321 | 11 | 81 |
| $F_5$ | 41 | 641 | 7 | 97 |
| $F_6$ | 41 | 1281 | 5 | 129 |
| $F_7$ | 41 | 2561 | 3 | 129 |
| $F_8$ | 41 | 5121 | 1 | 1 |
| | symmetric filter length | 10201 | symmetric filter length | 151 |
| | wave-shaping filter length | 10229 | wave-shaping filter length | 179 |

**Table 1.1 Effect of the limit of significance for the subfilter coefficients on the length of the subfilters , symmetric filter and wave-shaping filter**

insignificant terms were not discarded, this filter would contain 10201 points after the same number of subfilters. In this case, zeroing the subfilter and symmetric filter coefficients smaller than $10^{-12}$ has saved 10050 storage locations and the remaining symmetric filter coefficients are sufficient to define a good quality wave-shaping filter, as proven by the actual output obtained in Figure 1.3.

Dropping the insignificant subfilter and symmetric filter coefficients has the practical effect of:

1. gradually reducing the number of non-zero coefficients remaining in the subfilters

2. avoiding useless calculations with insignificantly small terms

3. reducing the length of the wave-shaping filter, F, and consequently reducing the number of required computer memories.

## 1.4 SOURCES OF ERROR IN THE EXACT WAVE-SHAPING FILTER

An important criterion for judging the quality of a filter involves the distribution of the error between the actual and desired outputs. In the filtering method defined by Mereu (1976), the distribution of the error depends on the time length between the non-zero terms of the time series given by ($W*F_0*G_N$) called the spike sequence.

Figure 1.4 shows a sequence of diagrams illustrating how the spike sequence, the actual output, H(t) and the error evolve with the number of subfilters. The series of diagrams shows how the number of inserted zeros gradually increases the time length between the non-zero terms of the spike sequence. Note that the actual output can be regarded as the result of the convolution between the spike sequence and the desired output. As shown, there are as many wavelets in the output as there are spikes in the spike sequence and the time length between these wavelets depends on the number of zeros separating the spikes. As the number of subfilters increases, the spikes become separated by a larger amount of zeros, the time length between the wavelets increases, and the error is pushed further away from the central position. Increasing the number of subfilters has the effect of redistributing the error in a region further away from the area of interest. Because the shaping errors can be pushed in a zone outside of the area of interest, Mereu (1976) refers to this filter as the "exact" wave-shaping filter.

Three other types of errors are known to affect the quality of the filtered output. These include errors caused by dropping insignificant subfilter coefficients, round-off errors and truncation errors. Because the subfilters contain fewer terms than they theoretically should, the shape of the filter F is slightly altered. However, when the threshold

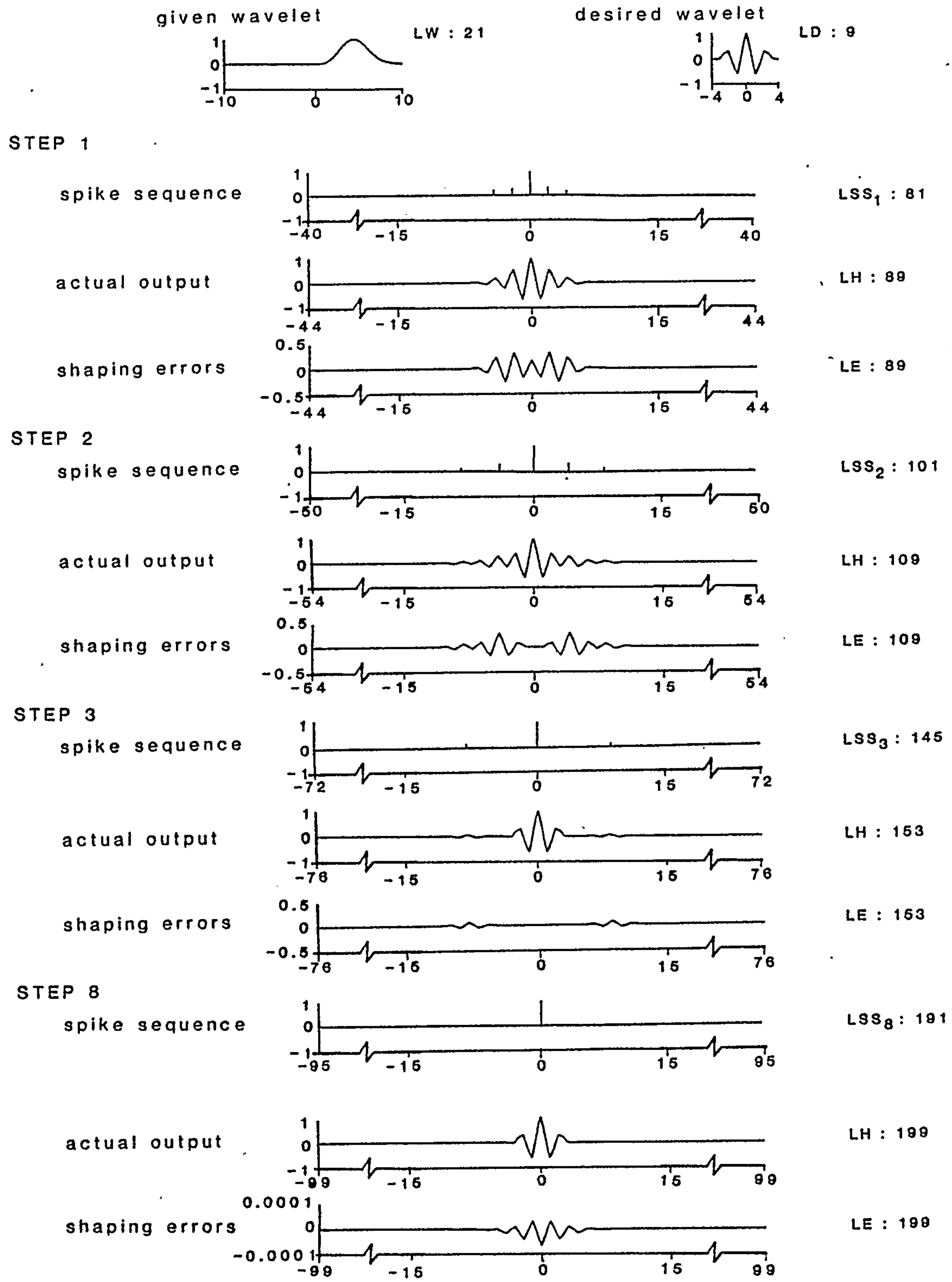



Figure 1.4 Series of diagrams showing part of the evolution of the spike sequence , actual output and shaping errors as N increases

of significance for the subfilter is properly chosen, this error stays very small. This will be discussed in more detail in Chapter 3. The round-off error is generally very small but is known to become more important as the number of subfilters increases. The error caused by truncating the filter depends on the truncation length and on the chosen truncation technique. This will be reviewed in Chapter 4.

## 1.5 CONCLUSION

Mereu (1976) defined a wave-shaping filtering method in the time domain based on a zero insertion procedure. The filter, defined through a sequence of convolutions, is efficiently shortened by discarding the insignificant coefficients of the subfilters and symmetric filter.

Mereu's technique can produce "exact" wave-shaping filters by using the shapes of the given and desired wavelets. The stability characteristics of this filtering method will be reviewed in Chapter 2.

The distribution of the shaping errors in Mereu's technique make this technique attractive. In other filtering techniques the errors are distributed through the output and affect the zone of interest. For instance, in the Wiener filtering method, the shaping errors are distributed throughout the output in such a way that the error energy is minimum in a least-squares sense. Therefore, the output is not "exact" in the zone of interest. In Mereu's filtering method, the error is distributed in a region that can be separated from the zone of

The following characteristics of the "exact" wave-shaping filtering method make this technique attractive for the definition of a wave-shaping filter designed to attenuate the correlation noise caused by harmonic distortion on Vibroseis data:

- use of the shape of the given and desired wavelets
- production of an "exact" output in the zone of interest.

# CHAPTER 2

# STABILITY CHARACTERISTICS OF THE EXACT WAVE-SHAPING FILTER

## 2.1 INTRODUCTION

A filter is stable when the energy of its impulse response is finite. Defining the conditions under which a filtering technique produces a stable output is critical as these conditions often limit the applications of the technique. The stability characteristics of the exact wave-shaping filter are studied by Mereu (1977). This chapter presents a review of these characteristics.

## 2.2 FILTER STABILITY

Mereu (1977) argued that the exact wave-shaping filter is generally stable by using the following two properties of discrete time series:

1. "Any wavelet may be broken down into a series of 2 point or 3 point components, and its corresponding autocorrelation function into 3 point and 5 point components." (Mereu, 1977, p. 73)

2. "Factoring the wavelet or autocorrelation function into its components and the zero insertion procedure through the use of the subfilters $F_1$, $F_2$,...$F_N$ are two sets of operations

which may be reversed without affecting the results." (Mereu, 1977, p. 73)

Mereu studied the stability of the "exact" wave-shaping filter in the following way:

1. Any wavelet can be broken down into a series of 2 point or 3 point components (property 1). The autocorrelation function of any 2 point or 3 point component may be expressed as:

| 2 point component | 3 point component |
|---|---|
| $R = (r_1, -1) * (-1, r_1)$ where $\lvert r_1 \rvert \leq 1$ | $R' = (r^2, -2r\cos\theta, 1) * (1, -2r\cos\theta, r^2)$ where $\lvert r \rvert \leq 1$ |

2. Using property 2, the coefficients of $F_1$ and $F_1'$ are derived from the above autocorrelation. This is:

| 2 point component | 3 point component |
|---|---|
| $F_1 = (-r_1, -1) * (-1, -r_1)$ | $F_1' = (r^2, 2r\cos\theta, 1) * (1, 2r\cos\theta, r^2)$ |

3. The coefficients of $F_2$ and $F_2'$ become:

| 2 point component | 3 point component |
|---|---|
| $F_2 = (r_1^2, 0, 1) * (1, 0, r_1^2)$ | $F_2' = (r^4, 0, 4r^2\cos^2\theta - 2r^2, 0, 1) * (1, 0, 4r^2\cos^2\theta - 2r^2, 0, r^4)$ <br> $= (r^4, 0, 2r^2\cos 2\theta, 0, 1) * (1, 0, 2r^2\cos 2\theta, 0, r^4)$ |

By writing only the non-zero coefficients, $F_2$ and $F_2'$ become:

| 2 point component | 3 point component |
|---|---|
| $F_2 = (r_1^2, 1) * (1, r_1^2)$ | $F_2' = (r^4, 2r^2\cos 2\theta, 1) * (1, 2r^2\cos 2\theta, r^4)$ |

4. The non-zero coefficients of the subsequent subfilters are defined using the same method. They are:

| 2 point component | 3 point component |
|---|---|
| $F_3 = (r_1^4, 1) * (1, r^4)$ | $F_3' = (r^8, 2r^4\cos 4\theta, 1) * (1, 2r^4\cos 4\theta, r^8)$ |
| $F_4 = (r_1^8, 1) * (1, r_1^8)$ | $F_4' = (r^{16}, 2r^8\cos 8\theta, 1) * (1, 2r^8\cos 8\theta, r^{16})$ |
| $F_N = (r_1^k, 1) * (1, r_1^k)$ | $F_N' = (r^{2k}, 2r^k\cos k\theta, 1) * (1, 2r^k\cos k\theta, r^{2k})$ |
| where $k = 2^{(N-1)}$ | where $k = 2^{(N-1)}$ |

5. Then, Mereu examines how the subfilter coefficients behave as N increases for $|r| < 1$, (or $|r_1| < 1$) and for $r = 1$ (or $r_1 = 1$)

CASE 1: $|r| < 1$ (or $|r_1| < 1$) - no roots lying on the unit circle or the Fourier transform of the given wavelet contains no zero spectral values.

When N increases, K increases and $r^K$ (or $r_1{}^k$) approaches zero. In this case, $F_N$ and $F'_N$ become:

| 2 point component | 3 point component |
|---|---|
| $F_N = (\ldots 0,0,0,1) * (1,0,0,0,\ldots)$<br>$= (\ldots 0,0,0,1,0,0,0,\ldots)$ | $F'_N = (\ldots 0,0,0,1) * (1,0,0,0\ldots)$<br>$= (\ldots 0,0,0,1,0,0,0\ldots)$ |

The subfilter $F_N$ (and $F'_N$) contains a finite number of coefficients. The symmetric and wave-shaping filters will then also have a finite number of coefficients and will be stable.

CASE 2: $r = 1$ (or $r_1 = 1$) - roots lying on the unit circle of the complex Z-plane or the Fourier transform of the given wavelet has some zero spectral values

In this case, $F_N$ (and $F'_N$) becomes

| 2 point component | 3 point component |
|---|---|
| $F_N = (1^k, 1) * (1, 1^k)$<br>where $k = 2^{(N-1)}$ | $F'_N = (1, 2\cos k\theta, 1) * (1, 2\cos k\theta, 1)$<br>where $k = 2^{(N-1)}$ |

The subfilter coefficients keep significant values as N increases. In the case of a 2 point component, Mereu explains that the filter "is stable in the sense that the weights do not increase in magnitude as N increases. However, it may be considered badly behaved in that the weights remain finite all the way to infinity" (Mereu, 1976, p. 669). In the case of the 3 point component Mereu states that "the non-zero subfilter weights repeat themselves in a cyclical manner with increasing N. The resulting filter weights remain finite (i.e., do not blow up) but have significant values all the way to infinity." (Mereu, 1977, p. 78)

In order to prevent this situation from happening, Mereu (1976) suggests adding some white noise to the central term of the autocorrelation function. This has the effect of attenuating the amplitude of the off-centered terms more rapidly with respect to the central term. More detail about the effect of adding some white noise on the output is given in Section 3.4.3.

## 2.3 CONCLUSION

According to Mereu's work, the exact wave-shaping filter is stable provided the Z-transform of the given wavelet has no roots lying on the unit circle of the complex Z plane. In other words, the filter is stable when the Fourier transform of the given wavelet has no zero spectral values (or notches). Limitations in the applications of the exact wave-shaping filtering method can then be predicted by analyzing the frequency spectrum of the given wavelet. When notches are present in the frequency spectrum of the given wavelet, that is when the Z-transform of W has roots lying on the unit circle, the subfilter coefficients significant as N increases. In this case, the wave-shaping filter contains some significant coefficients all the way to infinity. In order to avoid such situations, Mereu (1976) suggests adding some white noise to the central term of the autocorrelation function. This has the effect of attenuating more rapidly the amplitude of the off-centered terms with respect to the central term. Examples of "badly behaving" filters can be found in Mereu (1976) and Mereu (1977).

In the case of filtering the correlation noise due to harmonic distortion, the spectrum of the given wavelet is defined by the starting and ending frequencies of the pilot sweep and of its harmonics. This will be presented in section 5.2. In theory, no zero spectral values should occur in the frequency spectrum of the harmonically distorted sweep. Therefore, no problem of stability is expected.

# CHAPTER 3

# COMPUTATIONS

## 3.1 INTRODUCTION

The coefficients of the subfilters, symmetric filter, wave-shaping filter, actual output and shaping error are computed by a program designed by Mereu (1978). This chapter describes the structure of the program and reviews some routines used by Mereu (1978) to shorten the size of the arrays and the time spent on the calculations. The description of each control parameters and its effect on the filter quality is included. Recommendations for choosing appropriate values for each control parameters in a specific case are also given.

This program will be used in Chapter 5 to compute the coefficients of a filter designed to attenuate the correlation noise caused by harmonic distortion.

## 3.2 STRUCTURE OF THE COMPUTER PROGRAM

A copy of the original version of the program designed by Mereu (1978) is contained in Appendix 1 and the basic structure for this program is presented in a simplified manner in Figure 3.1 The main routine, named here "MEREU1", calls for two subroutines "FOLDM" and "SHAPEW". "FOLDM" is a standard convolution routine and "SHAPEW"

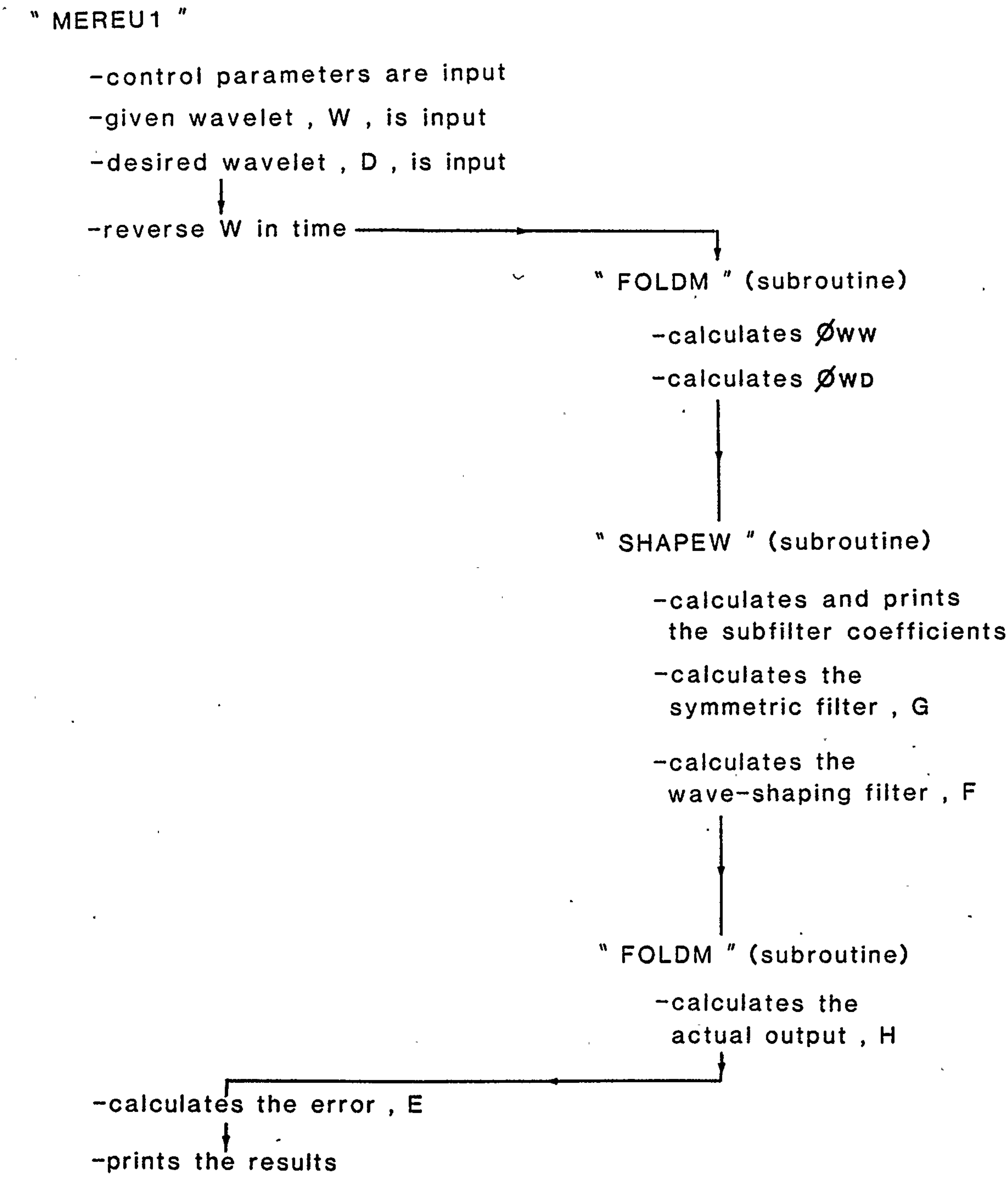



Figure 3.1 Simplified structure of the program " MEREU1 "

calculates the coefficients of the subfilters, symmetric filter and wave-shaping filter.

As shown in Figure 3.1, the control parameters, the given wavelet and the desired wavelet are specified in "MEREU1". The given wavelet is reversed in time and fed into "FOLDM" to generate the autocorrelation of the given wavelet, $\emptyset_{WW}$ and the crosscorrelation between the given and desired wavelets, $\emptyset_{WD}$. These two correlation functions and other parameters are used as input to "SHAPEW" where the coefficients of the subfilters, symmetric filter and wave-shaping filter are calculated. The wave-shaping filter is then input to "FOLDM" where it is convolved with the given wavelet to give the actual output. The actual and desired outputs are compared in "MEREU1" and the difference between the two, the shaping error, is calculated. Finally, the results are printed according to the format defined in the main routine. More information about each one of these steps can be found in Mereu (1978).

## 3.3 COMPUTATION METHOD

The subfilter and symmetric filter coefficients may be calculated using a standard convolution routine. However, because of their symmetry, many redundant calculations will be performed by such a routine entailing a relatively large amount of required memory locations. Instead of using the standard convolution routine, Mereu (1976) suggests using some routines that take advantage of the characteristics of the subfilters and symmetric filter. These routines, used in the subroutine

"SHAPEW", are reviewed in some detail. The method used for computing the coefficients of the exact wave-shaping filter is also included.

### 3.3.1 Computation of the subfilter coefficients

Mereu (1976) defined a routine for computing the subfilter coefficients based on the following observations:

1. The number of zeros that exist between the non-zero terms is known for each subfilter,

2. The subfilters $F_1$, $F_2 \ldots F_N$ are derived from a symmetric time series and are also symmetric,

3. The non-zero terms of the Nth subfilter can be derived from the non-zero terms of the (N-1)th subfilter.

In his routine, the zeros are first ignored and the non-zero terms of each subfilter are calculated using the following formula:

$$"b_k = b_{-k} = (-1)^k \sum_{i=k}^{M} (-1)^i \, pa_{2k-i} \, a_i \qquad (3.1)$$

where $b_k$ is the kth non-zero term in the Nth subfilter

$a_i$ is the ith non-zero term of the (N-1)th subfilter

$a_i = a_{-i}$

$p = 1$ if $i = k$

$p = 2$ if $i \neq k$

$k = 0, 1, 2, \ldots M$

M = the number of points in the given wavelet"
(Mereu, 1976, p. 664)

Note that in (3.1), $b_k = b_{-k}$ implies that only the non-zero coefficients of one half of each subfilter (including the central term) are calculated. These coefficients are then normalized about their central term $b_0$ and the theoretical number of zeros, $N_Z$, that exist between each of these non-zero coefficients is calculated. Finally, the insignificant subfilter coefficients are discarded and the subfilter length is updated.

Mereu's routine considerably reduces the number of required memories to store the subfilters. Table 3.1 presents the number of memories needed to store the subfilter coefficients when computed using a standard convolution routine and Mereu's routine for a specific example. As shown, the required number of memories differs between the two methods. When using a standard routine, the required amount of memories augments rapidly as the number of subfilters increases. Mereu's routine, on the other hand, demands a maximum amount of memories for each subfilter equal to the number of points in the given wavelet.

Length of the given wavelet : 21 points

| Subfilter | Number of locations required to store the subfilter coefficients | |
|---|---|---|
| | routine 1 | routine 2 |
| $F_0$ | 21 | 21 |
| $F_1$ | 41 | ≤ 21 |
| $F_2$ | 81 | ≤ 21 |
| $F_3$ | 161 | ≤ 21 |
| $F_4$ | 321 | ≤ 21 |
| $F_5$ | 641 | ≤ 21 |

Table 3.1 Example showing the number of locations required to store the subfilters coefficients when computed using a standard convolution routine (routine 1) and Mereu's routine (routine 2)

### 3.3.2 Computation of the coefficients of the symmetric filter

Mereu (1976) suggests using the following routine to compute the coefficients of the symmetric filter:

> "The weights of $G_N$ may be computed from the weights of $G_{N-1}$ using a standard convolution formula with a modification involving j which keeps track of the number of zeros which theoretically are present" (Mereu, 1976, p. 665)

This is done by:

$$"g_l = g_{-l} = \sum_{i=-M}^{M} b_i \, C_{l-ij} \qquad (3.2)$$

where $l = 0, 1, 2, 3, \ldots L$

$L = (LB + LC-2)/2$

with $LB = 2M.j + 1$

$LC$ = number of points in $G_{N-1}$

$g_l$ = the lth coefficient of $G_N$

$C_l$ = the lth coefficient of $G_{N-1}$

$b_i$ = the ith coefficient of $F_N$"

(Mereu, 1976, p. 665)

Once again, because $G_N$ is symmetric, only the coefficients of the positive half of $G_N$ (including the central term) are calculated.

The coefficients of $G_N$ are then normalized about their central term $G_0$, and the insignificant coefficients of $G_N$ are discarded. The length of the symmetric filter is then updated accordingly.

### 3.3.3 Computation of the coefficients of the wave-shaping filter

The coefficients of the wave-shaping filter, F, are simply computed by using a standard convolution routine. The filter F is given by:

$$F = \emptyset_{WD} * G_N \tag{3.3}$$

where $G_N$ is the symmetric filter

$\emptyset_{WD}$ is the crosscorrelation of W with D.

Figure 3.2 presents a simplified flowchart of the subroutine "SHAPEW" in which the three computation methods just described are used. The inputs for this subroutine are:

MXN = maximum number of subfilters to be used
BMIN = limit of significance for the subfilter coefficients
MXLG = maximum working space for the symmetric filter
PRNT = 1 specifies the values are to be printed out
PRNT = 0 specifies the values are not to be printed out
NPT = number of central values to be printed out
R = $\emptyset_{WW}$
LR = length of the autocorrelation function $\emptyset_{WW}$
S = $\emptyset_{WD}$
LS = length of the crosscorrelation function $\emptyset_{WD}$

Input to the subroutine "SHAPEW" :

R , LR , S , LS , MXN , BMIN , MXLG , WEIGHT , PRNT , NPT

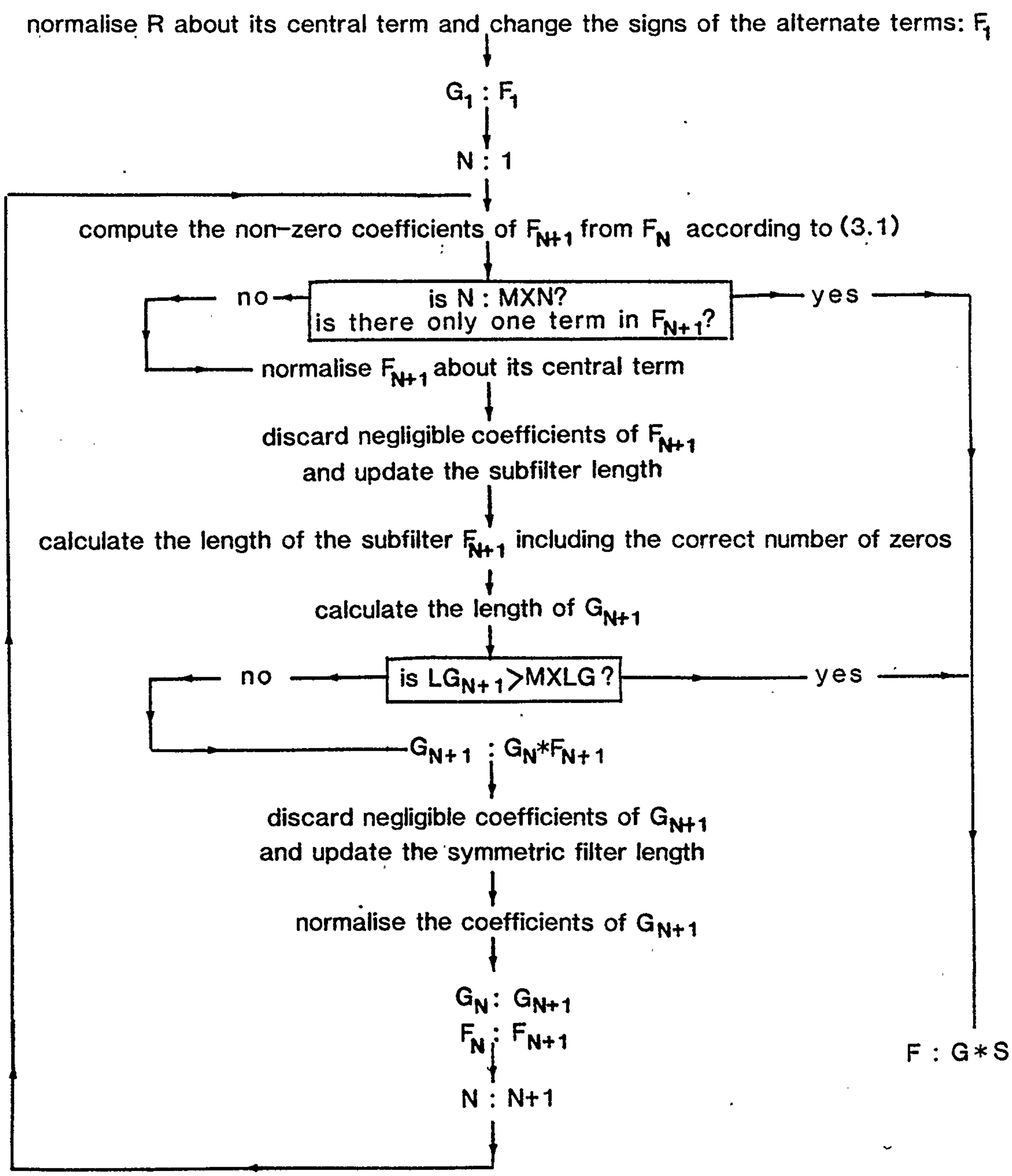



Figure 3.2 Simplified flowchart of the subroutine "SHAPEW'

The time series $R(\emptyset_{WW})$ and $S(\emptyset_{WD})$ as well as their respective length are computed in some previous steps of the program. The other parameters are specified in "MEREU1".

Subfilters and symmetric filter coefficients are computed as shown in Figure 3.2 until one of the following conditions is met:

1. the maximum number of subfilters to be used is reached
2. there is only one significant value left in a subfilter
3. the length of the symmetric filter becomes larger than MXLG.

In any one of these cases, the computation of the subfilter and symmetric filter coefficients stops and the wave-shaping filter is computed using the current coefficients of the symmetric filter.

## 3.4 CONTROL PARAMETERS

The list of control parameters for Mereu's program appears at the beginning of the main routine "MEREU1" (Appendix 1). Some of these parameters such as "PRNT" and "NPT" are only required for the printing step and have nothing to do with the calculations. On the other hand, the parameters "MXN," "BMIN", "MXLG" and "WEIGHT" are not only essential to the computation but can also significantly affect the filter quality.

In the following, a description of each control parameter (except "PRNT" and "NPT") is given along with some examples showing the effect of each of these parameters on the filter quality.

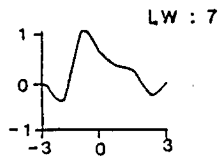
### 3.4.1 "MXN"

"MXN" sets the maximum number of subfilters to be used.

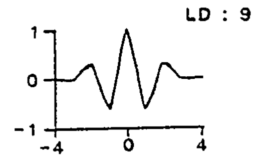
An appropriate value to specify for "MXN" in a particular case depends on the size and shape of the given wavelet as well as on the values attributed to other parameters such as "BMIN" and "WEIGHT". Because of this, determining the correct value for "MXN" can be problematic. However, a reasonable value can be estimated and input in the program. The output will show if "MXN" was correctly chosen.

Figures 3.3 and 3.4 show two examples in which the number of subfilters has been set too large in the first case and too small in the second. In the first case, "MXN" is set to 15 but after the 8th subfilter the number of significant terms has already decreased down to one. There is then no need for subfilters 9 to 15. The computation of the subfilter coefficients stopped after the 8th subfilter. Recall that, in this program, the computation of the subfilter coefficients stops whenever one subfilter is composed of a single weight, even though the maximum number of subfilters to be used has not been reached. Consequently, specifying a value for "MXN" larger than necessary does not adversely affect the computation. Figure 3.4 shows a case where "MXN"

given wavelet




desired wavelet




The following subfilter table shows only the non-zero weights of the positive half including the central point

subfilter 1

1.00000 -0.24000 -0.00571 0.09142 -0.23428 -0.05714
0.01714

subfilter 2

1.00000 0.02026 -0.46531 -0.00120 0.06702 0.01162
0.00030

subfilter 3

1.00000 0.68936 0.24227 0.04320 0.00294 0.00006
0.00000

subfilter 4

1.00000 0.29176 0.03164 0.00322 0.00001 0.00000

subfilter 5

1.00000 0.02850 -0.00101 0.00001 0.00000

subfilter 6

1.00000 0.00284 0.00000 0.00000

subfilter 7

1.00000 0.00000

subfilter 8

1.00000

actual output

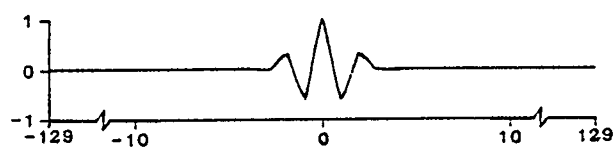
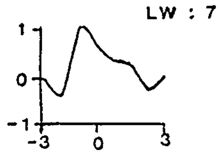
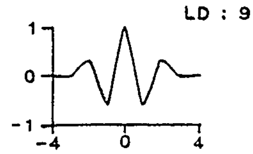



Figure 3.3 Example showing the effect of setting "MXN" to a larger value than necessary "MXN" : 15

given wavelet




desired wavelet




The following subfilter table shows only the non-zero weights of the positive half including the central point

subfilter 1

1.00000 -0.24000 -0.00571 0.09142 -0.23428 -0.05714
0.01714

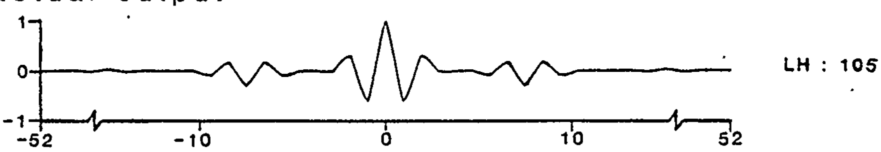
subfilter 2

1.00000 0.02062 -0.46531 -0.00120 0.06702 0.01182
0.00030

subfilter 3

1.00000 0.68936 0.24279 0.04320 0.00294 0.00006
0.00000

actual output




shaping errors

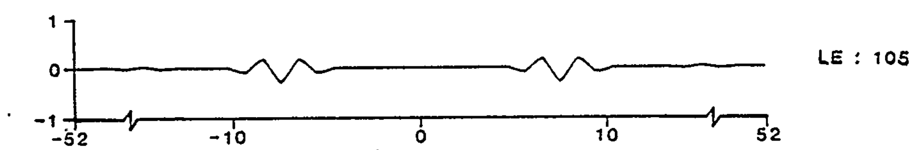



Figure 3.4 Example showing the effect of setting "MXN" to a smaller value than necessary "MXN" : 3

has been set too small. This is indicated by the number of significant coefficients left in the last subfilter. In this case, the filter F computed with only 3 subfilters will contain some unwanted terms. As discussed in Section 1.4, this has the effect of increasing the error between the actual and desired outputs. The magnitude of this error depends on the quantity and relative importance of the unwanted terms. Figure 1.4 showed how the magnitude of the error and its distribution varies with the number of subfilters being used. In general, the error increases as the number of subfilters being used decreases.

In order to get a good quality filter, "MXN" should be chosen large enough to let the number of significant coefficients in the subfilters decrease to one. Because specifying a value larger than necessary does not adversely affect the computation, it is suggested to input a relatively large value. The output will show if this value was correctly chosen.

### 3.4.2 "BMIN"

"BMIN" sets the minimum threshold of significance for the subfilter coefficients. The subfilter coefficients having an absolute value smaller than "BMIN" are discarded.

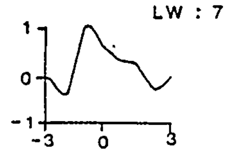
The minimum threshold of significance for the symmetric filter coefficients "GMIN" is also specified through "BMIN":

"GMIN" = "BMIN"/100000

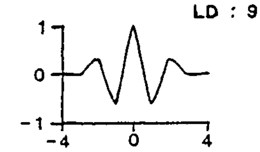
According to Mereu (1976), the amplitude of the off-centered coefficients normally decreases faster than the amplitude of the central term. Zeroing the subfilter coefficients whose absolute value is smaller than "BMIN" then has the effect of reducing the number of coefficients left in each subfilter and consequently reducing the length of the wave-shaping filter. Zeroing the symmetric filter coefficients whose absolute value is smaller than "GMIN" has the effect of reducing the length of the wave-shaping filter. An example showing the difference between the theoretical (BMIN = 0) and actual (BMIN = $10^{-10}$) length of the subfilters and wave-shaping filter is presented in Figure 3.5. As shown, instead of 2725 coefficients, the actual wave-shaping filter contains only 253 coefficients, and the shaping errors are still relatively small. Specifying a value of $10^{-10}$, in this case, has saved 2472 storage locations without significantly altering the quality of the filter.

The length of the filter would even be shorter if "BMIN" were set larger. However, when "BMIN" is set larger, more terms are left out of the computations and the quality of the filter is affected. Table 3.2 shows the effect of "BMIN" on the length of the filter as well as its effect on the shaping errors. As indicated, the error generally

given wavelet




desired wavelet

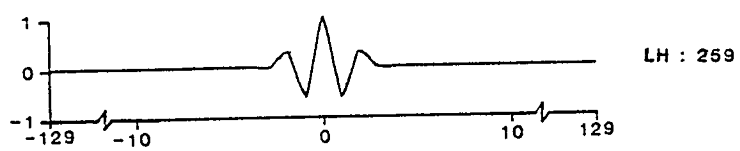



| subfilter | theoretical length (BMIN : 0) #coefficients | actual length (BMIN : $10^{-10}$) #coefficients | symmetric filter | theoretical length (BMIN : 0) #coefficients | actual length (BMIN : $10^{-10}$) #coefficients |
|---|---|---|---|---|---|
| $F_0$ | 7 | 7 | | | |
| $F_1$ | 13 | 13 | $G_1$ | 13 | 13 |
| $F_2$ | 25 | 25 | $G_2$ | 37 | 37 |
| $F_3$ | 49 | 49 | $G_3$ | 85 | 85 |
| $F_4$ | 97 | 81 | $G_4$ | 181 | 147 |
| $F_5$ | 193 | 97 | $G_5$ | 373 | 217 |
| $F_6$ | 385 | 129 | $G_6$ | 757 | 273 |
| $F_7$ | 769 | 129 | $G_7$ | 1525 | 315 |
| $F_8$ | 1537 | 1 | $G_8$ | 2061 | 239 |

theoretical length of the exact wave-shaping filter (BMIN : 0) : 2725 coefficients

actual length of the wave-shaping filter (BMIN : $10^{-10}$) : 253 coefficients

actual output (with BMIN : $10^{-10}$)




shaping errors (with BMIN : $10^{-10}$)

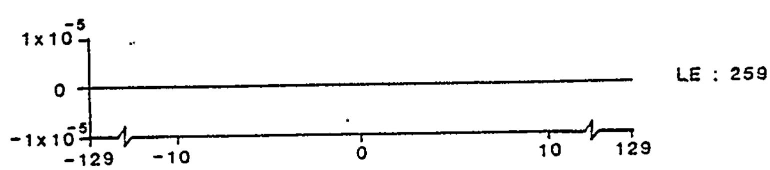



Figure 3.5 Example showing the effect of "BMIN" on the length of the subfilters, symmetric filter and wave-shaping filter and on the filter quality

| BMIN | N | LF | Shaping errors (only 5 central values are shown) | | | | |
|---|---|---|---|---|---|---|---|
| 1.0E-40 | 10 | 547 | 1.6E-10 | 2.8E-10 | 3.9E-10 | 4.6E-10 | 4.6E-10 |
| 1.0E-15 | 8 | 217 | 1.6E-10 | 2.8E-10 | 3.9E-10 | 4.6E-10 | 4.6E-10 |
| 1.0E-14 | 8 | 205 | 1.6E-10 | 2.8E-10 | 3.9E-10 | 4.6E-10 | 4.6E-10 |
| 1.0E-13 | 8 | 189 | 1.5E-10 | 2.7E-10 | 3.8E-10 | 4.5E-10 | 4.5E-10 |
| 1.0E-12 | 8 | 177 | 1.6E-10 | 2.8E-10 | 3.9E-10 | 4.6E-10 | 4.6E-10 |
| 1.0E-11 | 8 | 165 | 1.6E-10 | 2.8E-10 | 3.9E-10 | 4.6E-10 | 4.6E-10 |
| 1.0E-10 | 8 | 151 | 1.5E-10 | 2.7E-10 | 3.8E-10 | 4.4E-10 | 4.4E-10 |
| 1.0E-09 | 7 | 135 | 6.6E-11 | 1.5E-10 | 2.4E-10 | 2.9E-10 | 2.9E-10 |
| 1.0E-08 | 7 | 123 | 6.6E-11 | 1.5E-10 | 2.4E-10 | 2.9E-10 | 2.9E-10 |
| 1.0E-07 | 7 | 111 | 1.5E-10 | 2.4E-10 | 3.1E-10 | 3.6E-10 | 3.6E-10 |
| 1.0E-06 | 7 | 95 | 4.9E-09 | 4.9E-09 | -1.5E-09 | -1.5E-09 | -1.5E-09 |
| 1.0E-05 | 7 | 83 | 7.9E-08 | 8.1E-08 | -3.9E-08 | -3.9E-08 | -3.9E-08 |
| 1.0E-04 | 5 | 73 | 1.3E-06 | 1.3E-06 | 5.3E-07 | 5.3E-07 | 5.3E-07 |
| 1.0E-03 | 5 | 59 | 3.9E-05 | 3.9E-05 | -8.4E-06 | -8.4E-06 | -8.4E-06 |
| 1.0E-02 | 5 | 43 | 9.3E-01 | 2.2E-01 | 3.6E-02 | 2.7E-03 | 2.7E-03 |

N : number of subfilters needed to let the number of coefficients decrease to one.

LF : length of the wave-shaping filter

Table 3.2 Table showing how the value of "BMIN" affects the shaping errors and the length of the wave-shaping filter (after Mereu(1978),p.201) increases, and the quality of the filter decreases as "BMIN" is set to larger values.

The quality and length of the filter depend on the value selected for "BMIN". Mereu suggests using an initial value in the order of $10^{-12}$. If, when using this value, the errors are relatively large, "BMIN" should be set to a smaller value. On the other hand, if the errors are relatively small, a larger value can be tried for "BMIN". This will have the advantage of reducing the length of the filter, and saving some storage locations. However, the errors should not increase significantly. A series of tests may be necessary to determine the correct value to use for "BMIN" in a practical situation.

### 3.4.3 "WEIGHT"

The parameter "WEIGHT" specifies the level of white noise to be added to the autocorrelation of the given wavelet. It is defined as a fraction which may range from 0.0 (noiseless case) up to 0.99 (99% white noise).

Because this filtering method is defined in the time domain, the white noise is introduced by adding the value of "WEIGHT" to the central term of the normalized autocorrelation, $\phi_{WW}$.

According to Mereu (1976), the addition of white noise increases the magnitude of the shaping errors and reduces the length of the shaping filter. The errors are larger because "the filter does not redistribute the statistical error in the same manner as it redistributes the shaping errors, and the filter will not produce an error-free output." (Mereu, 1976, p. 671). Figure 3.6 shows an example where 10% of white noise (WEIGHT = 0.1) is added. The wave-shaping filter and the actual output for the noiseless case are also presented for comparison. As shown, the errors are more important in the case where white noise is added. The length of the filter is also shorter in this case. This is because the "off-centered terms are attenuated much more rapidly with respect to the central term". (Mereu, 1976, p. 669)

Adding some white noise to the autocorrelation of the given wavelet also means adding a positive constant value to the frequency spectrum of the given wavelet. As shown in Chapter 2, filter instability is expected when the Fourier transform of the given wavelet has some zero spectral values. Adding some white noise may then be useful in the cases where the wave-shaping filter encounters some stability difficulties.

Because the statistical errors are not redistributed in a region that can be separated from the zone of interest, it is recommended to set "WEIGHT" to zero. If the wave-shaping filter encounters some stability problems, then "WEIGHT" can be gradually set to some larger values (0.05, 0.1, 0.2,...). However, one should remember that adding some white noise to the autocorrelation of the given wavelet will cause some errors throughout the output.

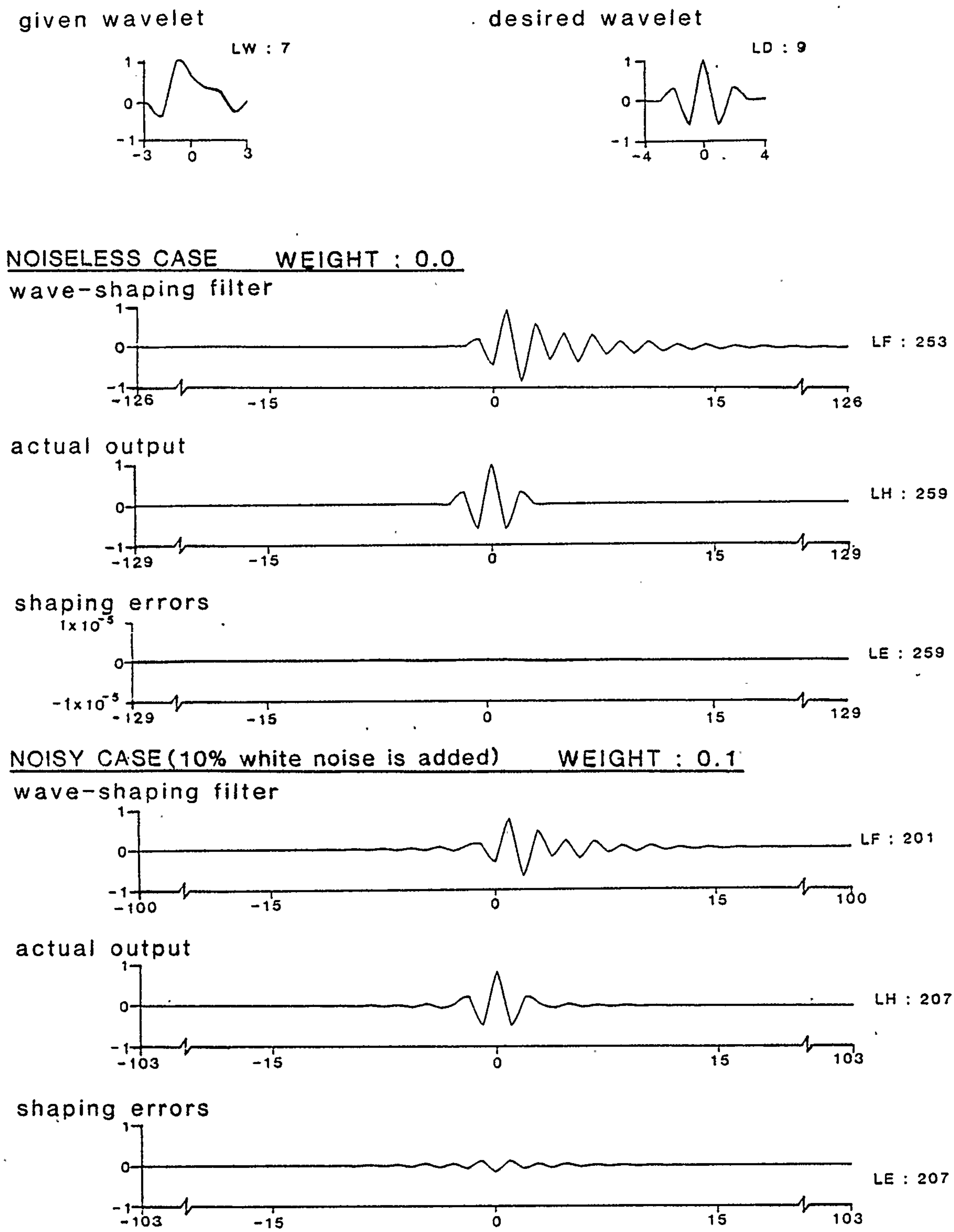



Figure 3.6 Example showing the effect of adding 10% white noise ("WEIGHT" : 0.1) on the filter quality

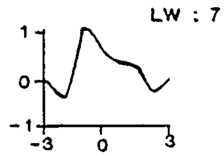
### 3.4.4 "MXLG"

"MXLG" specifies the working space for the arrays containing the subfilters, the symmetric filter and the wave-shaping filter.

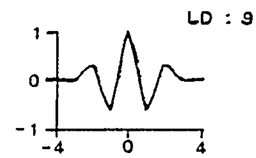
An appropriate value to specify for "MXLG" in a particular situation depends on the size and the shape of the given wavelet and on the size of the desired wavelet as well as on the values attributed to "MXN", "BMIN" and "WEIGHT". "MXLG" can become a relatively large number which cannot be known in advance. However, a reasonable value can be input to be tested. If this value is too small, there will be insufficient working space for G. In this case, only a limited number of subfilters will be used to calculate the symmetric filter and the wave-shaping filter will be truncated to "MXLG" (see Figure 3.2). An example is presented in Figure 3.7. In this example, the same given wavelet, desired wavelet and control parameters as in Figure 3.3 are used except that "MXLG" which is now set to 101. As shown in Figure 3.7, there is not enough working space to let the symmetric filter develop with eight subfilters. Instead of eight subfilters, only four subfilters are used to generate the symmetric filter. This creates an error because not enough subfilters are used and because the wave-shaping filter is truncated to "MXLG".

The program written by Mereu (1978) is designed to work with several time series that contain an odd number of coefficients. Effectively, the main routine "MEREU1" makes sure that the given and

given wavelet




desired wavelet




The following subfilter table shows only the non-zero weights of the positive half including the central point

subfilter 1

1.00000 -0.24000 -0.00571 0.09142 -0.23428 -0.05714
0.01714

subfilter 2

1.00000 0.02062 -0.46531 -0.00120 0.06702 0.01162
0.00030

subfilter 3

1.00000 0.68936 0.24279 0.04320 0.00294 0.00006
0.00000

subfilter 4

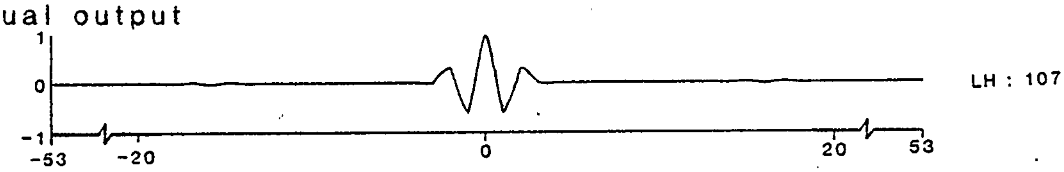
1.00000 0.29176 0.03164 0.00322 0.00001 0.00000

Warning! There is insufficient working space for G.

The maximum number of subfilters which can be used : 4

filter F was truncated to the length LF :101

actual output




shaping errors

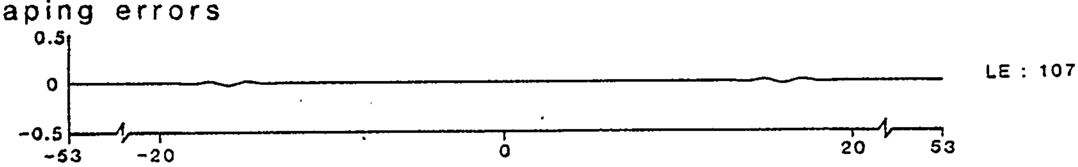



Figure 3.7 Example showing the effect of setting "MXLG" too small on the filter quality "MXLG" : 101

desired wavelets both have an odd number of points by adding one zero if necessary. When a truncation occurs caused by too small a value for "MXLG", the number of coefficients in the wave-shaping filter is equal to "MXLG". If "MXLG" is set to an even value, then the actual output contains an even number of coefficients. All the coefficients on the actual output are then assigned to incorrect time sample. This translates into a shift of the actual output by one sample in the positive direction. The consequence of this shift is better appreciated when considering the shaping errors which usually become large. Figure 3.8 shows an example in which "MXLG" is set to an even value in case 1 and to an odd value in case 2. As shown, the actual output is shifted by one sample in case 1 and the resulting shaping errors are very large. Note the change is the scale of amplitude for the shaping errors. In order to avoid such a situation, it is recommended setting "MXLG" to an odd value (especially when truncation occurs).

In order to get a good quality filter, "MXLG" should be set large enough to let the symmetric filter develop using "MXN" subfilters and also cover the length of the wave-shaping filter. When truncation occurs, "MXLG" should be set to an odd value.

## 3.5 CONCLUSION

The program "MEREU1" computes the coefficients of the subfilters, symmetric filter and wave-shaping filter in a very efficient manner. The quality of the filter greatly depends on the values chosen for each of

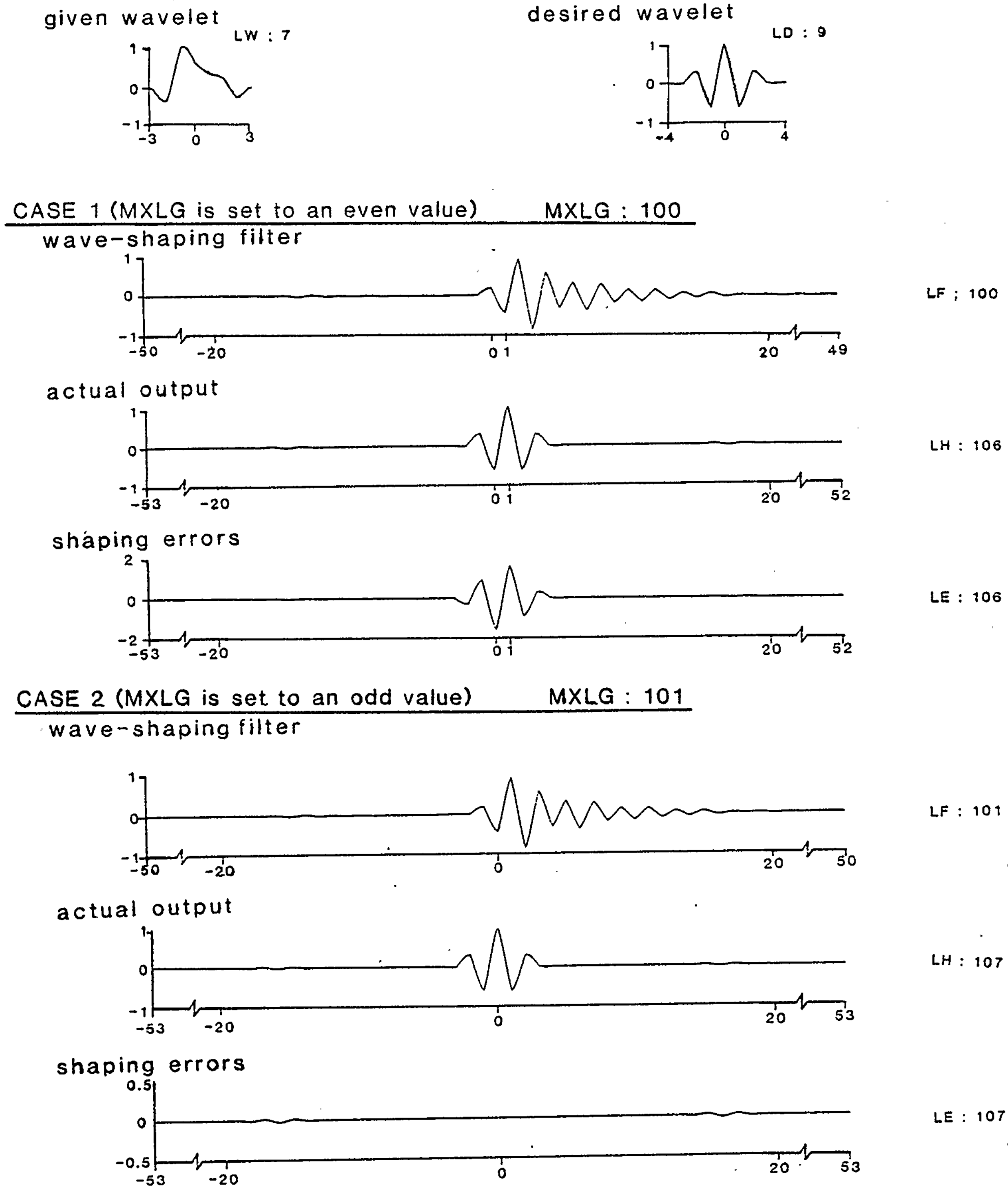



Figure 3.8 Example showing how setting "MXLG" to an even value affects the wave-shaping filter and the actual output

the control parameters. In a specific case, a series of tests might be necessary to determine the appropriate values to input. Changing the value of a particular control parameter may have an effect on the other parameters. For instance, decreasing the value of "BMIN" may have the effect of making the subfilters, the symmetric filter, and the wave-shaping filter longer. Increasing "MXLG" may then also be necessary. A logical order in which the tests can be performed is shown in Figure 3.9. When the shaping errors are small, a good quality filter is obtained.

In a specific case, one should set :

- "MXLG" to a relatively large odd value
- "MXN" to a relatively large value
- "BMIN" to a small value (typically $10^{-12}$)
- "WEIGHT" equals to zero

then make the following checks on the output :

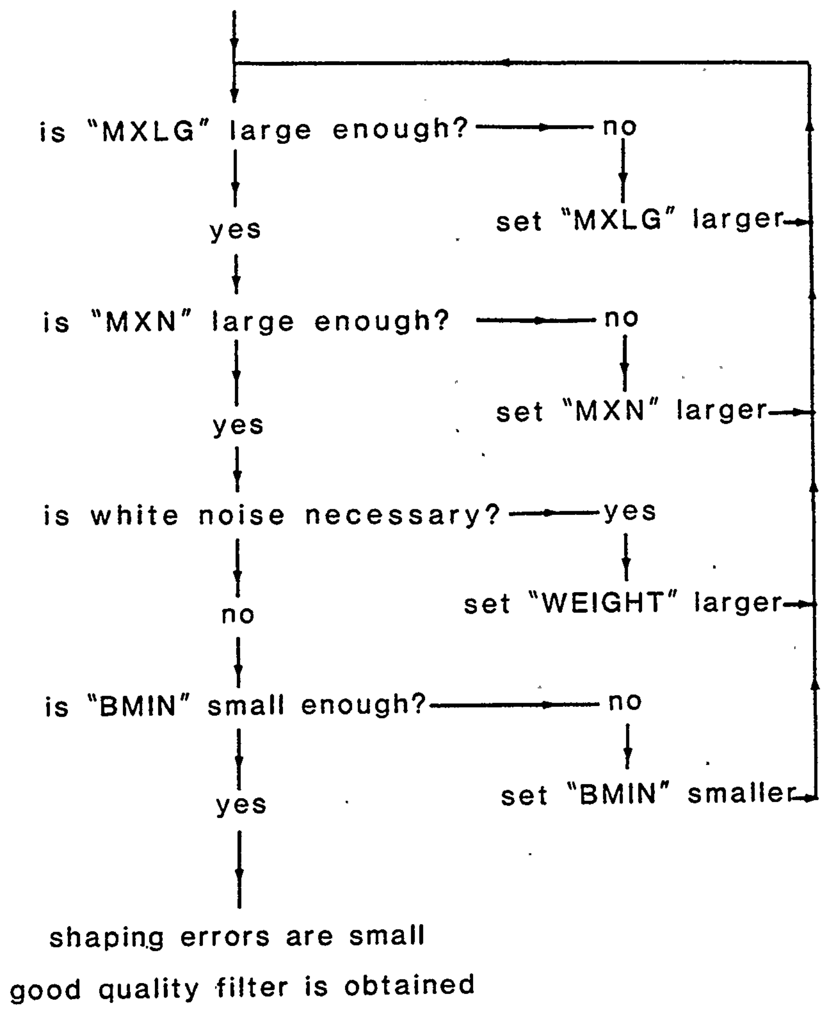



Figure 3.9 Chart showing the order in which the tests can be run to determine an appropriate value for each control parameter in a given case

# CHAPTER 4

# TRUNCATION TECHNIQUES

## 4.1 INTRODUCTION

The "exact" wave-shaping filter can become very long compared to the length of the data set to be filtered. However the significant portion of the "exact" wave-shaping filter is usually contained in a restricted time interval. Therefore, before applying the filter to a dataset, it is suggested truncating the "exact" wave-shaping filter. As truncating a filter alters its response, the truncation should be done very carefully. This chapter presents two truncation techniques: the standard technique and the alternate technique. In the first technique, the standard technique, some coefficients are discarded at both ends of the exact wave-shaping filter. In the second technique, the alternate technique, the length of the symmetric filter is limited during the calculations. This second technique can be particularly useful when the filter becomes extremely long with respect to the length of the dataset to be filtered. In order to compute the filter coefficients truncated with the second technique, some modifications were brought to the original version of the computer program. These modifications are given. The precautions one should take to get the best possible results in each case are also discussed.

## 4.2 STANDARD TRUNCATION TECHNIQUE

The standard truncation technique corresponds here to the technique described by Mereu (1976). It simply consists of discarding some coefficients at both ends of the filter F.

Figure 4.1 shows schematically how the length of the symmetric filter progressively increases making the length of the filter F relatively long before truncation. It also shows the sequence of operations. First, the symmetric filter coefficients are computed using all the coefficients in $G_{N-1}$ and $F_N$. Then the wave-shaping filter coefficients are computed using all coefficients in $G_N$, W and D. Finally, the truncation is done. It is assumed when using this technique that enough memory space is available to contain all the filter coefficients.

The points at which the filter should be truncated depend mainly on the length of the dataset to be filtered. When the dataset is long relative to the filter, the truncation should be done at points where the filter amplitude is negligible. According to Mereu (1976), the shaping errors introduced by the truncated filter remain very small if the significant coefficients left in the truncated filter are able to generate a single desired wavelet in the output sequence. When the length of the dataset to be filtered is relatively short, the filter can be truncated at points where its amplitude is large. In order to get

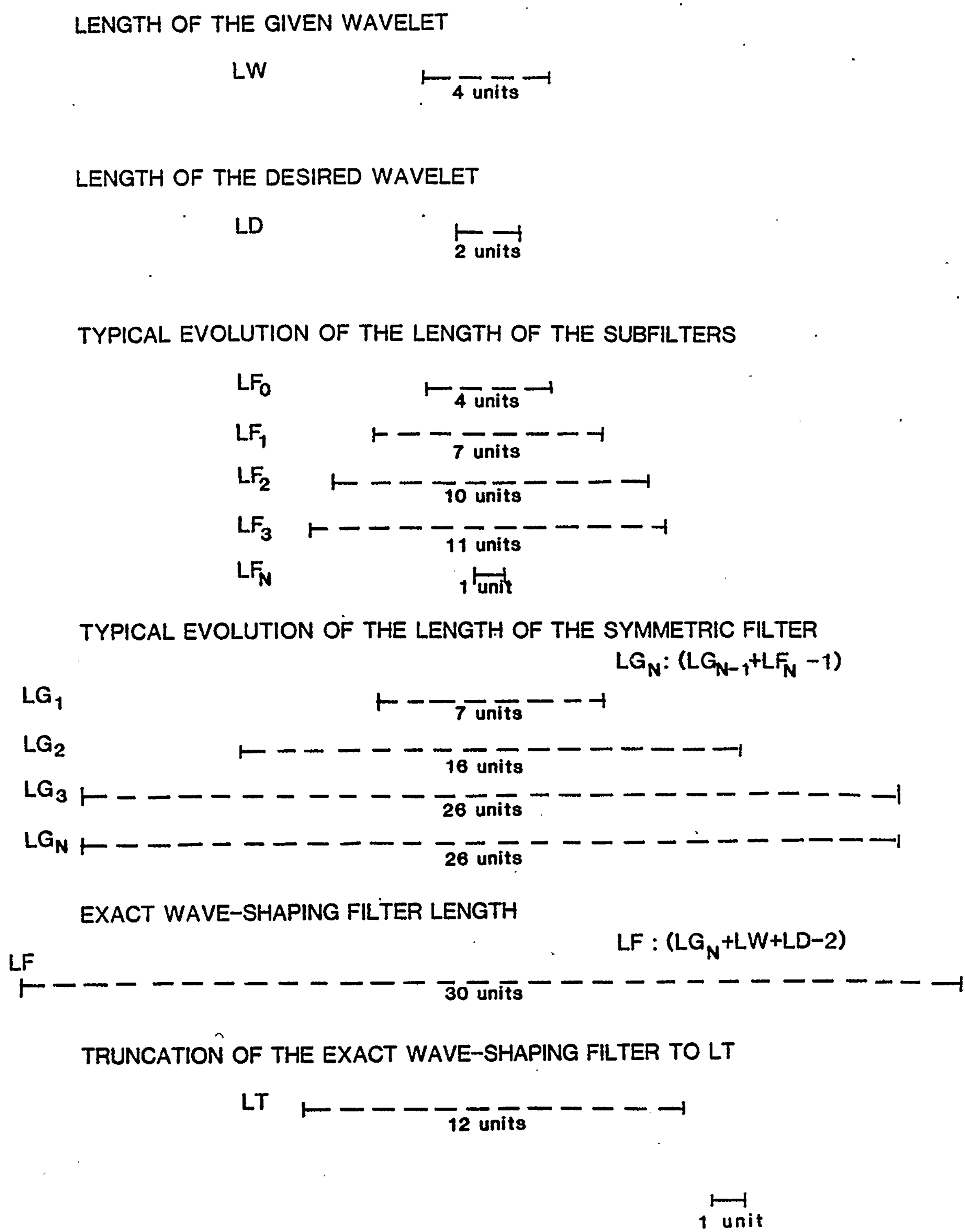



Figure 4.1 Schematic representation of the standard truncation technique

good results in this case, the two following conditions must be met:

"1. the number of inserted zeros between the wavelets in the wavelet output sequence (F*W) should exceed the length of the data to be filtered;

2. the length of the truncated filter should exceed $2t_d + t_w$ where $t_d$ = the data length and $t_w$ = the desired wavelet length.

If these two conditions are met, all the errors generated by the filter will lie outside the region of interest leaving that region effectively error-free, except for the small round-off errors which may still be present." (Mereu, 1976, p. 666)

Examples of truncated filters are presented in Figures 4.2 and 4.3. In each case, the "exact" wave-shaping filter contains 253 coefficients before truncation and is obtained using eight subfilters. The subfilter coefficients are shown in Table 4.1. Figure 4.2 presents an example of a truncated filter at points where its amplitude is relatively negligible. In this case, the amplitude of the filter at the truncation points 50 and -50 is very small and the output obtained using the truncated filter is very similar to the desired output as proven by the small magnitude of the shaping errors. In this example, the "exact" wave-shaping filter contains 253 points. However, the significant

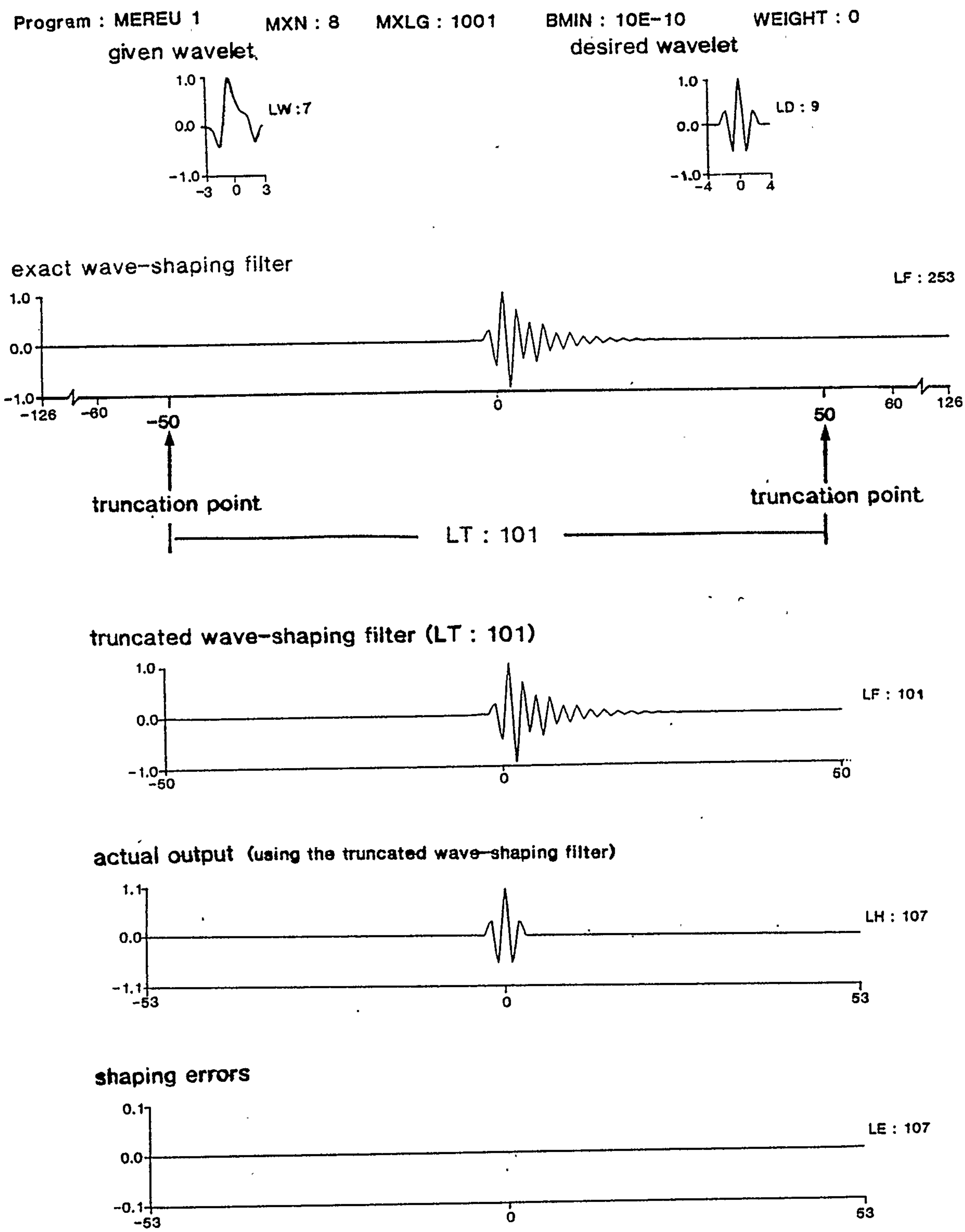



Figure 4.2 Example of a truncated filter at points where its amplitude is relatively negligible

portion of the filter is contained within the central 101 coefficients, as proven by the small magnitude of the shaping errors obtained when using the truncated filter.

Figure 4.3 presents an example of a filter truncated at points where its amplitude is relatively large. As shown, the errors produced at one end of the output sequence are relatively large because the filter amplitudes are large at one truncation point. This filter could be used with satisfaction if the dataset to be filtered contained no more than 11 coefficients (assuming the sampling rate of the dataset equals the sampling rate of the filter). This would effectively satisfy both conditions mentioned previously:

1. The number of inserted zeros ($2^8$-1) between the wavelets in the output sequence is larger than 11.

2. $LT > 2t_d + tw$

   $31 > 22 + 7$

The standard truncation technique consists of discarding some coefficients at both ends of the filter, F. The points where the filter should be truncated depends on the length of the dataset to be filtered. When the dataset to be filtered is long relative to the filter, the truncation should be done at points where the amplitude is negligible. If the remaining filter coefficients generate a single spike in the spike sequence, or a single desired wavelet in the output sequence, then the

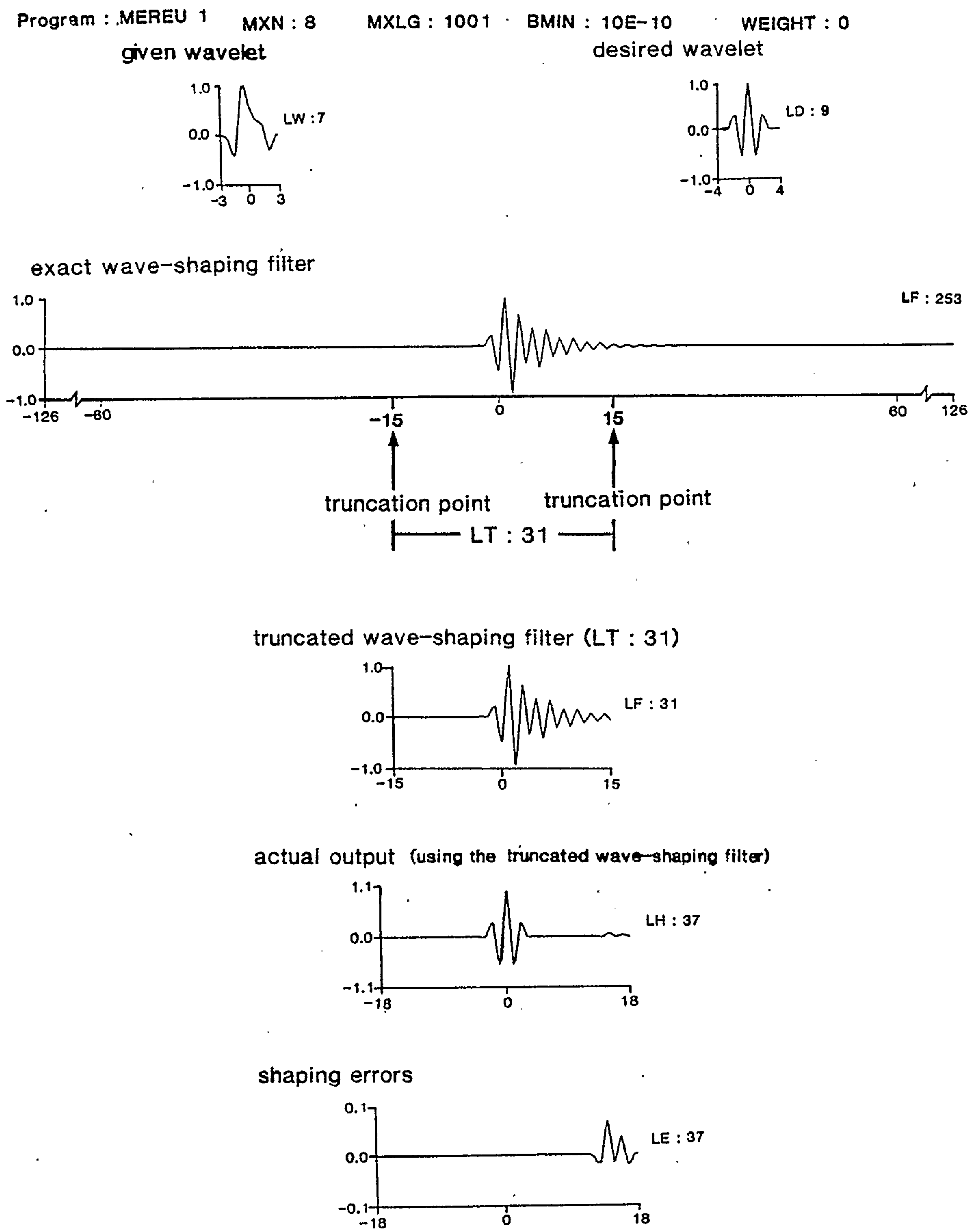



Figure 4.3 Example of a truncated filter at points where its amplitude is relatively large

The following subfilter table shows only the non-zero weights of the positive half including the central point

subfilter 1 — LG : 7

1.00000 -0.24000 -0.00571 0.09142 -0.23428 -0.05714
0.01714

subfilter 2 — LG : 19

1.00000 0.02026 -0.46531 -0.00120 0.06702 0.01162
0.00030

subfilter 3 — LG : 43

1.00000 0.68936 0.24227 0.04320 0.00294 0.00006
0.00000

subfilter 4 — LG : 74

1.00000 0.29176 0.03164 0.00322 0.00001 0.00000

subfilter 5 — LG : 109

1.00000 0.02850 -0.00101 0.00001 0.00000

subfilter 6 — LG : 137

1.00000 0.00284 0.00000 0.00000

subfilter 7 — LG : 158

1.00000 0.00000

subfilter 8 — LG : 120

1.00000

Table 4.1 Subfilters coefficients for the examples presented in figures 4.2 and 4.3

shaping errors remain small and the truncated wave-shaping filter is good quality. When the dataset to be filtered is short relative to the filter, the truncation can be done at points where the amplitude is large. In this case, the two conditions quoted in page 55 should be respected.

## 4.3 ALTERNATE TRUNCATION TECHNIQUE

In every example shown up to now, the given wavelet is short (7-21 points) and the wave-shaping filters, although long with respect to the given wavelet, have a length that stays within the limit of the working space. In other words, there has always been enough storage allocated for each array to allow the full development of the subfilters, symmetric filter and wave-shaping filter.

In practical situations, the given wavelet can become longer than the ones used in the previous examples. For instance, Vibroseis sweeps used in the field can typically contain thousands of samples. If such a sweep were to be input as a given wavelet, then the wave-shaping filter would become extremely long. In some cases, the length of the filter could even overcome the working space available. In these cases, no good quality filter could be defined as early truncation would occur.

In order to solve for this kind of restriction, a new truncation technique was developed: the alternate truncation technique. This new technique differs from the standard technique in that only a limited portion of the coefficients of the symmetric filter is used to calculate the coefficients of the next symmetric filter. The length of the symmetric filter is limited during the calculation of its coefficients. In the new technique part of the truncation occurs during the calculations rather than after as was the case in the standard technique.

Figure 4.4 illustrates schematically the process involved in the alternate truncation technique. The first truncation occurs during the calculation of the symmetric filter coefficients. After each step, the size of the symmetric filter is compared to a preset limit defined here by LT. As shown, the symmetric filter develops normally until it reaches a length larger than LT. This happens here after the second step. In this case, the length of $G_2$ is truncated to LT, and only a limited portion of the coefficients of $G_2$ will be used to generate the coefficients of $G_3$ whose length will also be compared to LT. This process of length comparison, truncation and calculation of the coefficients of the next symmetric filter is repeated until N subfilters have been used. The coefficients of the wave-shaping filter are then calculated and the length of the wave-shaping filter is also truncated to "LT".

The original program written by Mereu (1978) was modified to allow the truncation of the symmetric filter. A copy of the modified version is contained in Appendix 2. The major modification occurs at Line 132 of the subroutine "SHAPEW" where "lggb" is compared to "MXLG". The term

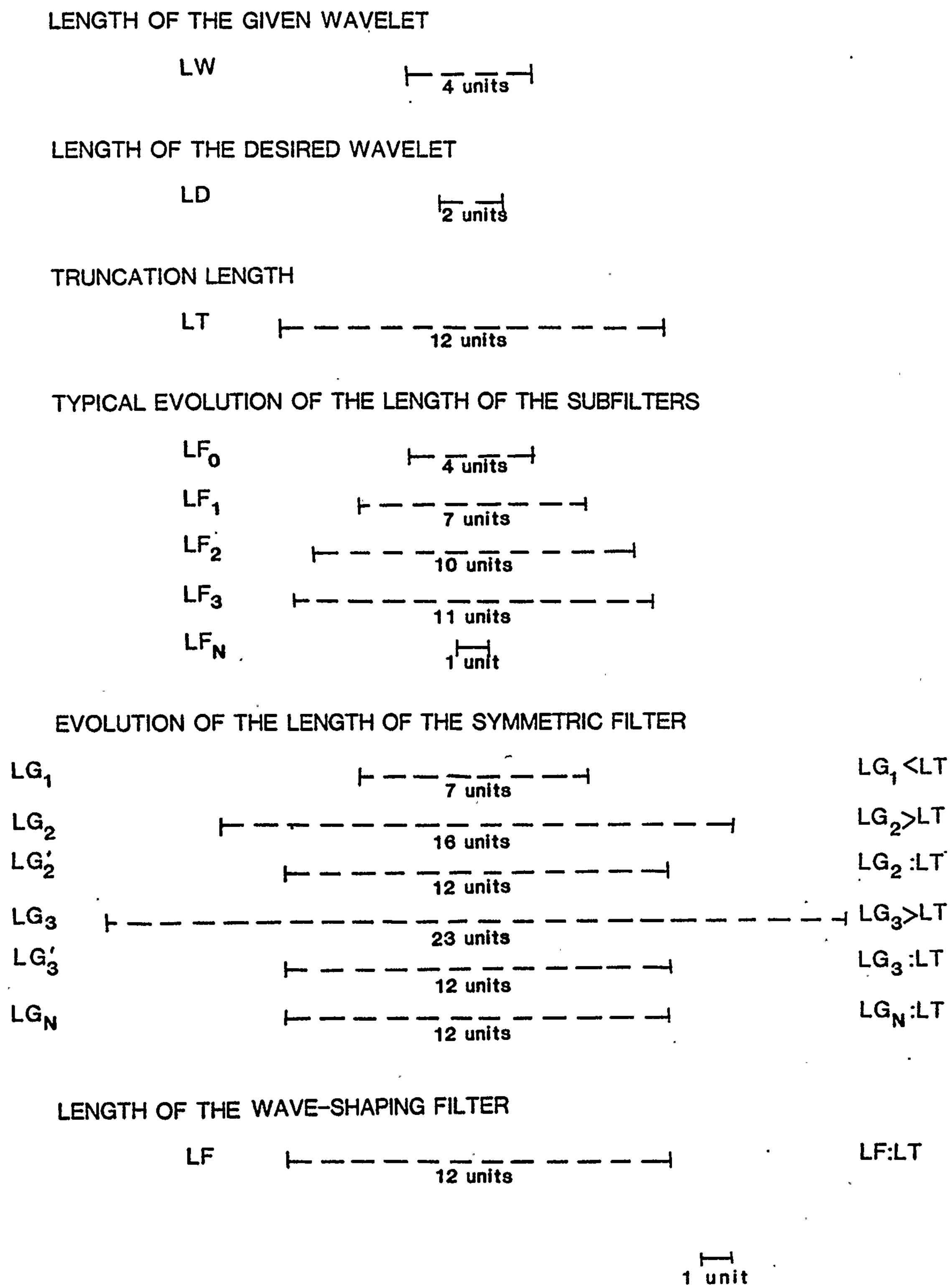



Figure 4.4 Schematic representation of the alternate truncation technique

"lggb" represents the length of one side of the symmetric filter including its central point. If "lggb" happens to be larger than "MXLG" then "lggb" is limited to "MXLG." This new value for "lggb" is then carried out into the rest of the subroutine, as only (2*lggb-1) coefficients of $G_{N-i}$ will be used to generate the coefficients of the next symmetric filter $G_{N-i+1}$. This implies that there will always be sufficient storage space for $G_N$. The wave-shaping filter calculated with a limited portion of the symmetric filter is then also truncated "MXLG".

In this modified version of Mereu's program, the parameter "MXLG" sets the maximum length over which one side of the symmetric filter can develop. It represents the truncation distance and also corresponds to the length of the wave-shaping filter.

Very similar outputs to those of the standard technique are produced by the modified version of the program. An example of a filter truncated using the alternate technique is presented in Figure 4.5. The given wavelet, desired wavelet and control parameters are the same as in Figure 4.2 except for "MXLG" which is here set to 101. Table 4.2 shows the subfilter coefficients obtained in this case. As indicated by the value of "LG" (half the length of the symmetric filter), the truncation of the symmetric filter occurs after the 5th subfilter. Figure 4.5 presents the wave-shaping filter truncated to 101 points. This truncated filter still gives a satisfactory output as proven by the small magnitude of the shaping errors.

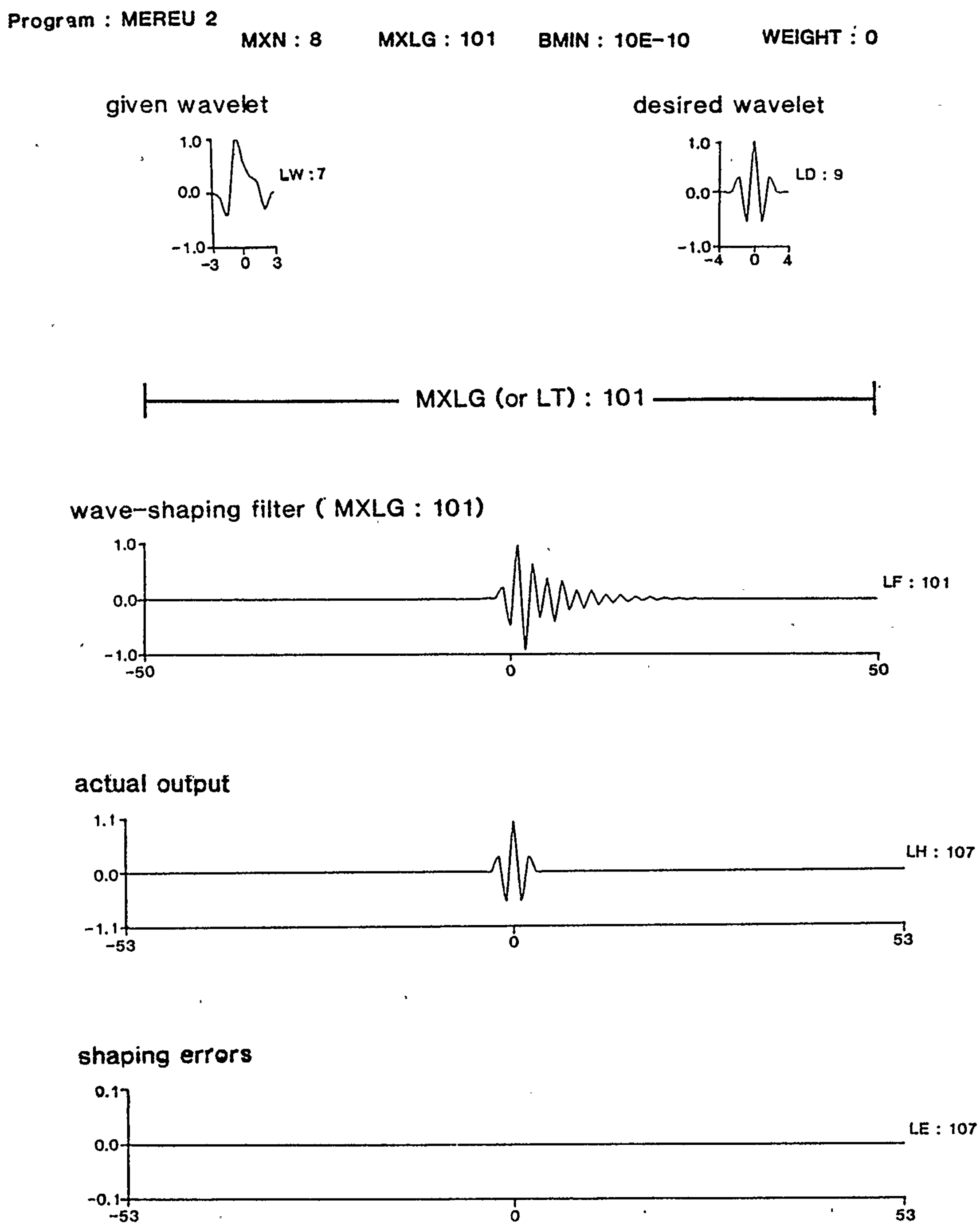



Figure 4.5 Example of a filter truncated using the alternate truncation technique

The following subfilter table shows only the non-zero weights of the positive half including the central point.

subfilter 1 LG : 7

1.00000 -0.24000 -0.00571 0.09142 -0.23428 -0.05714
0.01714

subfilter 2 LG : 19

1.00000 0.02026 -0.46531 -0.00120 0.06702 0.01162
0.00030

subfilter 3 LG : 43

1.00000 0.68936 0.24227 0.04320 0.00294 0.00006
0.00000

subfilter 4 LG : 74

1.00000 0.29176 0.03164 0.00322 0.00001 0.00000

subfilter 5 LG : 101

1.00000 0.02850 -0.00101 0.00001 0.00000

subfilter 6 LG : 101

1.00000 0.00284 0.00000 0.00000

subfilter 7 LG : 101

1.00000 0.00000

subfilter 8 LG : 101

1.00000

Table 4.2 Subfilters coefficients for the example presented in figure 4.5

Because the subfilter coefficients are calculated prior to the truncation (see Figure 4.4), they are not affected by the truncation. This can be verified by comparing the subfilter coefficients obtained in the cases presented in Table 4.1 versus Table 4.2. As shown, the coefficients are the same and therefore are not affected by the alternate truncation technique.

In this new truncation technique, the magnitude and the distribution of the shaping errors depend on the truncation distance, the number of symmetric filters affected by the truncation, and the magnitude of the discarded coefficients in the symmetric and wave-shaping filters. Discarding some coefficients in $G_{N-1}$ affects the quality of $G_N$ and consequently affects the quality of F. In general, the shorter the truncation distance, the earlier in the sequence the truncation occurs and the larger is the magnitude of the shaping errors. Tables 4.3, 4.4 and 4.5 present the subfilter coefficients obtained when the truncation distance gradually decreases from 81 to 61 to 31. As indicated by the value of LG, the number of symmetric filter affected by the truncation increases as the truncation distance decreases. The wave-shaping filter and actual output in each one of these cases are presented in Figure 4.6 As the truncation distance decreases the error in the actual output increases with the main error occurring at the truncation points. In order to get good results when using this technique. These conditions are quoted on p. 55.

The following subfilter table shows only the non-zero weights of the positive half including the central point

subfilter 1 LG : 7

1.00000 -0.24000 -0.00571 0.09142 -0.23428 -0.05714 0.01714

subfilter 2 LG : 19

1.00000 0.02026 -0.46531 -0.00120 0.06702 0.01162 0.00030

subfilter 3 LG : 43

1.00000 0.68936 0.24227 0.04320 0.00294 0.00008 0.00000

subfilter 4 LG : 74

1.00000 0.29176 0.03164 0.00322 0.00001 0.00000

subfilter 5 LG : 81

1.00000 0.02850 -0.00101 0.00001 0.00000

subfilter 6 LG : 81

1.00000 0.00284 0.00000 0.00000

subfilter 7 LG : 81

1.00000 0.00000

subfilter 8 LG : 81

1.00000

Table 4.3 Subfilters coefficients for the example presented in figure 4.6
"MXLG" : 81

The following subfilter table shows only the non-zero weights of the positive half including the central point

subfilter 1 — LG : 7

1.00000 -0.24000 -0.00571 0.09142 -0.23428 -0.05714
0.01714

subfilter 2 — LG : 19

1.00000 0.02026 -0.46531 -0.00120 0.06702 0.01162
0.00030

subfilter 3 — LG : 43

1.00000 0.68936 0.24227 0.04320 0.00294 0.00006
0.00000

subfilter 4 — LG : 61

1.00000 0.29176 0.03164 0.00322 0.00001 0.00000

subfilter 5 — LG : 61

1.00000 0.02850 -0.00101 0.00001 0.00000

subfilter 6 — LG : 61

1.00000 0.00284 0.00000 0.00000

subfilter 7 — LG : 61

1.00000 0.00000

subfilter 8 — LG : 61

1.00000

Table 4.4 Subfilters coefficients for the example presented in figure 4.6
"MXLG" : 61

The following subfilter table shows only the non-zero weights of the positive half including the central point

| | | | | | | |
|---|---|---|---|---|---|---|
| **subfilter 1** | | | | LG : 7 | | |
| 1.00000 | -0.24000 | -0.00571 | 0.09142 | -0.23428 | -0.05714 | 0.01714 |
| **subfilter 2** | | | | LG : 19 | | |
| 1.00000 | 0.02026 | -0.46531 | -0.00120 | 0.06702 | 0.01162 | 0.00030 |
| **subfilter 3** | | | | LG : 31 | | |
| 1.00000 | 0.68936 | 0.24227 | 0.04320 | 0.00294 | 0.00006 | 0..00000 |
| **subfilter 4** | | | | LG : 31 | | |
| 1.00000 | 0.29176 | 0.03164 | 0.00322 | 0.00001 | 0.00000 | |
| **subfilter 5** | | | | LG : 31 | | |
| 1.00000 | 0.02850 | -0.00101 | 0.00001 | 0.00000 | | |
| **subfilter 6** | | | | LG : 31 | | |
| 1.00000 | 0.00284 | 0.00000 | 0.00000 | | | |
| **subfilter 7** | | | | LG : 31 | | |
| 1.00000 | 0.00000 | | | | | |
| **subfilter 8** | | | | LG : 31 | | |
| 1.00000 | | | | | | |

Table 4.5 Subfilters coefficients for the example presented in figure 4.6
"MXLG" : 31

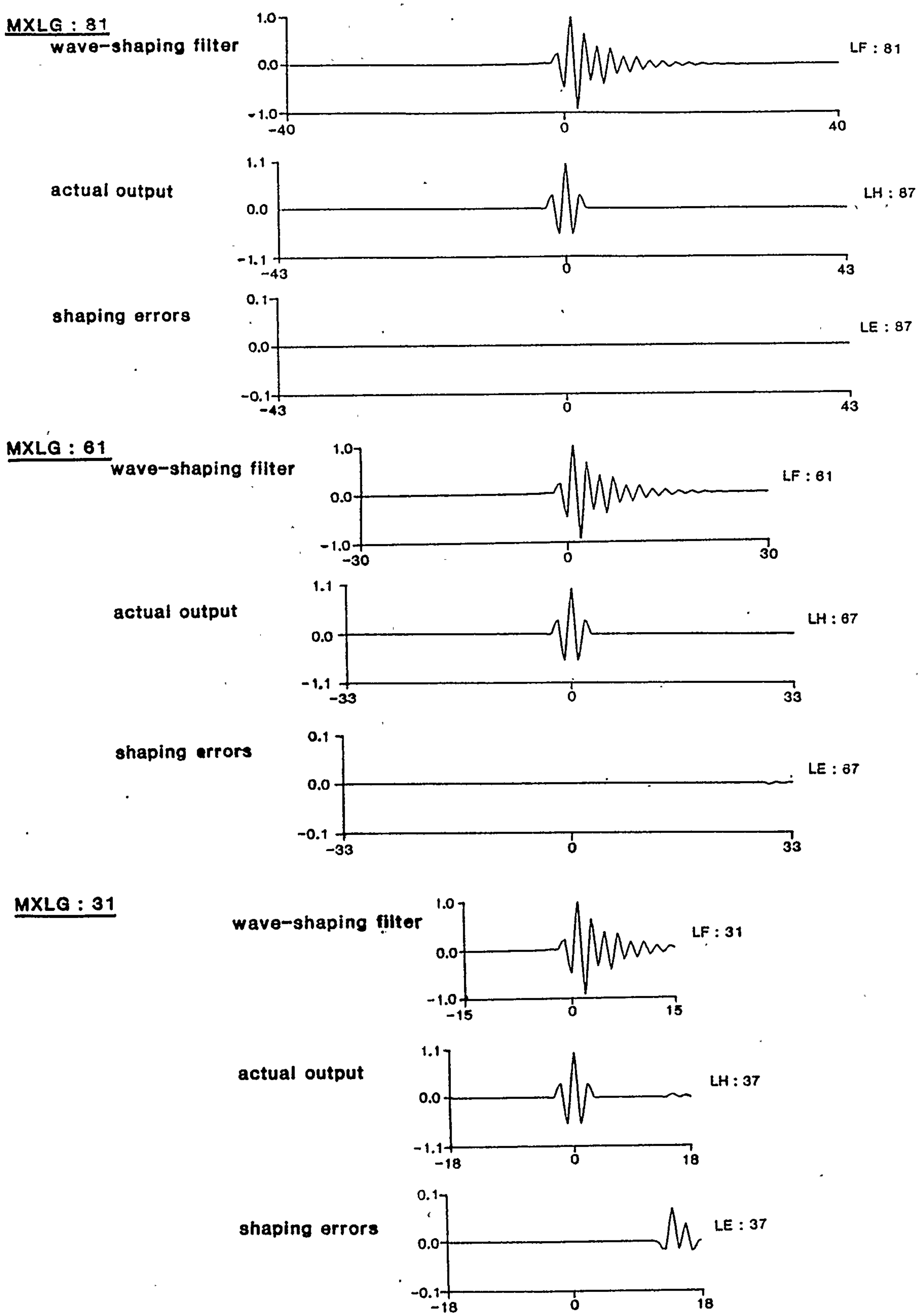



Figure 4.6 Sequence of diagrams showing the effect of the truncation distance on the wave-shaping filter,actual output and shaping errors

In the alternate truncation technique, only a limited range of the coefficients of the symmetric filter is used to calculate the coefficients of the next symmetric filter. The truncation distance, specified by "MXLG" in the modified version of the program (Mereu 2), should be chosen carefully in a specific case as it affects the filter quality.

## 4.4 COMPARISON OF THE TWO TECHNIQUES

Both truncation techniques can be used to define good quality wave-shaping filters. However, one technique may be preferred over the other under certain conditions.

When the length of the symmetric filter does not overcome the storage space available, the standard truncation technique may be preferred as it generally gives slightly better results. Table 4.6 compares the shaping errors obtained in Figure 4.3 (standard truncation technique) with the ones obtained in Figure 4.6, "MXLG" = 31 (alternate truncation technique). In both cases, the wave-shaping filter is truncated to 31 points. As shown, the errors in the case of the standard truncation technique are slightly smaller. The early truncation process involved in the alternate technique produces some errors of larger magnitude than those caused by discarding some negligible coefficients at the end of the "exact" wave-shaping filter.

However, in some cases the difference between the two sets of errors can be very small. This is the case in Table 4.6. The actual output and shaping errors obtained when the wave-shaping filter is truncated using both techniques are plotted in Figure 4.7. As shown, there is no apparent differences in the output or in the errors. One or the other truncation technique can be used with satisfaction in this case.

In some cases, the length of the symmetric filter becomes so long that it tends to overcome the storage space available. In these cases, the alternate truncation technique should be chosen as it is the only way to define a good quality wave-shaping filter. An example showing G becoming excessively long will be presented in Chapter 5.

## 4.5 CONCLUSION

The length of the wave-shaping filter tends to become very long compared to the length of the dataset to be filtered. Because the significant portion of the "exact" wave-shaping filter is usually contained in a restricted time interval, the wave-shaping filter can be truncated and still give a satisfactory output. Two different techniques can be used with success: the standard truncation technique developed by Mereu (1976) and a new technique called the alternate technique developed here. In the standard technique, some coefficients are discarded at both ends of the "exact" wave-shaping filter. It is then assumed that enough memory space is available to contain all the filter coefficients. In the

| time sample | shaping errors case 1 | shaping errors case 2 |
|---|---|---|
| -18 | -4.4E-07 | -1.6E-07 |
| -17 | -2.8E-06 | -9.8E-07 |
| -16 | 2.4E-07 | -5.6E-07 |
| -15 | 1.0E-05 | -2.3E-07 |
| -14 | -4.1E-06 | -5.1E-08 |
| -13 | 4.7E-06 | 2.3E-07 |
| -12 | 8.8E-07 | -2.9E-09 |
| -11 | -2.8E-06 | 8.6E-09 |
| -10 | 2.6E-06 | -8.6E-09 |
| -09 | -2.0E-06 | 1.4E-08 |
| -08 | 2.6E-07 | -9.9E-09 |
| -07 | 1.1E-07 | 1.0E-08 |
| -06 | -3.5E-07 | -1.4E-09 |
| -05 | 7.8E-07 | 1.6E-09 |
| -04 | -1.5E-06 | 9.5E-09 |
| -03 | 2.1E-06 | 1.5E-08 |
| -02 | 6.0E-07 | -3.7E-09 |
| -01 | -2.2E-06 | 1.5E-08 |
| 00 | 7.5E-06 | 1.0E-10 |
| 01 | -2.1E-06 | 1.0E-10 |
| 02 | 5.9E-07 | -1.1E-08 |
| 03 | 2.1E-06 | 3.7E-09 |
| 04 | -1.5E-06 | 1.9E-08 |
| 05 | 7.7E-07 | -3.7E-09 |
| 06 | -3.5E-07 | -1.9E-09 |
| 07 | 1.1E-07 | 8.4E-09 |
| 08 | 2.6E-07 | -7.5E-09 |
| 09 | -2.0E-06 | 1.2E-08 |
| 10 | 2.6E-06 | -1.1E-08 |
| 11 | -2.8E-06 | 7.5E-09 |
| 12 | 8.8E-07 | -3.3E-09 |
| 13 | -5.3E-03 | -5.3E-03 |
| 14 | -1.7E-02 | -1.7E-02 |
| 15 | 6.5E-02 | 6.5E-02 |
| 16 | -1.6E-02 | -1.6E-02 |
| 17 | 3.4E-02 | 3.4E-02 |
| 18 | -2.0E-02 | -2.0E-02 |

Table 4.6 Comparison between the shaping errors obtained using wave-shaping filters truncated with the alternate technique (case1) and the standard technique (case2) . In both cases the filter contains 31 points.

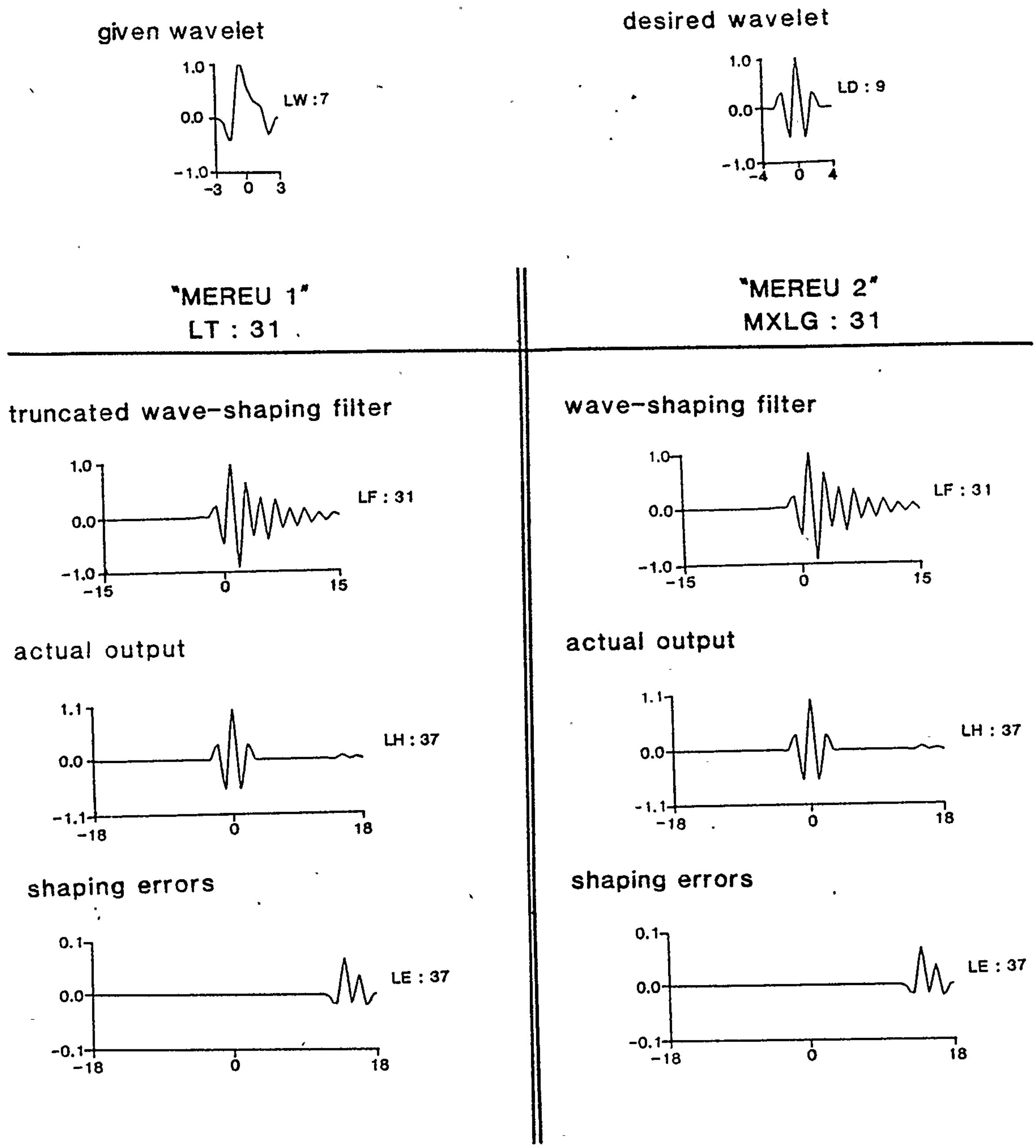



Figure 4.7 Example showing the similarity between the actual outputs when obtained with wave-shaping filters truncated using the standard truncation technique (MEREU 1) and the alternate truncation technique (MEREU 2)

new technique, the length of the symmetric filter is limited during the calculation of its coefficients. In this technique, there is always enough storage space to contain the symmetric filter and the wave-shaping filter. This becomes an advantage when the wave-shaping filter tends to exceed the maximum storage space available.

It has been shown that both truncation techniques can be used with success and that very few differences could be observed between the two techniques in some cases. In both techniques, precautions regarding the truncation distance should be taken as the quality of the filter may be seriously affected.

# CHAPTER 5

# PRACTICAL APPLICATION OF THE EXACT WAVE-SHAPING FILTER

## 5.1 INTRODUCTION

This chapter presents some examples of exact wave-shaping filters that are designed to attenuate the effect of harmonic distortion on Vibroseis data. The characteristics of this noise are presented and several examples of filters are shown. The efficiency of the wave-shaping filter and the difficulties encountered in defining a good quality filter in some specific cases are also discussed.

## 5.2 CORRELATION NOISE

The Vibroseis system of seismic data acquisition uses hydraulically operated vibrators to transmit a signal into the ground. The signal, called a sweep, is a long wavetrain whose instantaneous frequency varies with time. The input sweep, travelling through the ground, is reflected from each subsurface interface and is eventually recorded at the surface. Mathematically, it is expressed by this formula:

$$X(t) = R(t) * S_p(t)$$

where $X(t)$ = recorded trace

$R(t)$ = reflectivity series

$S_p(t)$ = pilot sweep

$*$ is the convolution operator (5.1)

The recorded trace, is composed of a series of reflected sweeps, each reflected sweep having an amplitude, polarity and arrival time related to the subsurface geology. As the sweep length is normally much greater than the arrival time between reflections, the delayed sweeps overlap. The resulting recorded trace has very little resemblance with the conventional recorded output obtained when using dynamite or any impulsive source.

In order to convert the Vibroseis recorded trace into an interpretable trace, the correlation process is used. The crosscorrelation of the recorded trace with the input sweep compresses each reflected sweep into a narrower wavelet and, consequently, removes most of the overlap. The effectiveness of the correlation process in Vibroseis data depends on the degree of resemblance between the input sweep and recorded wavetrain. The best result would be obtained if the shape of the recorded waveform were undistorted compared to the pilot sweep. In such a case, the crosscorrelation of the pilot sweep with the recorded wavetrain would result in a series of perfectly symmetrical zero-phase wavelets.

This is:

$$
\begin{aligned}
X'(t) &= X(t) * S_p(-t) \\
&= [R(t) * S_p(t)] * S_p(-t) \\
&= R(t) * A(t)
\end{aligned}
$$

where $X'(t)$ = correlated trace

$A(t)$ = autocorrelated pilot sweep (5.2)

In real situations, differences between the shape of the pilot and recorded waveforms have been observed. Part of these differences have been attributed to the distortion of the input signal at the source (Seriff and Kim, 1970). Non-linear effects in the vibrator hydraulics and non-linear reaction of the ground to the force exerted are presumed to be responsible part of the distortion of the input signal at the source. According to Seriff and Kim (1970), these non-linear processes have the effect of generating a series of waveforms whose frequencies are harmonics of those of the input sweep. The distortion resulting from the superposition of these harmonics on the fundamental input sweep is known as harmonic distortion.

The sweep travelling into the ground is therefore composed of the pilot sweep plus its harmonics. The recorded trace can now be written as:

$$X(t) = R(t) * [S_p(t) + S_{2H}(t) + S_{3H}(t) + ..S_{nH}(t)]$$

where $S_{2H}(t)$ = second harmonic

$S_{3H}(t)$ = third harmonic

$S_{nH}(t) = n^{th}$harmonic (5.3)

Usually,harmonics of second degree are the most important ones in magnitude and consequently the most detrimental (Seriff and Kim, 1970). For this reason, the above equation can be reduced to:

$$X(t) = R(t) * [S_p(t) + S_{2H}(t)] \qquad (5.4)$$

During the correlation process, both components of the input sweep, $S_p(t)$ and $S_{2H}(t)$ get correlated with the pilot sweep to give the following:

$$
\begin{aligned}
X'(t) &= X(t) * S_p(-t) \\
&= R(t) * [S_p(t) + S_{2H}(t)] * S_p(-t) \\
&= R(t) * [S_p(t) * S_p(-t)] + [S_{2H}(t) + S_p(-t)] \\
&= R(t) * [A(t) + C(t)]
\end{aligned}
$$

where C(t) = pilot sweep correlated with its second harmonic

(5.5)

The correlated trace is then made of a total correlated wavelet composed of the autocorrelated pilot sweep, A(t), and of an extra term C(t), which corresponds to the correlation between the pilot sweep and its second harmonic. An example of a fundamental sweep and its second harmonic is presented in Figure 5.1. The autocorrelation of the fundamental sweep, the crosscorelation of the fundamental sweep with its second harmonic and the total correlated wavelet are also plotted. As shown, the correlation between the second harmonic and the pilot sweep produces a relatively long wavetrain which added to the autocorrelated pilot sweep forms the total correlated wavelet. Because the wavetrain resulting from the correlation between the pilot sweep and its second harmonic could interfere with some of the main reflections, it represents an undesirable component of the total correlated wavelet. This wavetrain is called correlation noise.

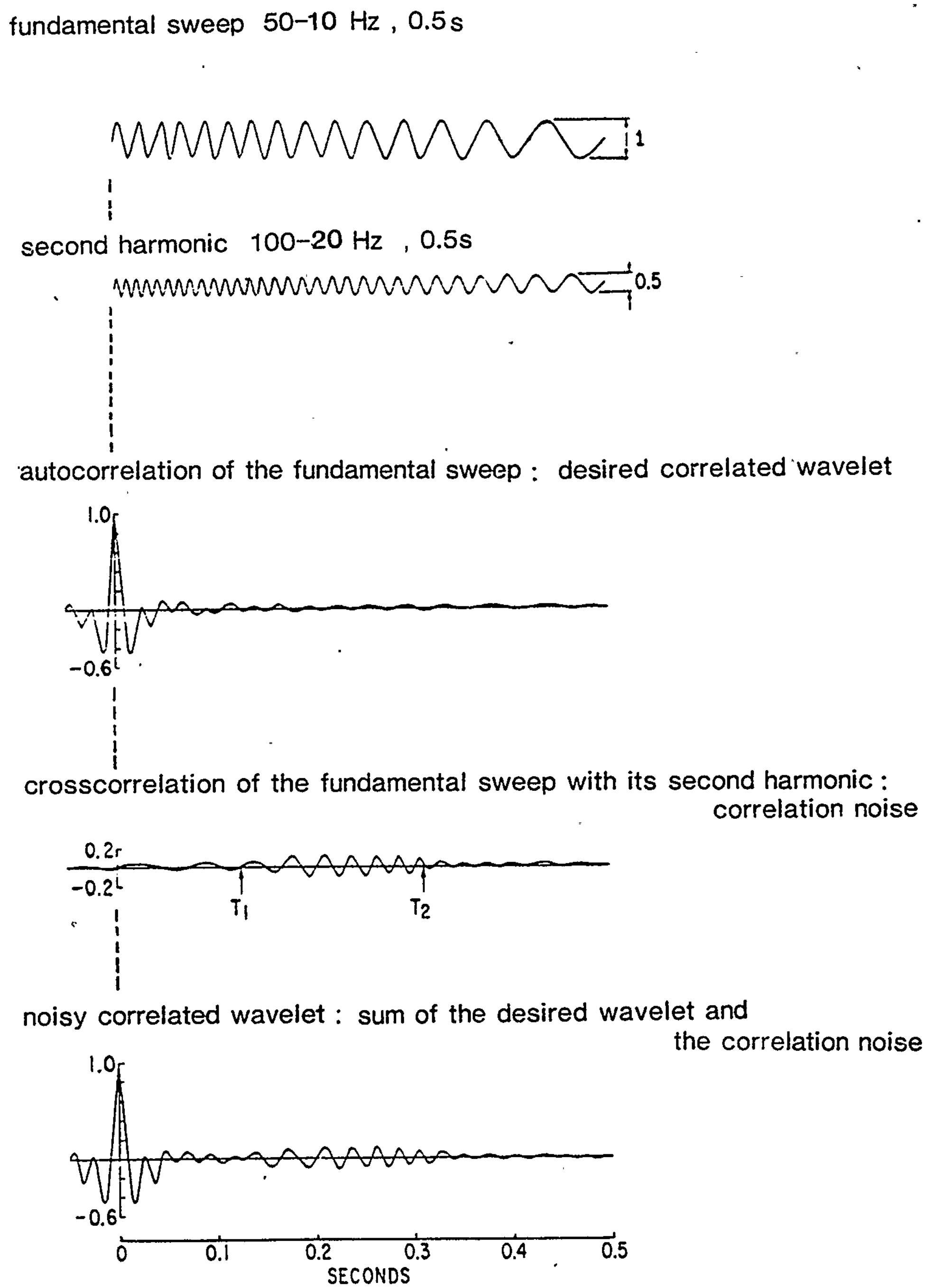



Figure 5.1 Example of correlation noise (after Seriff and Kim (1970),p.238)

The occurrence of the correlation noise, C(t), relative to the autocorrelated pilot sweep, A(t), depends on the pilot sweep used, whether is is a down-sweep or an up-sweep. Down-sweeps are sweeps in which the frequency decreases with time and up-sweeps are sweeps in which the frequency increases with time. Figure 5.2 shows how the correlation takes form as a pilot down-sweep is shifted relative to its second harmonic. Note that the direction of relative shifting corresponds to the conventional direction of relative shifting inherent to the Vibroseis correlation process. As shown, the correlation noise occurs during the time interval $T_1$, $T_2$ and, the autocorrelated pilot sweep is centered at zero time. It means that the correlation noise appears at a later time than the main peak of the autocorrelated pilot sweep. The correlation noise appears then after a main event when down-sweeps are used.

Figure 5.3 shows the occurrence of the correlation noise relative to the autocorrelated pilot sweep in the case of an up-sweep. In this case, the correlation noise occurs during the time interval $-T_1$, $-T_2$. Again, the autocorrelated pilot sweep is centered at zero time. The correlation noise, for an up-sweep, appears then earlier than the main peak of the autocorrelated pilot sweep.

Seriff and Kim (1970) studied in detail the correlation noise due to harmonic distortion. The characteristics of this noise are the following:

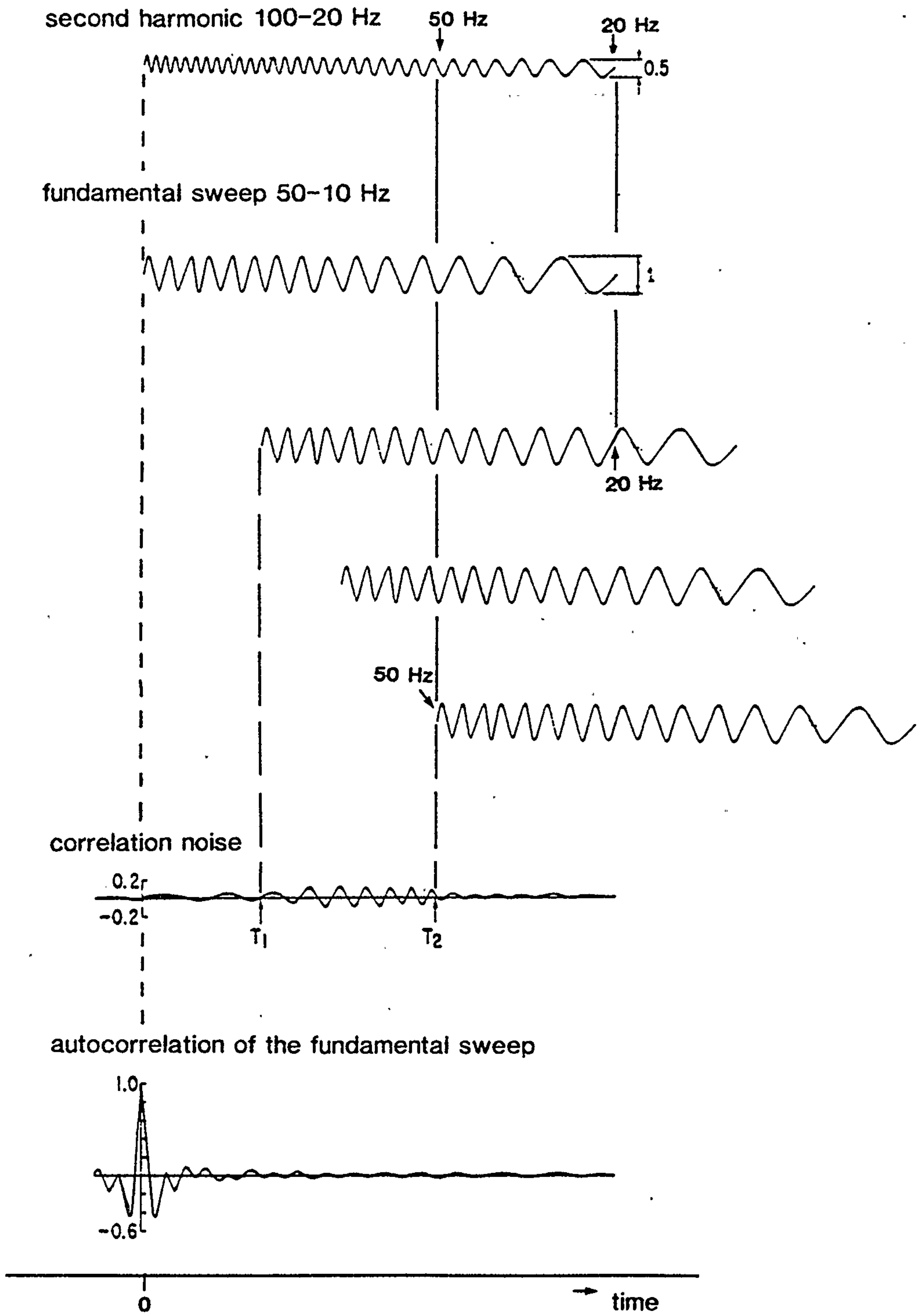



Figure 5.2 Example of correlation noise when using a down-sweep
(after Seriff and Kim (1970), p.237)

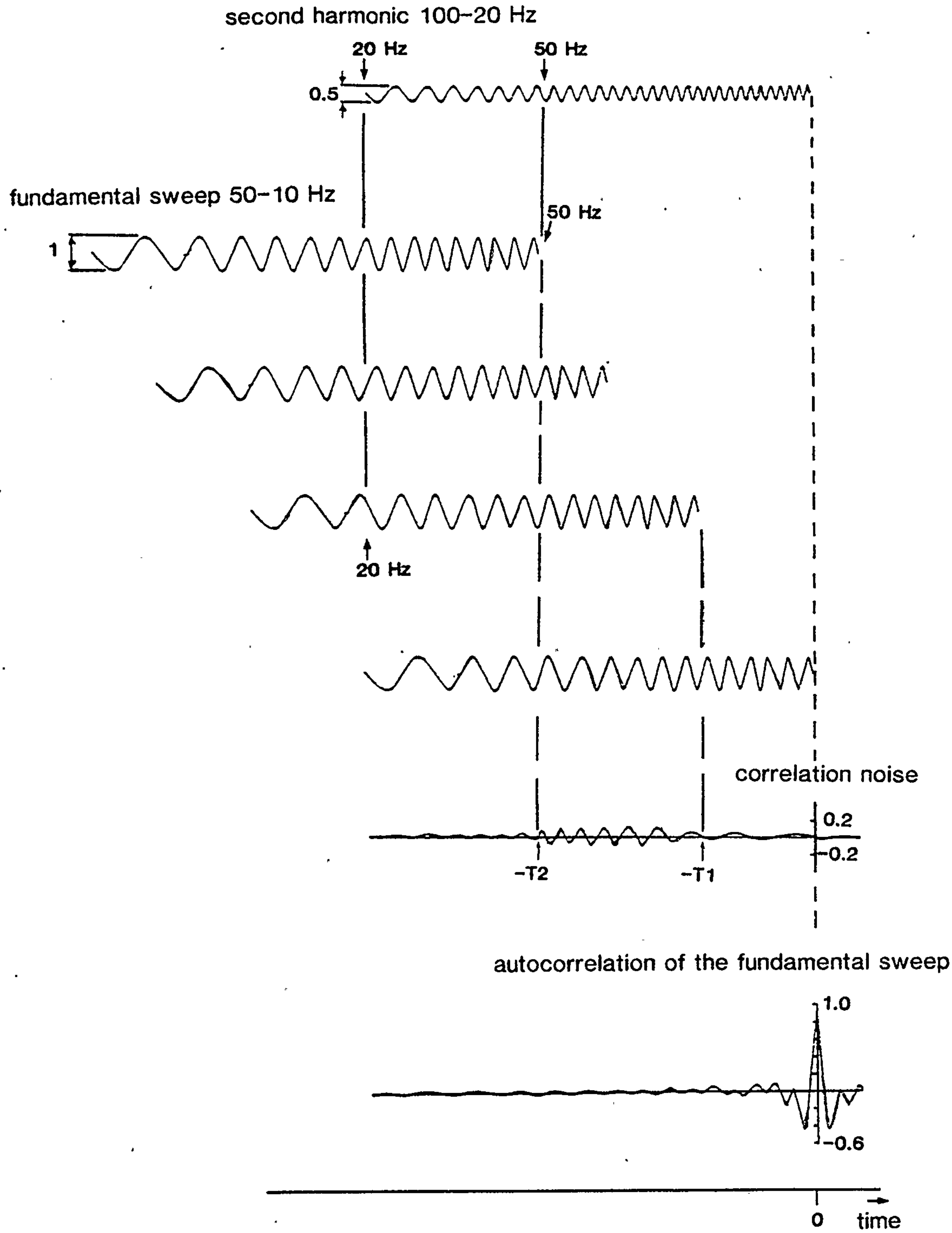



Figure 5.3 Example of correlation noise when using an up-sweep

° The frequencies contained in the noise are the frequencies common to the pilot sweep and its harmonics.

° The amplitude of the noise depends on the relative amplitude of the harmonic signal which in turn depends on the harmonic degree and the importance of the non-linear processes at the input.

° Harmonics of second degree can have a relatively high amplitude. According to Seriff and Kim (1970), second harmonics can be 30% to 100% of the amplitude of the fundamental sweep (below 24 Hz).

° The correlation noise appears during a time interval $T_1$, $T_2$ which is related to the starting and ending frequencies of the pilot sweep and the sweep length. This is:

$$"T_1 = \frac{(k-1)\ Tf_L}{W} \qquad T_2 = \frac{(k-1)\ Tf_H}{kW} \tag{5.6}$$

where k is the harmonic degree

T is the sweep length

W = (starting frequency - ending frequency)

$f_L$ = low frequency of the pilot sweep

$f_H$ = high frequency of the pilot sweep."

(Seriff and Kim, 1970, p. 238)

° The correlation noise appears after a main event when down-sweeps are used and it appears before a main event when up-sweeps used. The correlation noise will then interfere with the earlier events when an up-sweep is used and with the later events when a down-sweep is used. Because the later events have generally a much smaller amplitude than the early ones, the correlation noise is more detrimental when down-sweeps are used.

## 5.3 APPLICATION OF THE EXACT WAVE-SHAPING FILTERING METHOD

Several authors, for example, Seriff and Kim (1970), Eisner (1974) Cunningham (1979), and Rietsch (1981) have suggested different methods for attenuating the effect of harmonic distortion on Vibroseis data. All these methods involve the attenuation of the harmonic distortion during the acquisition. The two methods most commonly used are the ones proposed by Seriff and Kim (1970). The first one consists of using up-sweeps instead of down-sweeps. The relative importance of the correlation noise to the events with which it interferes is reduced when using up-sweeps. The second method consists of designing the input sweep in such a way that the correlation noise appears outside the area of interest. This can be done by choosing appropriate sweep bandwidth and duration.

These methods, although efficient, have not always been used and consequently important correlation noise is present on some data. Cook (1984) presents an example of correlation noise interfering with some reflections.

It is suggested here that the effect of harmonic distortion may be attenuated during the processing step using filtering methods. The "exact" wave-shaping filtering method was preferred to some other traditional techniques for the following reasons:

- it produces an "exact" output in the zone of interest,
- it uses the noise waveform which is known from Seriff and Kim study (1970),
- the filter is, according to Mereu, generally stable.

In the following, some examples of wave-shaping filters specifically defined to attenuate the effect of harmonic distortion are presented. Figure 5.4 summarizes the order in which these examples will be presented. In the first case, the fundamental sweep is short and contains a narrow band of frequencies. In the second case, a longer sweep of wider frequency bandwidth is used to study the quality of the filter obtained with longer time series. Two different approaches are presented in each case:

1st Approach: The harmonic distortion may be attenuated prior to the correlation of the data. In this approach the harmonically distorted sweep and the fundamental sweep are used as given and desired wavelets.

2nd Approach: The correlation noise may be attenuated after the correlation of the data. The noisy correlated wavelet and

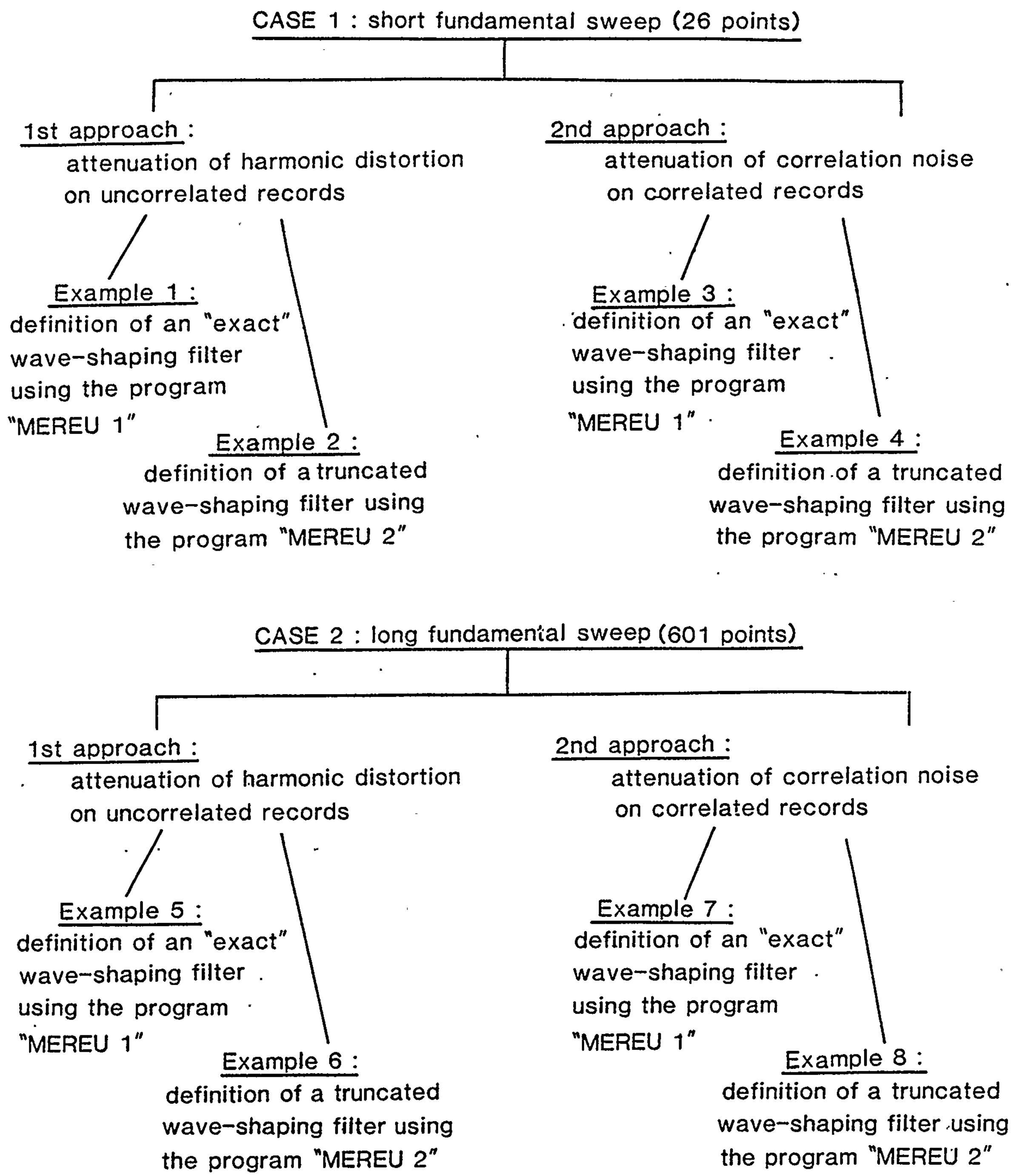



Figure 5.4 Diagram showing the order in which the examples of wave-shaping filters are run

the autocorrelation function of the input sweep are used in this second approach as the given and desired wavelets.

For each approach, the definition of an "exact" wave-shaping filter is attempted using the program "MEREU1". The definition of a truncated wave-shaping filter is also attempted through the program "MEREU2". In both cases, the computations are done using the Multics computer system at the University of Calgary.

### 5.3.1 Case 1: Short fundamental sweep (26 points)

In this first case, the fundamental or pilot sweep (SWEEP1) is characterized by the following:

bandwidth: 5-1 Hz
time length: 1 s
amplitude: 1 unit

Its second harmonic (HSWEEP1) is defined by:

bandwidth: 10-2 Hz
time length: 1 s
amplitude: 0.5 unit

"SWEEP1" and "HSWEEP1" are shown in Figures 5.5 A and B. The amplitude of the harmonic is arbitrarily chosen to half the amplitude of the fundamental sweep. The result of their addition, corresponding to the harmonically distorted signal, "NOISY-SWEEP1", is plotted in Figure 5.5 C. The crosscorrelation of "NOISY-SWEEP1" with the pilot sweep "SWEEP1" results in the waveform "W1" which is plotted in Figure 5.5 D. This waveform is composed of two parts: the desired zero-phase wavelet and the correlation noise. In this case, the major part of the noise appears 0.25 s after the main event and lasts for 0.375 s. The desired zero-phase wavelet, "A1", is plotted in Figure 5.5 E).

1st Approach: Attenuation of harmonic distortion on uncorrelated records.

In this first approach, the given and desired wavelets are:

- given wavelet: "NOISY-SWEEP1"
- desired wavelet: "SWEEP1"

Both time series contain 26 points.

Example 1: Definition of an "exact" wave-shaping filter using the program "MEREU1".

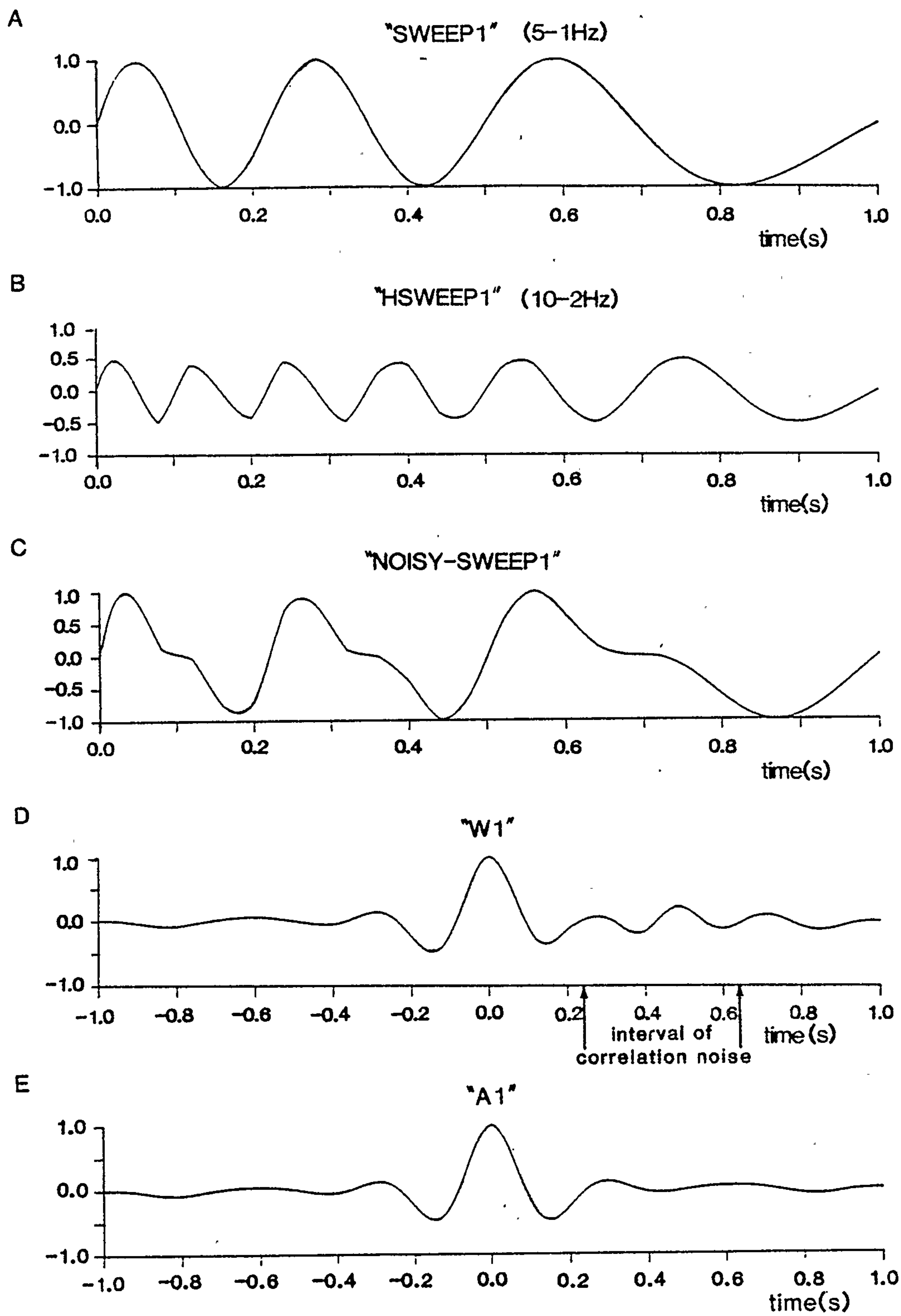



Figure 5.5 Plots of "SWEEP1", "HSWEEP1", "NOISY-SWEEP1", "W1" and "A1"

In order to respect the conditions mentioned in Chapter 3, the following values are input to "MEREU1":

- MXN: 20
- MXLG: 5001
- BMIN: $10^{-10}$
- WEIGHT: 0

Table 5.1 gives the subfilters coefficients obtained in this case. As shown, the 12th subfilter contains only one coefficient. Therefore, setting "MXN" to 20 is large enough. Setting "MXLG" to 5001 is also large enough to let the symmetric filter develop using these 12 subfilters. It can also be noted that the coefficients of each subfilter behave normally. That is, the amplitude of the coefficients in each subfilter decreases as the time from the center point increases. The addition of white noise is then not necessary.

Table 5.2 gives the coefficient values over the central portion of the input and output (25 central points are shown). The total number of coefficients in each time series is also given. The symmetric filter contains 3435 points and the exact wave-shaping filter contains 3487 coefficients. Note that the shaping errors over the shown portion of the output are small indicating that "BMIN" is set small enough. The "exact" wave-shaping filter, actual output and shaping errors are plotted over a larger portion (501 points) in Figure 5.6. As shown the shaping errors remain small over this portion (20 s long).

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given wavelet : "NOISY-SWEEP1"  desired wavelet : "SWEEP1"

```
subfilter  1      no. of weights =   27        lg =   27

b    1.000000   -0.579675    0.041073    0.250992   -0.288647    0.113360    0.0
44561    -0.072219    -0.019627    0.147727    -0.132894    -0.047860    0.175140
-0.107959    -0.035248    0.085224    -0.027897    -0.050655    0.078036    -0.02171
4    -0.072642    0.110518    -0.085503    0.041830    0.000000    0.000000    0.0
00000

subfilter  2      no. of weights =   24        lg =   70

b    1.000000    0.178637   -0.363772   -0.163521    0.040611    0.228171    0.2
70255    0.107883    -0.033981    -0.118369    -0.099109    0.080181    0.106294
 0.007895    -0.021172    -0.014152    -0.014716    0.000505    0.011615    0.00828
7    0.002704    -0.005646    -0.005397    0.004880

subfilter  3      no. of weights =   24        lg =  162

b    1.000000    0.576277    0.005912   -0.310438   -0.077586    0.198006    0.2
11405    0.053688    -0.058195    -0.040244    0.013117    0.032503    0.012626
-0.002863    -0.005952    0.000855    0.002823    0.001757    0.000033    -0.00025
3    -0.000025    0.000121    0.000072    0.000020

subfilter  4      no. of weights =   24        lg =  317

b    1.000000   -0.532960    0.218249   -0.007479    0.100301    0.026330   -0.0
11678    0.010202    -0.005925    0.001775    -0.002834    0.000238    -0.000545
 0.000199    -0.000061    0.000052    -0.000011    0.000006    -0.000001    0.00000
1    -0.000000    0.000000    0.000000    0.000000

subfilter  5      no. of weights =   18        lg =  402

b    1.000000   -0.340539    0.480216   -0.102902    0.009014    0.016927   -0.0
06323    0.001321    -0.000076    -0.000030    0.000012    -0.000003    0.000001
-0.000000    -0.000000    0.000000    -0.000000    0.000000

subfilter  6      no. of weights =    9        lg =  477

b    1.000000   -0.651088    0.152577    0.001845   -0.001466    0.000166   -0.0
00002    -0.000000    0.000000

subfilter  7      no. of weights =    6        lg =  641

b    1.000000    0.587779    0.115567    0.001198    0.000004    0.000000

subfilter  8      no. of weights =    5        lg =  847

b    1.000000    0.344783    0.035605    0.000002   -0.000000

subfilter  9      no. of weights =    3        lg = 1283

b    1.000000    0.062328    0.001656

subfilter 10      no. of weights =    3        lg = 2103

b    1.000000    0.000578    0.000003

subfilter 11      no. of weights =    2        lg = 2508

b    1.000000   -0.000005

subfilter 12      no. of weights =    1        lg = 1718

b    1.000000
```

Table 5.1 Example 1 . Table of the subfilters coefficients (only the non-zero coefficients of the positive half of the subfilters including the centre point are shown)

given wavelet : "NOISY-SWEEP1"    desired wavelet : "SWEEP1"

25 central points of the input and the output data

| time | given wavelet | desired wavelet | correlation functions | | symmetric filter | shaping filter | actual output | shaping errors |
|---|---|---|---|---|---|---|---|---|
| | w | d | r | s | g | f | h | e |
| -12 | 0.967 | 0.94 | 1.643 | 2.132 | -0.106 | 0.056 | 0.945 | 6.8E-06 |
| -11 | 0.121 | 0.65 | 0.449 | 0.599 | 0.003 | 0.016 | 0.651 | 5.8E-06 |
| -10 | -0.033 | -0.43 | -1.247 | -1.563 | 0.114 | -0.075 | -0.433 | 3.4E-07 |
| -9 | -0.765 | -1.00 | -1.386 | -1.663 | 0.043 | 0.024 | -1.000 | -3.9E-06 |
| -8 | -0.697 | -0.48 | -0.184 | -0.125 | -0.081 | 0.085 | -0.482 | -3.2E-06 |
| -7 | 0.729 | 0.51 | 0.678 | 0.628 | -0.068 | -0.095 | 0.508 | 2.2E-06 |
| -6 | 0.803 | 1.00 | 0.418 | 0.117 | 0.034 | -0.084 | 0.999 | 6.4E-06 |
| -5 | 0.099 | 0.61 | -1.064 | -1.281 | 0.049 | 0.158 | 0.612 | 4.8E-06 |
| -4 | -0.006 | -0.25 | -2.709 | -3.235 | 0.102 | 0.014 | -0.254 | -1.3E-06 |
| -3 | -0.401 | -0.90 | -2.355 | -2.778 | -0.118 | -0.226 | -0.905 | -7.5E-06 |
| -2 | -0.985 | -0.92 | 0.385 | 1.656 | 0.037 | 0.119 | -0.923 | -8.6E-06 |
| -1 | -0.554 | -0.37 | 5.440 | 6.917 | -0.233 | 0.262 | -0.373 | -4.1E-06 |
| 0 | 0.541 | 0.36 | 9.384 | 9.636 | 0.502 | 0.677 | 0.363 | 1.9E-06 |
| 1 | 1.000 | 0.88 | 5.440 | 6.609 | -0.233 | 0.348 | 0.885 | 3.5E-06 |
| 2 | 0.615 | 0.98 | 0.385 | 0.627 | 0.037 | -0.002 | 0.982 | 3.8E-07 |
| 3 | 0.140 | 0.68 | -2.355 | -3.822 | -0.118 | -0.256 | 0.681 | -3.1E-06 |
| 4 | 0.001 | 0.16 | -2.709 | -4.483 | 0.102 | -0.090 | 0.155 | -2.9E-06 |
| 5 | -0.023 | -0.39 | -1.064 | -2.341 | 0.049 | 0.101 | -0.387 | 5.2E-07 |
| 6 | -0.235 | -0.79 | 0.418 | 0.160 | 0.034 | 0.057 | -0.789 | 3.0E-06 |
| 7 | -0.615 | -0.98 | 0.678 | 1.306 | -0.068 | -0.094 | -0.982 | 2.1E-06 |
| 8 | -0.929 | -0.97 | -0.184 | 0.978 | -0.081 | -0.060 | -0.970 | -1.2E-06 |
| 9 | -0.989 | -0.80 | -1.386 | 0.082 | 0.043 | 0.112 | -0.805 | -1.9E-06 |
| 10 | -0.779 | -0.55 | -1.247 | -0.494 | 0.114 | 0.154 | -0.551 | 1.0E-06 |
| 11 | -0.406 | -0.27 | 0.449 | -0.499 | 0.003 | -0.009 | -0.268 | 4.9E-06 |
| 12 | 0.000 | 0.00 | 1.643 | -0.196 | -0.106 | -0.185 | 0.000 | 5.2E-06 |
| n | lw | ld | lr | ls | 2*lg-1 | lf | lh | le |
| 12 | 27 | 27 | 53 | 53 | 3435 | 3487 | 3513 | 3513 |

Table 5.2 Example 1 . Table of the input and output coefficients (only the central 25 coefficients are shown)

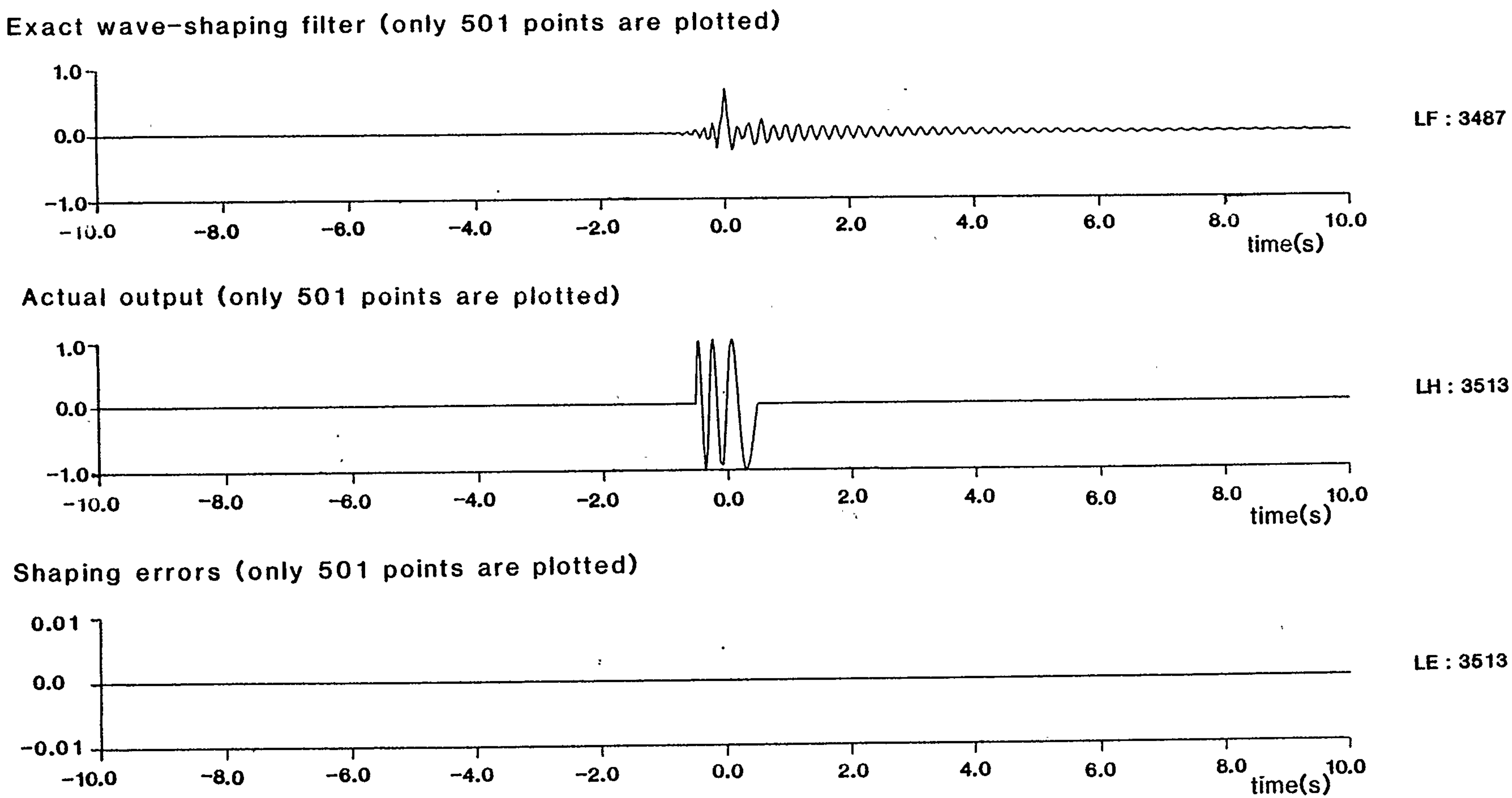


Figure 5.6 Example 1 . Plots of the central portion of the exact wave-shaping filter,actual output and shaping errors

The small magnitude of the shaping errors indicates that this "exact" wave-shaping filter has significantly attenuated the harmonic distortion that was affecting the input sweep, "NOISY-SWEEP1".

Example 2: Definition of a truncated wave-shaping filter using the program "MEREU2"

The major portion of the harmonic distortion can also be attenuated from uncorrelated records by using a truncated wave-shaping filter defined through "MEREU2". Tables 5.3 and 5.4 give the results obtained using "MEREU2" in which the given wavelet, the desired wavelet and the control parameters are the same as in the previous example except for "MXLG" which is set to 501 instead of 5001. This implies that only ((2 x 501)-1) coefficients of $G_{N-i}$ are used to generate $G_{N-i+1}$ and that the wave-shaping filter is truncated to 501 points. As shown in Table 5.3, the truncation of the symmetric filter first occurs after subfilter 7. A comparison of Tables 5.1 and 5.3, shows that the subfilter coefficients are not affected by this truncation. This is because the calculation of the subfilters occur before the successive truncations of the symmetric filter. Table 5.4 shows that the shaping errors, although slightly larger than in the first example (Table 5.2), remain very small over the central portion of the output. Figure 5.7 shows the complete wave-shaping filter actual output, and shaping errors. As shown, the shaping errors remain small in the central portion of the output but increase significantly around the truncation point. This is caused by the early truncation of the symmetric filter as well as the

given wavelet : "NOISY-SWEEP1"        desired wavelet : "SWEEP1"

```
subfilter 1      no. of weights =   27        lg =   27

b    1.000000   -0.579675    0.041073    0.250992   -0.288647    0.113360    0.0
44561   -0.072219   -0.019627    0.147727   -0.132894   -0.047860    0.175140
-0.107959   -0.035248    0.085224   -0.027897   -0.050655    0.078036   -0.02171
4   -0.072642    0.110518   -0.085503    0.041830    0.000000    0.000000    0.0
00000

subfilter 2      no. of weights =   24        lg =   70

b    1.000000    0.178637   -0.363772   -0.163521    0.040611    0.228171    0.2
70255    0.107883   -0.033981   -0.118369   -0.099109    0.080181    0.106294
 0.007895   -0.021172   -0.014152   -0.014716    0.000505    0.011615    0.00828
7    0.002704   -0.005646   -0.005397    0.004880

subfilter 3      no. of weights =   24        lg =  162

b    1.000000    0.576277    0.005912   -0.310438   -0.077586    0.198006    0.2
11405    0.053688   -0.058195   -0.040244    0.013117    0.032503    0.012626
-0.002863   -0.005952    0.000855    0.002823    0.001757    0.000033   -0.00025
3   -0.000025    0.000121    0.000072    0.000020

subfilter 4      no. of weights =   24        lg =  317

b    1.000000   -0.532960    0.218249   -0.007479    0.100301    0.026330   -0.0
11678    0.010202   -0.005925    0.001775   -0.002834    0.000238   -0.000545
 0.000199   -0.000061    0.000052   -0.000011    0.000006   -0.000001    0.00000
1   -0.000000    0.000000    0.000000    0.000000

subfilter 5      no. of weights =   18        lg =  402

b    1.000000   -0.340539    0.480216   -0.102902    0.009014    0.016927   -0.0
06323    0.001321   -0.000076   -0.000030    0.000012   -0.000003    0.000001
-0.000000   -0.000000    0.000000   -0.000000    0.000000

subfilter 6      no. of weights =    9        lg =  477

b    1.000000   -0.651088    0.152577    0.001845   -0.001466    0.000166   -0.0
00002   -0.000000    0.000000

subfilter 7      no. of weights =    6        lg =  501

b    1.000000    0.587779    0.115567    0.001198    0.000004    0.000000

subfilter 8      no. of weights =    5        lg =  501

b    1.000000    0.344783    0.035605    0.000002   -0.000000

subfilter 9      no. of weights =    3        lg =  501

b    1.000000    0.062328    0.001656

subfilter 10     no. of weights =    3        lg =  501

b    1.000000    0.000578    0.000003

subfilter 11     no. of weights =    2        lg =  501

b    1.000000   -0.000005

subfilter 12     no. of weights =    1        lg =  501

b    1.000000
```

Table 5.3 Example 2 : Table of the subfilters coefficients (only the non-zero coefficients of the positive half of the subfilters including the centre point are shown)

given wavelet : "NOISY-SWEEP1" desired wavelet : "SWEEP1"

25 central points of the input and the output data

| time | given wavelet | desired wavelet | correlation functions | | symmetric filter | shaping filter | actual output | shaping errors |
|---|---|---|---|---|---|---|---|---|
| | w | d | r | s | g | f | h | e |
| -12 | 0.967 | 0.94 | 1.643 | 2.132 | -0.106 | 0.056 | 0.945 | 8.3E-06 |
| -11 | 0.121 | 0.65 | 0.449 | 0.599 | 0.003 | 0.016 | 0.651 | 1.2E-05 |
| -10 | -0.033 | -0.43 | -1.247 | -1.563 | 0.114 | -0.075 | -0.433 | 6.2E-06 |
| -9 | -0.765 | -1.00 | -1.386 | -1.663 | 0.043 | 0.024 | -1.000 | -2.3E-06 |
| -8 | -0.697 | -0.48 | -0.184 | -0.125 | -0.081 | 0.085 | -0.482 | -5.7E-06 |
| -7 | 0.729 | 0.51 | 0.678 | 0.628 | -0.068 | -0.095 | 0.508 | -4.8E-07 |
| -6 | 0.803 | 1.00 | 0.418 | 0.117 | 0.034 | -0.084 | 0.999 | 5.5E-06 |
| -5 | 0.099 | 0.61 | -1.064 | -1.281 | 0.049 | 0.158 | 0.612 | 3.4E-06 |
| -4 | -0.006 | -0.25 | -2.709 | -3.235 | 0.102 | 0.014 | -0.254 | -5.2E-06 |
| -3 | -0.401 | -0.90 | -2.355 | -2.778 | -0.118 | -0.226 | -0.905 | -1.1E-05 |
| -2 | -0.985 | -0.92 | 0.385 | 1.656 | 0.037 | 0.119 | -0.923 | -7.6E-06 |
| -1 | -0.554 | -0.37 | 5.440 | 6.917 | -0.233 | 0.262 | -0.373 | 2.0E-06 |
| 0 | 0.541 | 0.36 | 9.384 | 9.636 | 0.502 | 0.677 | 0.363 | 7.7E-06 |
| 1 | 1.000 | 0.88 | 5.440 | 6.609 | -0.233 | 0.348 | 0.885 | 3.3E-06 |
| 2 | 0.615 | 0.98 | 0.385 | 0.627 | 0.037 | -0.002 | 0.982 | -6.7E-06 |
| 3 | 0.140 | 0.68 | -2.355 | -3.822 | -0.118 | -0.256 | 0.681 | -1.3E-05 |
| 4 | 0.001 | 0.16 | -2.709 | -4.483 | 0.102 | -0.090 | 0.155 | -8.2E-06 |
| 5 | -0.023 | -0.39 | -1.064 | -2.341 | 0.049 | 0.101 | -0.387 | 4.3E-06 |
| 6 | -0.235 | -0.79 | 0.418 | 0.160 | 0.034 | 0.057 | -0.789 | 1.4E-05 |
| 7 | -0.615 | -0.98 | 0.678 | 1.306 | -0.068 | -0.094 | -0.982 | 1.3E-05 |
| 8 | -0.929 | -0.97 | -0.184 | 0.978 | -0.081 | -0.060 | -0.970 | 2.6E-06 |
| 9 | -0.989 | -0.80 | -1.386 | 0.082 | 0.043 | 0.112 | -0.805 | -5.0E-06 |
| 10 | -0.779 | -0.55 | -1.247 | -0.494 | 0.114 | 0.154 | -0.551 | -3.2E-06 |
| 11 | -0.406 | -0.27 | 0.449 | -0.499 | 0.003 | -0.009 | -0.268 | 4.9E-06 |
| 12 | 0.000 | 0.00 | 1.643 | -0.196 | -0.106 | -0.185 | 0.000 | 9.3E-06 |
| n | lw | ld | lr | ls | 2*lg-1 | lf | lh | le |
| 12 | 27 | 27 | 53 | 53 | 1001 | 501 | 527 | 527 |

Table 5.4 Example 2 . Table of the input and output coefficients (only the central 25 coefficients are shown)

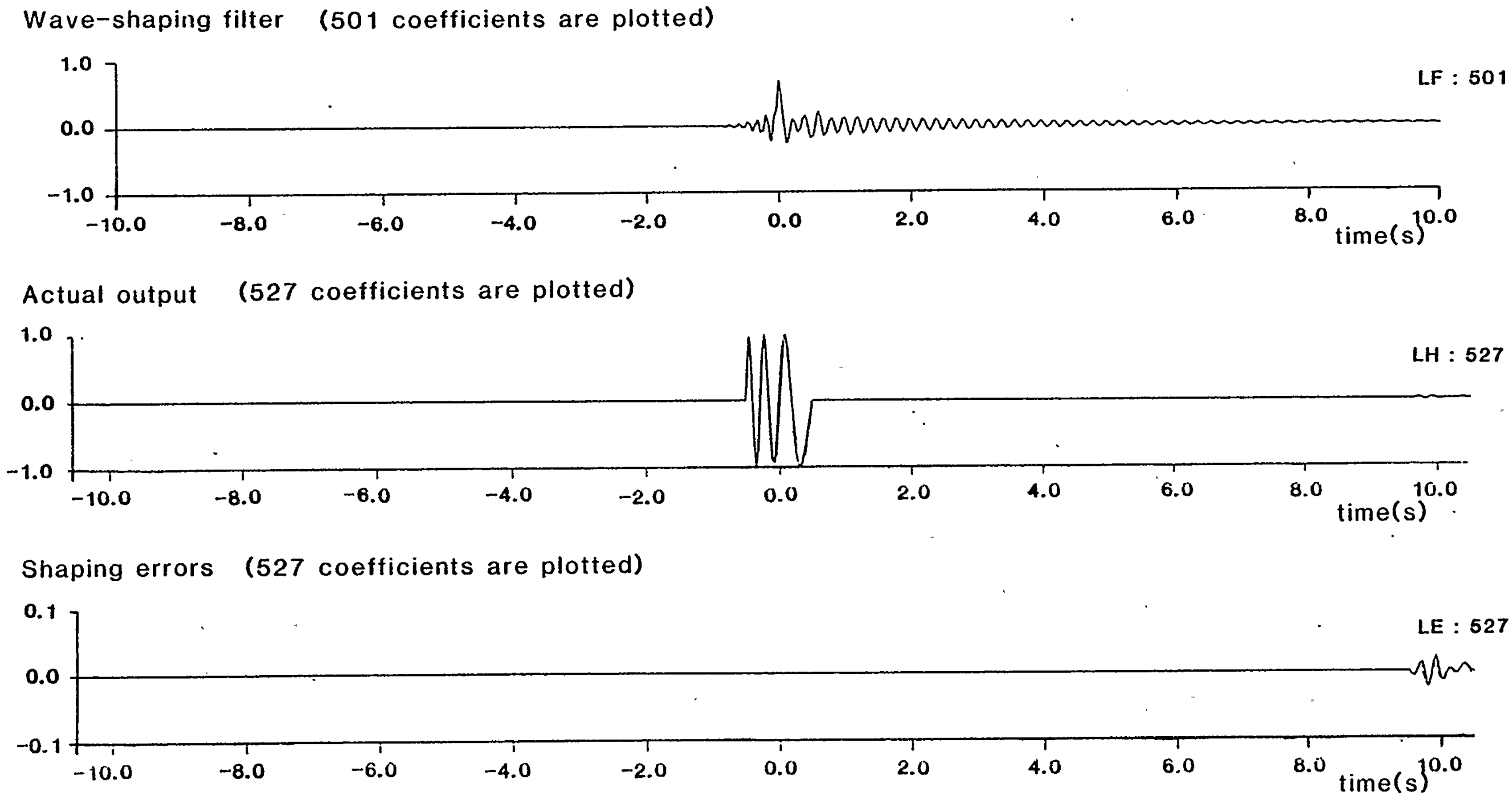


Figure 5.7 Example 2 . Plots of the wave-shaping filter,actual output and shaping errors

relatively large amplitude of the wave-shaping filter at the truncation point. If this filter were to be practically used, the length of the dataset to be filtered should not exceed 9.5 s. (assuming the sampling rate of the dataset is the same as the sampling rate of the wavelets used to define the filter). This is to respect the condition previously quoted in Chapter 4 (p. 55):

$LT > 2t_d + t_w$ where LT: length of the truncated filter

td: length of the dataset

tw: length of the desired wavelet

In this example:

LT = 20 s (501 coefficients),

tw = 1 s (26 coefficients),

then, $t_d < 9.5$ s (237 coefficients)

When the conditions quoted on p. 55 are respected, the errors caused by the truncation appear later than the end of the records and consequently do not affect the zone of interest.

This last example shows that efficient wave-shaping filters can be defined using "MEREU2" and that satisfactory results are obtained when the conditions quoted in Chapter 4 (p. 55) are respected.

The harmonic distortion affecting uncorrelated records can thus be significantly attenuated by using a properly defined wave-shaping filter. In this first approach, "exact" wave-shaping filter and truncated wave-shaping filter gave some satisfactory and encouraging results.

2nd Approach: Attenuation of correlation noise on correlated records

In this second approach, the given and desired wavelets are:

- given wavelet: "W1"
- desired wavelet: "A1"

Both time series contain 51 points.

Example 3: Definition of an "exact" wave-shaping filter using the program "MEREU1".

In order to get a good quality wave-shaping filter, the following values are input to "MEREU1":

- MXN: 20
- MXLG: 5001
- BMIN: $10^{-10}$
- WEIGHT: 0

The subfilter coefficients obtained in this case are given in Table 5.5. Here again, the 12th subfilter contains only one term.

given wavelet : "W1"     desired wavelet : "A1"

```
subfilter  1      no. of weights =   51 .       lg =   51

b    1.000000   -0.707966    0.059357    0.475377   -0.578749    0.302092    0.0
43538   -0.181801    0.079352    0.085147   -0.124589    0.017513    0.104795
-0.120370    0.040554    0.026428   -0.006565   -0.065406    0.093498   -0.02498
1   -0.096864    0.177739   -0.161139    0.071123    0.021149   -0.062825    0.0
52705   -0.021663   -0.000477    0.004628    0.000319   -0.002996    0.000405
 0.003003   -0.003282    0.001878   -0.002449    0.005314   -0.007065    0.00477
6    0.000683   -0.006481    0.008889   -0.007582    0.004458   -0.001748    0.0
00365    0.000000    0.000000    0.000000    0.000000

subfilter  2      no. of weights =   47         lg =  139

b    1.000000    0.221607   -0.596100   -0.359443    0.095566    0.289039    0.2
41301    0.060910   -0.091315   -0.227638   -0.139726    0.219648    0.237118
-0.085099   -0.158456   -0.017482    0.049599    0.042859    0.014665   -0.00690
5   -0.018586   -0.030856   -0.000445    0.037652    0.011551   -0.016881   -0.0
09628    0.001079    0.003266    0.002964    0.001649    0.000576   -0.001634
-0.002559    0.000735    0.001880   -0.000007   -0.000519   -0.000069   -0.00001
9    0.000000    0.000020    0.000039    0.000066   -0.000038   -0.000052    0.0
00035

subfilter  3      no. of weights =   47         lg =  308

b    1.000000    0.749022    0.265795    0.002501    0.124841    0.374040    0.4
33940    0.265899    0.063699   -0.006834    0.042647    0.100305    0.095322
 0.047468    0.008045    0.000561    0.010567    0.016738    0.012480    0.00450
7    0.000109    0.000225    0.001655    0.001905    0.001035    0.000190   -0.0
00045    0.000089    0.000190    0.000140    0.000046   -0.000000    0.000002
 0.000012    0.000011    0.000005    0.000001   -0.000000    0.000000    0.00000
1    0.000000    0.000000   -0.000000    0.000000    0.000000    0.000000    0.0
00000

subfilter  4      no. of weights =   36 .       lg =  440

b    1.000000   -0.616918    0.389031   -0.280620    0.236834   -0.027374    0.0
04760   -0.016192   -0.022036    0.013187   -0.009041    0.006202   -0.003727
 0.002454   -0.000611    0.000488   -0.000201   -0.000029   -0.000009   -0.00001
6    0.000017   -0.000009    0.000007   -0.000003    0.000002   -0.000001    0.0
00000   -0.000000    0.000000   -0.000000    0.000000   -0.000000    0.000000
-0.000000    0.000000   -0.000000

subfilter  5      no. of weights =   22         lg =  484

b    1.000000   -0.448538    0.464937   -0.096302   -0.017201    0.041772   -0.0
16182    0.002471    0.000925   -0.000587    0.000148   -0.000009   -0.000006
 0.000002   -0.000000    0.000000   -0.000000    0.000000   -0.000000   -0.00000
0    0.000000   -0.000000

subfilter  6      no. of weights =   12         lg =  567

b    1.000000   -0.628994    0.117229    0.017047   -0.003003    0.000096    0.0
00029   -0.000002    0.000000   -0.000000   -0.000000    0.000000

subfilter  7      no. of weights =    7         lg =  693

b    1.000000    0.595942    0.124371    0.003473    0.000044    0.000001    0.0
00000

subfilter  8      no. of weights =    5         lg =  900

b    1.000000    0.344753    0.035609    0.000005   -0.000000

subfilter  9      no. of weights =    4         lg = 1303

b    1.000000    0.062290    0.001653    0.000000

subfilter 10      no. of weights =    3         lg = 2150

b    1.000000    0.000578    0.000003

subfilter 11      no. of weights =    2         lg = 2222

b    1.000000   -0.000005

subfilter 12      no. of weights =    1         lg = 1434

b    1.000000
```

Table 5.5 Example 3 . Table of the subfilters coefficients (onlythe non-zero coefficients of the positive half of the subfilters including the centre point are shown)

Setting "MXN" to 20 is then large enough. Setting "MXLG" to 5001 is also large enough to let the symmetric filter develop using these 12 subfilters. Also, because the subfilter coefficients behave normally, the addition of white noise is not necessary.

The coefficient values for the central portion of the input and output are given in Table 5.6. The total number of coefficients in each time series is also given. The symmetric filter contains 2867 coefficients and the "exact" wave-shaping filter contains 2967 points. Note that the error, although slightly larger than in example 1, remains relatively small over the shown portion. This indicates that the value input for "BMIN" is set small enough. The exact wave-shaping filter, actual output and shaping errors are plotted over a larger portion (551 points) in Figure 5.8. As shown, the shaping errors remain small over this portion of the output (22 s long).

The small magnitude of the shaping errors indicates that this "exact" wave-shaping filter has significantly attenuated the correlation noise present on the wavelet "W1."

Example 4: Definition of a truncated wave-shaping filter using the program "MEREU2".

An efficient wave-shaping filter can also be defined using "MEREU2" in which the same parameters and wavelets as in example 3 are input except for "MXLG" which is now set to 501 instead of 5001. The

given wavelet : "W1"          desired wavelet : "A1"

25 central points of the input and the output data

| time | given wavelet w | desired wavelet d | correlation functions r | s | symmetric filter g | shaping filter f | actual output h | shaping errors e |
|---|---|---|---|---|---|---|---|---|
| -12 | -0.020 | -0.02 | 0.307 | 0.277 | -2.768 | 0.044 | -0.022 | -2.9E-04 |
| -11 | -0.052 | -0.05 | -0.051 | -0.029 | -0.720 | 0.013 | -0.051 | -5.6E-04 |
| -10 | -0.051 | -0.04 | -0.365 | -0.296 | 3.676 | -0.059 | -0.042 | -2.0E-04 |
| -9 | 0.009 | 0.02 | -0.250 | -0.186 | -0.145 | 0.019 | 0.019 | 2.9E-04 |
| -8 | 0.102 | 0.10 | 0.233 | 0.223 | -2.175 | 0.066 | 0.101 | 2.9E-04 |
| -7 | 0.136 | 0.12 | 0.533 | 0.433 | -3.262 | -0.075 | 0.121 | -3.1E-04 |
| -6 | 0.017 | 0.00 | 0.128 | 0.011 | 13.579 | -0.066 | 0.000 | -1.0E-03 |
| -5 | -0.243 | -0.25 | -0.886 | -0.903 | -26.201 | 0.123 | -0.247 | -1.2E-03 |
| -4 | -0.465 | -0.45 | -1.697 | -1.564 | 44.476 | 0.011 | -0.454 | -7.7E-04 |
| -3 | -0.397 | -0.38 | -1.394 | -1.182 | -73.621 | -0.177 | -0.379 | 1.4E-05 |
| -2 | 0.065 | 0.08 | 0.174 | 0.314 | 120.786 | 0.093 | 0.080 | 6.7E-04 |
| -1 | 0.686 | 0.69 | 2.076 | 2.045 | -174.667 | 0.205 | 0.696 | 1.0E-03 |
| 0 | 1.000 | 1.00 | 2.932 | 2.787 | 200.386 | 0.531 | 1.001 | 1.1E-03 |
| 1 | 0.718 | 0.69 | 2.076 | 1.952 | -174.667 | 0.272 | 0.696 | 1.0E-03 |
| 2 | 0.172 | 0.08 | 0.174 | 0.093 | 120.786 | -0.001 | 0.080 | 6.7E-04 |
| 3 | -0.288 | -0.38 | -1.394 | -1.540 | -73.621 | -0.200 | -0.379 | 1.1E-05 |
| 4 | -0.336 | -0.45 | -1.697 | -1.983 | 44.476 | -0.071 | -0.454 | -7.7E-04 |
| 5 | -0.133 | -0.25 | -0.886 | -1.212 | -26.201 | 0.079 | -0.247 | -1.2E-03 |
| 6 | 0.012 | 0.00 | 0.128 | 0.004 | 13.579 | 0.044 | 0.000 | -1.0E-03 |
| 7 | 0.065 | 0.12 | 0.533 | 0.800 | -3.262 | -0.074 | 0.121 | -3.0E-04 |
| 8 | -0.013 | 0.10 | 0.233 | 0.820 | -2.175 | -0.047 | 0.101 | 2.9E-04 |
| 9 | -0.173 | 0.02 | -0.250 | 0.307 | -0.145 | 0.088 | 0.019 | 2.9E-04 |
| 10 | -0.162 | -0.04 | -0.365 | -0.236 | 3.676 | 0.121 | -0.042 | -2.0E-04 |
| 11 | 0.062 | -0.05 | -0.051 | -0.462 | -0.720 | -0.007 | -0.051 | -5.6E-04 |
| 12 | 0.221 | -0.02 | 0.307 | -0.332 | -2.768 | -0.145 | -0.022 | -2.9E-04 |
| n | lw | ld | lr | ls | 2*lg-1 | lf | lh | le |
| 12 | 51 | 51 | 101 | 101 | 2867 | 2967 | 3017 | 3017 |

Table 5.6 Example 3 .Table of the input and output coefficients (only the central 25 coefficients are shown)

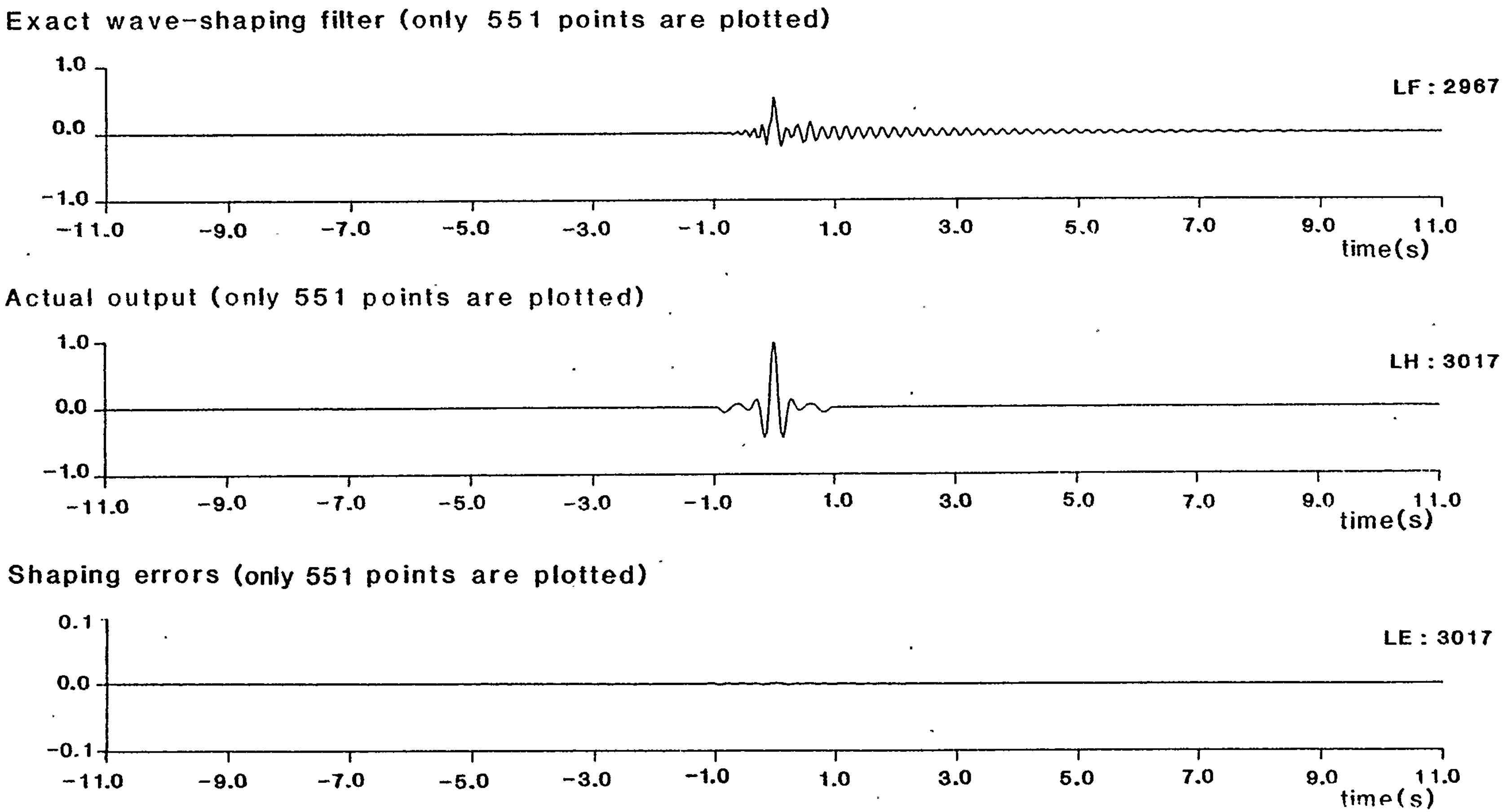


Figure 5.8 Example 3 . Plots of the central portion of the exact wave-shaping filter,actual output and shaping errors

results obtained are presented in Tables 5.7 and 5.8. As shown in Table 5.7, the truncation of the symmetric filter first happens after the 6th subfilter. Table 5.8 shows that the shaping errors remain relatively small over the central portion of the output. Figure 5.9 shows the complete wave-shaping filter, actual output, and shaping errors. Once again, the shaping errors remain small in the central portion of the output but increase significantly around the truncation point. In this case, if the truncated filter were to be practically used, the length of the filter should be compared to the length of the dataset to be filtered. In order to keep the large errors caused by the truncation of the filter in a zone outside the area of interest, the conditions quoted on p. 55 should be respected.

Here, LT = 20 s (501 coefficients)

$t_w$ = 2 s (51 coefficients)

then, $t_d$ <9 s (225 coefficients)

In this example, the length of the dataset to be filtered should not exceed 9 s (assuming the sampling rate of the dataset is the same as the sampling rate of the wavelets used to define the filter).

The major part of the correlation noise affecting "W1" can be removed using a truncated wave-shaping filter. If the conditions stated in Chapter 4, p. 55 are respected, then the errors generated by the

given wavelet : "W1"          desired wavelet : "A1"

```
subfilter  1      no. of weights =   51 .        lg =   51

b   1.000000   -0.707966    0.059357    0.475377   -0.578749    0.302092    0.0
43538   -0.181801 .  0.079352    0.085147   -0.124589    0.017513    0.104795
-0.120370    0.040554    0.026428   -0.006565   -0.065406    0.093498   -0.02498
1   -0.096864    0.177739   -0.161139    0.071123    0.021149   -0.062825    0.0
52705   -0.021663   -0.000477    0.004628    0.000319   -0.002996    0.000405
 0.003003   -0.003282    0.001878   -0.002449    0.005314   -0.007065    0.00477
6    0.000883   -0.006481    0.008889   -0.007582    0.004458   -0.001748    0.0
00365    0.000000    0.000000    0.000000    0.000000

subfilter  2      no. of weights =   47          lg =  139

b    1.000000    0.221607   -0.596100   -0.359443    0.095566    0.259039    0.2
41301    0.060910   -0.091315   -0.227638   -0.139726    0.219648    0.237118
-0.085099   -0.158456   -0.017482    0.049599    0.042859    0.014665   -0.00690
5   -0.018586   -0.030856   -0.000445    0.037652    0.011551   -0.016861   -0.0
09628    0.001079    0.003266    0.002964    0.001649    0.000576   -0.001634
-0.002559    0.000735    0.001880   -0.000007   -0.000519   -0.000069   -0.00001
9    0.000000    0.000020    0.000039    0.000066   -0.000038   -0.000052    0.0
00035

subfilter  3      no. of weights =   47          lg =  308

b    1.000000    0.749022    0.265795    0.002501    0.124841    0.374040    0.4
33940    0.265899    0.063699   -0.006834    0.042647    0.100305    0.095322
 0.047468    0.008045    0.000561    0.010567    0.016738    0.012480    0.00450
7    0.000109    0.000225    0.001655    0.001905    0.001035    0.000190   -0.0
00045    0.000089    0.000190    0.000140    0.000046   -0.000000    0.000002
 0.000012    0.000011    0.000005    0.000001   -0.000000    0.000000    0.00000
1    0.000000    0.000000   -0.000000    0.000000    0.000000    0.000000    0.0
00000

subfilter  4      no. of weights =   36          lg =  440

b    1.000000   -0.616918    0.389031   -0.280620    0.236834   -0.027374    0.0
04760   -0.016192   -0.022036    0.013187   -0.009041    0.006202   -0.003727
 0.002454   -0.000611    0.000458   -0.000201   -0.000029   -0.000009   -0.00001
6    0.000017   -0.000009    0.000007   -0.000003    0.000002   -0.000001    0.0
00000   -0.000000    0.000000   -0.000000    0.000000   -0.000000    0.000000
-0.000000    0.000000   -0.000000

subfilter  5      no. of weights =   22          lg =  484

b    1.000000   -0.448538    0.464937   -0.096302   -0.017201    0.041772   -0.0
16182    0.002471    0.000925   -0.000587    0.000148   -0.000009   -0.000006
 0.000002   -0.000000    0.000000   -0.000000    0.000000   -0.000000   -0.00000
0    0.000000   -0.000000

subfilter  6      no. of weights =   12          lg =  501

b    1.000000   -0.628994    0.117229    0.017047   -0.003003    0.000096    0.0
00029   -0.000002    0.000000   -0.000000   -0.000000    0.000000

subfilter  7      no. of weights =    7          lg =  501

b    1.000000    0.595942    0.124371    0.003473    0.000044    0.000001    0.0
00000

subfilter  8      no. of weights =    5 .        lg =  501

b    1.000000    0.344753    0.035609    0.000005   -0.000000

subfilter  9      no. of weights =    4          lg =  501

b    1.000000    0.062290    0.001653    0.000000

subfilter 10      no. of weights =    3          lg =  501

b    1.000000    0.000578    0.000003

subfilter 11      no. of weights =    2          lg =  501

b    1.000000   -0.000005

subfilter 12      no. of weights =    1          lg =  501

b    1.000000
```

Table 5.7 Example 4 . Table of the subfilters coefficients (only the non-zero coefficients of the positive half of the subfilters including the centre point are shown)

given wavelet : "W1" desired wavelet : "A1"

25 central points of the input and the output data

| time | given wavelet | desired wavelet | correlation functions | | symmetric filter | shaping filter | actual output | shaping errors |
|---|---|---|---|---|---|---|---|---|
| | w | d | r | s | g | f | h | e |
| -12 | -0.020 | -0.02 | 0.307 | 0.277 | -2.768 | 0.044 | -0.022 | -2.7E-04 |
| -11 | -0.052 | -0.05 | -0.051 | -0.029 | -0.720 | 0.013 | -0.051 | -5.6E-04 |
| -10 | -0.051 | -0.04 | -0.365 | -0.296 | 3.676 | -0.059 | -0.042 | -2.2E-04 |
| -9 | 0.009 | 0.02 | -0.250 | -0.186 | -0.145 | 0.019 | 0.019 | 2.6E-04 |
| -8 | 0.102 | 0.10 | 0.233 | 0.223 | -2.175 | 0.066 | 0.101 | 2.8E-04 |
| -7 | 0.136 | 0.12 | 0.533 | 0.433 | -3.262 | -0.075 | 0.121 | -3.0E-04 |
| -6 | 0.017 | 0.00 | 0.128 | 0.011 | 13.578 | -0.066 | 0.000 | -1.0E-03 |
| -5 | -0.243 | -0.25 | -0.886 | -0.903 | -26.201 | 0.123 | -0.247 | -1.2E-03 |
| -4 | -0.465 | -0.45 | -1.697 | -1.564 | 44.476 | 0.011 | -0.454 | -7.8E-04 |
| -3 | -0.397 | -0.38 | -1.394 | -1.182 | -73.621 | -0.177 | -0.379 | -1.7E-06 |
| -2 | 0.065 | 0.08 | 0.174 | 0.314 | 120.786 | 0.093 | 0.080 | 6.7E-04 |
| -1 | 0.686 | 0.69 | 2.076 | 2.045 | -174.666 | 0.205 | 0.696 | 1.0E-03 |
| 0 | 1.000 | 1.00 | 2.932 | 2.787 | 200.385 | 0.531 | 1.001 | 1.1E-03 |
| 1 | 0.718 | 0.69 | 2.076 | 1.952 | -174.666 | 0.272 | 0.696 | 1.0E-03 |
| 2 | 0.172 | 0.08 | 0.174 | 0.093 | 120.786 | -0.001 | 0.080 | 6.7E-04 |
| 3 | -0.288 | -0.38 | -1.394 | -1.540 | -73.621 | -0.200 | -0.379 | -1.6E-07 |
| 4 | -0.336 | -0.45 | -1.697 | -1.983 | 44.476 | -0.071 | -0.454 | -7.8E-04 |
| 5 | -0.133 | -0.25 | -0.886 | -1.212 | -26.201 | 0.079 | -0.247 | -1.2E-03 |
| 6 | 0.012 | 0.00 | 0.128 | 0.004 | 13.578 | 0.044 | 0.000 | -1.0E-03 |
| 7 | 0.065 | 0.12 | 0.533 | 0.800 | -3.262 | -0.074 | 0.121 | -3.0E-04 |
| 8 | -0.013 | 0.10 | 0.233 | 0.820 | -2.175 | -0.047 | 0.101 | 2.8E-04 |
| 9 | -0.173 | 0.02 | -0.250 | 0.307 | -0.145 | 0.038 | 0.019 | 2.6E-04 |
| 10 | -0.162 | -0.04 | -0.365 | -0.236 | 3.676 | 0.121 | -0.042 | -2.2E-04 |
| 11 | 0.062 | -0.05 | -0.051 | -0.462 | -0.720 | -0.007 | -0.051 | -5.6E-04 |
| 12 | 0.221 | -0.02 | 0.307 | -0.332 | -2.768 | -0.145 | -0.022 | -2.7E-04 |
| n | lw | ld | lr | ls | 2*lg-1 | lf | lh | le |
| 12 | 51 | 51 | 101 | 101 | 1001 | 501 | 551 | 551 |

Table 5.8 Example 4 . Table of the input and output coefficients (only the central 25 coefficients are shown)

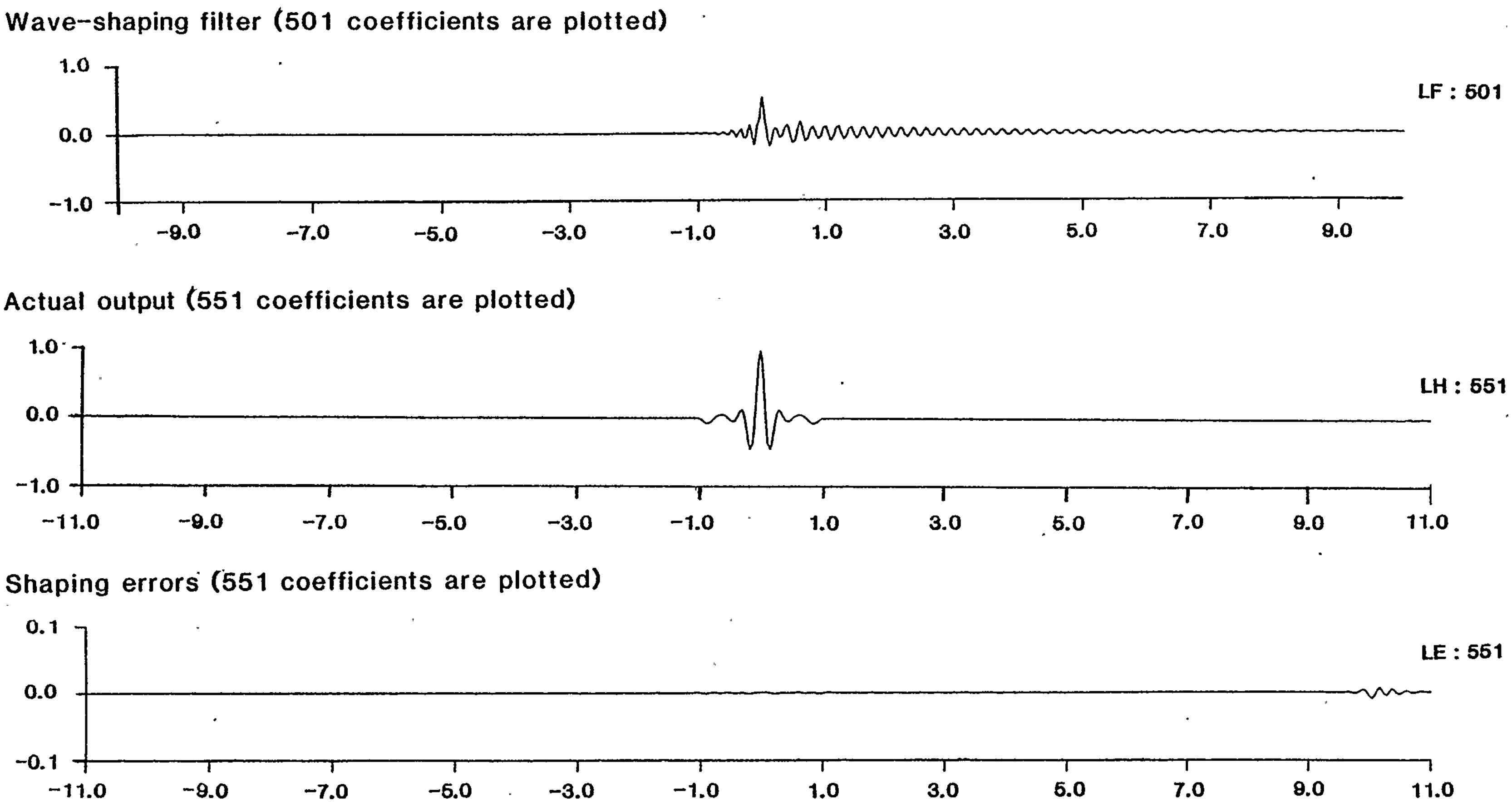


Figure 5.9 Example 4 . Plots of the wave-shaping filter,actual output and shaping errors

truncation appear later than the end of the records and satisfactory results are obtained.

The correlation noise affecting correlated records may be significantly attenuated using a properly defined wave-shaping filter. In this second approach, "exact" wave-shaping filter and truncated wave-shaping filters, both gave some satisfactory results.

Examples 1 to 4 have shown that the harmonic distortion affecting the uncorrelated records as well as the correlation noise affecting the correlated records could be attenuated by using the "exact" wave-shaping filtering method. Examples 1 and 3 have shown that "exact" wave-shaping filter could be defined using the program "MEREU1". Examples 2 and 4 have shown that efficient truncated wave-shaping filter could be defined using the program "MEREU2".

Although in this first case, the characteristics of the input sweep (5-1 Hz, 1s long) do not correspond to the ones practically used in the field, the results obtained are encouraging. This filtering technique could be used on longer sweeps.

### 5.3.2 Case 2: Long fundamental sweep (601 points)

In this second case, the fundamental sweep (SWEEP2) is characterized by the following:

bandwidth: 50-10 Hz

time length: 3 s

amplitude: 1 unit

Its second harmonic (HSWEEP2) is defined by:

bandwidth: 100-20 Hz

time length: 3 s

amplitude: 0.5 unit

"SWEEP2" and "HSWEEP2" are shown in Figures 5.10 A and 5.10 B. Here again, the amplitude of the harmonic is arbitrarily chosen to be half the amplitude of the fundamental sweep. The result of their addition, which corresponds to the harmonically distorted signal, "NOISY-SWEEP2", is plotted in Figure 5.10 C. The crosscorrelation of "NOISY-SWEEP2" with the fundamental sweep "SWEEP2" results in the waveform "W2" which is plotted in Figure 5.11 A. This waveform is composed of two parts: the desired zero-phase wavelet and the correlation noise. In this case, the major part of the noise appears 0.75 s after the main event and lasts for 1.25 s. The desired correlated wavelet, "A2", is plotted in Figure 5.11 B).

1st Approach: Attenuation of harmonic distortion on uncorrelated records.

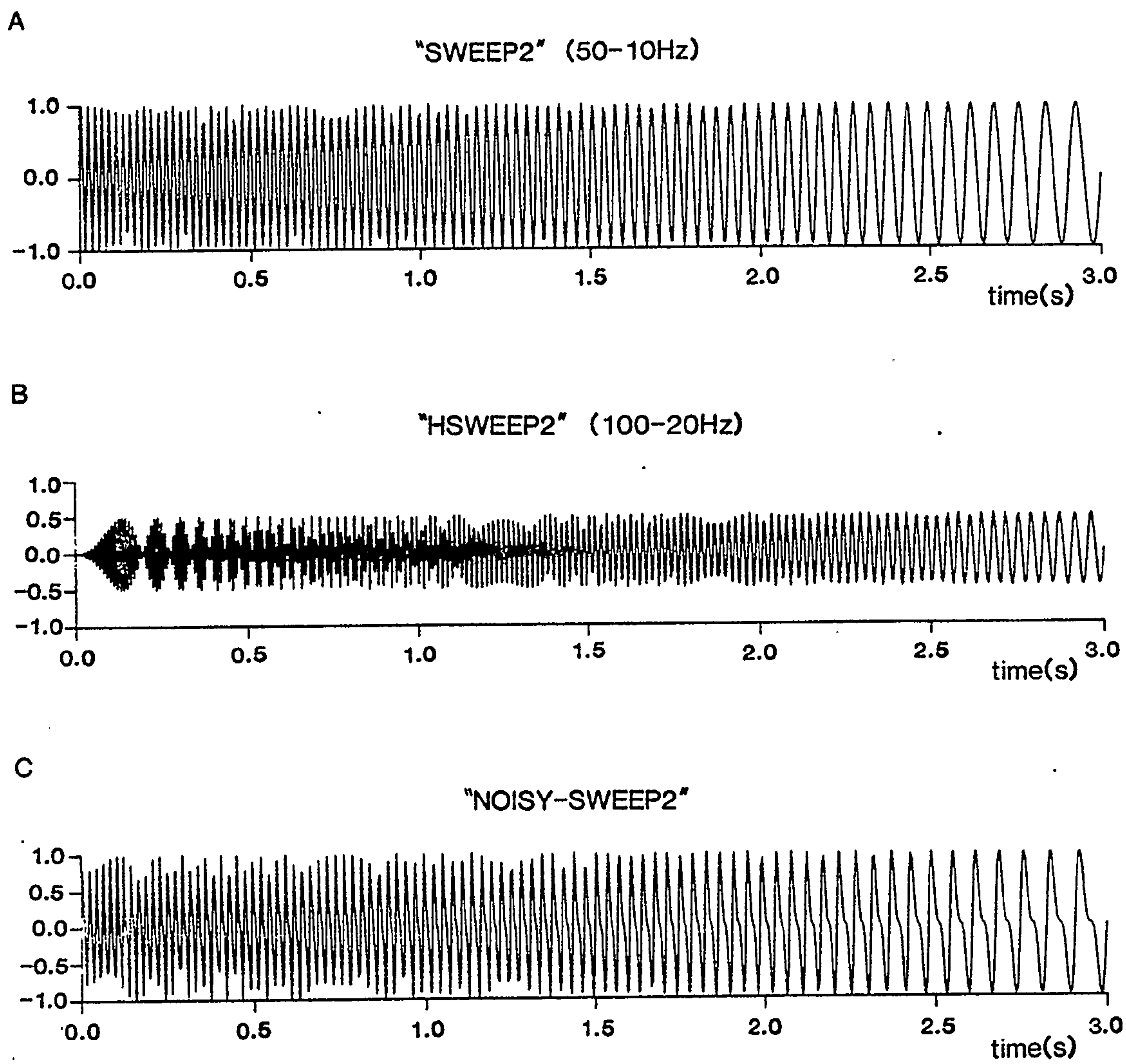


Figure 5.10 Plots of "SWEEP2", "HSWEEP2", "NOISY-SWEEP2"

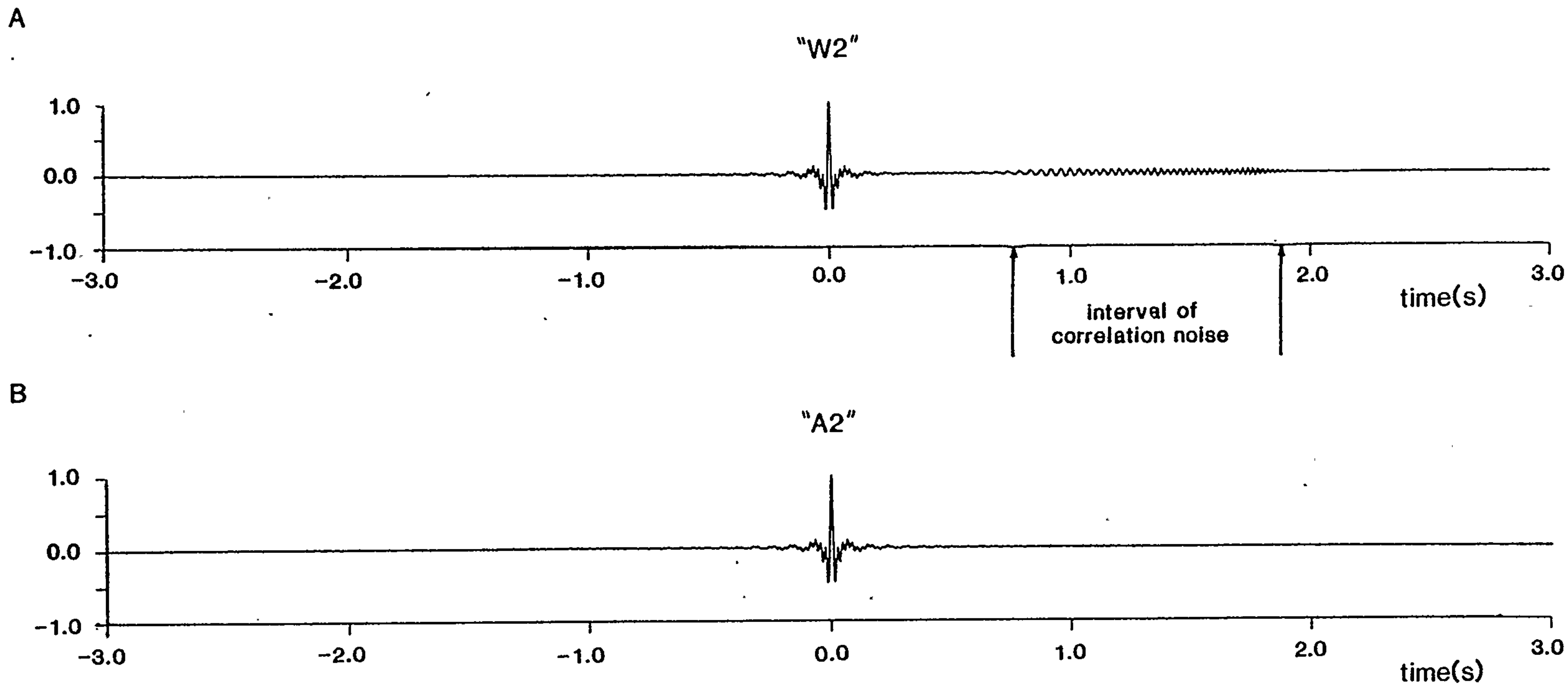


Figure 5.11 Plots of "W2" and "A2"

In this first approach, the given and desired wavelets are:

- given wavelet: "NOISY-SWEEP2"
- desired wavelet: "SWEEP2"

Both time series contain 601 points.

Example 5: Definition of an "exact" wave-shaping filter using the program "MEREU1".

In order to respect the conditions mentioned in Chapter 3, the following values are input to "MEREU1":

- MXN: 30
- MXLG: 9999
- BMIN: $10^{-10}$
- WEIGHT: 0

Parts of the results obtained in this example are presented in Tables 5.9 and 5.10. Table 5.9 gives the number of non-zero coefficients left in each subfilter and shows the evolution of the length of the symmetric filter. As shown, there is insufficient working space and only 5 subfilters can be used to generate the symmetric filter. Because of the limited amount of subfilters being used and also because of the truncation of the wave-shaping filter to 9999 coefficients, the shaping

given wavelet : "NOISY-SWEEP2" desired wavelet : "SWEEP2"

| subfilter | number of non-zero coefficients | lg |
|---|---|---|
| 1 | 601 | 601 |
| 2 | 599 | 1795 |
| 3 | 597 | 3874 |
| 4 | 417 | 5425 |
| 5 | 284 | 7265 |

Warning! There is insufficient working space for G.
The maximum number of subfilters which can be used : 5
filter F was truncated to the length : 9999

Table 5.9 Example 5 . Number of non-zero coefficients in each subfilter and value of "lg" after each subfilter

given wavelet : "NOISY-SWEEP2" desired wavelet : "SWEEP2"

51 central points of the input and the output data

| time | given wavelet w | desired wavelet d | correlation functions r | correlation functions s | symmetric filter g | shaping filter f | actual output h | shaping errors e |
|---|---|---|---|---|---|---|---|---|
| -25 | 0.983 | 0.79 | 0.000 | 0.957 | 0.001 | -0.019 | 1.922 | 1.1E-00 |
| -24 | 0.484 | 0.94 | 0.000 | -10.122 | 0.001 | 0.019 | 2.417 | 1.5E-00 |
| -23 | 0.005 | 0.24 | 0.000 | -13.498 | 0.001 | -0.015 | 1.482 | 1.2E+00 |
| -22 | -0.141 | -0.68 | 0.000 | -2.041 | 0.001 | -0.003 | -0.924 | -2.4E-01 |
| -21 | -0.885 | -0.99 | 0.000 | 2.570 | -0.000 | 0.007 | -3.857 | -2.9E+00 |
| -20 | -0.599 | -0.41 | 0.000 | -6.738 | 0.000 | 0.020 | -4.985 | -4.6E+00 |
| -19 | 0.760 | 0.53 | 0.000 | -9.638 | -0.000 | -0.006 | -2.423 | -3.0E-00 |
| -18 | 0.789 | 1.00 | 0.000 | 6.035 | 0.000 | -0.037 | 2.302 | 1.3E+00 |
| -17 | 0.082 | 0.58 | 0.000 | 15.825 | -0.001 | 0.011 | 4.320 | 3.7E+00 |
| -16 | -0.017 | -0.35 | -0.000 | 5.467 | 0.000 | 0.040 | 1.407 | 1.8E+00 |
| -15 | -0.574 | -0.97 | -0.000 | -1.467 | -0.000 | -0.004 | -2.533 | -1.6E+00 |
| -14 | -0.953 | -0.74 | -0.000 | 14.393 | 0.001 | -0.013 | -2.504 | -1.8E+00 |
| -13 | 0.210 | 0.14 | -0.000 | 25.297 | 0.000 | 0.005 | 0.783 | 6.5E-01 |
| -12 | 0.997 | 0.89 | -0.000 | 7.519 | 0.001 | 0.002 | 2.488 | 1.6E+00 |
| -11 | 0.350 | 0.38 | -0.000 | -10.103 | 0.001 | -0.022 | 0.553 | -3.2E-01 |
| -10 | 0.000 | 0.10 | -0.000 | 3.725 | 0.002 | 0.003 | -0.757 | -8.6E-01 |
| -9 | -0.201 | -0.76 | -0.000 | 15.985 | 0.002 | 0.029 | 0.182 | 9.4E-01 |
| -8 | -0.929 | -0.97 | 0.000 | -13.001 | 0.003 | -0.005 | 0.953 | 1.9E+00 |
| -7 | -0.532 | -0.36 | 0.000 | -45.576 | 0.004 | -0.070 | 0.215 | 5.7E-01 |
| -6 | 0.785 | 0.56 | 0.000 | -26.275 | 0.005 | -0.005 | -0.332 | -8.9E-01 |
| -5 | 0.790 | 1.00 | 0.000 | 1.387 | 0.004 | 0.051 | 0.255 | -7.4E-01 |
| -4 | 0.093 | 0.60 | -0.000 | -43.357 | 0.005 | -0.017 | 0.458 | -1.4E-01 |
| -3 | -0.011 | -0.30 | -0.000 | -110.021 | 0.006 | -0.129 | -0.583 | -2.8E-01 |
| -2 | -0.502 | -0.95 | 0.000 | -53.111 | 0.007 | -0.004 | -1.480 | -5.3E-01 |
| -1 | -0.989 | -0.81 | 88.096 | 127.243 | 0.001 | 0.415 | -1.224 | -4.1E-01 |
| 0 | 0.000 | 0.00 | 221.530 | 230.951 | 0.015 | 0.616 | -0.765 | -7.6E-01 |
| 1 | 0.989 | 0.81 | 88.096 | 126.750 | 0.001 | 0.316 | -0.903 | -1.7E+00 |
| 2 | 0.509 | 0.95 | -50.278 | -53.977 | 0.007 | -0.033 | -0.875 | -1.8E+00 |
| 3 | 0.013 | 0.32 | -89.802 | -111.062 | 0.006 | -0.072 | -0.083 | -4.0E-01 |
| 4 | -0.080 | -0.57 | -36.706 | -44.370 | 0.005 | 0.037 | 0.732 | 1.3E+00 |
| 5 | -0.749 | -1.00 | 1.957 | 0.593 | 0.004 | 0.020 | 1.086 | 2.1E+00 |
| 6 | -0.850 | -0.62 | -18.858 | -26.813 | 0.005 | -0.052 | 1.743 | 2.4E+00 |
| 7 | 0.393 | 0.26 | -31.705 | -45.825 | 0.004 | -0.059 | 2.950 | 2.7E+00 |
| 8 | 0.980 | 0.93 | -8.472 | -13.121 | 0.003 | -0.003 | 3.239 | 2.3E+00 |
| 9 | 0.318 | 0.86 | 13.936 | 16.008 | 0.002 | 0.005 | 1.362 | 5.1E-01 |
| 10 | 0.000 | 0.10 | 1.820 | 3.721 | 0.002 | -0.002 | -1.608 | -1.7E+00 |
| 11 | -0.176 | -0.73 | -8.562 | -9.949 | 0.001 | 0.004 | -3.422 | -2.7E+00 |
| 12 | -0.883 | -0.99 | 3.494 | 7.721 | 0.001 | 0.011 | -3.227 | -2.2E+00 |
| 13 | -0.683 | -0.47 | 18.820 | 25.862 | 0.000 | 0.014 | -1.802 | -1.3E+00 |
| 14 | 0.604 | 0.41 | 10.196 | 15.086 | 0.001 | 0.027 | -0.299 | -7.1E-01 |
| 15 | 0.923 | 0.97 | 0.033 | -0.407 | -0.000 | -0.002 | 0.890 | -8.2E-02 |
| 16 | 0.226 | 0.78 | 4.894 | 6.374 | 0.000 | -0.037 | 1.457 | 6.8E-01 |
| 17 | 0.000 | -0.01 | 13.656 | 16.745 | -0.001 | -0.022 | 1.088 | 1.1E+00 |
| 18 | -0.240 | -0.79 | 4.834 | 6.322 | 0.000 | 0.022 | 0.181 | 9.8E-01 |
| 19 | -0.931 | -0.97 | -7.131 | -9.614 | -0.000 | 0.027 | -0.189 | 7.8E-01 |
| 20 | -0.599 | -0.41 | -6.380 | -7.481 | 0.000 | 0.014 | 0.486 | 8.9E-01 |
| 21 | 0.672 | 0.46 | 1.316 | 1.788 | -0.000 | -0.003 | 1.796 | 1.3E-00 |
| 22 | 0.900 | 0.98 | -2.660 | -3.251 | 0.001 | -0.002 | 1.364 | 3.8E-01 |
| 23 | 0.208 | 0.76 | -10.236 | -14.268 | 0.001 | -0.007 | -0.065 | -8.3E-01 |
| 24 | 0.000 | -0.03 | -7.560 | -11.006 | 0.001 | -0.007 | -0.567 | -5.4E-01 |
| 25 | -0.239 | -0.79 | 1.818 | 0.713 | 0.001 | -0.025 | -0.127 | 6.7E-01 |
| n | lw | ld | lr | ls | 2*lg-1 | lf | lh | le |
| 5 | 601 | 601 | 1201 | 1201 | 14529 | 9999 | 10599 | 10599 |

Table 5.10 Example 5 . Table of the input and output coefficients (only the central 51 coefficients are shown)

errors are relatively large (Table 5.10). The wave-shaping filter defined in this example is therefore not efficient.

A simple way to increase the efficiency of the wave-shaping filter consists of setting "MXLG" to a larger value. However, considering the rate at which the size of the symmetric filter increases and the rate at which the number of non-zero coefficients in each subfilter decreases, it becomes clear that "MXLG" should be set to a very large number (possibly in the order of 100,000). By setting the arrays to such a large number, one can expect some difficulties with the computer memory space. Also, if this filter is to have any practical use at all, it is likely to be truncated. Effectively, very long filters are not only impractical to work with but also tend to be expensive to apply. Note that in this case, a filter containing 100,000 coefficients is 500 s long.

In this example, the "exact" wave-shaping filter tends to become very long. This is an example for which the use of the program "MEREU2" is preferable.

Example 6: Definition of a truncated wave-shaping filter using the program "MEREU2".

In this example, the same parameters as in example 5 are input to "MEREU2". Only ((2 x 9999)-1) coefficients of $G_{N-i}$ are used to generate $G_{N-i+1}$ and the wave-shaping will only contain 9999 coefficients. Note that in the present case, a filter containing 9999 coefficients is 50 seconds long.

Table 5.11 gives the number of non-zero coefficients left in each subfilter. The evolution of the length of the symmetric filter is also shown. As one can see, the number of non-zero coefficients stays constant after the 17th subfilter. It does not decrease down to one. The reason for this behaviour appears in Table 5.12 where the subfilter coefficients for subfilters 15 to 30 are given. As shown, the amplitude of the second non-zero term remains large and is sometimes larger than the first non-zero term. Consequently, the second non-zero term is never discarded from the calculations and the number of non-zero terms remains constant after subfilter 17.

The bad behaviour of the subfilters indicates that the filter becomes unstable. As discussed in Chapter 2, instability is expected when the Fourier transform of the given wavelet has some zero spectral values or notches. Figure 5.12 shows the amplitude spectrum of the given wavelet "NOISY-SWEEP2". As shown, a very low amplitude value is present around 50 Hz. This very low amplitude value is attributed to the sharp truncation of the sweep in the time domain. "Ringing" of the amplitude spectrum can be expected whenever a signal is sharply truncated in the time domain. In this case, both signals "SWEEP2" and HSWEEP2" are sharply truncated. Their combination causes a very low amplitude value to occur at about 50 Hz. As it was shown in Chapter 2, the presence of a

given wavelet : "NOISY-SWEEP2"　　desired wavelet : "SWEEP2"

| subfilter | number of non-zero coefficients | lg |
|---|---|---|
| 1 | 601 | 601 |
| 2 | 599 | 1795 |
| 3 | 597 | 3874 |
| 4 | 417 | 5425 |
| 5 | 284 | 7265 |
| 6 | 190 | 9857 |
| 7 | 128 | 9999 |
| 8 | 79 | 9999 |
| 9 | 45 | 9999 |
| 10 | 41 | 9999 |
| 11 | 27 | 9999 |
| 12 | 19 | 9999 |
| 13 | 11 | 9999 |
| 14 | 7 | 9999 |
| 15 | 5 | 9999 |
| 16 | 3 | 9999 |
| 17 | 2 | 9999 |
| 18 | 2 | 9999 |
| 19 | 2 | 9999 |
| 20 | 2 | 9999 |
| 21 | 2 | 9999 |
| 22 | 2 | 9999 |
| 23 | 2 | 9999 |
| 24 | 2 | 9999 |
| 25 | 2 | 9999 |
| 26 | 2 | 9999 |
| 27 | 2 | 9999 |
| 28 | 2 | 9999 |
| 29 | 2 | 9999 |
| 30 | 2 | 9999 |

Table 5.11 Example 6 . Number of non-zero coefficients in each subfilter and value of "lg" after each subfilter

given wavelet : "NOISY-SWEEP2" desired wavelet : "SWEEP2"

| | | | | | |
|---|---|---|---|---|---|
| <u>subfilter 15</u> | no. of weights = | 5 | | lg = 9999 | |
| b 1.000000 | 1.129481 | 0.001408 | 0.000001 | 0.000000 | |
| <u>subfilter 16</u> | no. of weights = | 3 | | lg = 9999 | |
| b -1.000000 | 0.820467 | 0.000000 | | | |
| <u>subfilter 17</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 1.943702 | | | | |
| <u>subfilter 18</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 0.576267 | | | | |
| <u>subfilter 19</u> | no. of weights = | 2 | | lg = 9999 | |
| b 1.000000 | 0.988832 | | | | |
| <u>subfilter 20</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 1.023244 | | | | |
| <u>subfilter 21</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 0.957014 | | | | |
| <u>subfilter 22</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 1.101140 | | | | |
| <u>subfilter 23</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 0.850873 | | | | |
| <u>subfilter 24</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 1.616148 | | | | |
| <u>subfilter 25</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 0.618375 | | | | |
| <u>subfilter 26</u> | no. of weights = | 2 | | lg = 9999 | |
| b 1.000000 | 1.625625 | | | | |
| <u>subfilter 27</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 0.616678 | | | | |
| <u>subfilter 28</u> | no. of weights = | 2 | | lg = 9999 | |
| b 1.000000 | 1.588404 | | | | |
| <u>subfilter 29</u> | no. of weights = | 2 | | lg = 9999 | |
| b -1.000000 | 0.623577 | | | | |
| <u>subfilter 30</u> | no. of weights = | 2 | | lg = 9999 | |
| b 1.000000 | 1.749183 | | | | |

Table 5.12 Example 6 . Subfilters coefficients for subfilters 15 to 30 (only the non-zero coefficients of the positive half of the subfilters including the centre point are shown)

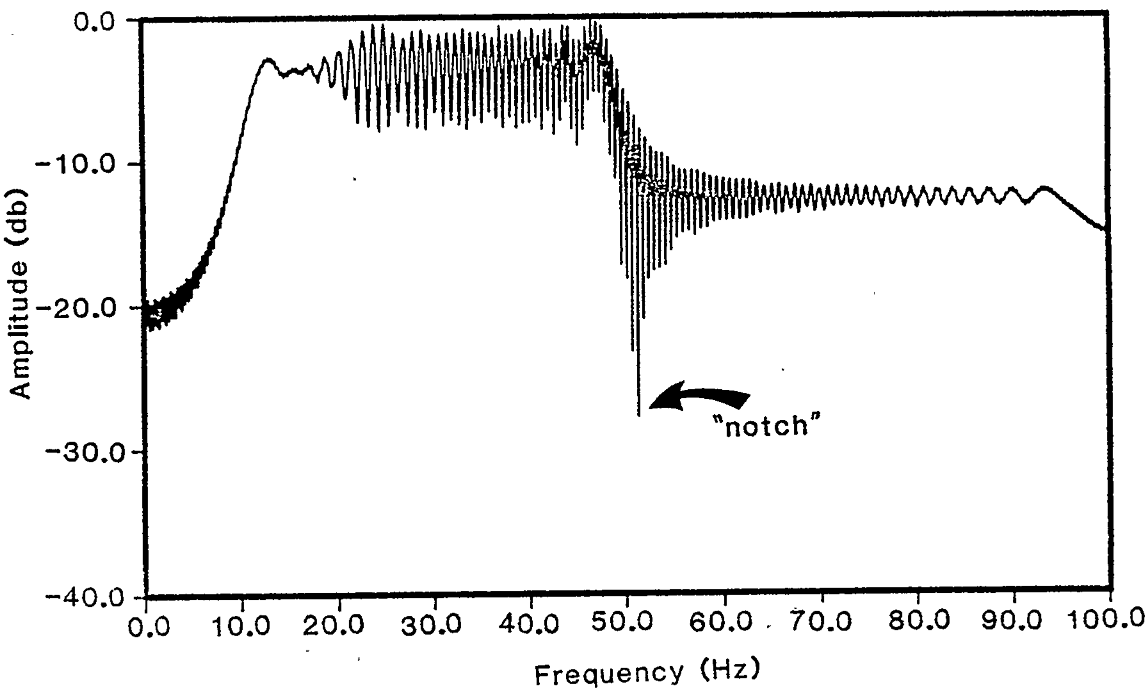


Figure 5.12 Amplitude spectrum of "NOISY-SWEEP2"

notch in the amplitude spectrum of the given wavelet brings instability of the wave-shaping filter.

In order to solve the stability problem, some white noise is added. Tables 5.13, 5.14, and 5.15 show the results obtained when 20% white noise is added (WEIGHT:0.2) without changing the other control parameters. As shown, the number of non-zero coefficients in the subfilters decreases gradually down to one. It can also be noted that adding 20% of white noise has the effect of attenuating the off-centered non-zero terms much more rapidly than in the noiseless case. Consequently, the number of non-zero coefficients left in the subfilters and the size of the symmetric filter are reduced compared to the noiseless case (Table 5.11). This, as discussed in Chapter 3, is expected when white noise is added.

Solving the instability problem by adding some white noise is done at the expense of increasing the shaping errors throughout the output. Figure 5.13 shows the central portion of the actual output and shaping errors (2001 points are plotted). Although the shaping errors are relatively large, the shape of the actual output and the relative magnitude and distribution of the shaping errors indicate that the wave-shaping filter has significantly removed the harmonic distortion affecting "NOISY-SWEEP2". Note that a better quality wave-shaping filter could possibly be defined by gradually decreasing the amount of white noise being added.

given wavelet : "NOISY-SWEEP2" desired wavelet : "SWEEP2"

| subfilter | number of non-zero coefficients | Ig |
|---|---|---|
| 1 | 601 | 601 |
| 2 | 599 | 1795 |
| 3 | 594 | 3703 |
| 4 | 387 | 4898 |
| 5 | 252 | 6209 |
| 6 | 158 | 7937 |
| 7 | 102 | 9985 |
| 8 | 62 | 9999 |
| 9 | 31 | 9999 |
| 10 | 26 | 9999 |
| 11 | 15 | 9999 |
| 12 | 10 | 9999 |
| 13 | 6 | 9999 |
| 14 | 3 | 9999 |
| 15 | 2 | 9999 |
| 16 | 1 | 9999 |

Table 5.13 Example 6 ("WEIGHT" : 0.2) . Number of non-zero coefficients in each subfilter and value of "Ig" after each subfilter

given wavelet : "NOISY-SWEEP2"  desired wavelet : "SWEEP2"

subfilter 10  no. of weights = 26  lg = 9999

b 1.000000 0.665518 0.248489 0.055203 0.008313 0.000389 -0.0
00438 -0.000243 -0.000048 0.000002 0.000007 0.000004 0.000002
-0.000000 0.000003 -0.000007 0.000007 -0.000003 0.000001 -0.00000
0 0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.000000

subfilter 11  no. of weights = 15  lg = 9999

b 1.000000 0.066169 0.018069 0.000038 0.000158 -0.000015 0.0
00021 -0.000089 0.000127 -0.000047 0.000007 -0.000001 0.000000
-0.000000 0.000000

subfilter 12  no. of weights = 10  lg = 9999

b 1.000000 -0.032020 0.000646 -0.000066 0.000276 -0.000025 0.0
00001 -0.000000 0.000000 0.000000

subfilter 13  no. of weights = 6  lg = 9999

b 1.000000 -0.000263 0.000548 0.000000 0.000000 0.000000

subfilter 14  no. of weights = 3  lg = 9999

b 1.000000 -0.001095 0.000000

subfilter 15  no. of weights = 2  lg = 9999

b 1.000000 0.000000

subfilter 16  no. of weights = 1  lg = 9999

b 1.000000

Table 5.14 Example 6 ("WEIGHT" : 0.2) . Subfilters coefficients for subfilters 10 to 16 (only the non-zero coefficients of the positive half of the subfilters including the centre point are shown)

given wavelet : "NOISY-SWEEP2"  desired wavelet : "SWEEP2"

51 central points of the input and the output data

| time | given wavelet | desired wavelet | correlation functions | | symmetric filter | shaping filter | actual output | shaping errors |
|---|---|---|---|---|---|---|---|---|
| | w | d | r | s | g | f | h | e |
| -25 | 0.983 | 0.79 | -0.000 | 0.957 | -0.000 | 0.004 | 0.767 | -2.6E-02 |
| -24 | 0.484 | 0.94 | -0.000 | -10.122 | 0.000 | 0.000 | 0.882 | -6.1E-02 |
| -23 | 0.005 | 0.24 | -0.000 | -13.498 | 0.000 | -0.015 | 0.186 | -5.2E-02 |
| -22 | -0.141 | -0.68 | -0.000 | -2.041 | -0.000 | 0.001 | -0.684 | -2.3E-03 |
| -21 | -0.885 | -0.99 | -0.000 | 2.570 | -0.000 | 0.015 | -0.935 | 5.2E-02 |
| -20 | -0.599 | -0.41 | 0.000 | -6.738 | -0.000 | 0.011 | -0.336 | 7.1E-02 |
| -19 | 0.760 | 0.53 | 0.000 | -9.638 | -0.000 | -0.010 | 0.568 | 3.3E-02 |
| -18 | 0.789 | 1.00 | -0.000 | 6.035 | -0.000 | 0.003 | 0.967 | -3.3E-02 |
| -17 | 0.082 | 0.58 | -0.000 | 15.825 | -0.000 | 0.021 | 0.520 | -5.8E-02 |
| -16 | -0.017 | -0.35 | -0.000 | 5.467 | -0.000 | 0.015 | -0.384 | -3.2E-02 |
| -15 | -0.574 | -0.97 | 0.000 | -1.467 | -0.000 | -0.009 | -0.978 | -5.9E-03 |
| -14 | -0.953 | -0.74 | 0.000 | 14.393 | -0.000 | 0.007 | -0.741 | -9.3E-04 |
| -13 | 0.210 | 0.14 | 0.000 | 25.297 | -0.000 | 0.028 | 0.147 | 1.0E-02 |
| -12 | 0.997 | 0.89 | 0.000 | 7.519 | 0.000 | 0.006 | 0.924 | 3.0E-02 |
| -11 | 0.350 | 0.88 | 0.000 | -10.103 | 0.000 | -0.036 | 0.914 | 3.7E-02 |
| -10 | 0.000 | 0.10 | 0.000 | 3.725 | 0.000 | -0.020 | 0.127 | 2.3E-02 |
| -9 | -0.201 | -0.76 | -0.000 | 15.985 | 0.000 | 0.013 | -0.764 | -8.0E-03 |
| -8 | -0.929 | -0.97 | -0.000 | -13.001 | 0.001 | -0.022 | -1.002 | -3.2E-02 |
| -7 | -0.532 | -0.36 | -0.000 | -45.576 | 0.001 | -0.082 | -0.384 | -2.6E-02 |
| -6 | 0.785 | 0.56 | -0.000 | -26.275 | 0.001 | -0.047 | 0.549 | -7.8E-03 |
| -5 | 0.790 | 1.00 | -0.000 | 1.387 | 0.000 | 0.015 | 0.995 | -5.1E-03 |
| -4 | 0.093 | 0.60 | -0.000 | -43.357 | 0.001 | -0.064 | 0.601 | -5.4E-04 |
| -3 | -0.011 | -0.30 | -0.000 | -110.021 | 0.002 | -0.201 | -0.284 | 1.6E-02 |
| -2 | -0.502 | -0.95 | 0.000 | -53.111 | 0.001 | -0.086 | -0.934 | 1.6E-02 |
| -1 | -0.989 | -0.81 | 88.096 | 127.243 | -0.001 | 0.297 | -0.811 | -1.3E-03 |
| 0 | 0.000 | 0.00 | 265.836 | 230.951 | 0.008 | 0.523 | -0.004 | -4.2E-03 |
| 1 | 0.989 | 0.81 | 88.096 | 126.750 | -0.001 | 0.298 | 0.814 | 5.4E-03 |
| 2 | 0.509 | 0.95 | -50.278 | -53.977 | 0.001 | -0.087 | 0.960 | 8.2E-03 |
| 3 | 0.013 | 0.32 | -89.802 | -111.062 | 0.002 | -0.204 | 0.327 | 8.9E-03 |
| 4 | -0.080 | -0.57 | -36.706 | -44.370 | 0.001 | -0.069 | -0.571 | 2.7E-03 |
| 5 | -0.749 | -1.00 | 1.957 | 0.593 | 0.000 | 0.011 | -1.025 | -2.5E-02 |
| 6 | -0.850 | -0.62 | -18.858 | -26.813 | 0.001 | -0.050 | -0.667 | -4.9E-02 |
| 7 | 0.393 | 0.26 | -31.705 | -45.825 | 0.001 | -0.083 | 0.230 | -2.9E-02 |
| 8 | 0.980 | 0.93 | -8.472 | -13.121 | 0.001 | -0.023 | 0.951 | 2.2E-02 |
| 9 | 0.318 | 0.86 | 13.936 | 16.008 | 0.000 | 0.012 | 0.924 | 6.8E-02 |
| 10 | 0.000 | 0.10 | 1.820 | 3.721 | 0.000 | -0.021 | 0.173 | 6.9E-02 |
| 11 | -0.176 | -0.73 | -8.562 | -9.949 | 0.000 | -0.036 | -0.718 | 1.1E-02 |
| 12 | -0.883 | -0.99 | 3.494 | 7.721 | 0.000 | 0.006 | -1.043 | -5.7E-02 |
| 13 | -0.683 | -0.47 | 18.820 | 25.862 | -0.000 | 0.032 | -0.533 | -6.2E-02 |
| 14 | 0.604 | 0.41 | 10.196 | 15.086 | -0.000 | 0.010 | 0.389 | -2.2E-02 |
| 15 | 0.923 | 0.97 | 0.033 | -0.407 | -0.000 | -0.003 | 0.971 | -1.0E-03 |
| 16 | 0.226 | 0.78 | 4.894 | 6.374 | -0.000 | 0.017 | 0.792 | 1.1E-02 |
| 17 | 0.000 | -0.01 | 13.856 | 16.745 | -0.000 | 0.024 | 0.035 | 4.7E-02 |
| 18 | -0.240 | -0.79 | 4.834 | 6.322 | -0.000 | 0.002 | -0.738 | 5.6E-02 |
| 19 | -0.931 | -0.97 | -7.131 | -9.614 | -0.000 | -0.009 | -0.972 | -3.1E-03 |
| 20 | -0.599 | -0.41 | -6.380 | -7.481 | -0.000 | 0.008 | -0.474 | -6.7E-02 |
| 21 | 0.672 | 0.46 | 1.316 | 1.788 | -0.000 | 0.016 | 0.403 | -6.0E-02 |
| 22 | 0.900 | 0.98 | -2.660 | -3.251 | -0.000 | -0.002 | 0.980 | -1.2E-03 |
| 23 | 0.208 | 0.76 | -10.236 | -14.268 | 0.000 | -0.015 | 0.806 | 4.3E-02 |
| 24 | 0.000 | -0.03 | -7.560 | -11.006 | 0.000 | -0.006 | 0.019 | 4.4E-02 |
| 25 | -0.239 | -0.79 | 1.818 | 0.713 | -0.000 | 0.002 | -0.776 | 1.8E-02 |
| n | lw | ld | lr | ls | 2*lg-1 | lf | lh | le |
| 16 | 601 | 601 | 1201 | 1201 | 19997 | 9999 | 10599 | 10599 |

Table 5.15 Example 6 ("WEGHT" : 0.2) . Table of the input and output coefficients (only the central 51 coefficients are shown)

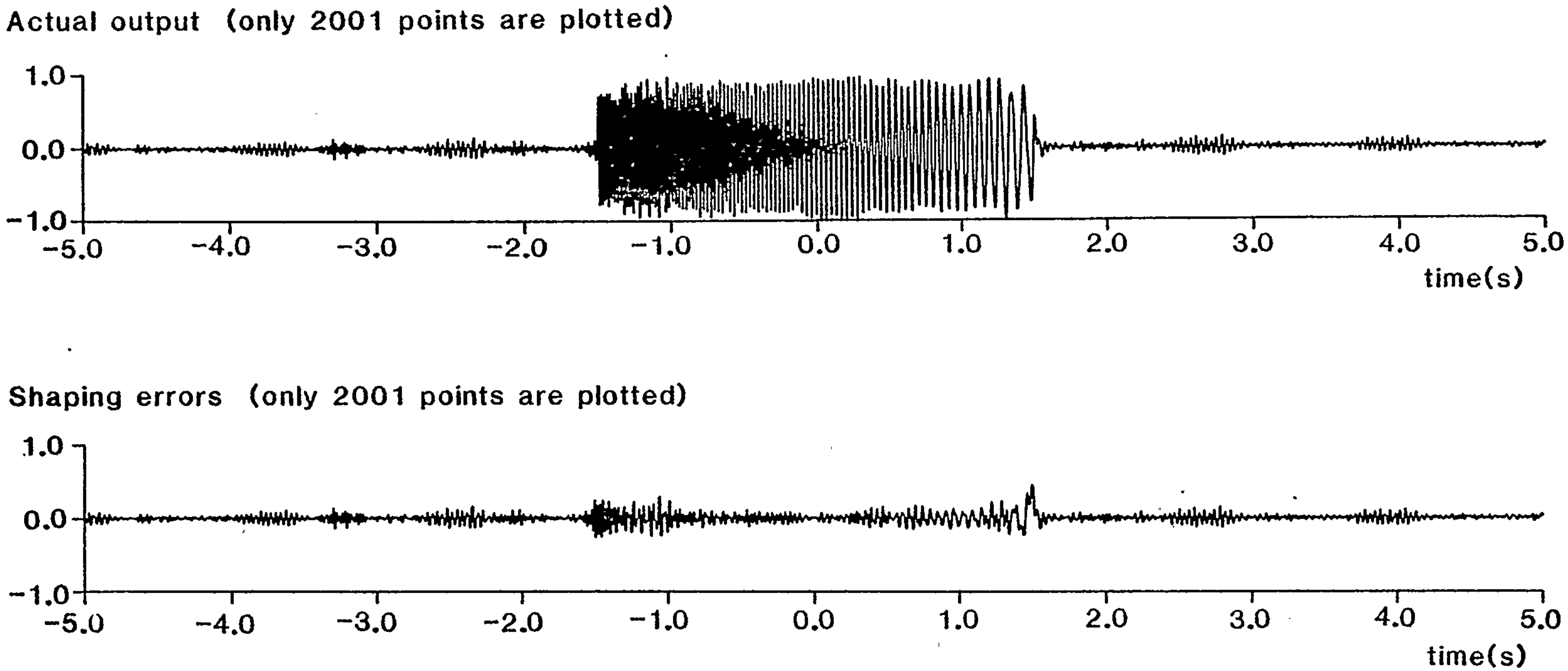


Figure 5.13 Example 6 ("WEIGHT" : 0.2) Plots of the central portion of the actual output and shaping errors

Here again, the length of the truncated wave-shaping filter should be compared to the length of the dataset to be filtered. In order to keep the errors caused by the truncation of the filter in a zone outside the area of interest, the conditions quoted in Chapter 4 (p. 55) should be respected. In this case, the length of the desired wavelet is 3 s (601 points) and the truncated wave-shaping filter is approximately 50 s (9999 points). Therefore, the length of the dataset to be filtered should not exceed 23.49 s (4698 points).

This example shows that a truncated wave-shaping filter can be defined to attenuate the harmonic distortion affecting long sweep (601 points). It also shows that the presence of notches in the amplitude spectrum of the given wavelet brings instability problems, and that adding some white noise can help solve these instability problems. In this case, a better quality wave-shaping filter could possibly be designed by gradually decreasing the amount of white noise being added.

In this first approach, the given wavelet is relatively long (601 points) and defining an "exact" wave-shaping filter is difficult. This is because the length of the "exact" wave-shaping filter tends to become extremely long. It is then strongly suggested using the program "MEREU2" for the definition of a truncated wave-shaping filter. Truncated filters are usually faster to compute, and consequently, their efficiency can be tested in a shorter time and at a lower cost. Nevertheless a significant portion of the harmonic distortion could be attenuated by using a truncated wave-shaping filter.

2nd Approach: Attenuation of correlation noise on correlated records.

In this second approach, the given and desired wavelets are:

- given wavelet: "W2"
- desired wavelet: "A2"

Both time series contain 1201 points.

Example 7: Definition of an "exact" wave-shaping filter using the program "MEREU1".

In order to respect the conditions discussed in Chapter 3, the following values are input to "MEREU1".

- MXN: 30
- MXLG: 14001
- BMIN: $10^{-10}$
- WEIGHT: 0

The number of non-zero coefficients left in each subfilter and the number of points in the symmetric filters $G_1$, $G_2$,... are given in Table 5.16. As shown, there is insufficient working space and only 6 subfilters can be used. Because of the limited amount of subfilters being used and also because of the truncation of the wave-shaping filter to 14001 coefficients, the shaping errors are expected to be relatively

given wavelet : "W2" desired wavelet : "A2"

| subfilter | number of non-zero coefficients | lg |
|---|---|---|
| 1 | 1201 | 1201 |
| 2 | 1197 | 3585 |
| 3 | 931 | 5689 |
| 4 | 615 | 8234 |
| 5 | 431 | 10914 |
| 6 | 267 | 14017 |

Warning
There is insufficient working space for G.
The maximum number of subfilters which can be used : 6
filter F was truncated to the length lf where lf : 14001

Table 5.16 Example 7 . Number of non-zero coefficients in each subfilter and value of "lg" after each subfilter.

large. Considering the rate at which the length of the symmetric filter increases and the rate at which the number of non-zero coefficients in each subfilter decreases, it is obvious that "MXLG" should be set to a very large number. Here again, setting the arrays to a very large number may cause some difficulties with the computer memory space. It is therefore suggested using the program "MEREU2" for the definition of a truncated wave-shaping filter. Note that in this case a filter containing 14001 points is 70 seconds long.

Example 8: Definition of a truncated wave-shaping filter using the program "MEREU2".

In this example, the same parameters as in example 7 are input to "MEREU2". It implies that only ((2 x 14001)-1) coefficients of $G_{N-i}$ will be used to generate the coefficients of $G_{N-i+1}$ and that the wave-shaping filter will contain 20001 coefficients, or will be 100 seconds long.

Table 5.17 gives the number of non-zero coefficients left in each subfilter. As shown, the number of non-zero coefficients does not decrease down to one but rather, stays constant after the 18th subfilter. Table 5.18 gives the subfilter coefficients for subfilters 15 to 30. As shown, the subfilter coefficients remain relatively large with the amplitude of some off-centered coefficients being sometimes larger than 1.0. This bad behaviour of the subfilters indicates that the filter becomes unstable.

given wavelet : "W2"    desired wavelet : "A2"

| subfilter | number of non-zero coefficients | lg |
|---|---|---|
| 1 | 1201 | 1201 |
| 2 | 1197 | 3585 |
| 3 | 931 | 5689 |
| 4 | 615 | 8234 |
| 5 | 431 | 10914 |
| 6 | 267 | 13977 |
| 7 | 168 | 14001 |
| 8 | 106 | 14001 |
| 9 | 63 | 14001 |
| 10 | 60 | 14001 |
| 11 | 43 | 14001 |
| 12 | 31 | 14001 |
| 13 | 25 | 14001 |
| 14 | 19 | 14001 |
| 15 | 16 | 14001 |
| 16 | 14 | 14001 |
| 17 | 13 | 14001 |
| 18 | 12 | 14001 |
| 19 | 12 | 14001 |
| 20 | 12 | 14001 |
| 21 | 12 | 14001 |
| 22 | 12 | 14001 |
| 23 | 12 | 14001 |
| 24 | 12 | 14001 |
| 25 | 12 | 14001 |
| 26 | 12 | 14001 |
| 27 | 12 | 14001 |
| 28 | 12 | 14001 |
| 29 | 12 | 14001 |
| 30 | 12 | 14001 |

Table 5.17 Example 8 . Number of non-zero coefficients in each subfilter and value of "lg" after each subfilter .

given wavelet : "W2" desired wavelet : "A2"

```
subfilter 17      no. of weights =  13       lg =14001

b   -1.000000    0.389840    0.621624   -0.790375    0.055176    0.541806   -0.3
62279   -0.091758    0.249815   -0.094173   -0.062782    0.045681   -0.000000

subfilter 18      no. of weights =  12       lg =14001

b   -1.000000    2.413400    9.505326   12.891577    8.321276    0.965089   -0.7
23813    3.514728    6.679726    5.218027    2.063259    0.343192

subfilter 19      no. of weights =  12       lg =14001

b   -1.000000   -0.801173   -0.294790    0.325280    0.866631    1.202294    1.2
58741    1.047476    0.680231    0.329246    0.107114    0.018677

subfilter 20      no. of weights =  12       lg =14001

b   -1.000000    0.981562   -0.847028    0.738609   -0.583456    0.581851   -0.5
25419    0.566535   -0.443380    0.349032   -0.159430    0.067364

subfilter 21      no. of weights =  12       lg =14001

b   -1.000000    0.987441   -0.946370    0.873791   -0.772279    0.648279   -0.5
09726    0.365538   -0.229659    0.118807   -0.045213    0.009496

subfilter 22      no. of weights =  12       lg =14001

b   -1.000000   -1.091074   13.676751  -15.728897    9.693914    9.653272  -42.5
01252   61.742653  -70.220365   61.144030  -44.706223   19.002967

subfilter 23      no. of weights =  12       lg =14001

b    1.000000   -0.565060   -0.100950    0.636554   -0.642450    0.413314   -0.0
03305   -0.463022    0.532716   -0.126602   -0.212878    0.236394

subfilter 24      no. of weights =  12       lg =14001

b    1.000000    0.702949    1.010990    1.466379    0.201110   -0.097013    1.2
04000    0.998850   -0.194927    0.169850    0.746704    0.396752

subfilter 25      no. of weights =  12       lg =14001

b   -1.000000    0.867359   -0.077731    2.430877    2.331504    4.676814    4.3
87918    4.993107    3.420354    2.418011    0.918895    0.342120

subfilter 26      no. of weights =  12       lg =14001

b   -1.000000    1.006347   -1.013476    0.991810   -0.910253    0.757134   -0.5
53084    0.343713   -0.174826    0.068635   -0.018675    0.002698

subfilter 27      no. of weights =  12       lg =14001

b   -1.000000    0.846435   -0.667124    0.433235   -0.215198    0.320092   -0.5
23234    0.838229   -1.009746    0.799692   -0.484487    0.163223

subfilter 28      no. of weights =  12       lg =14001

b    1.000000   -0.055924   -0.944019    0.196488    0.759297   -0.286567   -0.5
24248    0.315189    0.282650   -0.226532   -0.091374    0.092461

subfilter 29      no. of weights =  12       lg =14001

b    1.000000    0.969117    0.879651    0.744216    0.583662    0.423277    0.2
82411    0.171076    0.090708    0.039375    0.012267    0.002087

subfilter 30      no. of weights =  12       lg =14001

b   -1.000000    0.967962   -0.875152    0.738502   -0.581442    0.420979   -0.2
81917    0.169792   -0.091558    0.043439   -0.015445    0.004843
```

Table 5.18 Example 8 . Subfilters coefficients for subfilters 17 to 30 (only the non-zero coefficients of the positive half of the subfilters including the centre point are shown)

The amplitude spectrum of the given wavelet, "W2", is plotted in Figure 5.14. Here again, it shows that a notch is present around 50 Hz. This is also caused by the sharp truncation of the signal in the time domain.

In order to solve the instability problem, some white noise is added. Tables 5.19, 5.20 and 5.21 show the results obtained when 10% white noise is added (WEIGHT: 0.1) without changing the other parameters. As shown, the number of non-zero coefficients in the subfilters decreases gradually down to one. Here again, it can be noted that adding some white noise has the effect of decreasing more rapidly the number of non-zero coefficients in each subfilter as well as reducing the size of the symmetric filter. However, adding some white noise also has the effect of increasing the amplitude of the shaping errors throughout the output. Figure 5.15 shows the central portion of the actual output and shaping errors (2001 points are plotted). Although the shaping errors are relatively large, the shape of the output as well as the relative magnitude of the shaping errors indicate that the wave-shaping filter has significantly attenuated the correlation noise affecting "W2". A better quality wave-shaping filter could possibly be defined by gradually decreasing the amount of white noise being added.

The length of the truncated wave-shaping filter should be compared to the length of the dataset to be filtered. In order to keep the errors caused by the truncation of the filter in a zone outside the area of interest, the conditions quoted in Chapter 4 (p. 55) should be respected.

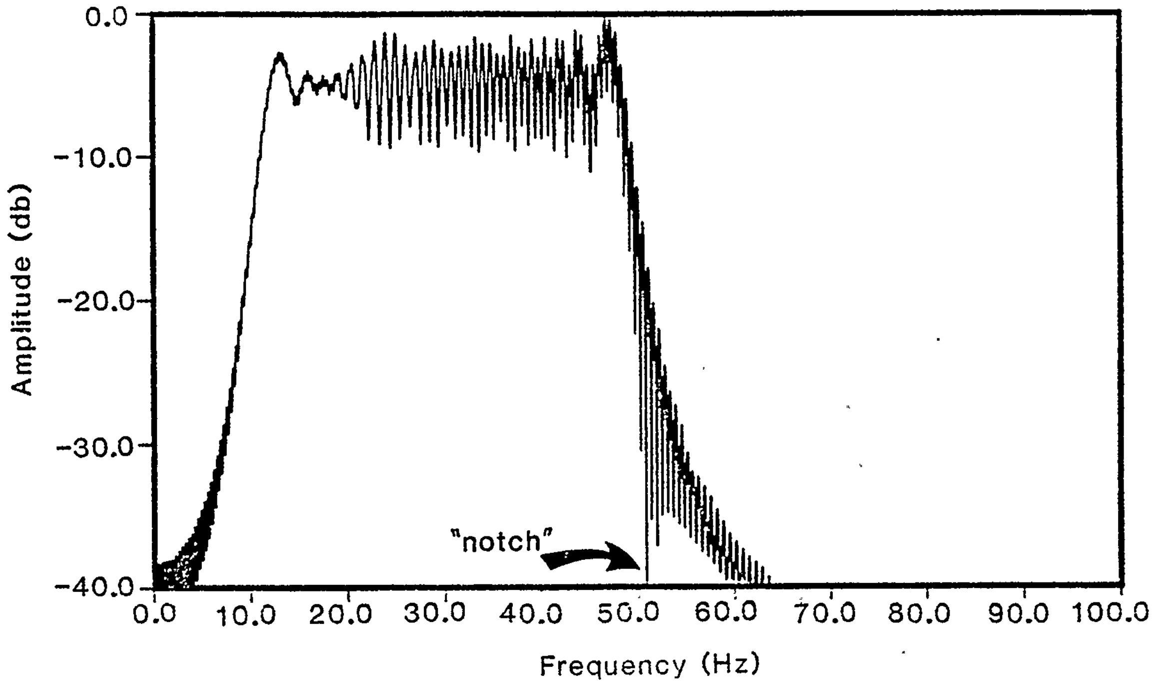


Figure 5.14 Amplitude spectrum of "W2"

given wavelet : "W2"    desired wavelet : "A2"

| subfilter | number of non-zero coefficients | lg |
|---|---|---|
| 1 | 1201 | 1201 |
| 2 | 1187 | 3357 |
| 3 | 644 | 4161 |
| 4 | 415 | 5329 |
| 5 | 253 | 6049 |
| 6 | 148 | 7425 |
| 7 | 94 | 9281 |
| 8 | 57 | 10753 |
| 9 | 32 | 13826 |
| 10 | 22 | 13990 |
| 11 | 13 | 14001 |
| 12 | 7 | 14001 |
| 13 | 3 | 14001 |
| 14 | 3 | 14001 |
| 15 | 2 | 14001 |
| 16 | 1 | 11778 |

Table 5.19 Example 8 (WEIGHT : 0.1) . Number of non-zero coefficients in each subfilter and value of "lg" after each subfilter

given wavelet : "W2"    desired wavelet : "A2"

```
subfilter 10     no. of weights =   22          lg =13990

b    1.000000    0.670490    0.248694    0.054537    0.007121    -0.000171    -0.0
00714    -0.000327    -0.000070    -0.000004    0.000001    0.000000    0.000000
 0.000000    0.000000    -0.000000    0.000000    -0.000000    0.000000    0.00000
0    0.000000    -0.000000

subfilter 11     no. of weights =   13          lg =14001

b    1.000000    0.099446    0.013099    0.001040    0.000085    0.000008    0.0
00001    -0.000001    0.000003    -0.000001    0.000000    -0.000000    0.000000

subfilter 12     no. of weights =    7          lg =14001

b    1.000000    -0.016423    0.000136    -0.000001    0.000007    -0.000000    0.0
00000

subfilter 13     no. of weights =    3          lg =14001

b    1.000000    -0.000003    0.000013

subfilter 14     no. of weights =    3          lg =14001

b    1.000000    -0.000026    0.000000

subfilter 15     no. of weights =    2          lg =14001

b    1.000000    0.000000

subfilter 16     no. of weights =    1          lg =11778

b    1.000000
```

Table 5.20 Example 8 (WEIGHT : 0.1) . Subfilters cefficients for subfilters 10 to 16 (only the non-zero coefficients of the positive half of the subfilters including the centre point are shown)

given wavelet : "W2" desired wavelet : "A2"

51 central points of the input and the output data

| time | given wavelet | desired wavelet | correlation functions | | symmetric filter | shaping filter | actual output | shaping errors |
|---|---|---|---|---|---|---|---|---|
| | w | d | r | s | g | f | h | e |
| -25 | 0.003 | 0.00 | 0.038 | 0.016 | 0.003 | 0.001 | 0.000 | -1.6E-03 |
| -24 | -0.048 | -0.05 | -0.182 | -0.187 | 0.019 | -0.007 | -0.052 | -4.0E-03 |
| -23 | -0.062 | -0.06 | -0.218 | -0.209 | 0.046 | -0.014 | -0.063 | -1.8E-03 |
| -22 | -0.014 | -0.01 | -0.035 | -0.029 | 0.017 | -0.002 | -0.013 | 1.2E-04 |
| -21 | 0.008 | 0.01 | 0.002 | 0.010 | -0.034 | 0.012 | 0.006 | -3.3E-03 |
| -20 | -0.032 | -0.03 | -0.172 | -0.151 | -0.022 | 0.003 | -0.037 | -5.9E-03 |
| -19 | -0.042 | -0.04 | -0.173 | -0.157 | 0.012 | -0.006 | -0.043 | -1.8E-03 |
| -18 | 0.027 | 0.03 | 0.104 | 0.088 | -0.028 | 0.010 | 0.029 | 2.0E-03 |
| -17 | 0.073 | 0.07 | 0.240 | 0.205 | -0.095 | 0.028 | 0.069 | -1.7E-03 |
| -16 | 0.028 | 0.03 | 0.058 | 0.041 | -0.069 | 0.015 | 0.022 | -4.6E-03 |
| -15 | -0.002 | -0.00 | -0.026 | -0.018 | -0.005 | -0.002 | -0.002 | 8.7E-04 |
| -14 | 0.065 | 0.07 | 0.219 | 0.224 | -0.043 | 0.014 | 0.071 | 5.6E-03 |
| -13 | 0.112 | 0.11 | 0.349 | 0.353 | -0.115 | 0.033 | 0.115 | 1.7E-03 |
| -12 | 0.033 | 0.03 | 0.067 | 0.097 | -0.049 | 0.010 | 0.035 | 1.9E-04 |
| -11 | -0.043 | -0.04 | -0.151 | -0.109 | 0.074 | -0.022 | -0.034 | 8.2E-03 |
| -10 | 0.016 | 0.02 | 0.084 | 0.085 | 0.040 | -0.005 | 0.028 | 1.1E-02 |
| -9 | 0.069 | 0.07 | 0.265 | 0.220 | -0.052 | 0.017 | 0.070 | 1.7E-03 |
| -8 | -0.057 | -0.06 | -0.120 | -0.153 | 0.074 | -0.025 | -0.060 | -1.3E-03 |
| -7 | -0.198 | -0.20 | -0.523 | -0.530 | 0.282 | -0.077 | -0.190 | 1.0E-02 |
| -6 | -0.116 | -0.12 | -0.255 | -0.291 | 0.203 | -0.048 | -0.105 | 1.2E-02 |
| -5 | 0.003 | 0.00 | 0.070 | 0.014 | 0.007 | 0.001 | -0.001 | -3.7E-03 |
| -4 | -0.192 | -0.19 | -0.546 | -0.508 | 0.226 | -0.066 | -0.195 | -4.2E-03 |
| -3 | -0.481 | -0.48 | -1.410 | -1.240 | 0.648 | -0.173 | -0.461 | 1.9E-02 |
| -2 | -0.234 | -0.23 | -0.727 | -0.597 | 0.337 | -0.081 | -0.217 | 1.5E-02 |
| -1 | 0.549 | 0.55 | 1.453 | 1.362 | -0.800 | 0.213 | 0.511 | -3.9E-02 |
| 0 | 1.000 | 1.00 | 2.970 | 2.473 | 2.227 | 0.383 | 0.924 | -7.6E-02 |
| 1 | 0.551 | 0.55 | 1.453 | 1.351 | -0.800 | 0.213 | 0.511 | -3.9E-02 |
| 2 | -0.230 | -0.23 | -0.727 | -0.612 | 0.337 | -0.082 | -0.217 | 1.5E-02 |
| 3 | -0.476 | -0.48 | -1.410 | -1.252 | 0.648 | -0.175 | -0.461 | 1.9E-02 |
| 4 | -0.188 | -0.19 | -0.546 | -0.513 | 0.226 | -0.069 | -0.195 | -4.2E-03 |
| 5 | 0.006 | 0.00 | 0.070 | 0.014 | 0.007 | -0.002 | -0.001 | -3.7E-03 |
| 6 | -0.114 | -0.12 | -0.255 | -0.290 | 0.203 | -0.050 | -0.105 | 1.2E-02 |
| 7 | -0.197 | -0.20 | -0.523 | -0.530 | 0.282 | -0.078 | -0.190 | 1.0E-02 |
| 8 | -0.056 | -0.06 | -0.120 | -0.153 | 0.074 | -0.025 | -0.060 | -1.3E-03 |
| 9 | 0.069 | 0.07 | 0.265 | 0.223 | -0.052 | 0.016 | 0.070 | 1.7E-03 |
| 10 | 0.016 | 0.02 | 0.084 | 0.088 | 0.040 | -0.006 | 0.028 | 1.1E-02 |
| 11 | -0.044 | -0.04 | -0.151 | -0.108 | 0.074 | -0.021 | -0.034 | 8.2E-03 |
| 12 | 0.033 | 0.03 | 0.067 | 0.094 | -0.049 | 0.011 | 0.035 | 1.9E-04 |
| 13 | 0.110 | 0.11 | 0.349 | 0.348 | -0.115 | 0.036 | 0.115 | 1.7E-03 |
| 14 | 0.062 | 0.07 | 0.219 | 0.224 | -0.043 | 0.017 | 0.071 | 5.6E-03 |
| 15 | -0.006 | -0.00 | -0.026 | -0.009 | -0.005 | -0.000 | -0.002 | 8.7E-04 |
| 16 | 0.024 | 0.03 | 0.058 | 0.057 | -0.069 | 0.016 | 0.022 | -4.6E-03 |
| 17 | 0.069 | 0.07 | 0.240 | 0.219 | -0.095 | 0.028 | 0.069 | -1.7E-03 |
| 18 | 0.026 | 0.03 | 0.104 | 0.093 | -0.028 | 0.010 | 0.029 | 2.0E-03 |
| 19 | -0.042 | -0.04 | -0.173 | -0.163 | 0.012 | -0.006 | -0.043 | -1.8E-03 |
| 20 | -0.029 | -0.03 | -0.172 | -0.163 | -0.022 | 0.003 | -0.037 | -5.9E-03 |
| 21 | 0.011 | 0.01 | 0.002 | -0.002 | -0.034 | 0.010 | 0.006 | -3.3E-03 |
| 22 | -0.009 | -0.01 | -0.035 | -0.038 | 0.017 | -0.005 | -0.013 | 1.2E-04 |
| 23 | -0.058 | -0.06 | -0.218 | -0.214 | 0.046 | -0.017 | -0.063 | -1.8E-03 |
| 24 | -0.044 | -0.05 | -0.182 | -0.190 | 0.019 | -0.008 | -0.052 | -4.0E-03 |
| 25 | -0.004 | 0.00 | 0.038 | 0.013 | 0.003 | 0.000 | 0.000 | -1.6E-03 |
| n | lw | ld | lr | ls | 2*lg-1 | lf | lh | le |
| 16 | 1201 | 1201 | 2401 | 2401 | 23555 | 14001 | 15201 | 15201 |

Table 5.21 Example 8 (WEIGHT : 0.1) . Table of the input and output coefficients (only the central 51 coefficients are shown)

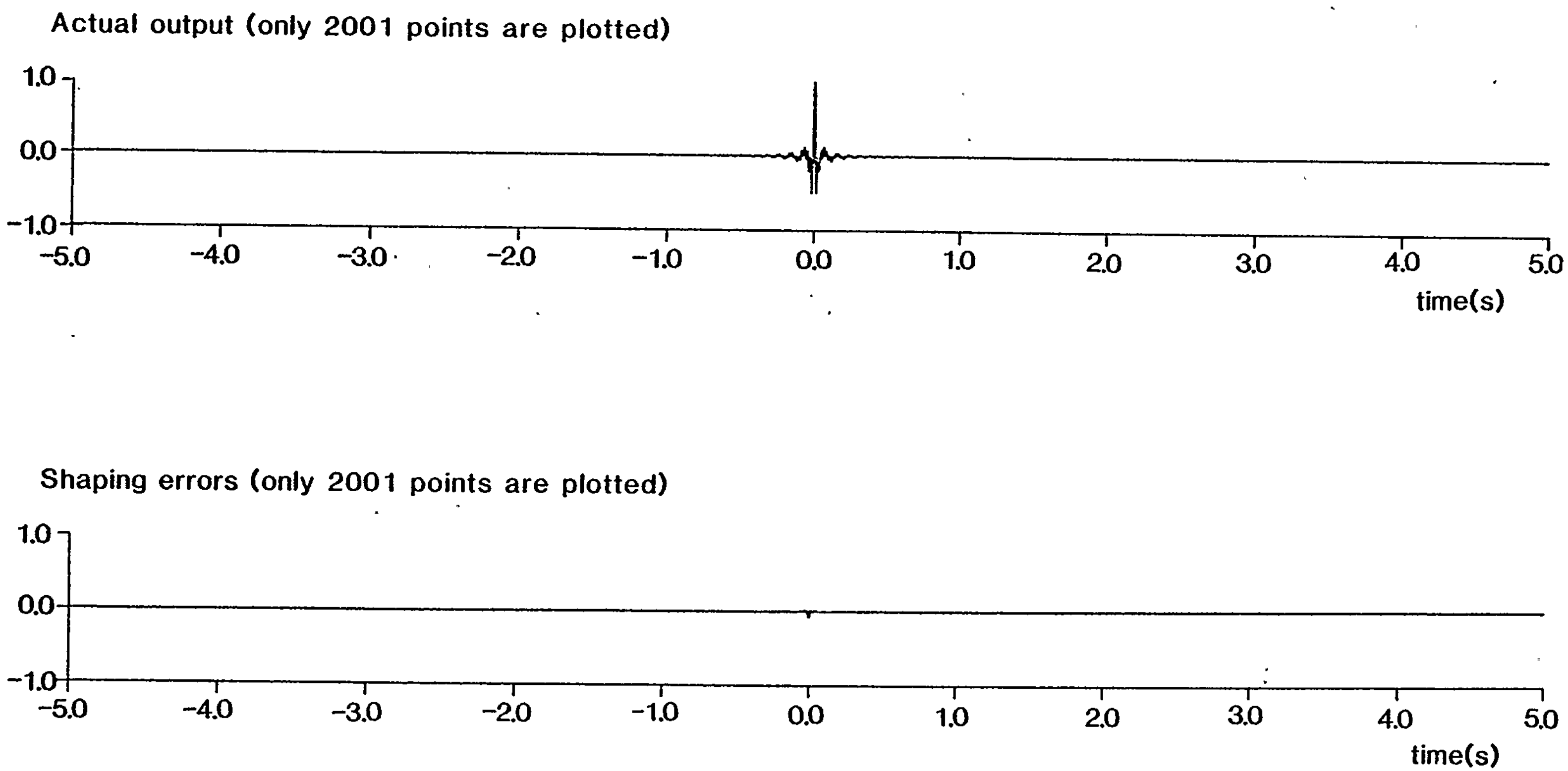


Figure 5.15 Example 8 ("WEIGHT": 0.1) Plots of the central portion of the actual output and shaping errors.

In this case, the length of the desired wavelet is 6 s (1201 points) and the truncated wave-shaping filter is 70 s (14001 points). Therefore, the length of the dataset to be filtered should not exceed 32 s (6400 points).

This last example shows that a truncated wave-shaping filter can be defined to attenuate the correlation noise affecting long correlated wavelet (1201 points). It also shows that adding some white noise can help solve instability problems. In this example, a better quality wave-shaping filter could possibly be defined by gradually decreasing the amount of white noise being added.

In this second approach, the given wavelet is relatively long (1201 points). Because the length of the wave-shaping filter tends to become extremely long, defining an "exact" wave-shaping filter is difficult. Using the program "MEREU2" for the definition of a truncated wave-shaping filter is then suggested. As it was presented, a significant portion of the correlation noise could be attenuated by using a truncated wave-shaping filter.

## 5.4 CONCLUSION

The effect of harmonic distortion on correlated Vibroseis data is to generate a series of wavetrains which may interfere with reflections and consequently affect the quality of the data. In order to reduce the importance of this effect, some methods were designed to attenuate the harmonic distortion at the source. Although efficient, these methods have not always been used, and some data are affected by correlation noise.

As demonstrated in this chapter, the effect of harmonic distortion can be attenuated during the processing step by using Mereu's wave-shaping filtering method. In fact, the harmonic distortion affecting the uncorrelated records as well as the correlation noise affecting the correlated records could be significantly reduced by this method. It was shown that when the size of the given wavelet is relatively small (26-51 points), good quality "exact" wave-shaping filter can be obtained. When the size of the given wavelet is relatively large (601-1201 points), only truncated wave-shaping filters obtained by the program "MEREU2" could give satisfactory output.

It was also shown that when a "notch" is present in the frequency spectrum of the given wavelet, the filter shows some instability problems. In these cases, some white noise can be added.

Typical sweeps used in the field contain thousands of points. For instance, a sweep 10 seconds long, sampled every 0.004 seconds contains 2501 points. If we assumed that the uncorrelated dataset is 20 seconds long, then a truncated wave-shaping filter designed to attenuate the harmonic distortion affecting these data should contain at least 12501 points ("MXLG" = 12501). This is to bring the errors that are caused by the truncation of the filter in a zone outside of the area of interest.

As shown in example 8, no storage problem was encountered by setting "MXLG" to a value of 20001. It implies that efficient, truncated wave-shaping filters could possibly be defined through "MEREU2" for affected data recorded in the field (assuming that the harmonically distorted waveform is known).

# CONCLUSION

The "exact" wave-shaping filtering method, defined by Mereu (1976), differs from most other techniques in that the filter is entirely defined in the time domain from a specific sequence of convolutions. Mereu's filtering technique has the advantage of producing an "exact" output in the zone of interest. The filter is also generally stable provided the Z-transform of the given wavelet has no roots lying on the unit circle. This filtering technique, as it was shown in this study, is very efficient at transforming a given wavelet into a desired wavelet, provided this given wavelet is relatively short.

The symmetric filter and the "exact" wave-shaping filter can become very long compared to the size of the given wavelet. However, the significant portion of both filters is usually contained in a restricted time interval. In order to take advantage of this characteristic and to keep the size of the wave-shaping filter within reasonable limits, modifications were brought to the design of the filter. These modifications include a series of successive truncations of the symmetric filter during the computations and the truncation of the final wave-shaping filter. Due to these truncations, the shaping errors increase and no "exact" wave-shaping filter can be produced. However, if the truncation length is properly chosen, the large errors caused by the truncations occur in a zone outside the area of interest. Examples have shown that when this condition is respected, very good quality wave-shaping filters are obtained. The new truncation technique, referred to as the "alternate" truncation technique, can then be very efficient at defining good quality wave-shaping filters for relatively long given wavelets.

The computer programs "MEREU1" and "MEREU2" calculate efficiently the coefficients of the subfilters, symmetric filter, wave-shaping filter, actual output and shaping errors. The control parameters for both programs should be chosen carefully as they affect directly the quality of the filter. In general the parameters "MXN" and "MXLG" should be set to some relatively large values. If truncation is expected, "MXLG" should be set to an odd value. This is to avoid the shifting of the output by one time sample. The parameter "BMIN" should be set to a relatively small number, typically $10^{-10}$. The amplitude spectrum of the given wavelet should be analyzed. If very low amplitude values or notches are present, filter instability problems can be expected. In these cases, some white noise can be added. It is however recommended to keep the amount of added white noise as low as possible.

Mereu's filtering technique was applied to the problem of harmonic distortion affecting Vibroseis data. It was shown that the harmonic distortion affecting the uncorrelated records as well as the correlation noise affecting the correlated records could be significantly attenuated. It was shown that when the size of the given wavelet is relatively long, only truncated wave-shaping filters obtained through the "alternate truncation technique" could give satisfactory output.

The effect of harmonic distortion on field data could be attenuated using Mereu's wave-shaping filtering method (assuming the harmonically distorted waveform is known). As the sweeps used in the field typically contain thousands of points, the alternate truncation technique should be used for the definition of a good quality wave-shaping filter.

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# APPENDICES

# APPENDIX 1

The following program computes the coefficients of the subfilters, symmetric filter, "exact" wave-shaping filter, actual output and shaping errors. It corresponds to the program designed by Mereu (1978).

Main Program

```
c     Program :  MEREU1

c     Input quatities for MEREU1
c           NEXP.....example number
c           MXN......maximum number of subfilters to be used
c           BMIN.....a minimum limit for size of subfilters weights
c           MXLG.....maximum working space for G,C and F
c           PRNT.....diagnostic print option
c           WEIGHT...ratio of white noise energy to signal energy
c           NPT......number of central points of F and H to be shown
c
c           W(i),i=1,lw...given wavelet
c           D(i),i=1,ld...desired wavelet
c
c
c     Quantities computed by progrm MEREU1 and subroutine FOLDM
c           WR(i),i=1,lw...wavelet W reversed in time
c           r(i),i=1,lr....the autocorrelation function of W
c           s(i),i=1,ls....the crosscorrelation function of W
c                          with D
c           H(i),i=1,lh....output of W convolved with F
c           E(i),i=1,le....shaping errors = difference between H
c                          and D
c
c
c     Quantities computed by subroutine SHAPEW
c           G(i),i=1,lg....the positive half of the symmetric filter
c           F(i),i=1,lf....the wave-shaping filter
c           N..............actual number of subfilters used

      open(10,form="formatted",file="autocl")
      open(11,form="formatted",file="ccorlhl")
      open(80,form="formatted",mode="out",file="ff5_2")
      open(81,form="formatted",mode="out",file="hh5_2")
      open(82,form="formatted",mode="out",file="ee5_2")


      dimension a(300),b(300),c(10000),d(52),f(10000),
     & g(5000),h(5000),r(300),s(300),w(300),wr(300)
      real autocl(60),tot_corrl(60),ccorlhl(50)
      real large1,large2,nt_c(100),nautocl(100)
      equivalence (c(1),f(1))

      do 60 i=1,51
      read(10,40) autocl(i)
 40   format(f10.4)
 60   continue

      do 61 i=1,51
      read(11,81) ccorlhl(i)
 81   format(f10.4)
 61   continue

      close(10)
      close(11)

      do 601 i=1,51
      tot_corrl(i)=autocl(i)+ccorlhl(i)
```

```
 601  continue

      large2=abs(autocl(1))
      do 73 i=2,51
      if(abs(autocl(i)).gt.large2) large2=abs(autocl(i))
  73  continue

      do 74 i=1,51
      nautocl(i)=autocl(i)/large2
  74  continue

      large1=abs(tot_corrl(1))
      do 602 i=2,51
      if(abs(tot_corrl(i)) .gt. large1) large1=abs(tot_corrl(i))
 602  continue

      do 66 i=1,51
      nt_c(i)=tot_corrl(i)/large1
  66  continue

      do 29 i=1,51
      w(i)=nt_c(i)
  29  continue

      do 600 i=1,51
      d(i)=nautocl(i)
  600 continue

      nexp=1
      mxn=20
      bmin=10.0**(-10)
      mxlg=501
      weight=0.0
      npt=25
      prnt=1.0
      lw=51
      ld=51

  1   continue
      if (nexp.eq.0) go to 4

c      the central points of the time series are the (time=0) points.
c      in order that W and D have a central point, lw and ld should
c      be odd numbers
c      the next 4 cards are inserted to make sure ld and lw are odd

      w(lw+1)=0.0
      d(ld+1)=0.0
      lw=(lw/2)*2+1
      ld=(ld/2)*2+1

c     create the wavelet W reversed in time

      do 2 i=1,lw
      k=lw-i+1
      wr(k)=w(i)
  2   continue
      if (weight.gt.0.0) write(6,10) weight

      write(6,17) mxn,bmin,mxlg
```

```
      call foldm (lw,w,lw,wr,lr,r)

      call foldm (lw,wr,ld,d,ls,s)

      call shapew (mxn,bmin,mxlg,prnt,lr,r,ls,s,a,b,c,lg,g,lf,f,n)

      call foldm (lw,w,lf,f,lh,h)



      if (weight.gt.0.0) write(6,10) weight
      write(6,17) mxn,bmin,mxlg
      write(6,18) npt

      lgg=2*lg-1
      le=lh
      im=(npt+1)/2
      it=-im
      do 3 i=1,npt
      it=it+1
      iw=(lw-npt)/2+i
      id=(ld-npt)/2+i
      ir=(lr-npt)/2+i
      is=(ls-npt)/2+i
      if (it.le.0) ig=im+1-i
      if (it.gt.0) ig=ig+1
      if=(lf-npt)/2+i
      ih=(lh-npt)/2+i
      ww=0.0
      dd=0.0
      rr=0.0
      ss=0.0
      gg=0.0
      ff=0.0
      hh=0.0
      ee=0.0
      mw=0
      md=0
      mr=0
      ms=0
      if (iw.gt.0.and.iw.le.lw) ww=w(iw)
      if (iw.gt.0.and.iw.le.lw) mw=1
      if (id.gt.0.and.id.le.ld) dd=d(id)
      if (id.gt.0.and.id.le.ld) md=1
      if (ir.gt.0.and.ir.le.lr) rr=r(ir)
      if (ir.gt.0.and.ir.le.lr) mr=1
      if (is.gt.0.and.is.le.ls) ss=s(is)
      if (is.gt.0.and.is.le.ls) ms=1
      if (ig.gt.0.and.ig.le.lg) gg=g(ig)
      if (if.gt.0.and.if.le.lf) ff=f(if)
      if (ih.gt.0.and.ih.le.lh) hh=h(ih)
      ee=hh-dd
      if (mw.eq.1.and.md.eq.1.and.mr.eq.1.and.ms.eq.1)
     & write(6,5) it,ww,dd,rr,ss,gg,ff,hh,ee
      if (mw.eq.0.and.md.eq.1.and.mr.ge.0.and.ms.eq.1)
     & write (6,19) it,dd,rr,ss,gg,ff,hh,ee
      if (mw.eq.1.and.md.eq.0.and.mr.eq.1.and.ms.eq.1)
     & write(6,20) it,ww,rr,ss,gg,ff,hh,ee
      if (mw.eq.0.and.md.eq.0.and.mr.eq.1.and.ms.eq.1)
```

```
     & write(6,21) it,rr,ss,gg,ff,hh,ee
       if (mw.eq.0.and.md.eq.0.and.mr.eq.1.and.ms.eq.0)
     & write(6,22) it,rr,gg,ff,hh,ee
       if (mw.eq.0.and.md.eq.0.and.mr.eq.0.and.ms.eq.0)
     & write(6,23) it,gg,ff,hh,ee
       if (mw.eq.0.and.md.eq.0.and.mr.eq.0.and.ms.eq.1)
     & write(6,24) it,ss,gg,ff,hh,ee


      write(80,111) ff
  111 format(f10.4)

      write(81,112) hh
  112 format(f10.4)

      write(82,113)  ee
 113  format(1pe9.1)

  3   continue
      write(6,25)
      write(6,26) n,lw,ld,lr,ls,lgg,lf,lh,le


      nexp=nexp-1
      go to 1
  4   continue
      close(80)
      close(81)
      close(82)

  5   format(1h0,i5,f9.3,f9.2,f9.3,f9.3,3f9.3,1pe9.1)
 10   format(1h0,37hwhite noise is assumed to be present.
     &,1h ,46hthe ratio of noise energy to signal energy is ,
     &f10.5)
 17   format(1h0,47hmaximum number of filters to be used     mxn =,
     & i5/1h,51hminimum limit set for sub-filter weights    bmin =,
     & 1pe9.2/1h,49hmaximum working space for  g,c,and f     mxlg =,
     & i5)
 18   format(1h0, 15x, i5, 3x,
     & 47hcentral points of the input and the output data//1h ,
     & 5h time,9h   given ,9h  desired,
     & 18h    correlation    ,
     & 9hsymmetric,
     & 9h  shaping,
     & 9h   actual,
     & 9h  shaping/1h ,
     & 5x,
     & 9h  wavelet,
     & 9h  wavelet,
     & 18h     functions    ,
     & 9h filter  ,
     & 9h   filter,
     & 9h   output,
     & 9h   errors//1h ,
     & 5h      ,
     & 9h      w  ,
     & 9h      d  ,
     & 9h      r  ,
     & 9h      s  ,
     & 9h      g  ,
```

```
     & 9h      f  ,
     & 9h      h  ,
     & 9h       e /)
 19  format(1h0,i5,9x,f9.2,f9.3,f9.3,3f9.3,1pe9.1)
 20  format(1h0,i5,f9.3,9x,f9.3,f9.3,3f9.3,1pe9.1)
 21  format(1h0,i5,9x,9x,f9.3,f9.3,3f9.3,1pe9.1)
 22  format(1h0,i5,9x,9x,f9.3,9x,3f9.3,1pe9.1)
 23  format(1h0,i5,9x,9x,9x,9x,3f9.3,1pe9.1)
 24  format(1h0,i5,9x,9x,9x,f9.3,3f9.3,1pe9.1)
 25 format(1h0,5h    n,4x,9h  lw     ,9h  ld     ,9h lr      ,9h  ls
,9h2*lg-1   ,9h  lf     ,9h  lh     ,9h  le     )
 26  format(1h0,i5,i8,i9,i8,i10,i9,i10,i9,i9)

     end
```

Subroutine shapew

```
c     This subroutine computes the weights of a wave-shaping
c     filter (F)
c     This filter is capable of transforming a given input signal
c     into a desired output signal in an optimum error
c     distribution sense.
c
c
c     Control parameters
c        MXN.....Maximum number of subfilters to be used
c        MXLG....Maximum working space for G,C and F
c                   Warning - If MXLG is not large enough, the
c                             number of subfilters N will be
c                             will be limited and filter F will be
c                             truncated automatically.
c        BMIN....A minimum limit for the size of the subfiter weights
c                Normalized subfilter weights which are less than
c                BMIN will be dropped from the computations.
c        PRNT....If (PRNT.eq.1.0) print information on weights
c                If (PRNT.eq.0.0) no information is printed
c
c     Input data
c        r(i),i=1,lr  The autocorrelation function of the input
c                     signal
c        s(i),i=1,ls  The crosscorrelation function between the input
c                     and the desired signals
c
c
c     Output
c        N             Actual number of subfilters used
c        G(i),i=1,lg   Weights of the positive half of the symmetric
c                      filter including the centre point
c        F(i),i=1,lf   Weights of the wave-shaping filter


      subroutine shapew (mxn,bmin,mxlg,prnt,lr,r,ls,s,
     &a,b,c,lg,g,lf,f,n)

      dimension a(50000),b(50000),c(50000),f(50000),
     &g(50000),r(6000),s(6000)

      do 1 i=1,mxlg
      c(i)=0.0
      g(i)=0.0
    1 continue
      lsf=(lr+1)/2
      gn=abs(r(lsf))


c     compute the normalized weights of the first subfilter
c     the alternate signs of r ear changed by means of sn

      sn=1.0
      do 2 i=1,lsf
      a(i)=r(i+lsf-1)*sn/gn
      g(i)=a(i)
      b(i)=0.0
      sn=-sn
    2 continue
```

```
      lg=lsf
      gmin=bmin/10000.0
      j=1
      if (prnt.eq.1.0) write(6,18)

c     in the first half of the next do-loop, compute the non-zero
c     weights of the subfilters including their centre points.
c     in the last half of this do-loop, compute the positive half
c     of the symmetric filter G including its centre point.
c     the computer leaves this main do-loop if
c              N=MXN
c              the number of significant subfilter weights lsf=1
c              the working space given by MXLG is not large enough
c


      do 13 n=1,mxn
      if (prnt.eq.1.0) write(6,19) n,lsf,lg
      if (prnt.eq.1.0) write(6,20) (a(i),i=1,lsf)

c     subfilter b is computed from subfilter a in the next do-loop
c     normalized values of b are stored back into a

      sm=-1.0
      do 4 kk=1,lsf
      k=kk-1
      sa=0.0
      sn=sm
      do 3 ii=kk,lsf
      i=ii-1
      p=2.0
      if (i.eq.k) p=1.0
      ll=2*k-i+1
      if (ll.lt.1) ll=2-ll
      sn=-sn
      sa=sa+sn*p*a(ll)*a(ii)
    3 continue
      if (kk.eq.1) saa=sa
      if (n.eq.mxn.or.lsf.eq.1) go to 14
      sm=-sm
      b(kk)=sa*sm
    4 continue

      bn=abs(b(1))
      do 5 i=1,lsf
      a(i)=b(i)/bn
      b(i)=0.0
    5 continue



c     the next do-loop discards the negligible weights of b

      lsm=lsf
      do 6 i=1,lsf
      k=lsf-i+1
      if (abs(a(k)).ge.bmin) go to 7
      lsm=lsm-1
      if (lsm.eq.1) go to 7
      a(k)=0.0
```

```
    6   continue
    7   continue
        lsf=lsm
        lrm=2*lsf-1
c       the spacing between the non-zero subfilter weights is given
c       by j
c       the weights of G are transferrd to C in the next do-loop
        j=j*2
        lb=j*(lrm-1)+1
        lggb=(2*lg-1+lb)/2
        if (lggb .gt. mxlg) sa=saa
        if (lggb .gt. mxlg) go to 14
        do 8 i=1,lg
        c(i)=g(i)
        g(i)=0.0
    8   continue
        lg=lggb
        gn=gn*bn
        do 10 kk=1,lg
        k=kk-1
        sbc=0.0
        do 9 iii=1,lrm
        i=iii-lsm
c       filter G is updated in the next do-loop
c       jj manipulates the subscripts to take care of the zero weights
        jj=k-i*j+1
        ii=i+1
        if (jj.lt.1) jj=2-jj
        if (ii.lt.1) ii=2-ii
        if (jj.gt.lg) go to 9
        sbc=sbc+a(ii)*c(jj)
    9   continue
        g(kk)=sbc
   10   continue
        if (lsf.eq.1) gmin=bmin
c     the next loop discards the negligible weights of G
c     lg is also updated
        lgm=lg
        do 11 i=1,lg
        k=lg-i+1
        gkl=g(k)/g(l)
        if (abs(gkl).gt.gmin) go to 12
        lgm=lgm-l
        if (lgm.eq.l) go to 12
        g(k)=0.0
   11   continue
   12   continue
        lg=lgm
   13   continue
c       the next do-loop normalizes the values of G
   14   continue
```

```
      gn=sa*gn
      do 15 i=1,lg
      g(i)=g(i)/gn
   15 continue

c     the remaining cards compute F by convolving s with G

      lf=2*lg+ls-2
      if (lf.gt.mxlg) lf=mxlg
      if (lggb.gt.mxlg.and.prnt.eq.1.0) write(6,21) n,lf
      do 16 i=1,mxlg
      f(i)=0.0
   16 continue
      lm=1+(lf+ls+1)/2-ls
      do 17 k=1,lf
      do 17 j=1,ls
      m=lm+j-k
      if (m.le.0) m=2-m
      f(k)=f(k)+s(j)*g(m)
   17 continue

      return

   18 format(lh0,40hthe following subfilter tables
     & show only/lh ,56hthe non-zero weights of the
     & positive half of the filter ,27hincluding the
     & centre point.)

   19 format(lh0,9hsubfilter,i3,5x,16hno. of weights =,
     &i5,9x,4hlg =,i5)
  20 format(lh0,lhb,200lf12.6)

   21 format(lh0,50hwarning     warning     warning
     &     warning      /lh ,50h*****************
     &******************************/lh ,50hthere
     & is insufficient working space for g.       /
     &lh ,52hthe maximum number of subfilters which
     & can be used =,i5/lh ,50hif more subfilters
     & are needed, increase mxlg.     /lh ,50hfilter
     & f was truncated to the length lf where lf =,i5)

     end
```

Subroutine foldm

```
      subroutine foldm (la,a,lb,b,lc,c)


c     This subroutine  convolves A with B to get C.
c
c     Reference
c     Robinson,E.A. Multichannel time series analysis page 29


      dimension a(50000),b(60000),c(70000)

      lc=la+lb-1
      do 1 i=1,lc
      c(i)=0.0
    1 continue

      do 2 i=1,la
      do 2 j=1,lb
      k=i+j-1
    2 c(k)=c(k)+a(i)*b(j)

      return
      end
```

# APPENDIX 2

The following program computes the coefficients of a wave-shaping filter truncated by the alternate truncation technique. The program also computes the coefficients of the subfilters, truncated symmetric filter, actual output and shaping errors.

It corresponds to a modified version of "MEREU 1" (Appendix 1).

Main Program

```
c      Program :  MEREU2

c      Input quatities for MEREU2
c            NEXP.....example number
c            MXN......maximum number of subfilters to be used
c            BMIN.....a minimum limit for size of subfilters weights
c            MXLG.....maximum number of weights in filter F.
c                     MXLG also corresponds to the truncation distance
c                     for the symmetric filter G
c            PRNT.....diagnostic print option
c            WEIGHT...ratio of white noise energy to signal energy
c            NPT......number of central points of F and H to be shown
c
c            W(i),i=1,lw...given wavelet
c            D(i),i=1,ld...desired wavelet
c
c
c      Quantities computed by progrm MEREU1 and subroutine FOLDM
c            WR(i),i=1,lw...wavelet W reversed in time
c            r(i),i=1,lr....the autocorrelation function of W
c            s(i),i=1,ls....the crosscorrelation function of W
c                           with D
c            H(i),i=1,lh....output of W convolved with F
c            E(i),i=1,le....shaping errors = difference between H
c                           and D
c
c
c      Quantities computed by subroutine SHAPEW
c            G(i),i=1,lg....the positive half of the symmetric filter
c            F(i),i=1,lf....the wave-shaping filter
c            N..............actual number of subfilters used

       open(10,form="formatted",file="autocl")
       open(11,form="formatted",file="ccorlhl")
       open(80,form="formatted",mode="out",file="ff5_2")
       open(81,form="formatted",mode="out",file="hh5_2")
       open(82,form="formatted",mode="out",file="ee5_2")


       dimension a(300),b(300),c(10000),d(52),f(10000),
      & g(5000),h(5000),r(300),s(300),w(300),wr(300)
       real autocl(60),tot_corrl(60),ccorlhl(50)
       real large1,large2,nt_c(100),nautocl(100)
       equivalence (c(1),f(1))

       do 60 i=1,51
       read(10,40) autocl(i)
 40    format(f10.4)
 60    continue

       do 61 i=1,51
       read(11,81) ccorlhl(i)
 81    format(f10.4)
 61    continue

       close(10)
       close(11)

```

```
      do 601 i=1,51
      tot_corrl(i)=autocl(i)+ccorlhl(i)
 601  continue

      large2=abs(autocl(1))
      do 73 i=2,51
      if(abs(autocl(i)).gt.large2) large2=abs(autocl(i))
  73  continue

      do 74 i=1,51
      nautocl(i)=autocl(i)/large2
  74  continue

      largel=abs(tot_corrl(1))
      do 602 i=2,51
      if(abs(tot_corrl(i)) .gt. largel) largel=abs(tot_corrl(i))
 602  continue

      do 66 i=1,51
      nt_c(i)=tot_corrl(i)/largel
  66  continue

      do 29 i=1,51
      w(i)=nt_c(i)
  29  continue

      do 600 i=1,51
      d(i)=nautocl(i)
  600 continue

      nexp=1
      mxn=20
      bmin=10.0**(-10)
      mxlg=501
      weight=0.0
      npt=25
      prnt=1.0
      lw=51
      ld=51

  1   continue
      if (nexp.eq.0) go to 4

c      the central points of the time series are the (time=0) points.
c      in order that W and D have a central point, lw and ld should
c      be odd numbers
c      the next 4 cards are inserted to make sure ld and lw are odd

      w(lw+1)=0.0
      d(ld+1)=0.0
      lw=(lw/2)*2+1
      ld=(ld/2)*2+1

c     create the wavelet W reversed in time

      do 2 i=1,lw
      k=lw-i+1
      wr(k)=w(i)
  2   continue
      if (weight.gt.0.0) write(6,10) weight
```

```
      write(6,17) mxn,bmin,mxlg

      call foldm (lw,w,lw,wr,lr,r)

      call foldm (lw,wr,ld,d,ls,s)

      call shapew (mxn,bmin,mxlg,prnt,lr,r,ls,s,a,b,c,lg,g,lf,f,n)

      call foldm (lw,w,lf,f,lh,h)



      if (weight.gt.0.0) write(6,10) weight
      write(6,17) mxn,bmin,mxlg
      write(6,18) npt

      lgg=2*lg-1
      le=lh
      im=(npt+1)/2
      it=-im
      do 3 i=1,npt
      it=it+1
      iw=(lw-npt)/2+i
      id=(ld-npt)/2+i
      ir=(lr-npt)/2+i
      is=(ls-npt)/2+i
      if (it.le.0) ig=im+1-i
      if (it.gt.0) ig=ig+1
      if=(lf-npt)/2+i
      ih=(lh-npt)/2+i
      ww=0.0
      dd=0.0
      rr=0.0
      ss=0.0
      gg=0.0
      ff=0.0
      hh=0.0
      ee=0.0
      mw=0
      md=0
      mr=0
      ms=0
      if (iw.gt.0.and.iw.le.lw) ww=w(iw)
      if (iw.gt.0.and.iw.le.lw) mw=1
      if (id.gt.0.and.id.le.ld) dd=d(id)
      if (id.gt.0.and.id.le.ld) md=1
      if (ir.gt.0.and.ir.le.lr) rr=r(ir)
      if (ir.gt.0.and.ir.le.lr) mr=1
      if (is.gt.0.and.is.le.ls) ss=s(is)
      if (is.gt.0.and.is.le.ls) ms=1
      if (ig.gt.0.and.ig.le.lg) gg=g(ig)
      if (if.gt.0.and.if.le.lf) ff=f(if)
      if (ih.gt.0.and.ih.le.lh) hh=h(ih)
      ee=hh-dd
      if (mw.eq.1.and.md.eq.1.and.mr.eq.1.and.ms.eq.1)
     & write(6,5) it,ww,dd,rr,ss,gg,ff,hh,ee
      if (mw.eq.0.and.md.eq.1.and.mr.ge.0.and.ms.eq.1)
     & write (6,19) it,dd,rr,ss,gg,ff,hh,ee
      if (mw.eq.1.and.md.eq.0.and.mr.eq.1.and.ms.eq.1)
```

```
     & write(6,20) it,ww,rr,ss,gg,ff,hh,ee
      if (mw.eq.0.and.md.eq.0.and.mr.eq.1.and.ms.eq.1)
     & write(6,21) it,rr,ss,gg,ff,hh,ee
      if (mw.eq.0.and.md.eq.0.and.mr.eq.1.and.ms.eq.0)
     & write(6,22) it,rr,gg,ff,hh,ee
      if (mw.eq.0.and.md.eq.0.and.mr.eq.0.and.ms.eq.0)
     & write(6,23) it,gg,ff,hh,ee
      if (mw.eq.0.and.md.eq.0.and.mr.eq.0.and.ms.eq.1)
     & write(6,24) it,ss,gg,ff,hh,ee


      write(80,111) ff
  111 format(f10.4)

      write(81,112) hh
  112 format(f10.4)

      write(82,113)  ee
 113  format(1pe9.1)

  3   continue
      write(6,25)
      write(6,26) n,lw,ld,lr,ls,lgg,lf,lh,le


      nexp=nexp-1
      go to 1
  4   continue
      close(80)
      close(81)
      close(82)

  5   format(1h0,i5,f9.3,f9.2,f9.3,f9.3,3f9.3,1pe9.1)
 10   format(1h0,37hwhite noise is assumed to be present.
     &,1h ,46hthe ratio of noise energy to signal energy is ,
     &f10.5)
 17   format(1h0,47hmaximum number of filters to be used     mxn =,
     & i5/1h,51hminimum limit set for sub-filter weights    bmin =,
     & 1pe9.2/1h,49hmaximum working space for  g,c,and f      mxlg =,
     & i5)
 18   format(1h0, 15x, i5, 3x,
     & 47hcentral points of the input and the output data//1h ,
     & 5h time,9h   given ,9h  desired,
     & 18h    correlation    ,
     & 9hsymmetric,
     & 9h  shaping,
     & 9h   actual,
     & 9h  shaping/1h ,
     & 5x,
     & 9h  wavelet,
     & 9h  wavelet,
     & 18h     functions    ,
     & 9h filter  ,
     & 9h   filter,
     & 9h   output,
     & 9h   errors//1h ,
     & 5h     ,
     & 9h     w  ,
     & 9h     d  ,
     & 9h     r  ,
```

```
      & 9h      s   ,
      & 9h      g   ,
      & 9h      f   ,
      & 9h      h   ,
      & 9h       e  /)
   19  format(1h0,i5,9x,f9.2,f9.3,f9.3,3f9.3,1pe9.1)
   20  format(1h0,i5,f9.3,9x,f9.3,f9.3,3f9.3,1pe9.1)
   21  format(1h0,i5,9x,9x,f9.3,f9.3,3f9.3,1pe9.1)
   22  format(1h0,i5,9x,9x,f9.3,9x,3f9.3,1pe9.1)
   23  format(1h0,i5,9x,9x,9x,9x,3f9.3,1pe9.1)
   24  format(1h0,i5,9x,9x,9x,f9.3,3f9.3,1pe9.1)
  25  format(1h0,5h    n,4x,9h  lw     ,9h  ld     ,9h lr      ,9h  ls
,9h2*lg-1   ,9h  lf     ,9h  lh     ,9h  le     )
   26  format(1h0,i5,i8,i9,i8,i10,i9,i10;i9,i9)

       end
```

Subroutine shapew

```
c     This subroutine computes the weights of a wave-shaping
c     filter (F)
c     This filter is truncated when the number of weights in
c     half the symmetric filter(including the central point)
c     becomes larger than MXLG.In such a case, the symmetric
c     filter is truncated to MXLG and the filter F is also
c     truncated to MXLG.
c
c
c     Control parameters
c        MXN.....Maximum number of subfilters to be used
c        MXLG....Maximum number of weights in filter F.
c                MXLG also corresponds to the truncation
c                distance
c        BMIN....A minimum limit for the size of the subfiter weights
c                Normalized subfilter weights which are less than
c                BMIN will be dropped from the computations.
c        PRNT....If (PRNT.eq.1.0) print information on weights
c                If (PRNT.eq.0.0) no information is printed
c
c     Input data
c        r(i),i=1,lr  The autocorrelation function of the input
c                     signal
c        s(i),i=1,ls  The crosscorrelation function between the input
c                     and the desired signals
c
c
c     Output
c        N             Actual number of subfilters used
c        G(i),i=1,lg   Weights of the positive half of the symmetric
c                      filter including the centre point
c        F(i),i=1,lf   Weights of the wave-shaping filter


      subroutine shapew (mxn,bmin,mxlg,prnt,lr,r,ls,s,
     &a,b,c,lg,g,lf,f,n)

      dimension a(50000),b(50000),c(50000),f(50000),
     &g(50000),r(6000),s(6000)

      do 1 i=1,mxlg
      c(i)=0.0
      g(i)=0.0
    1 continue
      lsf=(lr+1)/2
      gn=abs(r(lsf))


c     compute the normalized weights of the first subfilter
c     the alternate signs of r ear changed by means of sn

      sn=1.0
      do 2 i=1,lsf
      a(i)=r(i+lsf-1)*sn/gn
      g(i)=a(i)
      b(i)=0.0
      sn=-sn
    2 continue
```

```
      lg=lsf
      gmin=bmin/10000.0
      j=1
      if (prnt.eq.1.0) write(6,18)

c     in the first half of the next do-loop, compute the non-zero
c     weights of the subfilters including their centre points.
c     in the last half of this do-loop, compute the positive half
c     of the symmetric filter G including its centre point.
c     the computer leaves this main do-loop if
c              N=MXN
c              the number of significant subfilter weights lsf=1
c              the working space given by MXLG is not large enough
c


      do 13 n=1,mxn
      if (prnt.eq.1.0) write(6,19) n,lsf,lg
      if (prnt.eq.1.0) write(6,20) (a(i),i=1,lsf)

c     subfilter b is computed from subfilter a in the next do-loop
c     normalized values of b are stored back into a

      sm=-1.0
      do 4 kk=1,lsf
      k=kk-1
      sa=0.0
      sn=sm
      do 3 ii=kk,lsf
      i=ii-1
      p=2.0
      if (i.eq.k) p=1.0
      ll=2*k-i+1
      if (ll.lt.1) ll=2-ll
      sn=-sn
      sa=sa+sn*p*a(ll)*a(ii)
    3 continue
      if (kk.eq.1) saa=sa
      if (n.eq.mxn.or.lsf.eq.1) go to 14
      sm=-sm
      b(kk)=sa*sm
    4 continue

      bn=abs(b(1))
      do 5 i=1,lsf
      a(i)=b(i)/bn
      b(i)=0.0
    5 continue



c     the next do-loop discards the negligible weights of b

      lsm=lsf
      do 6 i=1,lsf
      k=lsf-i+1
      if (abs(a(k)).ge.bmin) go to 7
      lsm=lsm-1
      if (lsm.eq.1) go to 7
      a(k)=0.0
```

```
    6   continue
    7   continue
        lsf=lsm
        lrm=2*lsf-1
c       the spacing between the non-zero subfilter weights is given
c       by j
c       the weights of G are transferrd to C in the next do-loop
        j=j*2
        lb=j*(lrm-1)+1
        lggb=(2*lg-1+lb)/2
        if (lggb .gt. mxlg) sa=saa
        if (lggb .gt. mxlg) lggb=mxlg
        if (lggb .gt. mxlg) go to 14
        do 8 i=1,lg
        c(i)=g(i)
        g(i)=0.0
    8   continue
        lg=lggb
        gn=gn*bn
        do 10 kk=1,lg
        k=kk-1
        sbc=0.0
        do 9 iii=1,lrm
        i=iii-lsm
c       filter G is updated in the next do-loop
c       jj manipulates the subscripts to take care of the zero weights
        jj=k-i*j+1
        ii=i+1
        if (jj.lt.1) jj=2-jj
        if (ii.lt.1) ii=2-ii
        if (jj.gt.lg) go to 9
        sbc=sbc+a(ii)*c(jj)
    9   continue
        g(kk)=sbc
   10   continue
        if (lsf.eq.1) gmin=bmin
c      the next loop discards the negligible weights of G
c      lg is also updated
        lgm=lg
        do 11 i=1,lg
        k=lg-i+1
        gkl=g(k)/g(1)
        if (abs(gkl).gt.gmin) go to 12
        lgm=lgm-1
        if (lgm.eq.1) go to 12
        g(k)=0.0
   11   continue
   12   continue
        lg=lgm
   13   continue
c       the next do-loop normalizes the values of G
```

```
 14  continue
     gn=sa*gn
     do 15 i=1,lg
     g(i)=g(i)/gn
 15  continue

c    the remaining cards compute F by convolving s with G

     lf=2*lg+ls-2
     if (lf.gt.mxlg) lf=mxlg
     if (lggb.gt.mxlg.and.prnt.eq.1.0) write(6,21) n,lf
     do 16 i=1,mxlg
     f(i)=0.0
 16  continue
     lm=1+(lf+ls+1)/2-ls
     do 17 k=1,lf
     do 17 j=1,ls
     m=lm+j-k
     if (m.le.0) m=2-m
     f(k)=f(k)+s(j)*g(m)
 17  continue

     return

 18  format(1h0,40hthe following subfilter tables
    & show only/1h ,56hthe non-zero weights of the
    & positive half of the filter ,27hincluding the
    & centre point.)

 19  format(1h0,9hsubfilter,i3,5x,16hno. of weights =,
    &i5,9x,4hlg =,i5)
20  format(1h0,1hb,2001f12.6)

 21  format(1h0,50hwarning     warning    warning
    &     warning       /1h ,50h*****************
    &*********************************/1h ,50hthere
    & is insufficient working space for g.       /
    &1h ,52hthe maximum number of subfilters which
    & can be used =,i5/1h ,50hif more subfilters
    & are needed, increase mxlg.     /1h ,50hfilter
    & f was truncated to the length lf where lf =,i5)

    end
```

Subroutine foldm

```
      subroutine foldm (la,a,lb,b,lc,c)


c     This subroutine  convolves A with B to get C.
c
c     Reference
c     Robinson,E.A. Multichannel time series analysis page 29


      dimension a(50000),b(60000);c(70000)

      lc=la+lb-1
      do 1 i=1,lc
      c(i)=0.0
    1 continue

      do 2 i=1,la
      do 2 j=1,lb
      k=i+j-1
    2 c(k)=c(k)+a(i)*b(j)

      return
      end
```