

*The Paradox of Knowability and
Semantic Anti-Realism*

Julianne Chung

B.A. Honours Thesis

Supervisor:
Richard Zach

Department of Philosophy
University of Calgary

2007

UNIVERSITY OF CALGARY

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INTRODUCTION

The paradox of knowability is a paradox deriving from the work of Frederic Fitch in his 1963 paper, “A Logical Analysis of Some Value Concepts”. The paradox arises from the principle of knowability, which holds that all truths are knowable, and the claim that we are non-omniscient, which holds that there is at least one truth that is not known. The paradox occurs because one can use standard procedures of inference to show that these claims are inconsistent with each other. So, if all truths are knowable, then all truths are known. Given that the claim that “all truths are known” seems unacceptable, the paradox is traditionally viewed as endangering theories of truth or knowledge that rely on the claim that all truths are knowable. Such theories include verificationist or anti-realist theories of truth, which hold that a proposition is true only if it is provable.¹ An instance of such a theory is the theory of semantic anti-realism. Proponents of semantic anti-realism include prominent philosophers such as Michael Dummett, Crispin Wright, and Neil Tennant, to name a few.

This paper will be concerned with examining the paradox and its threat to semantic anti-realism in three chapters. In chapter one; I discuss the origins of the paradox in the work of Frederic Fitch before presenting two other proofs of the paradox. In chapter two; I explain the theory of semantic anti-realism and address the question of why Fitch’s result came to be considered paradoxical in nature. In chapter three; I survey four of the most compelling solutions that have been proposed to dissolve the paradox and the potential problems associated with each. Following that, I briefly comment on the solution that I find to be most palatable for one endorses semantic anti-realism.

¹ In this context, “proved” is to be understood as roughly meaning “verified to be true”.

CHAPTER 1

THE PARADOX OF KNOWABILITY

In this chapter, I first discuss the origins of the paradox of knowability in the work of Frederic Fitch before proceeding to prove the paradox independently of Fitch's theorems. Two proofs will be presented: one which is simpler, so that the reader can easily see how and why the paradox results, and one which is more complex, so as to assure the reader that the paradox is not a result of fallacious reasoning.

Fitch and Knowability

In his 1963 paper, “A Logical Analysis of Some Value Concepts,” Frederic Fitch states that his purpose, in that paper, is to provide a logical analysis of several concepts that may be classified as what he terms “value concepts,” or as concepts closely related to value concepts.² Among these concepts is the concept of “knowing”, which will be focused on here. Fitch claims that, just as the concepts of “necessity” and “possibility” as used in ordinary language correspond in some degree to the concepts of “necessity” and “possibility” as used in modal logic, so too may the ordinary concept of “knowing” correspond in some degree to a proper formalization of the concept.³ He states that we assume that “knowing” has some reasonably simple properties that can be described as follows (though he notes that he will leave the question open as to any further properties it has in addition):

- (i) “Knowing” is a *two-placed relation* between an agent and a proposition.
- (ii) “Knowing” is closed with respect to *conjunction elimination*, which is to say that, for

² Fitch, F. “A Logical Analysis of Some Value Concepts,” (*The Journal of Symbolic Logic* 28, 1963), 135

³ *Ibid.*, 135

any p and any q , necessarily, if an agent knows that p and q , then that agent knows that p and that agent knows that q .

(iii) “Knowing” can reasonably be assumed to denote a *truth class*, as it is the case that, for any p , necessarily, if an agent knows that p then p is true.⁴

Fitch then presents two theorems about truth classes that he will later apply to the concept of knowing in theorems that he presents later in the paper:

THEOREM 1. If α is a truth class which is closed with respect to conjunction elimination, then the proposition $(p \wedge \neg\alpha p)$, which asserts that p is true but not a member of α (where p is any proposition) is itself necessarily not a member of α .

Proof. Suppose that $(p \wedge \neg\alpha p)$ is a member of α ; that is, $\alpha(p \wedge \neg\alpha p)$. Since α is closed with respect to conjunction elimination, one can thus derive $(\alpha p \wedge \alpha \neg\alpha p)$. Since α is a truth class, and $\neg\alpha p$ is a member, we can infer that $\neg\alpha p$ is true. But this contradicts the result that αp is true. So the assumption, $\alpha(p \wedge \neg\alpha p)$ is necessarily false.⁵

THEOREM 2. If α is a truth class which is closed with respect to conjunction elimination, and if p is any true proposition which is not a member of α , then the proposition $(p \wedge \neg\alpha p)$ is a true proposition which is necessarily not a member of α .

Proof. The proposition $(p \wedge \neg\alpha p)$ is clearly true, and by Theorem 1 it is necessarily not a member of α .⁶

For the purposes of this paper, the next of Fitch’s theorems that will be presented is:

THEOREM 5. If there is some true proposition which nobody knows (or has known or

⁴ Ibid., 138

⁵ Ibid., 138

⁶ Ibid., 138

will know) to be true, then there is a true proposition which nobody can know to be true.⁷

Fitch perhaps did not consider this theorem to be of great importance, for his proof of Theorem 5 is a simple note, "similar to proof of Theorem 4". Though Theorem 4 does not need to be listed here, as it is irrelevant to the purposes of this paper, I will prove Theorem 5 in a similar fashion to the way Fitch proves Theorem 4.

Proof. Suppose that p is true but is not known by any agent at any time. Using the operator 'K' for 'is known that' (by someone at some time), we can state the supposition as $(p \wedge \neg Kp)$. However, since knowing is a truth class closed with respect to conjunction elimination, we can conclude from Theorem 2 that it cannot be the case that $K(p \wedge \neg Kp)$. But we assumed that $(p \wedge \neg Kp)$ is true. So there is a true proposition that nobody can know to be true, given the assumption.

Despite the fact that Theorem 5 directly contradicts the principle of knowability; that is, that all truths are knowable, Fitch himself does not seem to have been aware of this implication. This is not overwhelmingly surprising, given that his paper was not directed at refuting verificationist theories, or indeed any theory at all; rather, it was only intended as an investigation of the logical attributes of a variety of concepts. What *is* perhaps surprising is that no one seems to have realized that Fitch's theorems had potential implications for theories that rely on the principle of knowability until the 1970s, with the work of W.D. Hart in his 1979 paper "The Epistemology of Abstract Objects: Access and Inference".⁸ Since then, much work has been aimed at treating Fitch's result as a paradox; for many find Fitch's result surprising because it has the consequence that, if all truths are knowable, then all truths are known. (This consequence

⁷ Ibid., 139

⁸ Hart, W.D. "The Epistemology of Abstract Objects: Access and Inference" in *The Aristotelian Society Supplementary Volume LIII* (1979), 156

will become clearer in the next section of this chapter.) The paradox has come to be known as “Fitch’s paradox” or “the paradox of knowability”.

The Paradox of Knowability

It should be noted that it can be shown independently of Fitch's theorems that the claim that "all truths are knowable" and the claim that "there is at least one truth that is not known by anyone at anytime" are inconsistent with each other. The proof that I will show first is a proof adapted from the work of Berit Brogaard and Joe Salerno, as it is the most straightforward that I have encountered. The proof should help to clarify how it is that possible knowledge, as a characterization of truth, collapses into actual knowledge so easily.

Brogaard and Salerno's Proof⁹

In this proof, let “K” be the epistemic operator, “it is known by someone at some time that,” let “ \diamond ” be the modal operator, “it is possible that,” and let “ \square ” be the modal operator, “it is necessary that”.

Assume:

a) The Principle of Knowability, that is, the claim that all truths are knowable by someone at some time:

$$(KP) \forall p(p \rightarrow \diamond Kp)$$

and

b) That we are Non-Omniscient; that is, the claim that there is a truth that is not known by anyone at any time:

⁹ Brogaard, Berit and Salerno, Joe. “Fitch’s Paradox of Knowability,” in the *Stanford Encyclopedia of Philosophy*, (<http://plato.stanford.edu/entries/fitch-paradox/>), 2-3

(Non-O) $\exists p(p \wedge \neg Kp)$

If this existential is true, then so is an instance of it:

1. $q \wedge \neg Kq$

Now consider the instance of assumption a), the Principle of Knowability (KP); substituting line 1 for the variable p in (KP):

2. $(q \wedge \neg Kq) \rightarrow \Diamond K(q \wedge \neg Kq)$

It follows trivially (by *modus ponens*) that it is possible to know the conjunction expressed at line 1. Therefore:

3. $\Diamond K(q \wedge \neg Kq)$

The problem is that it can be shown independently that it is impossible to know this conjunction: line 3 is false. The independent result presupposes two epistemic inferences which are fairly uncontroversial:

1) A conjunction is known only if the conjuncts are known; that is, the knowledge is closed with respect to conjunction elimination (K-Dist)

c) $K(p \wedge s) \rightarrow (Kp \wedge Ks)$

and

2) A statement is known only if it is true; that is, knowledge implies truth (KIT)

d) $Kp \rightarrow p$

Also presupposed is the validity of two fairly uncontroversial modal inferences:

1) All theorems are necessarily true; that is, the Rule of Necessitation (RN):

e) 'if $\vdash p$ then $\vdash \Box p$

and

2) If it is necessary that not- p , then it is impossible that p ; that is, the definition of the \Box

operator in modal logic (Dual):

$$f) \Box \neg p = \neg \Diamond p$$

So, according to Brogaard and Salerno, the independent result proceeds as follows:

- | | |
|--|--|
| 4. $K(q \wedge \neg Kq)$ | Assumption for <i>reductio</i> |
| 5. $Kq \wedge K\neg Kq$ | From 4, by c) K-Dist |
| 6. $Kq \wedge \neg Kq$ | From 5, applying d) KIT to the right conjunct |
| 7. $\neg K(q \wedge \neg Kq)$ | From 4-6 by <i>reductio</i> , discharging assumption 4 |
| 8. $\Box \neg K(q \wedge \neg Kq)$ | From 7, by e) RN |
| 9. $\neg \Diamond K(q \wedge \neg Kq)$ | From 8, by f) Dual |

Since line 9 contradicts line 3, a contradiction follows from the principle of knowability and the claim that we are non-omniscient; thus, these two claims are inconsistent with each other. So, according to Brogaard and Salerno, an advocate of the view that all truths are knowable must deny that we are non-omniscient.

$$10. \neg \exists p(p \wedge \neg Kp)$$

However, it follows from this that all truths are actually known (by someone at some time):

$$11. \forall p(p \rightarrow Kp)$$

Hence, if all truths are knowable, then all truths are known. For any supporter of the principle of knowability, this is an obviously unacceptable conclusion.

To some it might seem as if this proof is potentially fallacious because it oversimplifies; for instance, there are two existential quantifiers embedded in the K-operator. What happens if the assumptions are spelled out more precisely? As Jonathan

Kvanvig has shown, the paradox still results fairly easily, even if the proof is made more complex.

*Kvanvig's Proof*¹⁰

Kvanvig's proof makes use of first-order quantifiers, \diamond , \square , a (one-place) truth predicate T, and a three-place relation K (where $KxTy$ is read 'x knows that y is true at time t'). Like Brogaard and Salerno, Kvanvig also makes use of the rules K-Dist, KIT, RN, and Dual (rules (c-f) above). His proof also proceeds similarly to theirs.

Assume:

a) The Principle of Knowability, that is, the claim that all truths are knowable by someone at some time:

$$(KP) \forall p(Tp \rightarrow \diamond \exists x \exists t KxTpt)$$

and

b) That we are Non-Omniscient; that is, the claim that there is a truth that is not known by anyone at any time:

$$(Non-O) \exists p(Tp \wedge \neg \exists y \exists s KyTps)$$

If this existential is true, then so is an instance of it :

$$1) Tq \wedge \neg \exists y \exists s KyTqs$$

Now consider the instance of assumption a), the Principle of Knowability (KP); substituting line 1 for the variable p in (KP):

$$2) Tq \wedge \neg \exists y \exists s KyTqs \rightarrow \diamond \exists x \exists t Kx(Tq \wedge \neg \exists y \exists s KyTqs)t$$

By *modus ponens*, we get:

$$3) \diamond \exists x \exists t Kx(Tq \wedge \neg \exists y \exists s KyTqs)t$$

¹⁰ Kvanvig, J. *The Knowability Paradox*. (Oxford University Press: 2006), 12-13

Assume:

- 4) $\exists x \exists t Kx(Tq \wedge \neg \exists y \exists s KyTqs)t$
- 5) $\exists x \exists t KxTqt \wedge \exists x \exists t Kx \neg \exists y \exists s KyTqs$ From 4, by K-Dist
- 6) $\exists x \exists t KxTqt \wedge \neg \exists y \exists s KyTqs$ From 5, by KIT
- 7) $\exists x \exists t KxTqt \wedge \neg \exists x \exists t KxTqt$ From 6, by First-Order Logic
- 8) $\neg \exists x \exists t Kx(Tq \wedge \neg \exists y \exists s KyTqs)t$ From 4-7, by *reductio*, discharging
assumption 4
- 9) $\Box \neg \exists x \exists t Kx(Tq \wedge \neg \exists y \exists s KyTqs)t$ From 8, by RN
- 10) $\neg \Diamond \exists x \exists t Kx(Tq \wedge \neg \exists y \exists s KyTqs)t$ From 9, by Dual

Since line 10 is the denial of line 3, once again, any defender of the principle of knowability is forced to admit that all truths are known by someone at some time.

CHAPTER 2**INTUITIONISTIC LOGIC, SEMANTIC ANTI-REALISM,
AND THE PARADOX**

At this point, one might be inclined to wonder why the result that the principle of knowability and the claim that we are non-omniscient are inconsistent with each other even qualifies as a paradox. A natural reaction, upon seeing the proofs, is to conclude that the principle of knowability is unsound and should simply be jettisoned; the thought being that there was perhaps little reason to think it true in the first place.¹¹

The problem with this, however, is that a number of prominent, plausible philosophical positions rely on the principle of knowability. Recently, it has been suggested that quite a wide variety of theories, from areas of philosophy as diverse as the philosophy of religion and the philosophy of science, are at least *tacitly* committed to the claim that all truths are knowable, and are thus threatened by Fitch's result.¹² Traditionally, however, Fitch's result was thought to only endanger anti-realist or verificationist theories of truth or meaning that *explicitly* rely on the principle of knowability.¹³ Perhaps the most well-known and important of such theories is semantic anti-realism, which has its origins in intuitionist mathematics and logic and first came onto the scene via the work of Michael Dummett. For the purposes of this paper, I have chosen to focus my discussion on semantic anti-realism in order to illustrate how it is that Fitch's result first came to be treated as paradoxical in nature.

In this chapter, I first discuss the origins of semantic anti-realism in intuitionistic

¹¹ Kvanvig, J. *The Knowability Paradox*. (Oxford University Press: 2006), 35

¹² *Ibid.*, 35

¹³ *Ibid.*, 2

logic before describing the theory of semantic anti-realism itself. A brief explanation of why Fitch's result came to be treated as a paradox, both by philosophers who endorse semantic anti-realism and philosophers who do not, will follow.

Intuitionistic Logic

In the following section, I present and discuss several of the main features of intuitionistic logic that differentiate it from classical logic. This, of course, is not intended to be a complete or comprehensive description of intuitionistic logic; rather, it is intended to simply convey its major tenets so that the uninitiated reader can better understand Michael Dummett's theory of semantic anti-realism and, later in the paper, Timothy Williamson's solution to the paradox.

Intuitionistic logic has its roots in the intuitionistic mathematics of L.E.J. Brouwer and was itself developed from Brouwer's work by A. Heyting. According to Heyting, the central philosophical claim of mathematical intuitionism is that mathematics has no unprovable truths; that is, to be true is to be provable.¹⁴ To put it another way, the idea is that, in mathematics, a proposition P is true only if it is provable. Intuitionistic logic is the result of applying this principle to the semantics of the logical connectives and quantifiers.¹⁵ It is also worth noting at this point that the notion of "truth in a model" as used in classical logic is replaced by the notion of "proof in an epistemic situation" or "assertability" in intuitionistic logic. This notion provides the philosophical basis for

¹⁴ Posy, C. "Intuitionism and Philosophy," in *The Oxford Handbook of Philosophy of Mathematics and Logic*. (Oxford University Press: 2005), 340

¹⁵ Dummett, M. *Elements of Intuitionism*. (Oxford University Press: 1977), 7. Dummett explains the need for this, noting that the classical mathematician claims that the objects of mathematics exist independently of human thought, whereas the intuitionist claims that mathematical objects are mental constructions that exist only in virtue of our mathematical activity, which consists in mental operations, and thus can have only those properties which they can be recognized by us as having. Thus the intuitionist reconstruction of mathematics has to question even the sentential logic employed in classical reasoning, as the two sides operate on two radically different conceptions of truth.

intuitionistic logic. Thus the conditions under which evidence in a particular (epistemic) situation will count as a proof of a proposition P are set out as follows:¹⁶

$P = (Q \wedge R)$ is proved in an epistemic situation iff the situation proves Q and R

$P = (Q \vee R)$ is proved in an epistemic situation iff either Q is proved or R is proved

$P = (Q \rightarrow R)$ is proved in an epistemic situation iff the situation contains a method for converting a proof of Q into a proof of R

$P = \neg Q$ is proved in an epistemic situation iff it is proved that Q can never be proved, which is to say that a proof of Q could be turned into a proof of a contradiction.¹⁷ This is also to say that it is impossible to prove that Q .

$P = \exists(x)Q(x)$ is proved in an epistemic situation iff $Q(t)$ is proved for some t

$P = \forall(x)Q(x)$ is proved in an epistemic situation iff the situation contains a method for converting any proof that a given object t is in the domain of discourse into a proof of $Q(t)$

It should be fairly clear that the interpretation of the logical particles in intuitionistic logic diverges sharply from their interpretation in classical logic. Given this alternate interpretation of the logical connectives and quantifiers, one can also see why some of the standard procedures of inference used in classical logic do not hold in intuitionistic logic. For instance, double-negation elimination is not allowed, since $\neg\neg P$, in intuitionistic logic should be read as saying something like "it can never be proved that P will never be proved" which does not amount to a proof of P itself. The law

¹⁶ Ibid., 340

¹⁷ Dummett, M. *Elements of Intuitionism*. (Oxford University Press: 1977), 13. Dummett also explains here why this is not just defining ' \neg ' in terms of itself; either a "contradiction" could be some other absurd statement, such as " $0=1$ ", so a proof of ' $\neg P$ ' could just be a proof that " $P \rightarrow 0=1$ "; or, ' \neg ' could be interpreted differently when applied to atomic statements.

of the excluded middle will fail, for, understood intuitionistically, $(P \vee \neg P)$ should be read as saying something like “either P or $\neg P$ is proved in an epistemic situation”.

However, since there are undoubtedly propositions for which, in some epistemic situation (i.e. the present one, for instance) there is no evidence that they will ever be decided, the law of the excluded middle does not always hold.¹⁸

Another important feature of intuitionistic logic that distinguishes it from classical logic is that it relies on a constructivist notion of proof. The distinction between constructive and non-constructive proofs is fully intelligible even from the perspective of classical mathematics. The distinction arises for proofs of existential and disjunctive statements. Any proof of such statements proves something in addition to the theorem which is its conclusion. To call a proof “constructive” is to say something very specific about this additional information. In the case of proofs of existential statements, a proof is constructive if and only if it yields a proof of a specific instance of the existential claim or provides an effective means, at least in principle, of finding such an instance. In the case of proofs of disjunctive statements, a proof is constructive if and only if it yields a proof of at least one of the disjuncts or provides an effective means, at least in principle, of obtaining a proof of at least one of the disjuncts.¹⁹ One also cannot prove a claim by *reductio*; which is to say that one cannot prove P by assuming $\neg P$, deriving a contradiction, and thus concluding that P . *Reductio* is not a constructively admissible form of proof because it is not the case in intuitionistic logic that $\neg\neg P \rightarrow P$.²⁰

Finally, it should be noted that though Heyting indeed develops intuitionistic logic

¹⁸ Ibid., 26. Dummett provides additional examples on pp. 26-31.

¹⁹ Ibid., 9

²⁰ Moschovakis, J. "Intuitionistic Logic", in *The Stanford Encyclopedia of Philosophy (Spring 2007 Edition)*, Edward N. Zalta (ed.), URL = <<http://plato.stanford.edu/archives/spr2007/entries/logic-intuitionistic/>>.

based on Brouwer's work in intuitionist mathematics, he does not include Brouwer's metaphysical grounds for intuitionistic mathematics as part of his account. The intuitionistic interpretation of the logical particles says nothing about the "objects" of mathematics; Heyting considers the assumption that a theory of truth must be referential to be an assumption that is made by the *classical* mathematician, but need not and perhaps should not be made by the intuitionist. As he sees it, it is this assumption that forces the classical mathematician to posit a potentially undesirable Platonistic world of objects with undecidable properties in order to meet the demands of classical logic. Heyting insists that it is to the detriment of classical mathematics that it is metaphysically weighted in this manner; and claims that intuitionism, in contrast, is metaphysically neutral.²¹

Intuitionism and Semantic Anti-Realism

Semantic anti-realism can accurately be described as a species of intuitionism. Through the work of Michael Dummett, intuitionism came to be generalized such that it was taken to apply to all language in general, not just the language of mathematics. The language of mathematics only represented a single, special case. Semantic anti-realism, simply described, is the result of generalizing intuitionist semantics to apply to all language. Semantic-anti realism holds that truth, in general, is determined by humans and their actions, and thus cannot transcend our capacities for knowledge. Thus the central philosophical claim of semantic anti-realism is that a proposition is true only if it is *knowable*, a clear generalization of the philosophical claim of intuitionistic mathematics

²¹ Posy, C. "Intuitionism and Philosophy," in *The Oxford Handbook of Philosophy of Mathematics and Logic* (ed. Stewart Shapiro). (Oxford University Press: 2005), 341

that a proposition is true only if it is provable.²² The essential difference between the former claim and the latter is that the former seems to more firmly emphasize the notion that truth is wholly determined by the cognitive capacities of humans, as it could be argued that “proof” is a notion that is best suited to mathematical discourse, whereas “knowledge” can be applied more generally.

So why is the name of “semantic anti-realism” bestowed upon this generalization? “Semantic realism,” as described by Dummett, has its major tenet the view that truth can transcend our capacities for knowledge, whereas his “semantic anti-realism” has as its major tenet the view that truth is based solely on our capacities for knowledge, and thus cannot transcend them. Semantic realism, then, can roughly be characterized as realism about truth, whereas semantic anti-realism can be roughly characterized as anti-realism about truth.²³

There is one additional characteristic of Dummett's semantic anti-realism that should perhaps be noted. As Carl Posy puts it in his 2005 article, “Intuitionism and Philosophy,” Dummett's semantic anti-realism is, essentially, “Heyting’s anti-metaphysical bent, writ large”; that is, Dummett claims that traditional metaphysical disputes about reality and objects are best described as modern *semantic* disputes.²⁴ That the realism debate is properly conducted within the scope of the philosophy of language is probably the most contentious of Dummett's claims.²⁵

At this point, it seems appropriate to inquire as to what could possibly provide the

²² Ibid., 343

²³ Tennant, N. *The Taming of the True*. (Oxford University Press: 2002), 15

²⁴ Posy, C. "Intuitionism and Philosophy," in *The Oxford Handbook of Philosophy of Mathematics and Logic* (ed. Stewart Shapiro). (Oxford University Press: 2005), 343

²⁵ Wright, C. *Realism, Meaning, and Truth*. (Blackwell: 1987), 2-3; Tennant, N. *The Taming of the True*. (Oxford University Press: 2002), 23

motivation for adopting such a sweeping and radical generalization about language and its corresponding consequences for truth, meaning, and a number of other philosophical positions. For, even if one accepts mathematical intuitionism, or that intuitionistic logic is appropriate for mathematics, it is far from clear as to whether or not generalizing it to apply to *all* language can be justified. Dummett, and others that follow him, have a number of arguments designed to support their case. Addressing this issue, however, is regrettably beyond the scope of this paper. For the purposes of this paper, it should suffice to say that Dummett's arguments are generally thought to provide compelling reasons to at least entertain the idea that the dominant logic, classical logic, may be misled.²⁶

Fitch's Result: A Paradox

Dummett's semantic anti-realism is not a fringe position, and has been endorsed by many prominent philosophers, including Crispin Wright and Neil Tennant. Those who endorse semantic anti-realism have obvious reason to treat Fitch's result as being paradoxical in nature. However, it should be noted that many philosophers who do *not* endorse semantic anti-realism have also found Fitch's result far too surprising to simply accept without further investigation. Some have expressed disbelief that what seemed like an at least plausible philosophical position (i.e. semantic anti-realism) could be so easily felled by such a swift natural deduction proof.²⁷ Others have wondered how it is that possible knowledge, as a characterization of truth, should collapse into actual knowledge

²⁶ Crispin Wright and Neil Tennant have argued for this; indeed, so has Jonathan Kvanvig, though he objects to the prospect of intuitionistic logic as being the correct logic.

²⁷ Brogaard, Berit and Salerno, Joe. "Fitch's Paradox of Knowability," in the *Stanford Encyclopedia of Philosophy*, (<http://plato.stanford.edu/entries/fitch-paradox/>), 1

so easily.²⁸ Others still have expressed concern that the paradox potentially threatens a logical distinction between actual and possible knowledge.²⁹

Since the paradox of knowability has intrigued philosophers of various theoretical persuasions, a wide variety of solutions to the paradox have been posited. Four of the most compelling are discussed in the next chapter.

²⁸ *Ibid.*, 1

²⁹ Kvanvig, J. *The Knowability Paradox*. (Oxford University Press: 2006), 2.

CHAPTER 3

PROPOSED SOLUTIONS TO THE PARADOX OF KNOWABILITY

In this chapter, I survey four of the most important solutions to the paradox of knowability: J.L. Mackie's solution (1980), Timothy Williamson's solution (1982), Dorothy Edgington's solution (1985), and Michael Dummett's solution (2001). Though these solutions have been traditionally thought to be among the most compelling solutions to the paradox of knowability, as they manage to successfully block the paradox, there are numerous potential problems with each that have led others to continue to seek out new solutions. This chapter will proceed by explaining each solution, as well as discussing the potential problems associated with each, in the sequence outlined above.

Before I begin, it is perhaps worth pointing out that there are a number of types of ways in which one can formulate a solution to a paradox. The solutions that are surveyed here fall into at least one of the following solution types:

(1a) The paradox is solved by arguing that the result is valid, though admittedly initially surprising, because at least one of the assumptions is false and should be discarded

(1b) The paradox is solved by arguing that the result is valid but that one of the assumptions as initially construed is false and should be amended

or

(2) The paradox is solved by arguing that the result is invalid because the logic used to derive the paradox should be revised

For the sake of clarity, for each solution, it will be noted as to what solution type or types it falls under.

J.L. Mackie

J.L. Mackie, in his paper, "Truth and Knowability" (1980) was among the first to comment on Hart's claim that the reasoning employed by Fitch can be used to disprove verificationist theories.³⁰ At the outset of the paper, Mackie notes that, though Hart believed that Fitch's result was an "unjustly neglected logical gem," many other philosophers at the time were not convinced by Fitch's reasoning; rather, many claimed that his argument was instead either fallacious or a paradox.³¹

Mackie does not believe that any of the above claims have it quite right. That is, though he claims that Fitch's result successfully refutes the principle of knowability, he does not think that it must follow directly from Fitch's work that all forms of verificationism are thus refuted also. He does, however, think that verificationism can be disproved using reasoning analogous to the reasoning employed by Fitch.

Mackie's solution to the paradox consists in an explication of why the unexpected result, that the claim that all truths are knowable is inconsistent with the claim that some truths are never known, occurs. Thus Mackie's solution to the paradox falls under solution type (1a) as outlined above. It perhaps should be noted that this approach is quite different from most of the well-known solutions that follow his, including Edgington's, Williamson's, and Dummett's, which either attempt to save the principle of knowability by amending it or the logic used to derive it in order to prevent it from falling victim to the paradox (and thus fall under solution types (1b) or (2)).

According to Mackie, a proper understanding of the argument perhaps requires

³⁰ Hart, W.D. "The Epistemology of Abstract Objects," *The Aristotelian Society Supplementary Volume LIII* (1979), 156

³¹ It perhaps should be noted that not much seems to have changed in this regard, as Fitch's result is viewed in much the same way today; as either a proof, the product of fallacious reasoning, or a paradox.

abstracting away from its implications for knowledge and knowability, at least to begin with. He thinks that once this is done, Fitch's result is only initially surprising; for it is clear that the result is derived simply because truth-entailing operators can be used to construct self-refuting expressions.³² Mackie gives the following example to illustrate this:

Let J be an operator variable that has any number of "innocent interpretations" (which is to say that for any p , it is possible that Jp and it is also possible that $\neg Jp$), including the interpretation, "it is written in green ink at t_1 that". Let W be the truth-entailing counterpart of J such that Wp is defined as $(Jp \wedge p)$. At this point, Mackie notes that it is tempting to say that, for any p , it is possible that Jp and thus for any p that is true it is possible that Wp . Mackie calls this latter claim inference rule R. He also notes one proviso: W distributes over conjunction. Mackie then proves that this inference rule is inconsistent with a statement of the form, $(p \wedge \neg Wp)$ in a similar fashion to the proofs presented in chapter one of this paper.

Thus, though it may be true that " p but it is not written in green ink at t_1 that p ", it does not follow from this that it can be truly written in green ink at t_1 that " p , but it is not written in green ink at t_1 that p ". Mackie thinks that this should be no more surprising than the fact that while I may be saying nothing at t_1 , I cannot truly say at t_1 that I am saying nothing at t_1 .³³ So inference rule R is unsound. Not everything that is true can be truly written in green ink at t_1 ; for there may be things that are true, and can be written in green in at t_1 , but which if they were written in green ink at t_1 , would not be true.³⁴

So, how does this help one to better understand the reasoning employed by Fitch?

³² Mackie, J.L. "Truth and Knowability," *Analysis* 40, (1980), 90

³³ *Ibid.*, 91

³⁴ *Ibid.*, 91

As Mackie notes, W could also possibly be interpreted as “it is known by someone at some time that”, which I will symbolize as K . Since on this interpretation, K is truth-entailing and distributes over conjunction, it can be shown analogously to the above example that the interpretation of R that this interpretation of K yields is unsound. This interpretation of R , however, just is the principle of knowability: if p is true, it can be known by someone at some time that p .

However, though Mackie affirms that the principle of knowability is unsound, he denies Hart’s claim that this automatically amounts to a refutation of verificationism.

As Mackie notes, Hart derives “what is true can be known” (by someone at some time”) from three premises:

- 1) What is true is meaningful
- 2) What is meaningful is verifiable
- 3) What is verifiable can be known³⁵

This is just a basic transitive argument, the conclusion of which is, “what is true can be known” (by someone at some time).

Since Hart thinks that the first and third premises are true, he takes the rejection of “what is true can be known” to require the rejection of “what is meaningful is verifiable”. This, however, only refutes a very strong form of verificationism in which “verified” entails “true”³⁶.

Mackie also claims that Fitch’s argument does not entail the rejection of the principle that what is true can be justifiably believed at some time. Thus, it does not entail the rejection of a form of verificationism that claims that what is meaningful is verifiable

³⁵ Hart, W.D. "The Epistemology of Abstract Objects" in *The Aristotelian Society Supplementary Volume LIII* (1979), 156

³⁶ Mackie, J.L. "Truth and Knowability," *Analysis* 40, (1980), 90

in the sense that it can be justifiably believed at some time. Mackie notes that, if K is interpreted as “it is justifiably believed by someone at some time that”, then no contradiction results; for it does not follow that if it is justifiably believed at any time that p is not justifiably believed at any time, then p is not justifiably believed at any time.

More formally, it is not the case that $(\neg Kp \rightarrow \neg Kp)$ if K is not truth-entailing and does not designate a specific time. For perhaps at some time, one could justifiably believe that p is false and will never be or never have been justifiably believed; yet, p might still be justifiably believed to be true at some other time.

However, if K is interpreted as “it is justifiably believed at t_1 that”, the proposal that whatever is true can be justifiably believed at t_1 can be shown to be false. As Mackie sees it, it is not possible to justifiably believe at t_1 that p and p is not justifiably believed at t_1 , for one cannot justifiably believe both that p and that no one justifiably believes that p !

More formally, it is not the case that $Kat_1(p \wedge \neg Kat_1p)$ because in order to justifiably believe that conjunction, one would have to simultaneously justifiably believe both that p and that it is not justifiably believed that p , which Mackie believes is absurd³⁷. However, $Kat_n(p \wedge \neg Kat_kp)$ is sound, because it only says that it can be justifiably believed at some time that p is true and is not justifiably believed at some other time.

Though Mackie contends that Fitch-style reasoning does not endanger the principle that whatever is true can be justifiably believed at some time, he claims that it indeed turns out to endanger the principle that whatever is meaningful is verifiable, just not due to the reasons advanced by Hart. If K is interpreted as, ‘it is true and verified at

³⁷ Ibid., 91

some time that”, and it is granted that something of the form ‘ p but it is never verified that p ’ is meaningful, then the principle that whatever is meaningful is verifiable should be rejected. For this interpretation of K is truth-entailing and distributes over conjunction; thus, the proof for the paradox of knowability succeeds under this interpretation of K . However, for the verificationist, it is in fact even worse than this, for a proposition of the form “ p but it is never verified that p ” simply cannot even be verified, let alone true and verified! For one would have to be able to verify both conjuncts together to verify the proposition. However, this is not possible, for one cannot verify that p whilst at the same time verifying that it is never verified that p . Thus, if “ p but it is never verified that p ” is meaningful, then it cannot be the case that what is meaningful is verifiable.

Thus, though Mackie believes that verificationism is indeed ultimately endangered by an analogue of the paradox, *contra* Hart he does not believe that the original version of the paradox entails this. Mackie’s solution then, is to simply abandon principles such as the principle of knowability and the verificationist principle that whatever meaningful is verifiable, for he uses reasoning analogous to Fitch’s to show that they are false.

Potential Problems with Mackie’s Solution

One problem with Mackie’s work on the paradox is that he does not consider what happens if we grant that there are truths that are never justifiably believed. If it is true that there are some truths that are never justifiably believed, then *contra* Mackie it cannot be the case that whatever is true can be justifiably believed at some time. One can employ reasoning analogous to Fitch’s reasoning to show that this is the case.

Let “B” stand for “it is justifiably believed by someone at some time that”.

Assume:

$\forall p(p \rightarrow \Diamond Bp)$ (That all truths can be justifiably believed by someone at some time)

$\exists p(p \wedge \neg Bp)$ (That some truths are never justifiably believed by anyone at any time)

It should be fairly clear that the formalization of these two assumptions are very similar to the formalization of the principle of knowability and the non-omniscience claim used to derive the paradox. However, since "B" is not truth entailing, one might expect the paradox to fail. It does not, however; for one can still derive $\Diamond B(p \wedge \neg Bp)$ which is bad enough; for it states that it is possible that one can justifiably believe both that p and it is never justifiably believed by anyone that p . Thus, if there are truths that are never believed by anyone, then the claim that all truths can be justifiably believed by someone at some time might also fall³⁸.

Dorothy Edgington has also pointed out that, if we restate the argument in terms of "evidence" rather than "justified belief" or "knowledge" (letting "E" stand for "someone at some time has evidence that"), we are able to derive $\Diamond E(p \wedge \neg Ep)$; that is, that it is possible that someone at some time has evidence both that p and that no one at anytime has evidence that p which is perhaps implausible. Thus it seems that even invoking the very weakest of epistemic attitudes might not help the situation, which is essentially just as paradoxical as it was in the case of knowledge³⁹.

As a result, some maintain that the multitude of paradoxes concerning epistemic attitudes weaker than knowledge that arise as a result of reasoning analogous to that

³⁸ Edgington, D. "The Paradox of Knowability," *Mind*. Vol. 94. No. 376 (1985), 558

³⁹ *Ibid.*, 558. It should be noted that these two examples of related paradoxes, along with Mackie's example that one could not consistently believe $Kt_1(p \wedge \neg Kt_1 p)$ face problems. For instance, it could be true that someone believes both that p and that no one will ever believe that p ; for one could perhaps be mistaken about his beliefs. In response to Edgington, it seems quite possible that it could be true that someone has evidence both that p and that no one ever has any evidence that p , and just is not aware that they have evidence for p .

employed by Fitch provide good reason to suspect that there is perhaps something amiss with the reasoning used to derive the paradox of knowability. For, though many are willing to discard the principle of knowability, far fewer are willing to abandon principles like, "if p is true, then it is possible that someone could have evidence that p ". Thus, many still harbor the suspicion that there is something fallacious about the result.

Moreover, some have suggested that Fitch's result shows us, at best, that there is structural unknowability, which is a function of logical considerations alone. They ask whether or not there is a more substantial kind of unknowability; for instance, unknowability that is a function of the recognition-transcendence of non-logical subject matter. Such critics insist that this question is the main point of contention between anti-realists and realists, and thus maintain that simply admitting that Fitch's result disproves the principle of knowability and with it, anti-realism, fails to address the main issue at hand⁴⁰.

Timothy Williamson

In his 1982 paper, "Intuitionism Disproved," Timothy Williamson suggests that, rather than giving the semantic anti-realist cause to abandon the principle of knowability, the paradox of knowability instead gives the anti-realist reason to embrace intuitionistic logic⁴¹. Thus, Williamson's solution falls under solution type (2) as outlined above, as his solution works by revising the logic that is used to derive the paradox, from classical to intuitionistic, which prevents the paradox from going through.

Williamson notes that, intuitionistically, the proof of the paradox is valid up until

⁴⁰ Berit Brogaard and Joe Salerno, "Fitch's Paradox of Knowability," in the *Stanford Encyclopedia of Philosophy*, p. 12 (<http://plato.stanford.edu/entries/fitch-paradox/>), 12

⁴¹ Williamson, T. "Intuitionism Disproved?" *Analysis* 42 (1982), 206

line 10, which is the assertion that:

$$\neg\exists p(p \wedge \neg Kp)$$

However, this is only classically, but not intuitionistically, equivalent to:

$$\forall p(p \rightarrow Kp)$$

Rather, since double-negation elimination is not permitted in intuitionistic logic, it is intuitionistically equivalent to:

$$\forall p(p \rightarrow \neg\neg Kp)^{42}$$

In Williamson's view, $\forall p(p \rightarrow \neg\neg Kp)$, or its intuitionistic equivalent, $\neg\exists p(p \wedge \neg Kp)$ is not evidently absurd; as it merely forbids intuitionists to produce claimed instances of truths that will never be known⁴³. In order to see this, it is crucial that one recall that the intuitionistic interpretation of the logical particles diverges significantly from their classical interpretation, as was discussed in chapter two. What should be especially emphasized is intuitionistic logic's replacement of classical logic's concept of "truth in a model" with the concept of "proof in an epistemic situation" or "assertability", as well as their special interpretations of the logical connectives and quantifiers. With this in mind, it is easy to see why intuitionists could grant that $\neg\exists p(p \wedge \neg Kp)$.

Recall that, in intuitionistic logic,

$P = (Q \wedge R)$ is proved in an epistemic situation iff the situation proves Q and R

$P = \neg Q$ is proved in an epistemic situation iff the situation contains evidence that Q can never be proved, which is to say that the situation contains evidence that shows that a proof of Q could be turned into a proof of a contradiction

and

⁴² Ibid., 205

⁴³ Ibid., 206

$P = \exists(x)Q(x)$ is proved in an epistemic situation iff $Q(t)$ is proved for some t

Additionally, recall that, in intuitionistic logic, proof must be constructive. Thus, a proof of an existential statement must yield a proof of a specific instance of the existential claim or provides an effective means, at least in principle, of finding such an instance.

With this in mind, let us try to prove, intuitionistically, $\exists p(p \wedge \neg Kp)$. To prove this, we must either find an instance of it or an effective method of finding an instance of it, as intuitionistic proofs must be constructive. Let us first consider the former. To find an instance of $\exists p(p \wedge \neg Kp)$ would involve finding some q such that $(q \wedge \neg Kq)$. To do this, one would have to prove both q and $\neg Kq$. However, if one proves that q , then one arguably knows that q ; that is, Kq . So Kq and $\neg Kq$. (Since this is a contradiction, it follows that it is not possible to find an instance of $\exists p(p \wedge \neg Kp)$; thus it is not possible to find an effective method of finding an instance of it, either.) Thus, since a proof of $\exists p(p \wedge \neg Kp)$ can be turned into a proof of a contradiction, the intuitionist can conclude $\neg \exists p(p \wedge \neg Kp)$.

At this point, one might ask how intuitionists could give credence to the almost certainly true claim that not all truths will be known (by someone at some time).

Williamson notes that they can do this in the formula:

$$\neg \forall p(p \rightarrow Kp)$$

Which is only classically, but not intuitionistically, equivalent to:

$$\exists p(p \wedge \neg Kp),$$

which, again, would compel intuitionists to produce *instances* of truths that cannot be proven to be known.

Since $\neg\forall p(p \rightarrow Kp)$, understood intuitionistically, is consistent with the principle of knowability, the paradox is thus averted.

Potential Problems with Williamson's Solution

The first potential problem with Williamson's solution that should be addressed is W.D. Hart's charge that $\forall p(p \rightarrow Kp)$ is "disastrously provable" in intuitionistic logic. The argument runs like this: for intuitionists, a proof of $(p \rightarrow q)$ is an evident way of converting any proof of p into a proof of q . So, if one is in possession of a proof of p , and one reviews and understands it as such, then it seems right to say that one also comes to know that p . That is, if one can prove that p , this is just a proof that p is known, or Kp , hence $\forall p(p \rightarrow Kp)$ is provable in intuitionistic logic⁴⁴.

Williamson is aware of Hart's argument and responds by noting that Hart does not understand proof in a way appropriate to intuitionism. Williamson grants that, though it may be the case that every proof *token* of p can be turned into a proof *token* that p is known, this does not entail that every proof *type* of p (as the permanent *possibility* of such a token) can be turned into a proof *type* that p is known.⁴⁵ That is, I cannot convert a way to prove that p into a way to prove that p is known, because a "way to prove that p is just a method that one can use to prove that p . I cannot simply convert this into a way to prove that p is known, because to prove that p is known would require being able to prove that someone actually has used or will use the method to prove p , which clearly cannot be deduced simply from the fact that there is a method to prove p , even if the

⁴⁴ Hart, W.D. "The Epistemology of Abstract Objects: Access and Inference" in *The Aristotelian Society Supplementary Volume LIII* (1979), 165

⁴⁵ Williamson, T. "Intuitionism Disproved?" *Analysis* 42 (1982), 206-207

particulars of the method are themselves known.⁴⁶

Second, Brogaard and Salerno express concern that, by admitting that $\neg\exists p(p \wedge \neg Kp)$ and that $\neg\forall p(p \rightarrow Kp)$, one who accepts Williamson's solution to the paradox admits both that no truths are unknown and that not all truths are known.⁴⁷ They also claim that the following cannot be accepted by intuitionists: $\forall p(\neg Kp \rightarrow \neg p)$, which follows intuitionistically from $\forall p(p \rightarrow \neg\neg Kp)$ (as contraposition is still permitted in intuitionistic logic), noting that it surely, the fact that nobody ever knows that p cannot be sufficient for the falsity of p !⁴⁸

These criticisms merely show that these claims are not being interpreted correctly from an intuitionistic standpoint. $\neg\exists p(p \wedge \neg Kp)$, interpreted intuitionistically, does not say “no truths are unknown”; rather, it reads something like, “it can never be proven that there a p such that one can prove both that p and that it can never be proven that p is known by someone at some time”. $\neg\forall p(p \rightarrow Kp)$, interpreted intuitionistically, does not say “not all truths are known”; rather, it reads something like, “it can never be proven that, for every p , there is a procedure that turns any proof of p into a proof that is p known by someone at some time.” $\forall p(\neg Kp \rightarrow \neg p)$, interpreted intuitionistically, does not say “for, all p , if p is never known, then p is false”; rather, it reads something like, “for every p , there is a procedure that turns any proof that it can never be proven that “ p is known by

⁴⁶ It is perhaps worth noting that even if one does not follow Williamson here, $\forall p(p \rightarrow Kp)$ is not necessarily “disastrous” if proven in intuitionistic logic. For it can plausibly be intuitionistically interpreted as reading “for every p , there is a procedure which one can use to turn any proof of p into a proof that p is known”. This is not, however, as implausible as saying that “all truths are known”, which is how $\forall p(p \rightarrow Kp)$ is interpreted in classical logic. Neil Tennant corroborates the view that $\forall p(p \rightarrow Kp)$ is perhaps acceptable in intuitionistic logic; see Tennant, N. *The Taming of the True*. (Oxford University Press: 2002), 272

⁴⁷ Brogaard, Berit and Salerno, Joe. “Fitch’s Paradox of Knowability,” in the *Stanford Encyclopedia of Philosophy*, (<http://plato.stanford.edu/entries/fitch-paradox/>), 5

⁴⁸ *Ibid.*, 7.

someone at some time” into a proof that it can never be proven that p ”. Interpreted intuitionistically, these three claims are not absurd; rather, they seem plausible, albeit complicated. Brogaard and Salerno’s error in thinking that they seem implausible lies in interpreting them *classically*, not *intuitionistically*.

Another possible complaint against Williamson’s solution is that it is *ad hoc*.⁴⁹ However, the anti-realist’s right to abandon classical logic in favour of intuitionistic logic has been defended independently by prominent philosophers such as Michael Dummett, Crispin Wright, and Neil Tennant.⁵⁰ Moreover, as was discussed in chapter two, semantic anti-realism seems to be borne out of intuitionistic logic and mathematics; thus it only seems natural that the anti-realist would opt for intuitionistic logic over classical.

However, if this is true, intuitionistic analysis of proof is not altogether innocent of anti-realist commitment; as Williamson puts it, without the corresponding anti-realist philosophy, intuitionistic logic would be nothing but a “dead formalism”.⁵¹ So, it seems that realists and anti-realists are perhaps arguing for two very different solutions to the paradox at least in part because each side is using (and perhaps, is *committed* to using) a different logic: realists argue that the principle of knowability can be shown to be false using classical logic; whereas anti-realists argue that this result does not occur if one uses intuitionistic logic instead.

As a result, this solution, by itself, does not seem to have convinced anyone who does not already have intuitionist or anti-realist leanings. For, as Williamson admits in a later paper, to many, it likely seems more probable that a theory of truth (that is, semantic

⁴⁹ Ibid., 6

⁵⁰ Ibid., 6. For instance, see Dummett, M. “The Philosophical Basis of Intuitionistic Logic,” in *Truth and Other Enigmas* (Duckworth: 1978)

⁵¹ Williamson, T. “Intuitionism Disproved?” *Analysis* 42 (1982), 207

anti-realism) is wrong, rather than our logic.⁵² This, of course, is no argument against Williamson's solution; it merely expresses the reluctance to change a well-entrenched logical system in order to salvage a theory of meaning, unless there are extremely powerful arguments in favour of such a change. The overall success of Williamson's solution to the paradox seems to rely on the existence and acceptance of such arguments. This issue will be revisited later in the paper.

Dorothy Edgington

In her 1985 paper, "The Paradox of Knowability", Dorothy Edgington attempts to solve the paradox of knowability by amending the principle of knowability. Thus her solution falls under solution type (1b) above. Her revised principle of knowability invokes the use of an actuality operator, which she claims renders it consistent with the non-omniscience claim.

Edgington begins her proposal by stating that she intends to show that in certain situations, while it is not possible to know that p , it is possible to know that actually p . Aware that this potentially violates readers' intuitions regarding the use of the term "actually", Edgington notes that she is invoking a stipulated, technical use of "actually", as the ordinary usage of "actually" cannot be appealed to as it lacks sufficient precision⁵³.

Basically, Edgington's solution works because she contends that, just as there can be actual knowledge about what is counterfactually the case, there can be counterfactual knowledge about what is actually the case. That is, there can be possible, non-actual knowledge that in a possible situation which is, in fact, the actual situation (though the possible knower would not describe it as such), p is true and unknown. As she sees it,

⁵² Ibid., 207

⁵³ Edgington, D. "The Paradox of Knowability," *Mind*. Vol. 94. No. 376 (1985), 563

possible, non-actual situations also contain people who have knowledge, including knowledge of other possible situations, some of which may be actual situations. If these claims are granted, then one can reformulate the principle of knowability using an actuality operator such that it is consistent with the claim that some truths are never known by anyone at any time.⁵⁴

The claim that "there can be possible, non-actual knowledge that in a possible situation which is, in fact, the actual situation[...], p is true and unknown" and the claim that "possible, non-actual situations also contain people who have knowledge [...]" will likely be quite puzzling to anyone who is previously unacquainted with them. Edgington attempts to both elucidate and defend the plausibility of these claims through the use of an example.⁵⁵ She asks the reader to imagine a case in which a comet is returning shortly. Given that the comet is in the process of breaking up, this will be our last chance to observe it. A spacecraft is being dispatched to investigate it and collect samples, in the hope that answers might be provided to certain questions, such as "does the comet contain pre-biotic molecules?"

Suppose the answer to this question is p ; and suppose that, if everything goes according to the plan, it will be known that p . However, if something goes wrong, it will not be known that p . Thus there are two possible outcomes; either the mission succeeds and it is known that p , or the mission fails and it is never known that p . Let us call the former outcome "situation one", or S_1 , and the latter outcome "situation two", or S_2 .

If S_1 obtains, then it will be known that, had the mission failed, it would never have been known that p . However, if S_1 obtains, it is also the case that p is known.

⁵⁴ Ibid., 565

⁵⁵ Ibid., 565

Suppose that, actually, S_2 obtains, and it is never known that p . Then, there is possible, non-actual knowledge in S_1 that in the possible situation S_2 (which is really the actual situation, though it would not be described as such is S_1), p and it is never known that p . Thus, according to Edgington, we can use the actuality operator A to formulate an instance of $\diamond KA(p \wedge \neg KAp)$, which is to say that, from the vantage point of a possible, non-actual situation, an agent could know that in the actual situation (though that agent would not describe it as such), p and it is never known by anyone in the actual situation that p .⁵⁶ So, Edgington suggests that the principle of knowability be revised as follows:

Edgington's Revised Knowability Principle (ERKP): If p is actually true, then it is possible to know that actually p .⁵⁷

Or, more formally, (ERKP): $Ap \rightarrow \diamond KAp$

Edgington claims that this principle is consistent, makes philosophical sense, and does not violate anti-realist or verificationist scruples about intelligibility. She also asserts that the appeal to the use of the actuality operator is not *ad hoc*, for it has been argued in independently motivated work that the use of the actuality operator is important for a variety of other purposes; for instance, it has been said to be essential for expressing certain modal thoughts, for properly understanding Kripke's work, and for developing a concept of epistemic necessity.⁵⁸

Potential Problems with Edgington's Solution

In his 1987 article, "On the Paradox of Knowability," Timothy Williamson claims that there does not seem to be a way of interpreting Edgington's revised knowability

⁵⁶ Ibid., 565-566

⁵⁷ Ibid., 566

⁵⁸ Ibid., 568

principle such that it does not entail an "obviously silly" form of verificationism.⁵⁹

Williamson first notes that Edgington's revised knowability principle entails a surprisingly weak form of verificationism. For, as Edgington herself was aware, '*Ap*' always entails ' $\Box Ap$ '. This may seem surprising given the indexical nature of "actually". However, if one claims that, "actually, *p*", this fixes the truth value of *p* to one possible world (or situation),⁶⁰ the world in which it is stated. In our case, stating that "actually, *p*" fixes the truth value of *p* to our situation, or as we see it, the "actual" situation. So, '*Ap*' designates rigidly. That is, if in the actual situation, *p* is true, then it is true in *all* possible situations that, in the actual situation, *p* is true. Furthermore, if ' in the actual situation, *p* is true ', or more formally, '*Ap*', is true in all possible situations, then it must follow that 'necessarily, *p* is true in the actual situation' or ' $\Box Ap$ ', as "necessary" in this context just means "true in all possible situations".⁶¹

Since '*Ap*' entails ' $\Box Ap$ ', Williamson contends that the only knowledge that ERKP requires is knowledge of necessary truths, noting that one might expect a robust verificationist theory to insist that at least some contingent truths are knowable. It should be mentioned, however, that though ERKP might require all knowable truths (actual truths) to be necessary, it does not require them to be *a priori* (for they could be necessary *a posteriori*) so this is not Williamson's main criticism. Instead, he turns to the matter of what could possibly constitute non-actual knowledge about what is actually the case.⁶²

⁵⁹ Williamson, T. "On the Paradox of Knowability," *Mind*. Vol. 96. No. 382 (1987), 256

⁶⁰ Edgington uses the term 'situation' rather than 'world' because she considers worlds to be too specific to figure into our ordinary modal talk. Edgington, D. "The Paradox of Knowability," *Mind*. Vol. 94. No. 376 (1985), 564

⁶¹ Williamson, T. "On the Paradox of Knowability," *Mind*. Vol. 96. No. 382 (1987), 257

⁶² *Ibid.*, 257

Williamson claims that the problem with Edgington's solution can be raised without the use of the actuality operator by focusing on the quantificational version of Edgington's revised knowability principle. He cashes this out as:

Edgington's Revised Knowability Principle (quantificational version) (ERKP_{qv}): For all situations, if p is true in a situation s , then there is another situation $s+$ in which it is known that in s , p is true.⁶³

Or, more formally, (ERKP_{qv}): $\forall s((\text{in } s, p) \rightarrow \exists s+(\text{in } s+, K(\text{in } s, p)))$

As Williamson sees it, in order to have knowledge in a situation $s+$ that in a situation s , p , an agent in $s+$ who is said to possess such knowledge must have somehow specified the situation s , as the situation variable s occurs inside the knowledge operator K . Williamson considers four possible ways of specifying a situation:⁶⁴

- i) by necessary and sufficient conditions
- ii) by counterfactuals
- iii) by space-time coordinates
- iv) by ostension

Williamson then argues that if s is specified in way (i) or (ii) in the knowledge that in s , p , this knowledge requires no more than knowledge of a trivial logical truth; and if s is specified in way (iii) or (iv) in the knowledge that in s , p , then it cannot count as non-actual knowledge of the actual. As a result, in his view, Edgington's revised knowability principle either yields a form of verificationism too weak to warrant its description as such, or it simply does not yield any form of verificationism at all.⁶⁵

Crucial to Williamson's argument is the observation that "In s , p ", if true, is

⁶³ Ibid., 257

⁶⁴ Ibid., 258

⁶⁵ Ibid., 258

necessarily true. As was discussed earlier, “actually, p ” designates rigidly. Analogously, so does, “In s , p ”; for this states that p is true in a given situation s . Thus, if true, “In s , p ” is true in *all* possible situations, and thus is a necessary truth.⁶⁶

Williamson notes that, given this, any attempt to specify situation s in $s+$ in way (i), by necessary and sufficient conditions, will result in no more than knowledge of a trivial logical truth. Assume that a set of necessary and sufficient conditions, q , specify situation s . However, since “In s , p ”, if true, is necessarily true, then a set of necessary and sufficient conditions ($p \wedge q$) also specify situation s . So, either situation s can be specified by the conditional, $\Box(q \rightarrow p)$, or by the conditional, $\Box((p \wedge q) \rightarrow p)$. However, the latter is clearly trivial. Edgington's solution needs to find a way (that is not simply *ad hoc*) to disallow the latter if knowledge of possible situations is going to be more substantial than just knowledge of a trivial logical truth.⁶⁷

Similarly, any attempt to specify situation s in $s+$ in way (ii), by counterfactuals will also yield no more than knowledge of a trivial logical truth. Say that s is specified by a counterfactual, so for some q , it would be the case that if q , then s . This of course, could not specify s uniquely, for s must also be the *most specific* situation that would obtain if q . So the assumption is, in some situation $s+$, s is the most specific situation which would obtain if q . So, it is also the case in $s+$ that $(q \rightarrow p)$, for in $s+$, $(q \rightarrow s)$, and necessarily, “In s , p ”. Williamson claims that, by standard counterfactual reasoning, the statements that “it would be the case that $(q \rightarrow s)$ ” and “it would be the case that $(p \wedge q \rightarrow s)$ ” have the same truth value in $s+$. So the statements that “it would be the case that $(q \rightarrow s)$ ” and “it would be the case that $(p \wedge q \rightarrow s)$ ” specify s equally well. Since “In s , p ” is

⁶⁶ Ibid., 258

⁶⁷ Ibid., 259

necessarily true, either it would be the case that $(q \rightarrow p)$ or it would be the case that $(p \wedge q \rightarrow p)$. As above, Edgington's solution has to find some way (that is not simply *ad hoc*) to disallow the latter.⁶⁸

Any attempt to specify s in way (iii) or (iv) will not yield non-actual knowledge of the actual, given that both of these ways seem to require some kind of causal link; however, it seems rather impossible that there could be a causal link between the actual situation and a merely possible one.⁶⁹

Finally, Williamson takes issue with the non-quantificational version of Edgington's revised principle of knowability, (EKRP): $(Ap \rightarrow \Diamond KAp)$, for similar reasons. He claims that, if (ERKP) is not to collapse into the schema $(Ap \rightarrow AKAp)$ (which reads "if p is actually true, then it is actually known that p is actually true"), non-actual knowledge that actually p is required. *A fortiori*, non-actual thought that actually p is required. However, it is hard to see how there can be non-actual thought about what is actually the case. For either non-actual thinkers have the concept we express by "actually" or they do not. If they do, they cannot express it by saying "actually, p " (for they are not located in the actual situation); rather, they must be able to somehow uniquely specify the actual situation using counterfactuals. However, this is implausible; as Edgington herself notes, knowledge of counterfactuals is never about one specific possible situation. If they do not, how can their thinking express the thought expressed when someone in the actual situation states that "actually, p "? Williamson claims that it cannot simply be necessarily equivalent to the thought expressed by "actually, p ", for as noted, "actually, p ", if true, is necessarily true. Anything that is necessarily true is

⁶⁸ Ibid., 259

⁶⁹ Ibid., 260

necessarily (logically) equivalent to any tautology. If all that is required of non-actual knowers is that they know a tautology, (ERKP) does not express a form of verificationism.⁷⁰

It should perhaps be noted that Williamson's article points out difficulties with Edgington's position but does not decisively refute it. Thus, others have attempted to elaborate on her proposal and address these difficulties.⁷¹ However, since it is not, in my view, the best solution to the paradox (for anyone who is inclined to endorse semantic anti-realism), I will not discuss Edgington's solution further.

Michael Dummett

In his 2001 paper, "Victor's Error", Michael Dummett claims that, rather than providing a blanket characterization of truth, the anti-realist should have provided an inductive one. He suggests that, in order to avoid the paradox, a proponent of anti-realism should distinguish some class of *basic statements*. Following that, he proposes that the anti-realist can restrict the principle of knowability as follows:

Dummett's Restricted Knowability Principle (DRKP) It is true that A iff it is possible to know that A (if A is a basic statement).⁷²

Or, more formally, (DRKP): $\text{Tr}(A) \leftrightarrow \Diamond K(A)$

He also includes the following clauses as part of his account:

(i) $\text{Tr}(A \text{ and } B) \text{ iff } \text{Tr}(A) \wedge \text{Tr}(B)$

(ii) $\text{Tr}(A \text{ or } B) \text{ iff } \text{Tr}(A) \vee \text{Tr}(B)$

⁷⁰ Ibid., 260

⁷¹ Brogaard, Berit and Salerno, Joe. "Fitch's Paradox of Knowability," in the *Stanford Encyclopedia of Philosophy*, (<http://plato.stanford.edu/entries/fitch-paradox/>), 9

⁷² Dummett, M. "Victor's Error," *Analysis* 61 (2001), 1

(iii) $\text{Tr}(\text{if } A \text{ then } B) \text{ iff } (\text{Tr}(A) \rightarrow \text{Tr}(B))$

(iv) $\text{Tr}(\text{it is not the case that } A) \text{ iff } \neg\text{Tr}(A)$

(v) $\text{Tr}(A(\text{something})) \text{ iff } \exists x\text{Tr}(A(x))$

(vi) $\text{Tr}(A(\text{everything})) \text{ iff } \forall x\text{Tr}(A(x))$

He also notes that the logical constant on the right hand side of each of these biconditionals is subject to the laws of intuitionistic logic.⁷³

Since Dummett's solution works by invoking the use of intuitionistic rather than classical logic and by revising the principle of knowability, his solution could be construed as falling under both solution type (1b) and solution type (2) above. However, since his solution primarily works by restricting the kinds of truths that can be known via a revised knowability principle, his solution is probably best characterized as falling under type (1b).

Though Dummett acknowledges that it must also be specified as to what *exactly* counts as a basic statement, that does not worry him at this juncture. Rather, he is simply concerned at this point with showing how his revised knowability principle avoids the paradox of knowability.

Dummett's solution to the paradox essentially works by blocking the distribution of the K-operator over conjunction and restricting the kinds of truths that can be known to basic statements. He notes that, under certain notions of truth, one may hold that "if A, then $\text{Tr}(A)$ ".⁷⁴ With this in mind, consider a variant of the problematic conjunction for the proof for the paradox shown earlier: $\diamond K(B \wedge \neg K(B))$. As Dummett puts it, though it is

⁷³ The merits of which he has argued for independently; for instance, see Dummett, M. "The Philosophical Basis of Intuitionistic Logic," in *Truth and Other Enigmas* (Duckworth: 1978)

⁷⁴ *Ibid.*, 2. Dummett notes that this of course does not hold for every possible conception of truth; for instance, not for one under which A's not being true does not imply the negation of A.

obviously impossible that anyone should know both that B and that it will never be known that B; under his characterization of truth, the K-operator, “it is known that” cannot be exported over conjunction. He then contends that, if B is a basic statement, then one would still be committed by this inductive characterization of truth to inferring from $(B \wedge \neg K(B))$, “both that it could have been or could later be known that B and that in fact it never has been and never will be known that B⁷⁵.” Though Dummett does not elaborate further, it seems that this follows from $(B \wedge \neg K(B))$ because of the clause $\text{Tr}(A \text{ and } B) \text{ iff } \text{Tr}(A) \wedge \text{Tr}(B)$ and his revised principle of knowability. Since $(B \wedge \neg K(B))$ is compound and not basic, we apply his rule with respect to conjunction to derive $(\text{Tr}(B) \wedge \text{Tr}(\neg K(B)))$. Since B is a basic statement, by $\text{Tr}(A) \leftrightarrow \Diamond K(A)$ on the left side of the conjunction, we derive $\Diamond K(B)$. Presumably, $\neg K(B)$ is non-basic, so $\text{Tr}(\neg K(B))$ cannot be altered further. Thus we have $(\Diamond K(B) \wedge \text{Tr}(\neg K(B)))$; which is the result that Dummett seeks. As should be clear, Dummett’s solution works primarily because of his revised principle of knowability, as the problematic conjunction $(B \wedge \neg K(B))$, being compound and therefore not basic, cannot replace the variable A in $(A \text{ iff } \Diamond K(A))$; thus avoiding the paradox.⁷⁶

Potential Problems With Dummett’s Solution: Brogaard and Salerno’s Response

In their 2004 article, “Clues to the Paradox of Knowability: A Reply to Dummett and Tennant,” Berit Brogaard and Joe Salerno suggest that Dummett’s solution potentially falls victim to related paradoxes. They begin their inquiry into Dummett’s solution by asking whether, under Dummett’s account, epistemic statements of the form

⁷⁵ Dummett, M. “Victor’s Error,” *Analysis* 61 (2001), 2

⁷⁶ Brogaard, Berit and Salerno, Joe. “Clues to the paradoxes of knowability: a reply to Dummett and Tennant,” *Analysis* 62, (2002), 143

‘K(B)’ qualify as being basic, for in their view, Dummett’s inductive account does not clearly determine this.⁷⁷ As they see it, either such epistemic statements are basic or they are not. In the event of the latter, the truth conditions must be given by supplementary clauses.⁷⁸ However, Brogaard and Salerno contend that in either case, Dummett’s account encounters difficulties.

First, let us suppose that statements of the form ‘K(B)’ are basic, which they plausibly are, given that they are not truth-functionally complex. This would allow substitutions of ‘K(B)’ for ‘A’ in Dummett’s Restricted Knowability Principle, which states that $\text{Tr}(A)$ iff $\Diamond K(A)$ (if A is a basic statement). Keeping in mind that this proof refers to the clauses laid out by Dummett in “Victor’s Error” provided above, and adding to it the bi-conditional form of Dummett’s claim that if A, then $\text{Tr}(A)$,⁷⁹ Brogaard and Salerno derive the following result:

- | | |
|---|--|
| 1. $B \wedge \neg K(B)$ | Assumption |
| 2. $\text{Tr}(K(B))$ iff $\Diamond KK(B)$ | by clause (i), Dummett’s restricted KP |
| 3. $\text{Tr}(B)$ iff $\Diamond K(B)$ | by clause (i) |
| 4. $\neg \Diamond KK(B)$ | from 1 and 2, by A iff $\text{Tr}(A)$ |
| 5. $\Diamond K(B)$ | from 1 and 3, by A iff $\text{Tr}(A)$ |

As Brogaard and Salerno note, Line 1 is the problematic ‘Fitch conjunction,’ which is true for some statement B, given that we are non-omniscient. Lines 2 and 3 are substitution instances of Dummett’s clause (i) on the assumption that K(B) and B are basic. Line 4 is reached by taking the right conjunct of 1 and applying ‘A iff $\text{Tr}(A)$ ’ to it,

⁷⁷ Ibid., 144

⁷⁸ Ibid., 144

⁷⁹ Ibid., 143

which gives $\neg\text{Tr}(K(B))$. From line 2, it follows that $\neg\Diamond KK(B)$. Similarly, line 5 follows from the left conjunct of 1 conjoined with line 3.⁸⁰

Brogaard and Salerno then ask the reader to consider the following closure principle: if a conditional is necessary, then if the antecedent is possible then so is the consequent. They contend that since semantic anti-realism (and thus the Principle of Knowability and $A \text{ iff } \text{Tr}(A)$) are often taken to be necessary theses, if the antecedent of these theses is possible then the consequent is possible as well. So if $A \rightarrow \Diamond K(A)$ is necessary, then $\Diamond A \rightarrow \Diamond \Diamond K(A)$. However, substituting ‘ $K(B)$ ’ for A , this principle entails $\Diamond K(B) \rightarrow \Diamond \Diamond KK(B)$. Applying this formula to line 5 then gives us $\Diamond \Diamond KK(B)$.

- | | |
|----------------------------------|--|
| 6. $\Diamond \Diamond KK(B)$ | from 5, by closure, clause (i) and $A \text{ iff } \text{Tr}(A)$ |
| 7. $\Diamond KK(B)$ at world w | from 6 |
| 8. $KK(B)$ at world v | from 7 |
| 9. $\Diamond KK(B)$ in actuality | by the transitivity of \Diamond |

However, at this point it should be noted that line 9 contradicts line 4.

So, how is this result supposed to make sense? Brogaard and Salerno contend that if line 6 is ‘actually’ true, then there is an accessible world w at which $\Diamond KK(B)$. That means that there is another world v which is accessible from w such that $v \Vdash KK(B)$. If \Diamond is transitive, then v is accessible from the ‘actual’ world since v is accessible from w and w is accessible from the ‘actual’ world. So, as Brogaard and Salerno see it, if \Diamond is transitive, then in ‘actuality’, $\Diamond KKB$. But this contradicts line 4. So, if statements of the form $K(B)$ are considered basic under Dummett’s inductive characterization of truth, it

⁸⁰ Ibid., 144

seems that this results in contradiction.⁸¹

So, what if statements of the form $K(B)$ are treated as *non*-basic statements, in which case, Dummett owes the reader a supplementary clause which outlines the truth conditions of such statements? As Brogaard and Salerno note, it is not ruled out *a priori* that such a supplementary clause will have as a consequence the KK-thesis:

(KK) $\Box(K(B) \rightarrow KK(B))$, which essentially contends that necessarily, if it is known that B then it is known that it is known that B . It surely is not implausible that the anti-realist might be committed to this; though the validity of the KK thesis has not been decisively established.⁸²

According to Brogaard and Salerno, what also should be noted is that the anti-realist considers $\Diamond K(A)$ to be factive, and thus embraces the principle (that they call (F)) which contends that if $\Diamond K(A)$, then A for all A . Thus, if $K(B)$ is non-basic and the anti-realist is committed both to KK and to F, a new version of Fitch's Paradox of Knowability can be formulated against Dummett's inductive characterization of truth.

The proof proceeds as follows:

- | | |
|---|---|
| 1. $B \wedge \neg K(B)$ | Assumption |
| 2. $\Box(K(B) \rightarrow K(K(B)))$ | from KK |
| 3. $\text{Tr}(B) \rightarrow \Diamond K(B)$ | by clause (i), Dummett's revised KP |
| 4. $\Diamond K(B)$ | from 1 and 3, by A iff $\text{Tr}(A)$ |
| 5. $\Diamond(K(K(B)))$ | from 4 and 2 by closure |
| 6. $K(B)$ | from 5 by F |
| 7. $K(B) \wedge \neg K(B)$ | from 1 and 6 |

⁸¹ Ibid., 145

⁸² Ibid., 145

However, line 7 is clearly absurd. So it seems that a new version of the paradox might arise that Dummett's inductive characterization of truth fails to block.

Other Potential Problems with Dummett's Solution

It should be noted that both of Brogaard and Salerno's proofs rely in part on the supposition that Dummett is committed to the claim that 'A iff Tr(A),' which strictly speaking, Dummett never directly accedes to. Rather, in "Victor's Error," Dummett claims that 'if A, then Tr(A)' but does not directly endorse the bi-conditional form used in the Brogaard and Salerno proofs.⁸³ Moreover, Brogaard and Salerno's work only illustrates *possible* problems with Dummett's account. The paradoxes that they derive rely on several suppositions that Dummett does not explicitly or even tacitly endorse; perhaps most contentiously, the possibility that K(B) is a basic statement and the KK-thesis, which is not only far from being established, but is widely thought to be false, despite Hintikka's "proof" of it.⁸⁴ Thus it seems as if Brogaard and Salerno simply provide Dummett with reasons not to consider statements such as K(B) to be basic, and to ensure that his supplementary clauses do not include or imply the KK-thesis.

The major problem with Dummett's solution, in my view, is that Dummett's talk of "basic statements" is a little perplexing, given that he does not specify or even discuss what "basic statements" are or could consist in. Moreover, he does not justify his move to restricting the principle of knowability such that it only includes basic statements except by stating that it is motivated by the threat that the paradox poses.⁸⁵ As a result, Dummett's proposal seems rather *ad hoc*. For he does not provide the reader with any

⁸³ Ibid., 143

⁸⁴ Hintikka, J. *Knowledge and Belief: an introduction to the logic of the two notions* (Cornell University Press: 1962), 77-79. For a criticism of Hintikka's proofs, see Chisholm, R. "The Logic of Knowing," *The Journal of Philosophy* Vol. 60, No. 25 (1963), 773- 795

⁸⁵ This is also noticed by Brogaard and Salerno, though they deny that is their main criticism.

reason to accept his account, other than it rescues the principle of knowability from paradox and allows Dummett to achieve the result he seeks. It is far from clear as to whether or not there are other reasons that might justify weakening the principle of knowability to only apply to basic statements. Dummett's account stands in need of such reasons to avoid the charge of being unprincipled. There also needs to be more explanation as to what these "basic statements" actually are.

CONCLUSION

In my view, Williamson's solution should be the most appealing to those who endorse semantic anti-realism.⁸⁶ For, in addition to using principles of logic that the anti-realist likely accepts, or in any event, perhaps ought to accept (given that semantic anti-realism is the result of generalizing intuitionist semantics to apply to all language),⁸⁷ Williamson's solution is the only solution, at least out of those surveyed here, that allows the principle of knowability) - to the main tenet of semantic anti-realism - to remain unscathed and unaltered.

Thus, though intuitionistic logic has not been decisively established as the correct logic, in my view, it nevertheless can be employed by the semantic anti-realist so as to prevent the principle of knowability, and semantic anti-realism with it, from succumbing to the paradox of knowability. For the paradox of knowability only arises if one uses principles of logic that a semantic anti-realist likely does not accept; or in any event, either should not or perhaps even cannot, accept. Given that intuitionistic logic is at least plausible alternative to classical logic, the paradox of knowability should not be considered to pose any threat to semantic anti-realism unless arguments are advanced that render intuitionistic logic generally untenable. In the absence of such reasons, proponents of semantic anti-realism and the principle of knowability can ensure that their view is unaffected by the paradox, simply by using a logic that is the natural counterpart of their

⁸⁶ Crispin Wright, for instance, appears to endorse a solution very much like Williamson's. See: Wright, C. *Realism, Meaning, and Truth*. (Blackwell: 1987), 311

⁸⁷ It should be noted that there is an ongoing debate as to whether or not one can be an anti-realist without endorsing intuitionistic logic. (See, for instance, Wright, C. *Realism, Meaning, and Truth*. (Blackwell: 1987), 317-362 (chapters 10 and 11)) I will not discuss this issue here; for I think it is clear why I claim that intuitionistic logic is at least, the natural choice for the anti-realist, though it is granted that it is perhaps not the only choice (though it may indeed turn out that it is for a variety of reasons, and especially since using intuitionistic logic allows one to avoid the paradox!)

philosophical position. For, as was discussed, the main objections to Williamson's intuitionistic revision solution to the paradox are flawed because they rely on a mistaken conception of how logical claims are interpreted in intuitionistic logic.

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